OCEAN FLUXES AND BUDGETS

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Abstract

An essential task in physical oceanography is to construct budgets of conserved quantities, like heat, salt and seawater mass, and the masses of trace chemicals. In the Arctic and subArctic oceans, for example, such budgets are routinely used to diagnose and understand the effects of natural variations and anthropogenic climate change on temperature and salinity. Traditionally, temperature variability is analyzed using a budget of heat fluxes relative to a reference temperature. Similarly, salinity variability is analyzed using so-called freshwater fluxes relative to a reference salinity. Well-documented pitfalls exist in the interpretation of these heat and freshwater fluxes, however. Yet, despite being well-documented, these pitfalls are not universally understood or accepted.

This contribution aims to improve understanding of, and to promote best-practices in, the interpretation of heat and freshwater fluxes, and the construction of their budgets. The contribution consists of: (i) A free, open-source, interactive, pedagogical software application called the Ocean-Flux-Budget tool. (ii) A tutorial YouTube video demonstrating the Ocean-Flux-Budget tool, the pitfalls mentioned above, and suggested workarounds. (iii) A document explaining the issues, with references to the original literature, and proposed best practices. To access these resources visit `https://github.com/ThomasHaine/Ocean-Flux-Budget`. At this website you can also seek advice, ask questions, and help refine understanding of ocean fluxes and budgets.

1. Introduction

This document discusses how to form budgets for conserved quantities in the ocean, like heat and salt. The use of reference temperatures and salinities, and the associated ambiguous fluxes of heat and freshwater, are explored. Equations are developed that highlight the ambiguities and pitfalls in interpretation. Guidance on how to avoid the pitfalls is offered. The discussion is based on primary literature by Schauer and Beszczynska-Möller [2009], Schauer and Losch [2019], Bacon et al. [2015] and Tsubouchi et al. [2012]. In a model context, Piecuch et al. [2017] is also useful. One prominent application of these concepts is to the Arctic and subArctic Oceans, although the discussion is generic.

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Date: May 5, 2023.

Consider the budgets of mass, volume, salt mass, heat (potential enthalpy[1]), and liquid freshwater content for a fixed control volume V. The control volume has bounding surface Ω, which includes the sea surface and the sea bed and vertical side walls. The arguments below are easily modified to relax these assumptions on the shape of V and Ω. We follow and extend McDonagh and the TICTOC Consortium [2022] (especially their Appendix A).

2. Seawater mass budget

The seawater mass budget is

$$\frac{dM}{dt} = -\int_{\Omega} \rho\, \mathbf{u} \cdot \hat{\mathbf{n}}\, d\Omega, \tag{1}$$

where $\mathbf{u}$ is the (three-dimensional) vector seawater velocity and $\hat{\mathbf{n}}$ is the unit outward normal vector to Ω. The seawater mass is

$$M = \int_V \rho\, dV, \tag{2}$$

with (variable) *in-situ* seawater density ρ. At the ocean free surface, $z = \eta(x, y, t)$, vertical transport arises from the boundary mass fluxes of precipitation, P, evaporation, E, river runoff, R, sea-ice melt M_{ice}, and sea-ice formation F_{ice}

$$\rho\, w(x, y, z = \eta, t) \equiv -Q^{\mathrm{m}} = -(P - E + R + M_{\text{ice}} - F_{\text{ice}}), \tag{3}$$

with the boundary mass flux, Q^{m}, having dimensions of mass per horizontal area per time, and with the minus sign indicating that $Q^{\mathrm{m}} > 0$ for mass entering the ocean.

Mass fluxes across the sea floor (e.g., at hydrothermal vents) are similarly included, although here they are neglected for simplicity.

Now consider that the vertical side walls consist of a set of N rectilinear faces, Ω_i, enumerated with index $i \in 1 \cdots N$. These faces are the abstraction of vertical sections across gateway straits, like Davis Strait and Bering Strait. Thus, (1) gives

$$\frac{dM}{dt} = \int_{\Omega_s} Q^{\mathrm{m}}\, d\Omega_s - \sum_{i=1}^{N} \int_{\Omega_i} \rho\, \mathbf{u} \cdot \hat{\mathbf{n}}\, d\Omega_i, \tag{4}$$

where Ω_s denotes the sea surface. Physically, (4) states that the mass in the control volume changes according to the sum of the mass fluxes across the sea surface and all of the vertical side walls.

3. Seawater Volume budget

The volume of seawater is

$$\mathcal{V} = \int_V dV \tag{5}$$

(where $\mathcal{V}$ is the volume in meters cubed, and V is the control volume occupied by seawater). In the Boussinesq approximation (which is accurate for Earth's ocean; see Klinger and Haine 2019, Chapter 1), seawater density is constant for the purposes of budgeting mass. Thus,

$$\mathcal{V} = \frac{M}{\rho_0}, \tag{6}$$

[1]Following McDougall [2003] and IOC et al. [2010].

where ρ_0 is the constant characteristic density. Therefore, volume is a conserved, extensive property[2] and the volume budget is simply related to the mass budget in equations (1)–(4). Without making the Boussinesq approximation, seawater density varies. Then, mass is still a conserved, extensive variable (section 2), but seawater volume is not. Therefore, volume does not possess an unambiguous physical budget. In this way, the mass budget is more fundamental.

4. Seawater Heat budget

The heat budget is

$$\frac{d\mathcal{H}}{dt} = -\int_\Omega \left[C_\mathrm{p}\rho\left(\mathbf{u}\Theta + \mathbf{F}\right) + \mathbf{F}_r\right] \cdot \hat{\mathbf{n}}\, d\Omega, \tag{7}$$

where $C_\mathrm{p} = 3991.87\ \mathrm{Jkg^{-1o}C^{-1}}$ is the constant heat capacity, Θ is the Conservative Temperature, and

$$\mathcal{H} = C_\mathrm{p}\int_V \rho\Theta\, dV \tag{8}$$

is the heat content for the volume V. In (7), $\mathbf{F}$ is the flux of heat due to subgridscale processes, typically parametrized as

$$\mathbf{F} = -\mathbb{K}\cdot\nabla\Theta, \tag{9}$$

with $\mathbb{K}$ a second order transport tensor (which could be the molecular diffusivity, or an eddy diffusivity with symmetric and anti-symmetric parts). The sensible (conductive) heat flux[3] across the sea surface is represented by this $\mathbf{F}$ term, for example. Also in (7), $\mathbf{F}_r$ is the flux of heat due to radiation (at all wavelengths) and $\mathbf{F}_r \cdot \hat{\mathbf{n}} < 0$ means ocean warming.

The sea-surface mass flux affects the heat budget (7) as the flux (across surface area $\Delta\Omega_s$) carries heat

$$\left[\frac{d\mathcal{H}}{dt}\right]_{P-E+R+M_\mathrm{ice}-F_\mathrm{ice}} = Q^\mathrm{adv}\,\Delta\Omega_s. \tag{10}$$

Here,

$$Q^\mathrm{adv} = C_\mathrm{p}(P\,\Theta^{\,\mathrm{precip}} - E\,\Theta_s + R\,\Theta^\mathrm{runoff} + M_\mathrm{ice}\,\Theta^\mathrm{melt} - F_\mathrm{ice}\,\Theta^\mathrm{freeze}) \tag{11}$$

defines the advective surface heat flux, and $\Theta^{\,\mathrm{precip}}$ is the Conservative Temperature of the precipitation, Θ_s is the surface ocean temperature, $\Theta^{\,\mathrm{runoff}}$ is the temperature of the runoff, Θ^melt is the temperature of the melt, and Θ^freeze is the local freezing temperature.

Heat fluxes across the sea floor (e.g., at hydrothermal vents and conductive geothermal processes) are similarly included, although here they are neglected for simplicity.

Consider again the case of N vertical rectilinear faces (gateway straits): (7) gives

$$\frac{d\mathcal{H}}{dt} = \int_{\Omega_s} Q^\mathrm{adv} - \mathbf{F}_r\cdot\hat{\mathbf{n}}\, d\Omega_s - C_\mathrm{p}\sum_{i=1}^{N}\int_{\Omega_i}\rho\left(\mathbf{u}\Theta + \mathbf{F}\right)\cdot\hat{\mathbf{n}}\, d\Omega_i, \tag{12}$$

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[2] An extensive property, like mass, is additive when subsystems are combined. In contrast, an intensive property, like temperature, is not additive in this way.

[3] Some authors use "flux" to mean air/sea exchange, and "transport" to mean horizontal advective exchange across a vertical section. Here, "flux" covers both possibilities: where necessary, we use qualifiers, like "advective," "diffusive," and "radiative."

which states physically that the total heat content in V changes according to the sum of all the boundary sources of heat.

Now consider the effect on the heat budget if all temperatures are measured relative to a (constant) reference temperature Θ_{ref}. The heat content relative to Θ_{ref} is

$$\mathcal{H}^{\Theta_{\text{ref}}} \equiv C_{\text{p}} \int_V \rho \left(\Theta - \Theta_{\text{ref}}\right)\, dV, \tag{13}$$

so

$$\begin{aligned}
\frac{1}{C_{\text{p}}} \frac{d\mathcal{H}^{\Theta_{\text{ref}}}}{dt} &= \frac{1}{C_{\text{p}}} \frac{d\mathcal{H}}{dt} - \Theta_{\text{ref}} \frac{dM}{dt}, \\
&= \int_{\Omega_s} P(\Theta^{\text{precip}} - \Theta_{\text{ref}}) - E(\Theta_s - \Theta_{\text{ref}}) + R(\Theta^{\text{runoff}} - \Theta_{\text{ref}}) + M_{\text{ice}}(\Theta^{\text{melt}} - \Theta_{\text{ref}}) - F_{\text{ice}}(\Theta^{\text{freeze}} - \Theta_{\text{ref}})\, d\Omega_s \\
&\quad - \frac{1}{C_{\text{p}}} \int_{\Omega_s} \mathbf{F}_r \cdot \hat{\mathbf{n}}\, d\Omega_s - \sum_{i=1}^{N} \int_{\Omega_i} \rho \left[\mathbf{u}(\Theta - \Theta_{\text{ref}}) + \mathbf{F}\right] \cdot \hat{\mathbf{n}}\, d\Omega_i.
\end{aligned} \tag{14}$$

Hence, rearranging and using (4),

$$\begin{aligned}
\frac{d\mathcal{H}^{\Theta_{\text{ref}}}}{dt} &= \int_{\Omega_s} Q^{\text{adv}} - \mathbf{F}_r \cdot \hat{\mathbf{n}}\, d\Omega_s - \sum_{i=1}^{N} \int_{\Omega_i} C_{\text{p}} \rho \left(\mathbf{u}\Theta + \mathbf{F}\right) \cdot \hat{\mathbf{n}}\, d\Omega_i \\
&\quad - C_{\text{p}} \Theta_{\text{ref}} \int_{\Omega_s} P - E + R + M_{\text{ice}} - F_{\text{ice}}\, d\Omega_s + C_{\text{p}} \Theta_{\text{ref}} \sum_{i=1}^{N} \int_{\Omega_i} \rho \mathbf{u} \cdot \hat{\mathbf{n}}\, d\Omega_i,
\end{aligned}$$

$$\implies \frac{d\mathcal{H}^{\Theta_{\text{ref}}}}{dt} = \int_{\Omega_s} Q^{\text{adv}} - \mathbf{F}_r \cdot \hat{\mathbf{n}}\, d\Omega_s - C_{\text{p}} \sum_{i=1}^{N} \int_{\Omega_i} \rho \left(\mathbf{u}\Theta + \mathbf{F}\right) \cdot \hat{\mathbf{n}}\, d\Omega_i - C_{\text{p}} \Theta_{\text{ref}} \frac{dM}{dt}. \tag{15}$$

This equation coincides with (12) for $\Theta_{\text{ref}} = 0$, and otherwise generalizes it.

From equation (15), the following conclusions can be drawn:

(1) The heat budget, meaning $d\mathcal{H}^{\Theta_{\text{ref}}}/dt$, is unambiguous (has physical significance because it does not depend on the reference temperature) provided the seawater mass is constant, $dM/dt = 0$. Moreover, the importance of the individual terms in (15) setting $d\mathcal{H}^{\Theta_{\text{ref}}}/dt$ is unambiguous provided the seawater mass is constant.

(2) Computation of advective heat fluxes, $\mathcal{H}_i^{\Theta_{\text{ref}}}$, across individual faces,

$$\mathcal{H}_i^{\Theta_{\text{ref}}} = C_{\text{p}} \int_{\Omega_i} \rho \mathbf{u}(\Theta - \Theta_{\text{ref}}) \cdot \hat{\mathbf{n}}\, d\Omega_i,$$

depends on the arbitrary reference temperature Θ_{ref} if a non-zero mass flux crosses the face, and therefore has no physical significance. The advective heat flux $\mathcal{H}_i^{\Theta_{\text{ref}}}$ is a linear combination of the advective mass flux and the covariance between the velocity and temperature. Changing the reference temperature changes the importance of these two terms in the linear combination.

In particular, when there is non-zero mass flux across the face (or faces):

(a) Changing Θ_{ref} changes $\mathcal{H}_i^{\Theta_{\text{ref}}}$. Thus, $\mathcal{H}_i^{\Theta_{\text{ref}}}$ has no physical significance because it depends on the arbitrary value of Θ_{ref}.

(b) For time-varying $\mathbf{u}, \Theta$, changing Θ_{ref} changes $\mathcal{H}_i^{\Theta_{\text{ref}}}$ by an arbitrary multiple of the mass flux time series. Therefore, there is no physical significance to the correlation between $\mathcal{H}_i^{\Theta_{\text{ref}}}$ and the mass flux time series across the face.

(c) Comparison of the relative importance of the heat flux across the same face for different times depends on the reference temperature. Therefore, it has no physical significance.

(d) The trend in an advective heat flux time series across a face has no physical significance.

(e) The extrema (maximum, minimum) in an advective heat flux time series across a face have no physical significance

(f) Comparison of the relative importance of heat fluxes across different faces (e.g., Fram Strait versus Davis Strait for the Arctic heat budget) depends on the reference temperature. Therefore, it has no physical significance.

(3) Comparison of advective heat fluxes $\mathcal{H}_i^{\Theta_{\text{ref}}}$ across faces from models with each other, and from models with observations, is a legitimate test of model realism. The test is legitimate in the sense that it's necessary (but not sufficient) for a model to be realistic to produce the same heat flux across a particular strait as seen in observations.

(4) The average temperature over the control volume V is

$$\overline{\Theta} \equiv \frac{\mathcal{H}}{C_{\text{p}} M}.$$

Therefore, provided the seawater mass M is constant, the change in average temperature can be unambiguously determined from the individual terms in (15).

5. Salt mass budget

The salt budget[4] is

$$\frac{dM_{\text{S}}}{dt} = -\int_{\Omega} \rho\ (\mathbf{u}S_A + \mathbf{F}_S) \cdot \hat{\mathbf{n}}\ d\Omega, \tag{16}$$

where S_A is the absolute salinity. The salt mass is

$$M_{\text{S}} = \int_V \rho S_A\ dV. \tag{17}$$

In (16), $\mathbf{F}_S$ is the flux of salt due to subgridscale processes:

$$\mathbf{F}_S = -\mathbb{K} \cdot \nabla S_A, \tag{18}$$

as for heat in (9), although in principle the diffusion tensors for heat and salt can differ. Notice that no equivalent surface radiative flux term to $\mathbf{F}_r$ appears in (16). This is a tacit assumption that salt mass cannot enter or leave the ocean, although exchange with sea ice is included. Exchange with sea ice is included in (16) via the $\mathbf{u}S_A \cdot \hat{\mathbf{n}}$ term. Also note that the salt mass (and hence the salinity) is a positive definite quantity. Heat is different in this respect, because it has no well-defined zero value.

[4]The arguments in this section apply to the masses of other dissolved constituents, like trace chemicals.

For these reasons, interpretation of the salt budget (dM_S/dt in (16)) is unambiguous: it doesn't depend on a reference salinity and it doesn't require a zero total mass tendency. The role of different processes can be obscure, however. For instance, an ocean region can freshen (decreased average salinity) because of increased precipitation with no change in salt mass. Similarly, salt mass can change because of changed advective or diffusive salt fluxes across the boundaries, but not because of changed precipitation. These obscurities are clarified by recalling how the (absolute) average salinity equals the ratio of salt mass to seawater mass:

$$\overline{S_A} = \frac{M_\text{S}}{M}. \tag{19}$$

Thus, $\overline{S_A}$ can change because of M_S change, M change, or both. For instance, anomalous precipitation freshens the surface ocean because seawater mass increases, without changing the salt mass.

6. Liquid freshwater volume budget

Liquid freshwater content (LFC) is defined as the volume-integrated fractional salinity anomaly from reference salinity S_ref[5]:

$$\text{Liquid freshwater content (LFC)} = \mathcal{V}^{S_\text{ref}} \equiv \int_V \frac{S_\text{ref} - S_A}{S_\text{ref}}\, dV. \tag{20}$$

This definition gives an LFC with units of meters cubed[6]. To grasp the physical meaning of LFC, consider the following thought-experiment: Begin with a volume $\mathcal{V}$ of incompressible seawater of salinity S_ref. Replace a volume $\mathcal{V}^{S_\text{ref}}$ of this seawater with freshwater (zero salinity). The resulting solution has a mean salinity of $(1/\mathcal{V})\int_V S_A\ dV$. Hence, liquid freshwater content is the volume $\mathcal{V}^{S_\text{ref}}$ of freshwater that replaces seawater in a volume V of salinity S_ref to yield an average salinity $\int_V S_A\ dV$.

LFC (20) relates to salt mass (17) via:

$$\mathcal{V}^{S_\text{ref}} \approx \mathcal{V} - \frac{M_\text{S}}{\rho_0 S_\text{ref}} \tag{21}$$

(recall from section 3 that ρ_0 is the constant characteristic seawater density). The approximation is an equality under the Boussinesq approximation.

Assuming the Boussinesq approximation[7], (16) is

$$\begin{aligned} \frac{d}{dt}\int_V S_A\ dV &= -\int_\Omega (\mathbf{u}S_A + \mathbf{F}_S)\cdot\hat{\mathbf{n}}\ d\Omega, \\ &= -\sum_{i=1}^{N}\int_{\Omega_i} (\mathbf{u}S_A + \mathbf{F}_S)\cdot\hat{\mathbf{n}}\ d\Omega_i. \end{aligned} \tag{22}$$

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[5]Unlike the reference temperature, S_ref is a positive definite number.

[6]Sometimes the integral is over vertical distance, not volume, so liquid freshwater content then has units of meters.

[7]The Boussinesq approximation is required because, otherwise, volume is not an extensive property. See section 3 and footnote 2.

Also, (4) is

$$\frac{d\mathcal{V}}{dt} = \frac{1}{\rho_0}\int_{\Omega_s} Q^{\mathrm{m}}\, d\Omega_s - \sum_{i=1}^{N}\int_{\Omega_i} \mathbf{u}\cdot\hat{\mathbf{n}}\, d\Omega_i, \tag{23}$$

and the LFC tendency from (20) is

$$\frac{d}{dt}\mathcal{V}^{S_{\mathrm{ref}}} = \frac{d\mathcal{V}}{dt} - \frac{1}{S_{\mathrm{ref}}}\frac{d}{dt}\int_V S_A\, dV. \tag{24}$$

Hence, the LFC budget is

$$\begin{aligned}\frac{d}{dt}\mathcal{V}^{S_{\mathrm{ref}}} &= \frac{1}{\rho_0}\int_{\Omega_s} Q^{\mathrm{m}}\, d\Omega_s - \sum_{i=1}^{N}\int_{\Omega_i} \mathbf{u}\cdot\hat{\mathbf{n}}\, d\Omega_i + \frac{1}{S_{\mathrm{ref}}}\left[\sum_{i=1}^{N}\int_{\Omega_i} (\mathbf{u}S_A + \mathbf{F}_S)\cdot\hat{\mathbf{n}}\, d\Omega_i\right], \\ &= \frac{1}{\rho_0}\int_{\Omega_s} Q^{\mathrm{m}}\, d\Omega_s - \sum_{i=1}^{N}\int_{\Omega_i} \left[\mathbf{u}\left(\frac{S_{\mathrm{ref}} - S_A}{S_{\mathrm{ref}}}\right) - \frac{\mathbf{F}_S}{S_{\mathrm{ref}}}\right]\cdot\hat{\mathbf{n}}\, d\Omega_i, \end{aligned} \tag{25}$$

This equation resembles the heat budget equations (14) and (15); specifically, $(S_{\mathrm{ref}} - S_A)/S_{\mathrm{ref}}$ associates with $C_{\mathrm{p}}\rho_0(\Theta - \Theta_{\mathrm{ref}})$ and $\mathbf{F}_S/S_{\mathrm{ref}}$ associates with $-C_{\mathrm{p}}\mathbf{F}$. Also, the surface volume flux Q_m/ρ_0 associates with the advective surface heat flux Q^{adv}. In the LFC budget, there is no corollary to the radiative surface heat flux term $\mathbf{F}_r$.

Similar to the heat budget in section 4, from (23) and (25) the following conclusions can be drawn:

(1) The LFC budget, meaning $d\mathcal{V}^{S_{\mathrm{ref}}}/dt$, is ambiguous (depends on the reference salinity) whether seawater mass is constant or not. Look at (23): Even if seawater volume is constant, $d\mathcal{V}/dt = 0$, $\mathcal{V}^{S_{\mathrm{ref}}}$ depends on S_{ref} through the denominator in the final term. Also, (23) requires the Boussinesq approximation. Still, for constant seawater volume and under the Boussinesq approximation, $\mathcal{V}^{S_{\mathrm{ref}}}$ isn't very sensitive to S_{ref} when S_{ref} is close to the mean salinity, $S_{\mathrm{ref}} \approx \overline{S_A}$.

(2) Computation of advective LFC fluxes, $\mathcal{V}_i^{S_{\mathrm{ref}}}$, across individual faces,

$$\begin{aligned}\mathcal{V}_i^{S_{\mathrm{ref}}} &\equiv \int_{\Omega_i} \mathbf{u}(S_{\mathrm{ref}} - S_A)/S_{\mathrm{ref}}\cdot\hat{\mathbf{n}}\, d\Omega_i, \\ &= \int_{\Omega_i} \mathbf{u}\cdot\hat{\mathbf{n}}\, d\Omega_i - \frac{1}{S_{\mathrm{ref}}}\int_{\Omega_i} \mathbf{u}S_A\cdot\hat{\mathbf{n}},\end{aligned}$$

depends on the arbitrary reference salinity S_{ref}. Therefore, $\mathcal{V}_i^{S_{\mathrm{ref}}}$ has no physical significance. This statement holds if a non-zero volume flux crosses the face, or not. Similarly, there is no physical significance to the correlation between the advective LFC flux $\mathcal{V}_i^{S_{\mathrm{ref}}}$ and the volume flux time series across the face. Still, for constant seawater volume and under the Boussinesq approximation, $\mathcal{V}_i^{S_{\mathrm{ref}}}$ isn't very sensitive to S_{ref} when S_{ref} is close to the mean salinity, $S_{\mathrm{ref}} \approx \overline{S_A}$. Also, the advective LFC flux is a linear combination of the mass flux across the face and the salt flux across the face (under the Boussinesq approximation). This is a useful relation; for example, to estimate the salt flux from the volume and LFC fluxes across a section.

(3) Comparison of the relative importance of the LFC flux across the same face for different times depends on the reference salinity. Therefore, it has no physical significance.

(4) The trend in an advective LFC flux time series across a face has no physical significance.

(5) The extrema (maximum, minimum) in an advective LFC flux time series across a face have no physical significance.

(6) Comparison of the relative importance of LFC fluxes across different faces (e.g., Fram Strait versus Davis Strait for the Arctic LFC budget) depends on the reference salinity. Therefore, it has no physical significance.

(7) Comparison of advective LFC fluxes $\mathcal{V}_i^{S_{\text{ref}}}$ across faces from models with each other, and from models with observations, is a legitimate test of model realism, as for heat fluxes.

(8) The average salinity over the control volume V is, from (19),

$$\overline{S_A} = \frac{M_S}{M} \approx \int_V S_A \, dV$$

(the final statement is an equality under the Boussinesq approximation). Therefore, provided the seawater volume $\mathcal{V}$ is constant (and making the Boussinesq approximation), the change in average salinity can be unambiguously determined from the individual terms in (25). This is a useful approach; for example, to interpret observations of LFC fluxes across different straits.

7. TEOS-10 Freshwater mass budget

IOC et al. [2010] defines a (dimensionless) freshwater mass density as:

$$\text{TEOS-10 freshwater mass density} = 1 - S_A,$$

where S_A is the absolute salinity (measured in kg/kg). Thus, the TEOS-10 freshwater mass is

$$\text{TEOS-10 freshwater mass} = \int_V \rho \left(1 - S_A\right) \, dV.$$

In words, the TEOS-10 freshwater mass is the mass in a seawater sample that is 100% pure freshwater. In other words, the TEOS-10 freshwater mass plus the salt mass equals the seawater mass. Therefore, like seawater mass and salt mass (sections 2 and 5), interpretation of the TEOS-10 freshwater mass is unambiguous. Under the Boussinesq approximation the TEOS-10 freshwater mass equals (minus) the liquid freshwater content (20) if $S_{\text{ref}} = 1000$ g/kg.

8. Proposed best practices

For reporting flux and budget data, either from field observations or numerical models, the following best practices are proposed:[8]

For fluxes across individual faces (straits), state:

(1) The face-averaged conservative temperature, absolute salinity, and seawater density timeseries.

(2) The mass flux timeseries (the terms in (4)) and salt flux timeseries (the terms in (16)).

(3) The advective relative heat flux timeseries $\mathcal{H}_i^{\Theta_{\text{ref}}}$ using at least two reference temperatures.

(4) Any tacit assumptions, such as a closed mass budget or the Boussinesq approximation.

[8] Comments welcome!

Also, optionally, state:

(1) The seawater volume flux timeseries.

(2) The advective LFC flux timeseries $\mathcal{V}_i^{S_{\mathrm{ref}}}$, as long as at least two reference salinities are used.

For control volumes, state:

(1) The volume-averaged seawater mass, conservative temperature, absolute salinity, and seawater density timeseries.

(2) Any tacit assumptions, such as a closed mass budget or the Boussinesq approximation.

Also, optionally, state:

(1) The seawater volume timeseries.

(2) The relative heat content $\mathcal{H}^{\Theta_{\mathrm{ref}}}$ and LFC timeseries $\mathcal{V}^{S_{\mathrm{ref}}}$, as long as at least two reference temperatures and salinities are used.

In all cases, provide access to the original data and the data-processing pipeline. Fluxes and budgets of trace chemicals are treated analogously to salt mass and salinity.

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