Marriage and Divorce Dynamics in Denmark

Thomas H. Jørgensen

2025

Plan for today

- Bruze, Svarer and Weiss (2015): "The Dynamics of Marriage and Divorce"
    - Full commitment!
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- **Reading guide:**
  1. What are the main *research questions*?

  2. What is the *(empirical) motivation*?

  3. What are the central *mechanisms in the model*?

  4. What is the *simplest model* in which we could capture these?

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    Danish data for cohorts 1960 (men), 1962 (women).

- **Reading guide:**
  1. What are the main *research questions*?
     - How does marriage and divorce behavior vary across age and educational groups?
     - How does educational differences influence intra-household inequality?
  2. What is the *(empirical) motivation*?
  3. What are the central *mechanisms in the model*?
  4. What is the *simplest model* in which we could capture these?

Empirical Motivation: I

- **Marriage and divorce**
  Age of female cohort = age of male cohort-2

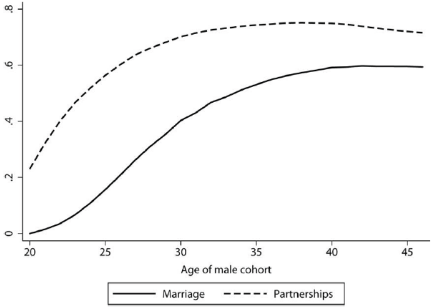

FIG. 1.—Fraction married or in partnerships (marriage plus cohabitation) by age. A color version of this figure is available online.

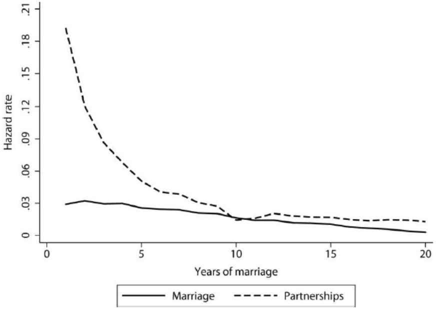

FIG. 3.—Divorce hazard for first marriage or partnership (marriage plus cohabitation). A color version of this figure is available online.

Empirical Motivation: II

- **Highly educated** people partner later but more “stable”.
  Especially if both highly educated

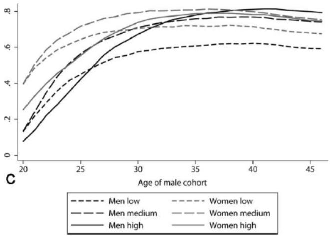

FIG. 4.—*A*, Fraction married men and women by age and education; *B*, fraction cohabiting men and women by age and education; *C*, fraction men and women in partnerships by age and education. A color version of this figure is available online.

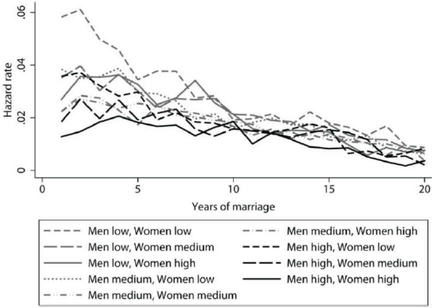

FIG. 6.—Divorce hazards for first marriages by education of the husband and wife. A color version of this figure is available online.

Empirical Motivation: III

- **Highly educated** people re-marry faster.
  And stay in second marriage longer.

FIG. 7.—Hazard rate into second marriage for men and women by education. A color version of this figure is available online.

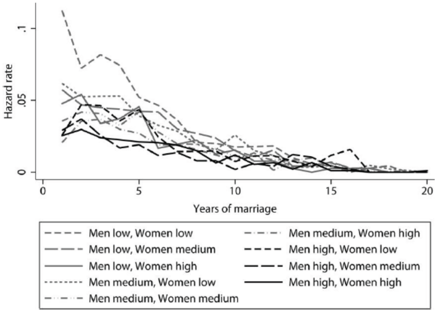

FIG. 8.—Divorce hazards when at least one spouse is in second marriage, by education of the husband and wife. A color version of this figure is available online.

Empirical Motivation: IV

- **Assortative matching:**
  people more likely to marry one with same education

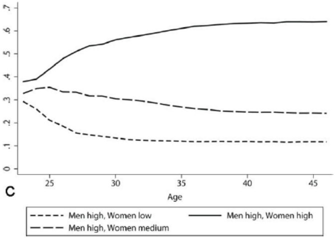

FIG. 9.—Distribution of marriages for men with low (top), medium (middle), or high (bottom) education. A color version of this figure is available online.

(a) $P(educ_w | educ_m = high)$

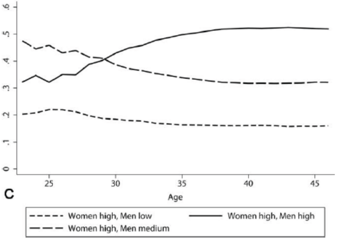

FIG. 10.—Distribution of marriages for women with low (top), medium (middle), or high (bottom) education. A color version of this figure is available online.

(b) $P(educ_m | educ_w = high)$

Empirical Motivation: V

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  - "Explain" the data
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    Important for understanding gender-inequality

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  - "Explain" the data
  - Structure $\rightarrow$ identify bargaining power $\rightarrow$ intra-household inequality
    Fundamentally unobserved
    Important for understanding gender-inequality

- **From Abstract:**
  Education raises the share of the marital surplus for men but not for women. As men and women get older, husbands receive a larger share of the marital surplus

Outline

1. Model and Mechanisms

2. Estimation

Model Overview

- **Full commitment:**
  Transferable utility
  Perfect foresight wrt bargaining power.
  Particular timing/expectation assumptions (get back)
- **Choices:**
  Marriage: which type IJ
  Divorce
- **States:**
  d_t: duration of marriage
  "TYPES":
  $e \in E = \{l, m, h\}$: Educational type of both members
  $p_t \in P = \{nm, pm\}$: never/previously married
  $u \in U = \{1, 2\}$ (unobserved type)
  $\rightarrow I \in E \times P \times U$ (men) and $J \in E \times P \times U$ (women)
  (love-shock, $\theta_t \sim iid\mathcal{N}(0, 1)$)

Bellman Equation: Married

- **Bellman equation** for type IJ (remaining) couple is

$$\underbrace{W_t^{IJ}(d_t)+\theta_t}_{V_t^{m\to m}} = \underbrace{\zeta^{IJ}+\theta_t}_{U^{IJ}} + R\mathbb{E}_t[\underbrace{\max\{\underbrace{W_{t+1}^{IJ}(d_{t+1})+\theta_{t+1}}_{V_{t+1}^{m\to m}}, \underbrace{V_{t+1}^{I}+V_{t+1}^{J}-s(d_{t+1})}_{V_{t+1}^{m\to s}}\}}_{V_{t+1}^{m}}]$$

where
ζ^{IJ}: type-specific utility
R: discount factor
$s(d_{t+1})$: divorce cost
$V_{t+1}^{I}+V_{t+1}^{J}$: sum of value of singlehood (TU)
(I would think that $d_{t+1}=d_t+1$, but they never write)
$\mathbb{E}_t[]$ is wrt. θ_{t+1}

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- **Probability** of observing divorce, $P_D(I,J,t,d_t)$:

$$\Pr(W_t^{IJ}(d_t)+\theta_t < V_t^I+V_t^J-s(d_t)) = \Phi(V_t^I+V_t^J-s(d_t)-W_t^{IJ}(d_t))$$

Bellman Equation: Single

- **Bellman equation** for type I man being single is

$$V_t^I = \varphi^I + R\mathbb{E}_t[V_{t+1}^I + \max_{J \in E \times P \times U}\{\varepsilon_{t+1}^0, \gamma_{t+1}^{IJ}[W_{t+1}(\underbrace{1}_{d_{t+1}}) - V_{t+1}^I - V_{t+1}^J] + \varepsilon_{t+1}^J\}]$$

where
ε_{t+1}^0: EV taste-shock wrt value of singlehood
ε_{t+1}^J: EV taste-shock wrt value of marriage with type J
γ_{t+1}^{IJ}: share of (new) marital surplus to man. Focus in a bit.
$\mathbb{E}_t[]$ is wrt. Extreme Value taste shocks over type of female match, J.
(See discussion on following slides.)

- **Value of marriage next period** is thus the value of being single + the share of the marital surplus he gets.
- **Symmetric** for women with share $1 - \gamma_{t+1}^{IJ}$.

Bellman Equation: Single, Marital Surplus!

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where

$$\begin{aligned} V_{t+1}^s &= V_{t+1}^I + \max_{J \in E \times P \times U}\{\varepsilon_{t+1}^0, \gamma_{t+1}^{IJ}[W_{t+1}(1) - V_{t+1}^I - V_{t+1}^J] + \varepsilon_{t+1}^J\} \\ &= \max_{J \in E \times P \times U}\{V_{t+1}^I + \varepsilon_{t+1}^0, V_{t+1}^I + \gamma_{t+1}^{IJ}[W_{t+1}(1) - V_{t+1}^I - V_{t+1}^J] + \varepsilon_{t+1}^J\} \end{aligned}$$

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- **Marital surplus** is "special" since future love-shock does not enter...:

$$W_{t+1}(1) - V_{t+1}^I - V_{t+1}^J = \mathbb{E}_t[W_{t+1}(1) + \theta_{t+1} - V_{t+1}^I - V_{t+1}^J]$$

since $\mathbb{E}_t[\theta_{t+1}] = 0$.

Bellman Equation: Single, Marital Surplus!

- They interpret this as a timing-thing (p. 140):
  "The quality of match ... is revealed to the partners only at the end of each period. ...
  In particular, single agents who marry at time t do not know the quality of their math θ_t and expect it to equal the mean, which is set to zero."

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- **"Wrong":** This means that the expected value is inserted in the max, rather than taking the expected value of the max...:

$$\max_{J \in E \times P \times U} \{\varepsilon^0_{t+1}, \gamma^{IJ}_{t+1} \mathbb{E}_t [W_{t+1}(1) + \theta_{t+1} - V^I_{t+1} - V^J_{t+1}] + \varepsilon^J_{t+1}\}$$

  vs "correct"

$$\mathbb{E}_t [\max_{J \in E \times P \times U} \{\varepsilon^0_{t+1}, \gamma^{IJ}_{t+1} [W_{t+1}(1) + \theta_{t+1} - V^I_{t+1} - V^J_{t+1}] + \varepsilon^J_{t+1}\}]$$

- **Their formulation removes a numerical integral wrt.** θ_{t+1}
  $\rightarrow$ Speeds up the solution...

Bellman Equation: Single, Marital Surplus!

- The expectation in

$$V_t^I = \varphi_t^I + R\mathbb{E}_t[V_{t+1}^I + \max_{J \in E \times P \times U}\{\varepsilon_{t+1}^0, \gamma_{t+1}^{IJ}[W_{t+1}(1) - V_{t+1}^I - V_{t+1}^J] + \varepsilon_{t+1}^J\}]$$

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- **Known in closed-form:** The log-sum!

$$\mathbb{E}_t[V_{t+1}^I + \max_{J \in E \times P \times U}\{\varepsilon_{t+1}^0, \gamma_{t+1}^{IJ}[W_{t+1}(1) - V_{t+1}^I - V_{t+1}^J] + \varepsilon_{t+1}^J\}]$$

$$=$$

$$\log\left[\exp(V_{t+1}^I) + \sum_{J \in E \times P \times U} \exp(V_{t+1}^I + \gamma_{t+1}^{IJ}[W_{t+1}(1) - V_{t+1}^I - V_{t+1}^J])\right]$$

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- **Probability** (logit) of entering marriage with type j, ($j = 0 \rightarrow$single)

$$P_M^I(j,t) = \frac{\exp(V_t^I + \gamma_t^{Ij}[W_t(1) - V_t^I - V_t^j])}{\exp(V_t^I) + \sum_{J \in E \times P \times U} \exp(V_t^I + \gamma_t^{IJ}[W_t(1) - V_t^I - V_t^J])}$$

Commitment

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- **Assume** the functional form

$$\gamma_t^{IJ} = \frac{\exp\{\rho^{IJ} + \kappa^{IJ} t + \lambda^{IJ} t^2\}}{1 + \exp\{\rho^{IJ} + \kappa^{IJ} t + \lambda^{IJ} t^2\}}$$

which has 108 estimated parameters.
(not reported)

Outline

Remaining Parameters

- **Cost of divorce** "non-parametric" (10)

$$s(d_t) = \sum_{k=1}^{9} \beta_k \mathbf{1}(d_t = k) + \beta_{10} \mathbf{1}(d_t \geq 10)$$

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- **Utility** for singles are $u \in \{1, 2\}$

$$\varphi_t^I = \mu_t^I + \eta_u^I$$
$$\varphi_t^J = \mu_t^J + \eta_u^J$$

and estimated parameters are

$$\zeta^{IJ} : 13 \text{ (education mix (9) or marital order mix (4))}$$
$$\mu_t^I, \mu_t^J : 2 \times 18 \text{ (gender, age, educ)}$$

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- **Unobserved types** (4):
estimate u_2^I, u_2^J (relative to type 1) and the share of type 2.

Estimation

- **Maximum likelihood**
  Dynamic logit due to EV taste-shocks wrt discrete types.

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- Let $S_{i,0}$ and $S_{j,0}$ be initial states. These states are taken as given.

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- The (conditional) likelihood function of the observed data is

$$L = \prod_{i=1}^{N^m} \Pr(O_i | S_{i,0}) \times \prod_{j=1}^{N^f} \Pr(O_j | S_{j,0})$$

assuming independence.

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  assuming independence.
- The EV assumption makes $\Pr(\bullet)$ conditional multinomial logit (MNL) Can be found in closed form.

Estimation

- **Likelihood of *sequence*** of choices given $S_{i,0}$, u_i

$$\Pr(O_i|S_{i,0}, u_i) = \prod_{t=2}^{T} \Pr(O_{i,t}|O_{i,t-1}, u_i) \Pr(O_{i,1}|S_{i,0}, u_i)$$

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- **Do not observe** u; we "integrate that out":

$$\begin{aligned} \Pr(O_i|S_{i,0}) &= \mathbb{E}[\Pr(O_i|S_{i,0}, u_i)] \\ &= q^m \Pr(O_i|S_{i,0}, u_i = 1) + (1 - q^m) \Pr(O_i|S_{i,0}, u_i = 2) \end{aligned}$$

where q^m and q^f are the shares of type 1 ($u = 1$)

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- **The likelihood** of observing the outcomes is then

$$\begin{aligned}L = &\prod_{i=1}^{N^m}[q^m \Pr(O_i|S_{i,0},u_i=1) + (1-q^m)\Pr(O_i|S_{i,0},u=2)] \\ \times &\prod_{j=1}^{N^f}[q^f \Pr(O_j|S_{j,0},u_j=1) + (1-q^f)\Pr(O_j|S_{j,0},u_j=1)]\end{aligned}$$

Identification (idea)

- **Identification** arguments in paper
  Only without unobserved types

- **Talk about some here**
  To give idea of arguments
  Ignores unobserved types, $u \in \{1, 2\}$
  $\rightarrow I, J \in E \times P$ (educ and pre-marital status)

Identification: Weights

- **From probability of marriage** of I with J, relative to remaining single

$$\log\left(\frac{P_M^I(J,t)}{P_M^I(0,t)}\right) = \gamma_t^{IJ}[W_t^{IJ}(1) - V_t^I - V_t^J]$$

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- similarly for women marrying type I:

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- such that taking ratios **identifies weights**, γ_t^{IJ}:

$$\frac{\gamma_t^{IJ}}{1-\gamma_t^{IJ}} = \underbrace{\log\left(\frac{P_M^I(J,t)}{P_M^I(0,t)}\right) / \log\left(\frac{P_M^J(I,t)}{P_M^J(0,t)}\right)}_{\text{data}}$$

Identification: Divorce costs

- **From probability of divorce**:

$$V_t^I + V_t^J - s(d_t) - W_t^{IJ}(d_t) = \underbrace{\Phi^{-1}(P_D(I, J, t, d_t))}_{\text{data}}$$

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We can then insert into last slide to get $s(1)$ (first-yr divorce cost):

$$\begin{aligned} \log\left(\frac{P_M^I(J, t)}{P_M^I(0, t)}\right) &= \gamma_t^{IJ}[W_t^{IJ}(1) - V_t^I - V_t^J] \\ &= \gamma_t^{IJ}[s(1) - \Phi^{-1}(P_D(I, J, t, 1))] \\ &\Updownarrow \end{aligned}$$

$$s(1) = \underbrace{\log\left(\frac{P_M^I(J, t)}{P_M^I(0, t)}\right)}_{\text{data}} / \underbrace{\gamma_t^{IJ}}_{\text{"known"}} + \underbrace{\Phi^{-1}(P_D(I, J, t, 1))}_{\text{data}}$$

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$$V_t^I + V_t^J - s(d_t) - W_t^{IJ}(d_t) = \underbrace{\Phi^{-1}(P_D(I, J, t, d_t))}_{\text{data}}$$

We can then insert into last slide to get $s(1)$ (first-yr divorce cost):

$$\log\left(\frac{P_M^I(J,t)}{P_M^I(0,t)}\right) = \gamma_t^{IJ}[W_t^{IJ}(1) - V_t^I - V_t^J]$$

$$= \gamma_t^{IJ}[s(1) - \Phi^{-1}(P_D(I, J, t, 1))]$$

$$\Updownarrow$$

$$s(1) = \underbrace{\log\left(\frac{P_M^I(J,t)}{P_M^I(0,t)}\right)}_{\text{data}} / \underbrace{\gamma_t^{IJ}}_{\text{"known"}} + \underbrace{\Phi^{-1}(P_D(I, J, t, 1))}_{\text{data}}$$

- Remaining $s(d)$: Noting that $W_t^{IJ}(d_t)$ depends on d_t through $s(d)$ and

$$\underbrace{\Phi^{-1}(P_D(I, J, t, d_t)) - \Phi^{-1}(P_D(I, J, t, d_t'))}_{\text{data}} = s(d_t') - s(d_t) + W_t^{IJ}(d_t') - W_t^{IJ}(d_t)$$

Identification: Utility Flow

- **Assume** that flow-utility in couple are constant
- **"Normalize"** value of singlehood for men over age 40 to zero (but more than normalization since several periods, $T = 71?$)
- **"Normalize"** value of singlehood for women over age 38 to zero (but more than normalization since several periods, $T = 71?$)

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- **Couples:** For $t > 39$:

$$V_t^I + V_t^J - s(d_t) - W_t^{IJ}(d_t) - \theta_t = s(d_t) - W_t^{IJ}(d_t) - \theta_t$$

and thus gets ζ^{IJ} from

$$P_D(I, J, t, d_t) = \Pr(\theta_t \leq W_t^{IJ}(d_t) - s(d_t)) = \Phi(s(d_t) - W_t^{IJ}(d_t))$$

$$\Updownarrow$$

$$W_t^{IJ}(d_t) = \underbrace{s(d_t)}_{\text{"known"}} - \underbrace{\Phi^{-1}(P_D(I, J, t, d_t))}_{\text{data}}$$

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- **Singles:** time-differences in likelihood gives φ_t^I and φ_t^J.

Results: Marriage Order

- **Estimates** suggest that second marriages are less "costly" for men

Table 3
Effects of Marriage Order on the Marital Output Flow

	Wife's First Marriage	Wife's Second Marriage
Husband's first marriage	.5166	.3891
Husband's second marriage	.4709	.5364

Results: Divorce Costs

- **Estimates** suggest that divorce costs are U-shaped
  Authors are surprised, but this could still be due to children.

Table 4
Costs of Divorce by Duration of Marriage

Marital Duration	Cost of Divorce
1 year	14.3
2 years	14.1
3 years	12.4
4 years	11.5
5 years	11.6
6 years	11.6
7 years	11.5
8 years	12.7
9 years	12.7

Results: Marital Surplus Shares

- Do not want to look at γ_t^{IJ} due to selection.

Results: Marital Surplus Shares

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- **Adjusted shares**: Want to add the Extreme Value taste shock and states

$$\mathbb{E}_t[V_{t+1}^I + \varepsilon_{t+1}^0] = V_{t+1}^I$$

is the *expected value of being forced to remain single*.

Results: Marital Surplus Shares

- Do not want to look at γ_t^{IJ} due to selection.
- **Adjusted shares**: Want to add the Extreme Value taste shock and states

$$\mathbb{E}_t[V^I_{t+1} + \varepsilon^0_{t+1}] = V^I_{t+1}$$

is the *expected value of being forced to remain single*.

- **If allowed on the marriage market**, the expected value is (log-sum):

$$C^I_{t+1} = \mathbb{E}_t[V^I_{t+1} + \max_{J \in E \times P \times U} \{\varepsilon^0_{t+1}, \gamma^{IJ}_{t+1}[W_{t+1}(1) - V^I_{t+1} - V^J_{t+1}] + \varepsilon^J_{t+1}\}$$

- **Expected gains** from entering the marriage market:

$$S^I_t = C^I_{t+1} - \mathbb{E}_t[V^I_{t+1} + \varepsilon^0_{t+1}]$$

and *similarly for women.*

Results: Marital Surplus Shares

- They define the total share of surplus of the husband be

$$\Gamma_t^{IJ} = \frac{S_t^I}{S_t^I + S_t^J}$$

Table 9
Estimated Average Total Surplus Share Γ for Husband by Education of Husband and Wife

	Wife's Education		
Husband's Education	Low	Medium	High
Low	.417	.387	.402
Medium	.496	.463	.490
High	.530	.493	.498

Results: Marital Surplus Shares

Table 11
Estimated Average Total Surplus Share Γ for Husband by Marital History of Husband and Wife

	Wife's First Marriage	Wife's Second Marriage
Husband's first marriage	.464	.353
Husband's second marriage	.563	.455

Table 12
Estimated Average Total Surplus Share Γ for Husband by Age of Husband

Age of Husband	Share of Gains to Marriage
25	.425
30	.465
35	.486
40	.505
45	.541

Next Time

- **Next time:**
  Fertility and Labor Supply.

- **Literature:**
  Jakobsen, Jørgensen and Low (2022): "Fertility and Family Labor Supply"
  [unitary]
  - **Read** before lecture
  - **Reading guide:**
    Section 1: Introduction – Read.
    Section 2: Data. Skim.
    Section 3: Empirical motivation. Get idea.
    Section 4: Model. Key. Get the idea.
    Section 5: Estimation results. skim/read.
    Section 6: Simulation results. Key - read.
    Section 7: Sensitivity/robustness. Can drop.

References I

BRUZE, G., M. SVARER AND Y. WEISS (2015): “The Dynamics of Marriage and Divorce,” *Journal of Labor Economics*, 33(1), 123–170.

JAKOBSEN, K., T. H. JØRGENSEN AND H. LOW (2022): “Fertility and Family Labor Supply,” Working paper, Centre for Economic Behavior and Inequality.