Fuzzy Fault Tree Analysis over the Continuous Time

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ABSTRACT: Fault Tree Analysis (FTA) is a method used to model and analyse the probability of a system failing and has seen a widespread use in economically important areas like nuclear power plants, transport and web shops. FTA over the Continuous Time (FTACT) can be used to not only model the current state of a system, but can also predict future states. Its main downside is that it requires a lot of data to model. This research proposes a method in which so called Fuzzy FTACT (FFTACT) can be performed, by hypothesizing three new methods and elaborating further on the most sufficient one. The new method allows the user to not only describe the chance of a certain event happening, but it also allows the user to use an uncertainty factor, which enables the possibility to model inaccurate data as well.

Key words: Fault Trees, Risk Assessment, Reliability, Fuzzy, Continuous Time

1 INTRODUCTION

Fault Tree Analysis (FTA) has been used in many different areas including the nuclear and transport sector and is a common way to analyse how likely a system is to fail and why. There are many ways to perform such analysis, but this research focusses on Fuzzy Fault Tree Analysis (FFTA) and Fault Tree Analysis over the Continuous Time (FTACT), as described by Ruijters and Stoelinga (2015). FFTA can be used to model systems where a lot of uncertainty is present in data, where FTACT can be used to model the probability that a system fails over time. Since FTACT requires a lot of accurate data in order to function properly, this research aims to create a fuzzy approach to FTACT.

The traditional way of FTA makes use of a Fault Tree as depicted in Fig 1, which is a top down graph, in which the failure paths of a system can be analysed. A failure path is a sequence of events that leads to the failure of a system. At the bottom of a Fault Tree the Basic events (BE) can be found. These are events that can occur spontaneously in a system and are usually denoted with a circle symbol. Events are connected with each other through gates. Examples of gates are OR-gates and AND-gates. An OR-gate indicates that the output of the gate occurs if any of the inputs occur. An AND-gate indicates that the output of the gate occurs if both inputs occur. Intermediate events (IE) occur when one or more other events occur and are found at the output of a gate. The event at the top of a Fault Tree is the top event, which is the event that the whole system fails. Constructing a Fault Tree using basic events, gates and intermediate events make it possible to determine what combinations of events can fail the system. Clifton A. Ericson II (2000) describes ways to construct Fault Trees.

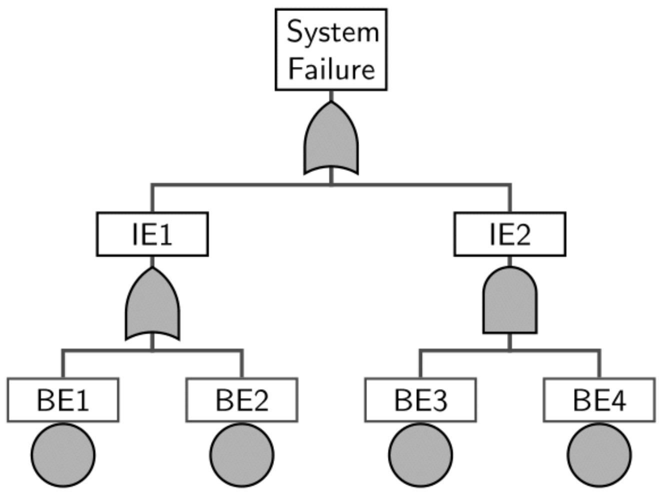

Fig 1. Example of a Fault Tree

The quantitative way of evaluating a Fault Tree in FTA is done by giving all basic events certain probabilities. By using defined calculations for an OR-gate and AND-gate, it is possible to calculate the probability the top event might occur, resulting in a failure of the system. This method requires the given probabilities to be numerical and precise. This can sometimes be undesirable, for example when there is not much information available to give a correct assessment, resulting in uncertain estimations. A fuzzy approach of FTA, Fuzzy Fault Tree Analysis (FFTA), can be used to prevent this problem.

With FFTA, instead of giving the basic events numerical probabilities, the basic events are given linguistic variables, as described by Mahmood et al. (2013). Linguistic variables are for example low, medium and high, which are easier to understand for humans making it easier for risk management experts to describe the occurrence of a basic event.

Another method of FTA is Fault Tree Analysis over the Continuous Time (FTACT). As described by Ruijters and Stoelinga (2015), continuous-time fault trees consider the evolution of the system failures over time.

Yuhua and Datao (2005), Senol et al. (2015), Lavasani et al. (2015a) and Lavasani et al. (2015b) describe a way for different experts to collaborate on deciding the failure rate of basic events. In these papers hired experts gave their opinions on a certain topic and averaged their opinions out to a single fuzzy value. These opinions were also rated on the background of the experts, with the opinion of a more educated expert rating higher than a lesser educated one.

While methods exist for practising FFTA and FTACT, no method exists yet for a fuzzy approach of a fault tree over the continuous time. That is what this research is focussed on; rather than analysing a regular fault tree over the continuous time, a fuzzy fault tree over the continuous time will be analysed.

2 METHODOLOGY

We explored several methods that could be used to model FFTACT and compared them against each other based on certain traits. This resulted in three main ideas: Fuzzy Words Method, Mountain Method and the Time Method. All methods are based on an exponential distribution with parameter λ depicting the chance something may happen at any time t, as described in Ruijters and Stoelinga (2015).

2.1 *Fuzzy Words Method*

This method is based on using words to describe the state of a basic event like with a regular Fuzzy Fault Tree. Since words like low, medium and high were not applicable when describing how fast a basic event fails, the idea is to use words like slow, medium and fast instead. As seen in Fig 2, each fuzzy word depicts a different probability range.

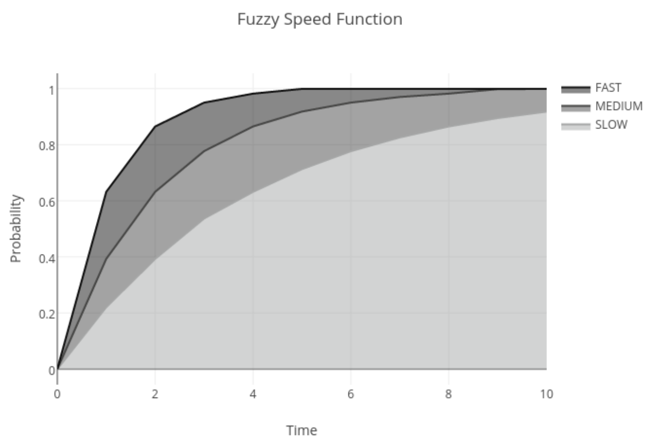

Fig 2. Fuzzy Words Method

2.2 *Mountain Method*

The Mountain Method primarily focuses on the uncertainty aspect when creating a Fuzzy Fault Tree. Rather than defining one function that describes the probability over time, a range of functions is used. The result of event A at time T is not just one value, but a range of values with a certain certainty factor. This can be seen in Fig 3 with the x axis depicting time, y depicting a chance between 0 and 1 and the z axis depicting a certainty value.

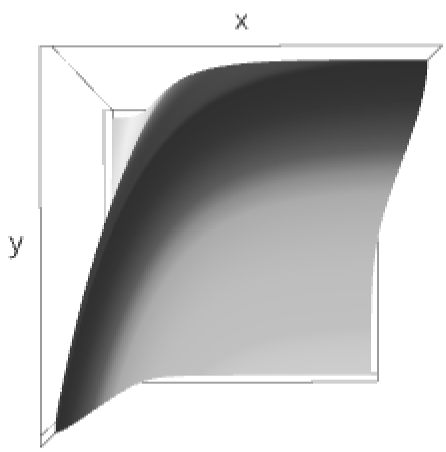

Fig 3. Mountain Method

2.3 Time Method

When asking how fast a certain event may fail, one can either reply by giving a speed or by giving the time at which it is most likely to fail. This method therefore revolves around the idea that an event doesn't fail slow, medium or fast, but that it is most likely fail in, for example, two months. In this method an exponential distribution will be stretched until the desired time results in a percentage of 95% or higher.

3 RESULTS

The Mountain Method ended up being used for its clear way of indicating fuzziness of a certain event. Further evaluating this method lead to the development of the following formulas.

The fuzziness of an event can be determined by using a normal distribution:

$$N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} \tag{1}$$

With $\mu \in [0,1]$ indicating the probability and $\sigma^2 \in [0,1]$ indicating the fuzziness.

The rate at which an event fails can be determined by using an exponential distribution with parameter $\lambda\left(1 - e^{-\lambda t}\right)$. Since it has shown its effectiveness in Ruijters and Stoelinga (2015).

The goal is to create a three-dimensional graph where the distribution function gets traced across the exponential distribution. Since changing the μ in a distribution function changes the location of the graph, μ can be replaced with the result of the exponential distribution, resulting in the following function:

$$f(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{1}{2}(\frac{y-(1-e^{-\lambda t})}{\sigma})^2} \tag{2}$$

Note that $f(t)$ does not return a value but a normal distribution.

Since $f(t)$ returns a normal distribution, the logic gates need to be able to work with them. An AND-gate works by generating each possible outcome and taking the sum of all probabilities per outcome (see equation (3) with $f_a(x)$ and $f_b(x)$ being two distribution functions). This can however be approximated by creating a new normal distribution with μ equals to the product of the input- μs and the σ being the average of the input-σs, as seen in Fig 4 (dotted graph depicts AND) and equation (4).

$$and(y) = \int_0^1 f_a(x) \cdot f_b\left(\frac{y}{x}\right) dx + \int_0^1 f_b(x) \cdot f_a\left(\frac{y}{x}\right) dx \tag{3}$$

$$approx: N(\mu_a, \sigma_a^2) \wedge N(\mu_b, \sigma_b^2) = N(\mu_a \cdot \mu_b, \frac{\sigma_b^2}{2}) \tag{4}$$

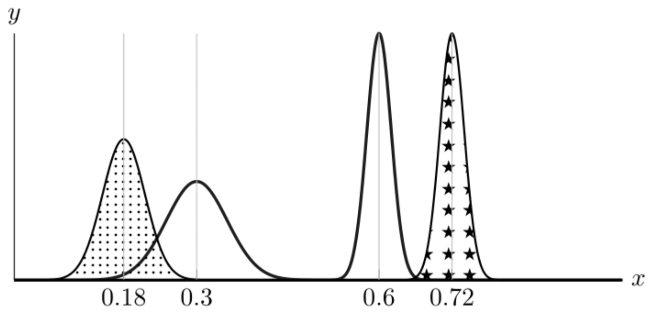

Fig 4. Fuzziness functions

In the same fashion an OR-gate takes the union between two sets, which can also be done by generating each possible outcome and taking the sum of all probabilities per outcome (see equation (5) with $f_a(x)$ and $f_b(x)$ being two distribution functions). However, this can also be approximated by creating a normal distribution with μ equal to the sum of the input μs minus the product of the μs and σ equals to the smallest σ, as depicted in Fig 4 (starry graph depicts OR) and equation (6).

$$or(y) = \int_0^1 f_a(x) \cdot f_b\left(\frac{y-x}{1-x}\right) dx + \int_0^1 f_b(x) \cdot f_a\left(\frac{y-x}{1-x}\right) dx \tag{5}$$

$$approx: N(\mu_a, \sigma_a^2) \vee N(\mu_b, \sigma_b^2) = N(\mu_a + \mu_b - \mu_a \cdot \mu_b, \min(\sigma_a, \sigma_b)^2) \tag{6}$$

To perform a logical NOT operation, one should mirror the normal distribution at time t across $x =$

0.5. This can be done by taking $\mu = 1 - \mu$.

$$\neg N(\mu, \sigma^2) = N(1 - \mu, \sigma^2) \quad (7)$$

Furthermore, a framework was developed in Node.js which can be used to perform such analysis. It does not support a graphical user interface, but the resulting graph can be plotted in plot.ly. The framework can be found on https://github.com/ThomasHakkers/FFTACT

4 DISCUSSION

FFTACT is a good way to describe the uncertainty one may have when analysing a system over the continuous time. This is mainly due to only having to add one parameter to the already existing method for performing FTACT. On top of that it is possible to mix uncertain probabilities with certain probabilities (see paragraph 3), making it possible to be integrated with existing models that apply FTACT. The main problem with this method is that it is hard to visualize it properly, due to its three dimensional nature.

5 CONCLUSION

To conclude, the Mountain Method to perform FFTACT presented in this paper can be used to model uncertainty in data when desired, bridging the gap for systems that need to be analysed without enough data present. It boasts a clear way to define BEs, whilst keeping the simplicity of logic gates present in regular FTs. The two main negative sides of this method is that it is computationally expensive with larger FTs and that the resulting graph is relatively difficult to read and visualize.

6 PROPOSED CONTINUATION

The resulting data of the FFTACT via the Mountain Method is hard to read, mainly due to it being plotted in a three-dimensional surface graph. A research into a better way of presenting this is proposed. Furthermore, since the Time Method and the Fuzzy Words Method described in paragraph 2 have not been worked out any further, it is proposed to explore these ideas and verify which method is the most appropriate for which situations. As described by Yuhua and Datao (2005), Basic Events in a Fuzzy Fault Tree could be described as the combined opinions of multiple experts (weighted by their expertise). This same principle could possibly applied to FFTACT, where the normal distribution is averaged over the opinions of multiple experts. This is a perfect use case for FFTACT, since uncertainty between employees can be modelled very well in this system.

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REFERENCES

Clifton A. Ericson II (2000). Fault tree analysis.

Lavasani, S.M., Ramzali, N., Sabzalipour, F., and Akyuz, E. (2015a). Utilisation of fuzzy fault tree analysis (ffta) for quantified risk analysis of leakage in abandoned oil and natural-gas wells. Ocean Engineering, 108, 729 – 737. http://dx.doi.org/10.1016/j.oceaneng.2015.09.008.

Lavasani, S.M., Zendegani, A., and Celik, M. (2015b). An extension to fuzzy fault tree analysis (ffta) application in petrochemical process industry. Process Safety and Environmental Protection, 93, 75 – 88. http://dx.doi.org/10.1016/j.psep.2014.05.001.

Mahmood, Y.A., Ahmadi, A., Verma, A.K., Srividya, A., and Kumar, U. (2013). Fuzzy fault tree analysis: a review of concept and application. International Journal of System Assurance Engineering and Management, 4(1), 19–32. 10.1007/s13198-013-0145-x. http://dx.doi.org/10.1007/s13198-013-0145-x.

Ruijters, E. and Stoelinga, M. (2015). Fault tree analysis: A survey of the state-of-the-art in modeling, analysis and tools. Computer Science Review, 15–16, 29 – 62. http://dx.doi.org/10.1016/j.cosrev.2015.03.001.

Senol, Y.E., Aydogdu, Y.V., Sahin, B., and Kilic, I. (2015). Fault tree analysis of chemical cargo contamination by using fuzzy approach. Expert Systems with Applications, 42(12), 5232 – 5244. http://dx.doi.org/10.1016/j.eswa.2015.02.027.

Yuhua, D. and Datao, Y. (2005). Estimation of failure probability of oil and gas transmission pipelines by fuzzy fault tree analysis. Journal of Loss Prevention in the Process Industries, 18(2), 83 – 88. http://dx.doi.org/10.1016/j.jlp.2004.12.003.