Introduction

Thomas H. Jørgensen

2025

Dynamic Programming

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Outline

- This Course

Introduction

• Teacher: Thomas H. Jørgensen (Associate Professor, University of Copenhagen)

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- Focus: "Dynamic labor and family economics"
- Topics:

Labor supply of couples Career costs of children Family formation and dissolution

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- Tools: Dynamic programming in Python
- Prerequisite: Introduction to Programming and Numerical Analysis

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Tools:

Dynamic programming in Python

• Prerequisite:

Introduction to Programming and Numerical Analysis

- Plan for today:
 - 1. Course description
 - 2. Programming in Python
 - 3. Introduction to Dynamic Programming

Teaching Methods

- **Lectures:** Tuesday 10-13
 - ~2 hours lecture
 - ~1 hour of problem-solving (e.g. programming)

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Code:

I will provide code.

You will be asked to modify it to answer certain questions.

Dynamic Programming

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Material:

https://github.com/ThomasHJorgensen/HouseholdBehaviorCourse

Introduction

This Course

- 1. Introduction
- 2. Dynamic Programming and Structural Estimation

Part 1: Labor Supply

- 3. Static and Dynamic labor supply
- 4. Dynamic labor supply and human capital
- .. No lecture: Work on assignment / read papers
- 5. Career costs of children
- 6. Household Labor Supply and Taxation
- 7. Household Labor Supply and Child-Related Transfers

Part 2: Family Formation and Dissolution

- 8. Models of Household Behavior
- 9. Divorce Law and Intra-Household Bargaining
- 10. Marriage and Divorce
- 11. Fertility and labor supply
- 12. [online] Taxes, Transfers and Intra-Household Inequality

Outroduction

13. Children and Time Allocation

Assignments and Exam

• Exam (Portfolio):

This Course

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- 1. 3 individual assignments.
 - + peer feedback.
- 2. 48 hour individual take-home exam. Model formulation, code modification, simulations, economic interpretations.

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- Exam (Portfolio):
 - 1. 3 individual assignments.
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 Model formulation, code modification, simulations, economic interpretations.
- 3 individual assignments (hand-in on Absalon)
 - 1. Based on our dynamic labor supply model

Deadline: March 14

- 2. Based on our dynamic **bargaining** model Deadline: April 11
- 3. **Free**: Formulate a research question + model + data.

Deadline: May 9

Deadline for peer feedback: May 16

• All feedback can be used to improve assignments before exam date

Outline

This Course

- Models

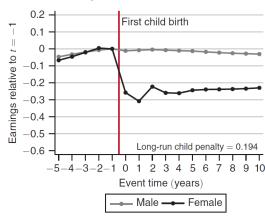
This Course

• We will use empirical regularities as motivation

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Example: "Child penalty" (Kleven, Landais and Søgaard, 2019)





Dynamic Programming

This Course

• We will use economic models to understand/quantify behavior

This Course

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Example: "Career costs of children" (Adda, Dustmann and Stevens, 2017) "We estimate a dynamic life cycle model of labor supply, fertility, and savings, incorporating occupational choices, with specific wage paths and skill atrophy that vary over the career. This allows us to understand the trade-off between occupational choice and desired fertility, as well as sorting both into the labor market and across occupations. We **quantify** the life cycle career costs associated with children, how they **decompose** into loss of skills during interruptions, lost earnings opportunities, and selection into more child-friendly occupations. We analyze the **long-run effects of policies** that encourage fertility and show that they are considerably smaller than short-run effects."

This Course

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or bluntly... Equations that describe a small part of reality...

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"All models are wrong, but some are useful"

George E.P. Box

Dynamic Programming

Models! What are they good for!?

• Why ever use a model then?

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This Course

• The Lucas critique: Behavioral rules change with policy ⇒ policy advice can not rely on estimated behavioral rules (reduced-form estimates using existing variation)

• Why ever use a model then?

- The Lucas critique: Behavioral rules change with policy ⇒ policy advice can not rely on estimated behavioral rules (reduced-form estimates using existing variation)
 - ⇒ we need to estimate structural (deep) parameters

"Invariance of parameters in an economic model is not, of course, a property which can be assured in advance, but it seems reasonable to hope that neither tastes nor technology vary systematically with variations in counter-cyclical policies." (Lucas, 1977)

Economic laboratory

The usefulness of a model lies in which types of mechanisms are included and thus which *counterfactual* scenarios the model can inform about.

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Counterfactual simulations:

Implications of not-yet implemented policy reforms (e.g. child subsidies) Responses to changes in the economic environment (e.g. wages) Quantifying the importance of mechanisms (e.g. risk)

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• Example:

A model of *retirement timing of couples* likely cannot inform us about the effects of changing *school fees*.

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→ Tight link between **research question** and model formulation.

See all at the Course page

• Knowledge:

This Course

Define, formulate and interpret *models* of household behaviour Account for backwards induction and how to solve dynamic programming models

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Skills:

This Course

Analyze counterfactual policy reform simulations from simple and more complex models of household behavior

Learning Objectives

See all at the Course page

• Knowledge:

Define, formulate and interpret *models* of household behaviour Account for backwards induction and how to solve dynamic programming models

Skills:

Analyze counterfactual policy reform simulations from simple and more complex models of household behavior

Competences:

Discuss and evaluate research on household behavior over the life cycle Modify computer *code* to analyze small changes to simple models

How to read a research paper (in this course)

 Each lecture will be based on 1 mandatory (*) research paper What is the main research question? What is the *(empirical)* motivation? What are the central mechanisms in the model? What is the *simplest model* in which we could capture these? **Challenging:** Research frontier.

- Each lecture will be based on 1 mandatory (*) research paper What is the main research question? What is the *(empirical)* motivation? What are the central mechanisms in the model? What is the *simplest model* in which we could capture these? **Challenging:** Research frontier.
- How to read a research paper in this course?
 - Focus on the questions above How do the questions interact and inform each other?
 - Try not to get stuck in too many details! (we can discuss some in class if you want)
 - Research papers often include many "bells and whistles"
 - Read ~40 min before each lecture. See reading-guide for each lecture

Outline

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- Programming in Python

Programming in Python

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- Only very simple/unrealistic models can be solved with pen/paper
 - \rightarrow We need numerical methods

Programming in Python

- Only very simple/unrealistic models can be solved with pen/paper
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- **Purpose** of you coding: Learn by implementing! Appreciate bottlenecks and challenges Better understanding of frontier research Set you free...!

- Only very simple/unrealistic models can be solved with pen/paper
 - \rightarrow We need numerical methods
- **Purpose** of you coding: Learn by implementing! Appreciate bottlenecks and challenges Better understanding of frontier research Set you free...!
- Goal is to keep things simple! No fancy numerical tricks (at least in beginning) Code should be "intuitive"
 - \rightarrow slow...

Programming in Python

- **Setup** Visual Studio Code as Introduction to Programming and Numerical Analysis https://sites.google.com/view/numeconcph-introprog/
- Installation guide: https://sites.google.com/view/numeconcph-introprog/guides/ installation
- Packages (all by Jeppe Druedahl): **EconModel** consav See "01. Introduction to EconModel and consav.ipynb"

Dynamic Programming

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Introduction to Dynamic Programming (DP)

• Agents maximize expected discounted sum of utility throughout life Maximize wrt. $\{\mathcal{C}_t\}_{t=1}^T$ Forward looking \rightarrow dynamic Assume *optimal* behavior in all periods

- **Agents maximize** expected discounted sum of utility throughout life Maximize wrt. $\{\mathcal{C}_t\}_{t=1}^T$ Forward looking \rightarrow dynamic Assume *optimal* behavior in all periods
- Bellman Equation:

$$\begin{aligned} V_t(\mathcal{S}_t) &= \max_{\mathcal{C}_t} U(\mathcal{C}_t, \mathcal{S}_t) + \beta \mathbb{E}[V_{t+1}(\mathcal{S}_{t+1}) | \mathcal{C}_t, \mathcal{S}_t] \\ \text{s.t.} \\ \mathcal{S}_{t+1} &\sim F(\mathcal{C}_t, \mathcal{S}_t) \end{aligned}$$

- $V_t(S_t)$: Indirect utility, Value today of states, S_t (all relevant info).
- $U(\mathcal{C}_t, \mathcal{S}_t)$: flow-utility
- $\beta \mathbb{E}[V_{t+1}(S_{t+1})|C_t,S_t]$: expected discounted value of next-period
- $S_{t+1} \sim F(C_t, S_t)$: transition *density* of states (fcn of C_t !) (there might be other constraints)

- Solved by backwards induction
 - 1. Start with last/terminal period, T (no future)

$$V_{\mathcal{T}}(\mathcal{S}_{\mathcal{T}}) = \max_{\mathcal{C}_{\mathcal{T}}} U(\mathcal{C}_{\mathcal{T}}, \mathcal{S}_{\mathcal{T}})$$

Solved by backwards induction

1. Start with last/terminal period, T (no future)

$$V_{\mathcal{T}}(\mathcal{S}_{\mathcal{T}}) = \max_{\mathcal{C}_{\mathcal{T}}} U(\mathcal{C}_{\mathcal{T}}, \mathcal{S}_{\mathcal{T}})$$

2. Go to second-to-last (we now know period-T optimal behavior for all S_T)

$$\begin{aligned} \mathbf{V_{T-1}}(\mathcal{S_{T-1}}) &= \max_{\mathcal{C_{T-1}}} U(\mathcal{C_{T-1}}, \mathcal{S_{T-1}}) + \beta \mathbb{E}[\mathbf{V_T}(\mathcal{S_T}) | \mathcal{C_{T-1}}, \mathcal{S_{T-1}}] \\ \text{s.t.} \\ \mathcal{S_T} &\sim F(\mathcal{C_{T-1}}, \mathcal{S_{T-1}}) \end{aligned}$$

Solved by backwards induction

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$$\begin{split} V_{T-1}(\mathcal{S}_{T-1}) &= \max_{\mathcal{C}_{T-1}} U(\mathcal{C}_{T-1}, \mathcal{S}_{T-1}) + \beta \mathbb{E}[V_T(\mathcal{S}_T) | \mathcal{C}_{T-1}, \mathcal{S}_{T-1}] \\ \text{s.t.} \\ \mathcal{S}_T &\sim F(\mathcal{C}_{T-1}, \mathcal{S}_{T-1}) \end{split}$$

3. Continue backwards...

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Introduction to Numerical Dynamic Programming (DP)

• On a computer, everything is discrete \rightarrow arrays + loops

Introduction to Numerical Dynamic Programming (DP)

- On a computer, everything is discrete → arrays + loops
- Numerical Dynamic Programming
 - Backwards induction on arrays
 - Grids

- Interpolation
- Integration
- See "02. Consumption-Saving Model.ipynb"

Next Time

Next time:

Dynamic programming with *uncertainty* Structural estimation

Literature:

Gourinchas and Parker (2002): "Consumption Over the Life Cycle"

- Read before lecture
- Reading guide:
 - Section 1: Introduction Key (page 50 is not that important)
 - Section 2: Model *Key*, we will discuss. Do not get stuck.
 - Section 3: Estimation method (SMM). Key, we will discuss.
 - Section 4: First stage calibrations. Skim fast.
 - Section 5: Data. Skim fast.
 - Section 6: Results. Focus on 6.1. Figures 5 and 7 are main results.

References I

- ADDA, J., C. DUSTMANN AND K. STEVENS (2017): "The Career Costs of Children," Journal of Political Economy, 125(2), 293–337.
- GOURINCHAS, P.-O. AND J. A. PARKER (2002): "Consumption Over the Life Cycle," Econometrica, 70(1), 47–89.
- KLEVEN, H. J., C. LANDAIS AND J. E. SØGAARD (2019): "Children and gender inequality: Evidence from Denmark," American Economic Journal: Applied Economics, 11, 181–209.