Electromagnetism Formula Sheet

Maxwell Equations:

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{\mathrm{B}} = 0 \qquad \nabla \times \vec{\mathrm{B}} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{\mathrm{E}}}{\partial t} \right)$$

Gauss Law:

$$\oiint_{\partial\Omega} \vec{E}(\vec{\mathbb{r}}) \cdot d^2\vec{\mathbb{r}} = \frac{Q[\Omega]}{\epsilon_0}$$

Ampere Law:

$$\oint_{\partial\Sigma} \vec{B}(\vec{\mathbb{r}}) \cdot d\vec{\mathbb{r}} = \mu_0 (I[\Sigma] + I_d[\Sigma])$$

Current Density: $I[\Sigma] = \iint_\Sigma \vec{J}(\vec{\mathbb{r}}) \cdot d^2\vec{\mathbb{r}}$

Displacement Current: $\vec{J}_d = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

Faraday Flux Rule:

$$\mathcal{E} = \frac{1}{q} \oint_{\partial\Sigma(t)} \vec{F}(\vec{\mathbb{r}}) \cdot d\vec{\mathbb{r}} = -\frac{d\Phi_{\mathrm{B}}[\Sigma(t)]}{dt}$$

Lenz Law: The induced Eddy current creates a field as to oppose the change in magnetic flux $\Phi_{\mathrm{B}}[\Sigma] = \iint_\Sigma \vec{\mathrm{B}}(\vec{r}) \cdot d^2\vec{\mathbb{r}}$

Continuity Equation: $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$

Lorentz Force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Coulomb Law:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \iiint_{\mathcal{V}} \frac{\vec{r} - \vec{\mathbb{r}}}{\|\vec{r} - \vec{\mathbb{r}}\|^3} \rho(\vec{\mathbb{r}})\, d^3\mathbb{r}$$

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \iiint_{\mathcal{V}} \frac{\rho(\vec{\mathbb{r}})}{\|\vec{r} - \vec{\mathbb{r}}\|} d^3\mathbb{r}$$

On statics: $\boldsymbol{\vec{E}} = -\boldsymbol{\nabla\phi}$

$$\boldsymbol{\phi(\vec{r})} = \boldsymbol{\phi(\vec{r_0}\}} - \int_{\vec{r_0} \to \vec{r}} \boldsymbol{\vec{E}} \cdot d\vec{\mathbb{r}}$$

Biot-Savart Law:

$$\vec{\mathrm{B}}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint_{\mathcal{V}} \vec{\mathrm{J}}(\vec{\mathbb{r}}) \times \frac{\vec{r} - \vec{\mathbb{r}}}{\|\vec{r} - \vec{\mathbb{r}}\|^3}\, d^3\mathbb{r}$$

Remark: $\lim_{\vec{r} \to \infty} \vec{E}(\vec{r}) = \lim_{\vec{r} \to \infty} \vec{B}(\vec{r}) = \vec{0}$

Table of Electric Fields

Charge Dist.	$\vec{E}$
Infinite Plane $(z > 0)$	$\frac{\sigma}{2\epsilon_0}\hat{z}$
Spherical Shell	$\frac{\sigma R^2}{\epsilon_0 r^2}\hat{r}(\text{outside})$ $\vec{0}$ (inside)
Solid Sphere	$\frac{\rho r}{3\epsilon_0}\hat{r}(\text{inside})$ $\frac{\rho R^3}{3\epsilon_0 r^2}\hat{\mathrm{r}}\ (outside)$
Cylindrical Shell	$\frac{\sigma R}{\epsilon_0 r}\hat{r}(\text{outside})$ $\vec{0}$ (inside)
Infinite Wire	$\frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$
Ring at Axis $(z > 0)$	$\frac{\lambda R z}{\epsilon_0 (R^2 + z^2)^{3/2}}\hat{z}$
Disk at Axis $(z > 0)$	$\frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{R^2 + z^2}}\right)\hat{z}$

Table of Magnetic Fields

Current Distribution	$\vec{B}$
Infinite Wire	$\frac{\mu_0 I}{2\pi r}\hat{\varphi}$
Ring at Axis	$\frac{\mu_0 I\, R^2}{2(R^2 + z^2)^{3/2}}\hat{z}$
Solenoid	$\mu_0 n I\ \hat{z}(\text{inside})$ $\vec{0}$ (outside)

Uniqueness Theorem: $\nabla^2\phi = -\frac{\rho}{\epsilon_0}$ given either $\phi|_{\partial\mathcal{V}}$ or $E_n|_{\partial\mathcal{V}}$, the solution is unique. Further, ϕ is continuous over interfaces.

Interface Equation:

$$\vec{E}_\uparrow - \vec{E}_\downarrow = \frac{\sigma}{\epsilon_0}\hat{n} \qquad \vec{B}_\uparrow - \vec{B}_\downarrow = \mu_0 \vec{K} \times \hat{n}$$

Dielectrics: For $\epsilon(\vec{r}) = \kappa(\vec{r}) \cdot \epsilon_0$

$$\vec{E}(\vec{r}) = \frac{1}{\kappa(\vec{r})} \cdot \vec{E}_{\text{vacc}}(\vec{r})$$

and $\rho_f = \nabla \cdot (\epsilon \cdot \vec{E})$

Interface Dielectric: $\epsilon_\uparrow \cdot \vec{E}_\uparrow - \epsilon_\downarrow \cdot \vec{E}_\downarrow = \sigma_f \hat{n}$

Energy stored:

$$U_E = \iiint_{\mathcal{V}} \frac{1}{2}\epsilon(\mathbb{r}) \cdot \left\|\vec{E}(\mathbb{r})\right\|^2 d^3\mathbb{r}$$

$$U_B = \iiint_{\mathcal{V}} \frac{\left\|\vec{B}(\mathbb{r})\right\|^2}{2\mu(\mathbb{r})} d^3\mathbb{r}$$

Work: $W = \frac{1}{2}\sum_{i=1}^{n} q_i \phi_i(\vec{r_i})$

Conductors: $\vec{E} = \vec{0}$ inside and all charges are on the surface.

Ohm's Law: $\vec{E}(\vec{r}) = \varrho(\vec{r}) \cdot \vec{J}(\vec{r})$

Capacitor: $Q_f = CV \Rightarrow U_E = \frac{Q^2}{2C} = \frac{1}{2}CV^2$

Resistor: $V = RI$

Table of Capacitance and Resistances

Geometry	C	R
Two Plates	$\frac{\epsilon_0 \mathrm{A}}{\mathrm{d}}$	$\frac{\varrho d}{A}$
Spherical Shell	$\frac{4\pi\epsilon_0}{\frac{1}{a} - \frac{1}{b}}$	$\frac{\varrho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$
Cylindrical Shell	$\frac{2\pi\epsilon_0 L}{ln\left(\frac{b}{a}\right)}$	$\frac{\varrho}{2\pi L} ln\left(\frac{b}{a}\right)$

Inductor: $\Phi_B = LI \Rightarrow U_B = \frac{LI^2}{2}$

Dipoles: $\vec{p} = \iiint_{\mathcal{V}} \mathbb{r}\, \rho(\mathbb{r})\, d^3\mathbb{r} = q\vec{d}$

$$\phi_{\text{dip}}(\vec{r}) = \frac{\vec{p} \cdot \hat{r}}{4\pi\epsilon_0|\vec{r}|^2};\ \vec{E}_{\text{dip}}(\vec{r}) = \frac{3(\vec{p} \cdot \hat{r})\,\hat{r} - \vec{p}}{4\pi\epsilon_0|\vec{r}|^3}$$

Force: $\vec{F} = (\vec{p} \cdot \nabla)\vec{E}$; Torque: $\vec{\tau} = \vec{p} \times \vec{E}$;
Energy: $U = -\vec{p} \cdot \vec{E}$

Dipole Moment: $\vec{\mu} = \frac{1}{2}\iiint_{\mathcal{V}} \mathbb{r} \times \vec{J}(\mathbb{r})\, d^3\mathbb{r} = I\,\vec{S}$

$$\vec{B}_{\text{dip}}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{3(\vec{\mu} \cdot \hat{r})\,\hat{r} - \vec{\mu}}{|\vec{r}|^3}$$

Force: $\vec{F} = \nabla\left(\vec{\mu} \cdot \vec{B}\right)$; Torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$;
Energy: $U = -\vec{\mu} \cdot \vec{B}$

Wave: $\vec{E} = \vec{E}_0\, \psi\left(\vec{r} \cdot \hat{k} - ct\right)$

$$\Rightarrow \vec{B} = \frac{\hat{k} \times \vec{E}_0}{c}\, \psi\left(\vec{r} \cdot \hat{k} - ct\right)$$

Poynting Vector: $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$

Poynting Theorem:

$$\frac{dW}{dt} + \frac{dU}{dt} = -\oiint_{\partial\mathcal{V}} \vec{S} \cdot d^2\vec{\mathbb{r}}$$

Lorentz Transformations:

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel} \qquad \vec{B}'_{\perp} = \gamma\left(\vec{B}_{\perp} - \frac{1}{c^2}\vec{v} \times \vec{E}\right)$$

$$\vec{E}'_{\perp} = \gamma\left(\vec{E}_{\perp} + \vec{v} \times \vec{B}\right) \qquad \vec{B}'_{\parallel} = \vec{B}_{\parallel}$$