Written Exam Economics Summer 2024
Household Behavior over the Life Cycle

19 June 2024 from 9 AM to 21 June at 9 AM

This exam question consists of 5 pages in total.

Answers only in English.
A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes.

You should hand in a single zip-file with all assignments and the exam. The zip-file should be named after your KU username (e.g. abs123) and have the folder and file structure:

Assignment_1
Assignment_1.pdf - with text and all results
files for reproducing the results

Assignment_2
Assignment_2.pdf - with text and all results
files for reproducing the results

Assignment_3
Assignment_3.pdf - with text and all results
files for reproducing the results

Exam
Exam.pdf - with text and all results
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Use of AI tools is permitted. You must explain how you have used the tools. When text is solely or mainly generated by an AI tool, the tool used must be quoted as a source.

Be careful not to cheat at exams!
Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text.
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts.
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism).
- Receive help from others in contrary to the rules in the Faculty of Social Science's common part of the curriculum.

You can read more about the rules on exam cheating on your Study Site and in the Faculty of Social Science's common part of the curriculum.

Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

Setup and Model Economy

We will study the effects of different taxes and transfers through extensions of the model and code from lecture 4 in the course. Baseline code is available online through Digital Exam.

We focus on individuals who choose how many hours to work, $h_t \geq 0$, and how much to consume, $c_t > 0$, each period of life, $t = 1, \ldots, T$. In each period, the worker accumulates human capital, k_t, and earns income $w_t \cdot h_t$ that is taxed with a flat rate of τ. Wages are a function of human capital, $w_t = w(1 + \alpha k_t)$, where w and α are known constants. Everything is normalized in such a way that w_t is interpreted as an annual full-time wage rate and $h_t = 1$ corresponds to a year of full-time work. In Denmark full time is 37 hours a week, but workers can work more if they want in the model. Resources that are not consumed can be carried over to the next period. In turn, wealth in the following period is given by $a_{t+1} = (1 + r)(a_t + (1 - \tau)w_t h_t - c_t)$ where r is the interest rate on wealth.

The recursive formulation (Bellman equation) for the model is, for $t < T$,

$$V_t(a_t, k_t) = \max_{c_t, h_t} U(c_t, h_t) + \rho V_{t+1}(a_{t+1}, k_{t+1}) \tag{1}$$

s.t.

$$a_{t+1} = (1 + r)(a_t + (1 - \tau)w_t h_t - c_t) \tag{2}$$

$$k_{t+1} = k_t + h_t \tag{3}$$

$$w_t = w(1 + \alpha k_t) \tag{4}$$

$$c_t > 0 \tag{5}$$

$$h_t \geq 0 \tag{6}$$

where preferences are

$$U(c_t, h_t) = \frac{c_t^{1+\eta}}{1 + \eta} - \beta \frac{h_t^{1+\gamma}}{1 + \gamma}. \tag{7}$$

There is no bequest motive such that for $t = T$,

$$V_T(a_T, k_T) = \max_{c_T, h_T} U(c_T, h_T). \tag{8}$$

s.t.

$$0 \leq (1 + r)(a_T + (1 - \tau)w_T h_T - c_T). \tag{9}$$

The government budget in this baseline economy only consists of taxes and is thus given by

$$G = \sum_{i=1}^{N} \sum_{t=1}^{T} \tau w_{i,t} h_{i,t}. \tag{10}$$

The baseline parameters are $T = 10$, $\rho = 0.99$, $\beta = 1.0$, $\eta = -2.5$, $\gamma = 2.5$, $\alpha = 0.1$, $w = 1.0$, $\tau = 0.1$, and $r = 0.03$. When simulating, we draw initial level of wealth from a standard Normal distribution, truncated at zero. Initial human capital is drawn from a Normal distribution with mean zero and standard deviation 0.5 and truncated at zero. We simulate $N = 1000$ individuals for $T = 10$ periods. Questions can be answered independently but the idea is that you answer question x building on the setup in question $x - 1$. Try to avoid copy-pasting code but rather generate one code base that can answer all questions.

Questions

1. Explain why optimal consumption in the last period of life will be

$$c_T^{\star}(a_T, k_T|h_T) = a_T + (1-\tau)w_T h_T$$

   in the problem (8)–(9) above, as a function of hours worked (to be chosen).

2. Add a human capital depreciation shock to the model. In each period, there is uncertainty about if human capital will depreciate when entering the following period. We thus modify eq. (3) to be

$$k_{t+1} = \begin{cases} (1-\delta)k_t + l_t & \text{with probability } p_k \\ k_t + l_t & \text{with probability } 1 - p_k \end{cases}$$

   where $p_k = 0.6$ is the probability that human capital will depreciate by 20% ($\delta = 0.2$). Be precise about how this changes the problem in eq. (1) above. Modify the code to allow for this modification and illustrate how this changes the average consumption, labor supply, human capital and savings over the life cycle.
   [*hint:* If either $\delta = 0$ or $p_k = 0$ behavior should be unchanged from the baseline.]

3. Add a flat tax rate of $\tau_a = 0.10$ on *positive wealth* in the model with human capital depreciation shocks. Be precise about which equations you need to modify to implement the wealth tax and show the modifications mathematically. Modify the code to allow for this tax and illustrate how this changes the average consumption, labor supply, human capital and savings over the life cycle.
   [*hint:* if $\tau_a = 0$ the model is identical to that in the previous question.]

4. Discuss the mechanisms through which a wealth tax affects incentives and behavior in the model.

5. Calculate and report the government budget, G, and the average consumer welfare,

$$W = \frac{1}{N}\sum_{i=1}^{N} W_i,$$

where

$$W_i = \sum_{t=1}^{T} U(c_{i,t}, h_{i,t})$$

is the *individual welfare* in the economy. Be precise about how you calculate these objects in Python. Show the distribution of welfare across individuals.

6. Perform two tax reforms. In the first reform, reduce the labor income tax rate, τ, to 0.1. In the second reform, reduce the wealth tax, τ_a, to 0.05.
   Show and discuss how these reforms affect *i)* hours worked throughout life, *ii)* human capital throughout life, *iii)* wealth accumulation throughout life, *iv)* government budget, and *v)* the *distribution* of individual welfare, W_i. Plot age profiles for the baseline model and the two reforms jointly in one graph for each outcome.

7. Determine what level of the labor income tax rate $\tau \in [0.10, 0.20]$ that generates a government budget of 1900 (with τ_a fixed at 0.10). Denote this tax-rate level as τ^G and this tax-system as (τ^G, τ_a). Likewise, determine the wealth tax rate $\tau_a \in [0.40, 0.80]$ that generates the same government budget of 1900 (with τ fixed at 0.12). Denote this rate as τ_a^G and the tax-system as (τ, τ_a^G). Report the two sets of tax rates and illustrate average consumption, labor supply, human capital, savings, and government budget over the life cycle in the two tax-systems/regimes

8. Analyze the individual welfare from the two alternative tax-systems found in the question above. Are there particular groups that prefer one system over the other?

9. Estimate, using simulated method of moments (SMM/SMD), the dis-utility from work $\beta \in (0.5, 1.5)$ that produces approximately the same labor supply as in Denmark. Concretely, match average weekly working hours of 35, corresponding to $h = 35/37$ in our model. The average is taken both over time (the life cycle in the model) and individuals. Report the estimated β parameter and the model fit.
   Use the model and parameters from question 3 (i.e. reset taxes to their baseline values before the reform experiments above) when estimating β.
   Show *i)* hours worked throughout life, *ii)* human capital throughout life, *iii)* wealth accumulation throughout life, and *iv)* government budget, for the model with $\beta = 1.0$ and for the model with the estimated value of $\hat{\beta}$.
   [*Hint:* Preferably, a numerical solver is used but alternatively a grid-search can be used. A good starting value could be $\beta_0 = 1.0$.]

10. Add disability to the model. Let $d_t = 0$ denote being not disabled and $d_t = 1$ disabled in period t. If a worker is disabled in period t, she/he will incur a reduction in the current wage rate of κ percent and receive a disability benefit of $b = 0.2$. The likelihood of being disabled

in period $t+1$ is a function of the disability status today,

$$P(d_{t+1} = 1 | d_t) = \begin{cases} p_d & \text{if } d_t = 0 \\ 2 \cdot p_d & \text{if } d_t = 1 \end{cases}$$

where $p_d = 0.1$. Let $\kappa = 0.5$. Write out the new recursive formulation of the extended model with human capital depreciation risk and disability risk.
[*Hint:* How should the government budget from equation (10) be modified?]

11. Implement the extended model with human capital depreciation risk and disability risk in Python. Assume that everyone starts without disability, i.e. $d_{i,0} = 0$ for all i in the simulation. Illustrate how this disability risk changes the average consumption, labor supply, human capital, savings, *disability*, government budget, and welfare over the life cycle.
[*Hint:* The original model is nested by setting $p_d = 0$ and focusing on individuals who have $d_t = 0$. You can also implement an explicit switch in the code.]

12. Explain how you would find the level of disability benefits, $b^\star$, that would exactly balance the government budget, setting $G = 0$.
Imagine now, that the government wants to remain on a balanced budget, but is interested in reducing taxes. Discuss the likely implications of such a policy in the final model.