Children and Time Allocation

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2025

Plan for today

- Blundell, Pistaferri and Saporta-Eksten (2018): "Children, Time Allocation and Consumption Insurance"
  - Unitary model - Combines several US data sources.

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- **Reading guide:**
  1. What are the main *research questions*?
  2. What is the *(empirical) motivation*?
  3. What are the central *mechanisms in the model*?
  4. What is the *simplest model* in which we could capture these?

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- **Reading guide:**
  1. What are the main *research questions*?
     - How do couples allocate time and consumption when having children?
     - How do children affect couples' abilities to smooth consumption?
  2. What is the *(empirical) motivation*?
  3. What are the central *mechanisms in the model*?
  4. What is the *simplest model* in which we could capture these?

Empirical Motivation: Siminski and Yetsenga (2022)

- Australian time-use data on panel of couples!

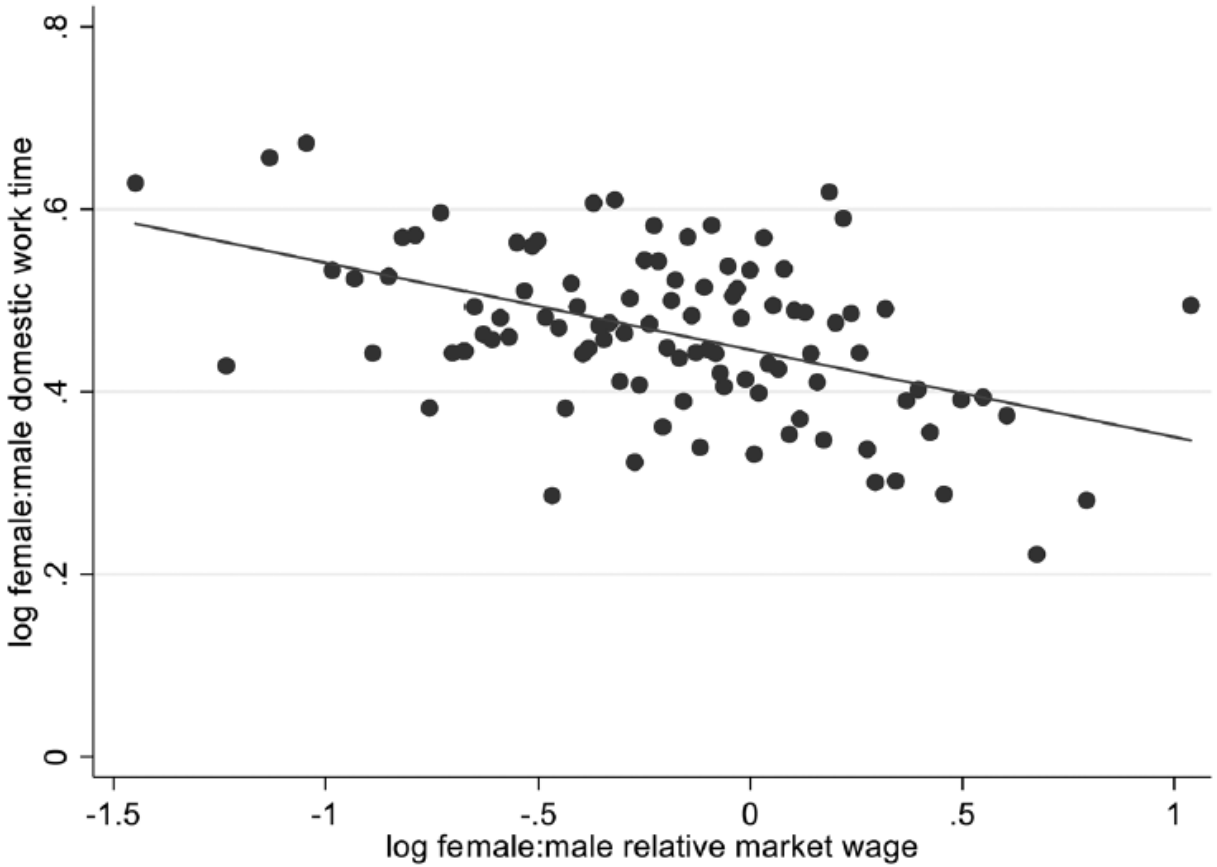

FIG. 1.—Relative domestic work time by relative wage. Each point represents 1 percentile of the female-to-male relative wage distribution among heterosexual couples. A color version of this figure is available online.

Outline

Model Overview

- **Write** out the *recursive formulation* of the model
  States
  Choices
  (transitions)

Model Overview

- **Choices:**
  $H_{j,t}$: work hours, $j \in \{1, 2\}$ (2=woman)
  $L_{j,t}$: leisure hours, $j \in \{1, 2\}$
  $T_{j,t}$: Parenting hours, $j \in \{1, 2\}$ (child care)
  C_t: Household consumption
- **States:**
  A_t: wealth
  $F_{j,t}$: permanent income shock, $j \in \{1, 2\}$
  $u_{j,t}$: transitory income shock, $j \in \{1, 2\}$
  ε: vector of 5 unobserved time-fixed taste-shifters.
  (only allow for ε_{L_2},wife's leisure, using two-point, fnt 27)
  z_t: child (50/50 prob. at age 28, young for 10years)

State Transitions

- **Budget**

$$A_{t+1} = (1+r)[A_t + \mathcal{T}(z_t, H_{1,t}W_{1,t} + H_{2,t}W_{2,t}) - C_t]$$

where joint taxation gives

$$\mathcal{T}(z_t, H_{1,t}W_{1,t} + H_{2,t}W_{2,t}) = \chi_t(b(z_t) + H_{1,t}W_{1,t} + H_{2,t}W_{2,t})^{1-\mu_t}$$

with $b(z_t)$ being a consumption floor.

- **Hours worked** are

$$H_{j,t} = \overline{L} - L_{j,t} - T_{j,t}$$

- **Wages** are

$$\log W_{j,t} = x'_{j,t}\beta^j_W + F_{j,t} + u_{j,t}$$
$$F_{j,t} = F_{j,t-1} + v_{j,t}$$

Preferences

- **Utility** is

$$\exp(\tilde{\phi}_C(z_t, \varepsilon_t)) \frac{[C_t - \gamma(z_t)\mathbf{1}(H_{2,t} > 0)]^{1-1/\eta}}{1-1/\eta}$$
$$- \frac{1}{1-\rho_L}\left[\exp(\tilde{\phi}_{L_1}(z_t, \varepsilon_t)) L_{1,t}^{1-1/\varphi_{L_1}} + \exp(\tilde{\phi}_{L_2}(z_t, \varepsilon_t)) L_{2,t}^{1-1/\varphi_{L_2}}\right]^{1-\rho_L}$$
$$- \frac{1}{1-\rho_T}\left[\exp(\tilde{\phi}_{T_1}(z_t, \varepsilon_t)) T_{1,t}^{1-1/\varphi_{T_1}} + \exp(\tilde{\phi}_{T_2}(z_t, \varepsilon_t)) T_{2,t}^{1-1/\varphi_{T_2}}\right]^{1-\rho_T}$$

where, for $x \in \{C, L_1, L_2, T_1, T_2\}$,

$$\tilde{\phi}_x(z_t, \varepsilon_t) = \phi_x^{nk} + \phi_x^k z_t + \varepsilon_{x,t}$$

are taste-shifters.
(only $var(\varepsilon_{L_2,t}) > 0$ so irrelevant in all other)

Preferences

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$$-\frac{1}{1-\rho_T}\left[\exp(\tilde{\phi}_{T_1}(z_t,\varepsilon_t))T_{1,t}^{1-1/\varphi_{T_1}}+\exp(\tilde{\phi}_{T_2}(z_t,\varepsilon_t))T_{2,t}^{1-1/\varphi_{T_2}}\right]^{1-\rho_T}$$

where
$\eta>0$ is the consumption Frisch elasticity ($1/\eta$ is the CRRA)
$\gamma(z_t)$ is cost of work (for women)
$\varphi_x\in(0,1)$ is the curvature wrt x. (Governs how sensitive x is to e.g. wage changes.)
Note the negative sign and that $1-1/\varphi_x<0$

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  where they claim that
  $\rho_x < 1$ is the complementarity ($\rho_x > 0$) / substitutability ($\rho_x < 0$) between men and women

- This is not true, I think. See note.
  If $\varphi_{x_1} = \varphi_{x_2} = \varphi_x$, then that parameter is the *elasticity of substitution*.
  $1 - \rho_x$ controls the *returns to scale*.

Preferences

- **Utility** is

$$\exp(\tilde{\phi}_C(z_t,\varepsilon_t))\frac{[C_t-\gamma(z_t)\mathbf{1}(H_{2,t}>0)]^{1-1/\eta}}{1-1/\eta}$$
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where
interpreting the last part as "home production of children"
$\rightarrow$ relative weight on j is their absolute advantage in child production
$\rightarrow$ if $\tilde{\phi}_{T_2}(z_t,\varepsilon_t) > \tilde{\phi}_{T_1}(z_t,\varepsilon_t)$ mothers have an absolute advantage

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- Illustrates the amount of hoops one could be willing to jump to reduce the parameter space in the SMD...

Data Sources

- **Panel Study of Income Dynamics (PSID)**
  labor income, and hours worked, $H_{j,t}$, $\longrightarrow$ $w_{j,t}$
  Non-durable consumption, c_t, and assets, A_t.

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  Time used for leisure, $L_{j,t}$, and child care, $T_{j,t}$
  Only for one respondent (not both partners)
  $\rightarrow$ Use responses of women and *impute* values for their partners:

$$X_{1,t} = f\left(cohort_1, educ_1\right),\ X \in \{L, T\}$$

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- **Consumer Expenditure Survey (CEX)**
  Non-durable consumption, c_t.
  (better quality than PSID)

MRS: 1

- **MRS between wife's and husband's leisure** can be found to give *[note: nothing about ρ_L!]*

$$L_2 = \left[\frac{(1-1/\varphi_{L_1})\exp(\tilde{\phi}_{L_1})}{(1-1/\varphi_{L_2})\exp(\tilde{\phi}_{L_2})}\right]^{-\varphi_{L_2}} \left[\frac{W_2}{W_1}\right]^{-\varphi_{L_2}} L_1^{\varphi_{L_2}/\varphi_{L_1}}.$$

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- Taking logs gives ($x = \log(X)$)

$$\begin{aligned} l_2 = &- \varphi_{L_2}[\log(1-1/\varphi_{L_1}) + \tilde{\phi}_{L_1} - \log(1-1/\varphi_{L_1}) - \tilde{\phi}_{L_2}] \\ &- \varphi_{L_2}(w_2 - w_1) + \frac{\varphi_{L_2}}{\varphi_{L_1}} l_1 \end{aligned}$$

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- We can write this as

$$l_2 = K_0 + \varphi_{L_2}(w_1 - w_2) + \frac{\varphi_{L_2}}{\varphi_{L_1}} l_1 + \varphi_{L_2}(\varepsilon_1 - \varepsilon_2)$$

where $\varepsilon_1 - \varepsilon_2$ is unobserved (random) and the constant is

$$K_0 = -\varphi_{L_2}[\log(1-1/\varphi_{L_1}) - \log(1-1/\varphi_{L_1}) + \phi_{L_1}^{nk} + \phi_{L_1}^{k} z - \phi_{L_2}^{nk} - \phi_{L_2}^{k} z]$$

MRS: 1

- **MRS between wife's and husband's leisure** (e.q. 7, $x = \log(X)$)

$$\mathbb{E}\left[l_{2,t} - K_0 - \varphi_{L_2}(w_{1,t} - w_{2,t}) - \frac{\varphi_{L_2}}{\varphi_{L_1}} l_{1,t} \middle| I_t\right] = 0$$

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  can give *three* moments to identify K_0, φ_{L_2} and φ_{L_1}.
- **Requires individual-level data** on leisure and wages.
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- **They use PSID**, people with *no children* younger than 10
  $\rightarrow L_{j,t} = \underbrace{\overline{L}}_{\text{4,160 ass.}} - \underbrace{T_{j,t}}_{\text{0 ass.}} - H_{j,t}$ observed through $H_{j,t}$.

MRS: 2

- **MRS between wife's leisure and consumption** (e.q. 8)

$$\mathbb{E}[l_{2,t} - K_1 + \varphi_{L_2} w_{2,t} - \mu\varphi_{L_2} y - \frac{\varphi_{L_2}}{\eta} c_t - \frac{\varphi_{L_2}}{\varphi_{L_1}} \rho_L (1 - \varphi_{L_1}) l_{1,t}$$
$$+ \varphi_{L_2} \rho_L \frac{\varphi_{L_2}(1 - \varphi_{L_2})}{\varphi_{L_1}(1 - \varphi_{L_1})} \frac{W_{2,t} L_{2,t}}{W_{1,t} L_{1,t}} | I_t] = 0$$

where μ is "known" tax parameter and y is household income.
Can likewise give *three* moments to identify K_1, η and ρ_L.

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  leisure, wages and consumption.

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  $\rightarrow L_{j,t} = \underbrace{\overline{L}}_{\text{4,160 ass.}} - \underbrace{T_{j,t}}_{\text{0 ass.}} - H_{j,t}$ observed through $H_{j,t}$.

MRS: 3

- **MRS between wife's and husband's parental time** (e.q. 9)

$$\mathbb{E}\big[t_{2,t} - K_2 - \varphi_{T_2}(w_{1,t} - w_{2,t}) - \frac{\varphi_{T_2}}{\varphi_{T_1}} t_{1,t} | I_t\big] = 0$$

- **MRS between wife's parental time and consumption** (e.q. 10)

$$\mathbb{E}\big[t_{2,t} - K_3 + \varphi_{T_2} w_{2,t} - \mu\varphi_{T_2} y - \frac{\varphi_{T_2}}{\eta} c_t - \frac{\varphi_{T_2}}{\varphi_{T_1}} \rho_T (1 - \varphi_{T_1}) t_{1,t}$$

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- **Solution:** Impute consumption from the CEX "into" the ATUS.
  1. **Estimate** avg. consumption in CEX: $\hat{C}(cohort, educ)$
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- Similarly for the time-use of men (as discussed above)

Parameter Estimates

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PARAMETER ESTIMATES

	A. MRS ESTIMATES		
	Leisure and Consumption (1)		Parental Time (2)
φ_{L_1}	.211	φ_{T_1}	.115
	(.037)		(.081)
φ_{L_2}	.162	φ_{T_2}	.503
	(.025)		(.201)
ρ_L	.535	ρ_T	−.197
	(.099)		(.123)
η	.903		
	(.049)		
Observations	11,195		2,901

	B. PREFERENCE SHIFTERS	
	With Children	Without Children
ϕ_{L_1}	−8.925	−7.680
	(1.108)	(1.013)
ϕ_{L_2}	−9.397	−8.816
	(1.036)	(1.024)
ϕ_{T_1}	−23.993	N/A
	(10.245)	
ϕ_{T_2}	−3.957	N/A
	(1.201)	
$\sigma^2_{\varepsilon L_2}$	1.476	.700
	(.174)	(.087)
γ	(see table 2)	4,794
		(438)
ϕ_C	.132	Normalized to 0
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leisure does not respond alot to wage-changes

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Child-care time of mothers reponds a bit to wages

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Observations	11,195		2,901

decreasing returns to scale
(them: lesure complements)

	B. Preference Shifters	
	With Children	Without Children
ϕ_{L_1}	−8.925	−7.680
	(1.108)	(1.013)
ϕ_{L_2}	−9.397	−8.816
	(1.036)	(1.024)
ϕ_{T_1}	−23.993	N/A
	(10.245)	
ϕ_{T_2}	−3.957	N/A
	(1.201)	
$\sigma^2_{\varepsilon_{L_2}}$	1.476	.700
	(.174)	(.087)
γ	(see table 2)	4,794
		(438)
ϕ_C	.132	Normalized to 0
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Parameter Estimates

TABLE 3
PARAMETER ESTIMATES

	A. MRS Estimates		
	Leisure and Consumption (1)		Parental Time (2)
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	(.037)		(.081)
φ_{L_2}	.162	φ_{T_2}	.503
	(.025)		(.201)
ρ_L	.535	ρ_T	−.197
	(.099)		(.123)
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increasing returns to scale (them: substitutes)

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children decrease the value of leisure

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women have a large abs. advantage in child-care

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random pref. shocks have more variance when children are present

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fixed cost (in cons.) of work 2,900

Parameter Estimates

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marg. util. of cons. is 13% higher when children present

Outline

Simulations

- Simulate **transitory and permanent** wage changes.
  Men and women separately
- **Transitory:** Approximate Frisch (since little income effect)
- **Permanent:** Approximate Marshall

Consumption and Labor Supply Responses

- Age 30 response from 10% increase in wage in two models
  With child from age 28 + Without child from age 28 (elasticities)

TABLE 5
CONSUMPTION AND LABOR SUPPLY RESPONSES TO TRANSITORY AND PERMANENT SHOCKS

	TOTAL RESPONSE						EXTENSIVE VS. INTENSIVE MARGIN			
	C		H_1		H_2		E_2		H_2|Employed	
	With Kids (1)	Without Kids (2)	With Kids (3)	Without Kids (4)	With Kids (5)	Without Kids (6)	With Kids (7)	Without Kids (8)	With Kids (9)	Without Kids (10)
Transitory:										
Husband	.119	.123	.180	.222	−.076	.001	−.051	.005	−.041	.006
Wife	.130	.135	.000	−.006	.703	.394	.574	.280	.329	.167
Permanent:										
Husband	.393	.410	.105	.116	−.296	−.140	−.193	−.065	−.170	−.088
Wife	.353	.375	−.070	−.106	.531	.304	.491	.266	.208	.086

NOTE.—Model-simulated responses for transitory and permanent shocks.

1. Consumption response consistent with buffer-stock theory: transitory shocks have little effect
2. Women have larger responses than men
3. Children increases response for women
4. Extensive margin important (for women)

Leisure and Parental Time Responses

TABLE 6
LEISURE AND PARENTAL TIME RESPONSES TO TRANSITORY AND PERMANENT SHOCKS

	L_1		L_2		T_1	T_2
	With Kids (1)	Without Kids (2)	With Kids (3)	Without Kids (4)	With Kids (5)	With Kids (6)
Transitory:						
Husband	−.230	−.231	−.003	−.001	−.095	.131
Wife	−.007	.006	−.217	−.309	.033	−.538
Permanent:						
Husband	−.131	−.120	.078	.110	−.067	.261
Wife	.085	.110	−.151	−.238	.058	−.443

NOTE.—Model-simulated responses for transitory and permanent shocks.

1. Leisure elasticities similar between men/women w/w.o. kids and compliments (same-sign cross trans ela)
2. Permanent $\rightarrow$ reduction in both own leisure and child care time and opposite sign cross elasticity $\rightarrow$ specialization.
3. Women have large responses on child-care time from own and male wages.

Consumption Insurance

TABLE 7
INSURANCE EFFECTS

Consumption	−3.9%	
After-tax and transfers household earnings	−5.0%	
Before-tax (after-transfers) household earnings	−5.6%	
	Husband	Wife
Earner's average share of before-tax earnings	.66	.34
Earner's before-tax and transfers earnings response:	−10.7%	+2.0%
Hours	−1.0%	+3.0%
Leisure	+1.3%	−.8%
Parental time	+.7%	−2.6%

NOTE.—Insurance decomposition calculations based on model-simulated responses to a 10 percent permanent decline in the husband's wage.

1. Some consumption insurance (3.9% drop from 10% drop in wages)
2. Substitution effect dominates (-1% in hours worked)
3. Sizable cross-effect (+3% in work hours of women)
4. Leisure margin most active for men, parent time most for women.

Counterfactual Simulations

Two counterfactuals with same budget effects:

1. unconditional child-subsidy, $b(z) \uparrow$
2. employment subsidy, $\gamma(z) \downarrow$

TABLE 10
POLICY EXPERIMENTS

	C (1)	H_1 (2)	H_2 (3)	E_2 (4)	L_1 (5)	L_2 (6)	T_1 (7)	T_2 (8)
	A. Experiment 1: Unconditional Subsidy for Families with Young Children							
Total	.6%	−.4%	−.7%	−.4%	.4%	.3%		
Before young children	.9%	−.4%	−.5%	−.2%	.4%	.4%		
With young children	1.3%	−.6%	−1.8%	−1.0%	.8%	.7%	.2%	1.0%
After young children	.1%	−.1%	−.1%	−.1%	.1%	.1%		
Consumption equivalent utility value	.95%							
	B. Experiment 2: Employment Subsidy for Wives with Young Children							
Total	.1%	−.2%	1.9%	4.6%	.2%	−.5%		
Before young children	.9%	−.4%	−.5%	−.1%	.4%	.4%		
With young children	−.3%	−.3%	6.5%	13.1%	.3%	−1.7%	.3%	−5.6%
After young children	.1%	−.1%	−.1%	~0%	.1%	.1%		
Consumption equivalent utility value	.17%							

Toy Model I

Unitary model, couples jointly **choose**

- labor supply of both, $l_{j,t} \in [0, 24]$, $j \in \{f, m\}$,
- home production of both, $h_{j,t} \in [0, 24]$, $j \in \{f, m\}$, subject to time-constraint $T_{j,t} = l_{j,t} + h_{j,t} \leq 24$

State variables

- human capital of both, $K_{j,t} \geq 0$
- child present, $n_t \in \{0, 1\}$

Toy Model II

- Couples care about market goods, C_t, and home produced goods, H_t, through the composite good

$$Q_t = C_t^{\omega} H_t^{1-\omega}$$

where $\omega = \omega_0 + \omega_n n_t \in (0,1)$ is the relative weight on market goods.

- Home production is a (CES) function,

$$H_t = \left(\alpha h_{f,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha) h_{m,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma \in (0,1)$ is the elasticity of substitution and $\frac{\alpha}{1-\alpha} \in (0,1)$ is the productivity in home production of females relative to males (her absolute advantage in home production, AAH).

Toy Model II

- Household preferences over work and consumption are

$$U(T_{f,t}, T_{m,t}, Q_t) = \frac{Q_t^{1-\rho}}{1-\rho} - \nu \left(\frac{T_{f,t}^{1+\frac{1}{\epsilon_f}}}{1+\frac{1}{\epsilon_f}} + \frac{T_{m,t}^{1+\frac{1}{\epsilon_m}}}{1+\frac{1}{\epsilon_m}} \right) \tag{1}$$

  in which the last term is the dis-utility from total work hours ($\nu > 0$) and $\epsilon_j > 0$ controls the curvature of the dis-utility. $\rho > 1$ is the constant relative risk aversion coefficient.

- There are no bequest motive and thus

$$V_{T+1}(n_{t+1}, K_{f,T+1}, K_{m,T+1}) = 0. \tag{2}$$

Toy Model III

- The recursive formulation of the model is for $t \leq T$:

$$V_t(n_t, K_{f,t}, K_{m,t}) = \max_{l_{f,t}, h_{f,t}, l_{m,t}, h_{m,t}} U(T_{f,t}, T_{m,t}, Q_t) + \beta \mathbb{E}_t[V_{t+1}(n_{t+1}, K_{f,t+1}, K_{m,t+1})] \quad (3)$$

$$\text{s.t.} \quad (4)$$

$$C_t = w_{f,t} l_{f,t} + w_{m,t} l_{m,t} + X_t \quad (5)$$

$$H_t = \left(\alpha h_{f,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha) h_{m,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (6)$$

$$Q_t = C_t^{\omega} H_t^{1-\omega} \quad (7)$$

$$\log w_{j,t} = \gamma_{j,0} + \gamma_{j,1} K_{j,t}, \; j \in \{f, m\} \quad (8)$$

$$K_{j,t+1} = (1-\delta) K_{j,t} + l_{j,t}/24, \; j \in \{f, m\} \quad (9)$$

$$n_{t+1} = \begin{cases} n_t + 1 & \text{with prob. } p \cdot (1 - n_t) \\ n_t & \text{with prob. } 1 - p \cdot (1 - n_t) \end{cases} \quad (10)$$

$$T_{j,t} = l_{j,t} + h_{j,t}, \; j \in \{f, m\} \quad (11)$$

$$l_{j,t}, h_{j,t} \geq 0, \; j \in \{f, m\} \quad (12)$$

$$T_{j,t} \leq 24, \; j \in \{f, m\} \quad (13)$$

Exam: Upload

You should hand in a single zip-file with all assignments and the exam. The zip-file should be named after your KU username (e.g. abs123) and have the following folder and file structure:

Assignment_1\
Assignment_1.pdf - with text and all results
files for reproducing the results
Assignment_2\
Assignment_2.pdf - with text and all results
files for reproducing the results
Assignment_3\
Assignment_3.pdf - with text and all results
files for reproducing the results
Exam\
Exam.pdf - with text and all results
files for reproducing the results

- **Individual exam!**

Exam: Tips

- **Try to answer all questions**
  48 hours, but thought of as 2×9 work days
  Make sure that your computer+Python works!
  Similar flavor as assignments

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- **Thanks for now - Good luck!**

References I

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