A Unified Reduced Order Model For Standing Captures Biological Balance Strategy Preferences

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Motivation

Extend reduced-order balancing models to include ankle, hip and toe balancing and elucidate their relative contributions of balancing strategies in push recovery

Introduction

Legged robots rely on active balancing to compensate and adjust for undesired disturbances. Humanoid standing balancing have been classified into discrete strategies [1]:

- **Ankle**: position center of pressure (CoP) within base of support
- **Hip**: generate angular momentum about the center of mass (CoM)
- **Toe**: use vertical dynamics of CoM to regulate magnitude of ground reaction force (GRF)

These balancing strategies can be modeled by reduced order models such as:

- **Linear Inverted Pendulum** [2] (LIP): ankle
- **Linear Inverted Pendulum Plus Flywheel** [3] (LIPPFW): ankle & hip
- **Variable Height Inverted Pendulum** [4] (VHIP): ankle & toe

Methods

Used trapezoidal direct collocation trajectory optimization to find the optimal set of controls to stabilize the system

- Cost function J penalized deviations from rest state & cost of control
- Weighting matrices Q and R normalized the components of state and the control, respectively, to be similar in magnitude

$$J(\boldsymbol{z}) = \sum_{i=1}^{N} \tilde{\boldsymbol{X}}_i^{\,T} Q \tilde{\boldsymbol{X}}_i + \boldsymbol{U}_i^T R \boldsymbol{U}_i$$

$$Q = \mathrm{diag}(x_{c_{\max}}^{-2}, \theta_{\max}^{-2}, z_{\max}^{-2}, 0.42^{-2}, 2.3^{-2}, 0.44^{-2}) \quad R = \mathrm{diag}(\boldsymbol{U}_{\max}^{-2})$$

Push recovery simulations of balancing models LIP, LIPPFW, VHIP, and VHIPPFW with a range of disturbances:

- Pushes modeled as constant horizontal force applied at CoM for 0.1s
- Recovery Performance evaluated on maximum capture point [3]

$$\xi = x + \sqrt{\frac{z}{g}}\dot{x}$$

- Relative control contribution each strategy evaluated by comparing each component's contribution to the cost of control

$$\sum_{i=1}^{N} \boldsymbol{U}_i^T R \boldsymbol{U}_i$$

Unified Model

The **variable height inverted pendulum plus flywheel (VHIPPFW) model** integrates all three balancing strategies. Its three components of control are:

- x_c: center of pressure (ankle strategy)
- τ : flywheel torque (hip strategy)
- $\ddot{z}$: vertical acceleration (toe strategy)

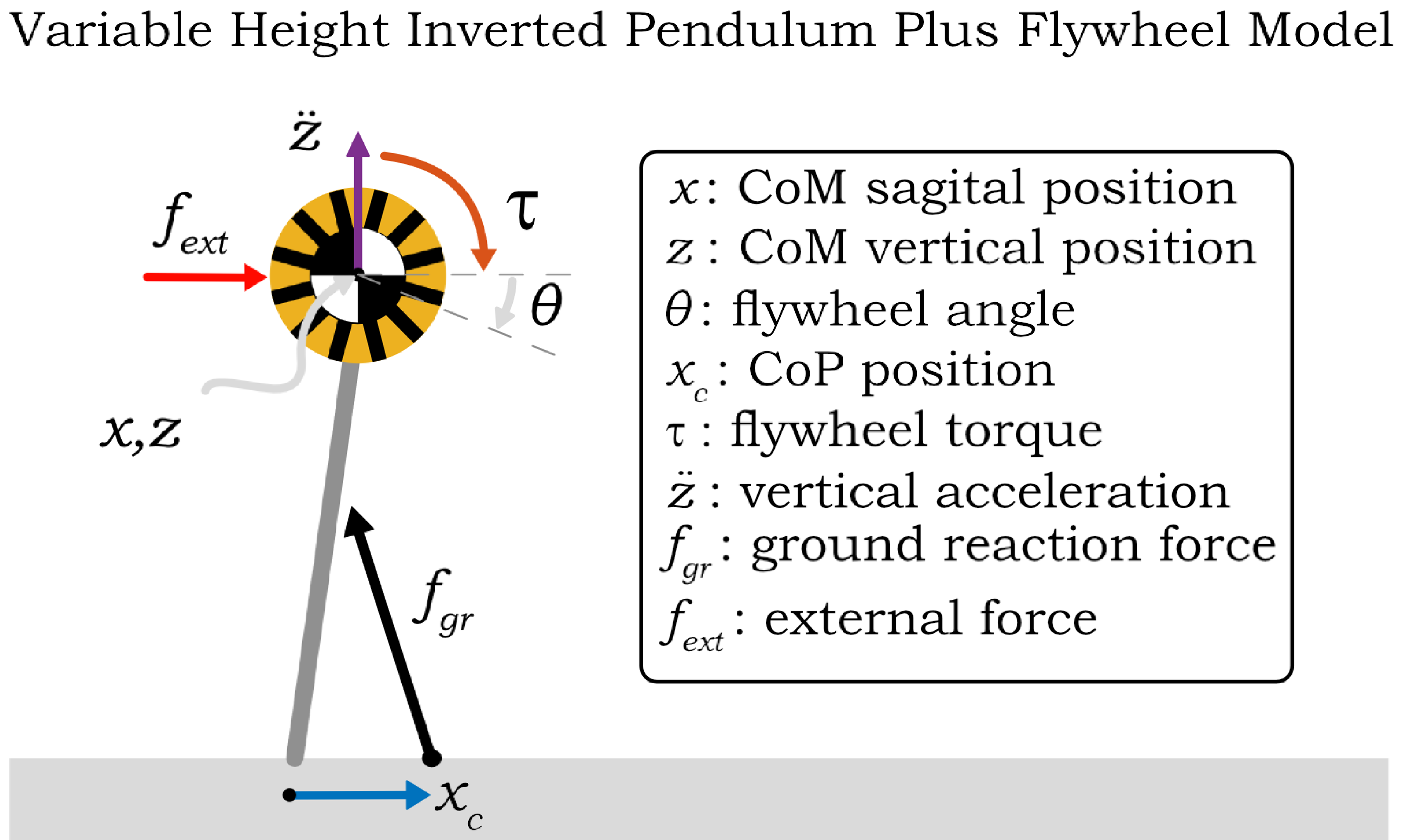
Dynamics $\ddot{x} = \dfrac{(g+\ddot{z})}{z}(x - x_c) + \dfrac{\tau}{mz}$

$I\ddot{\theta} = \tau$

State $\boldsymbol{X} = [x, \theta, z, \dot{x}, \dot{\theta}, \dot{z}]^T$

Control $\boldsymbol{U} = [x_c, \tau, \ddot{z}]^T$

Model parameters approximated a humanoid robot



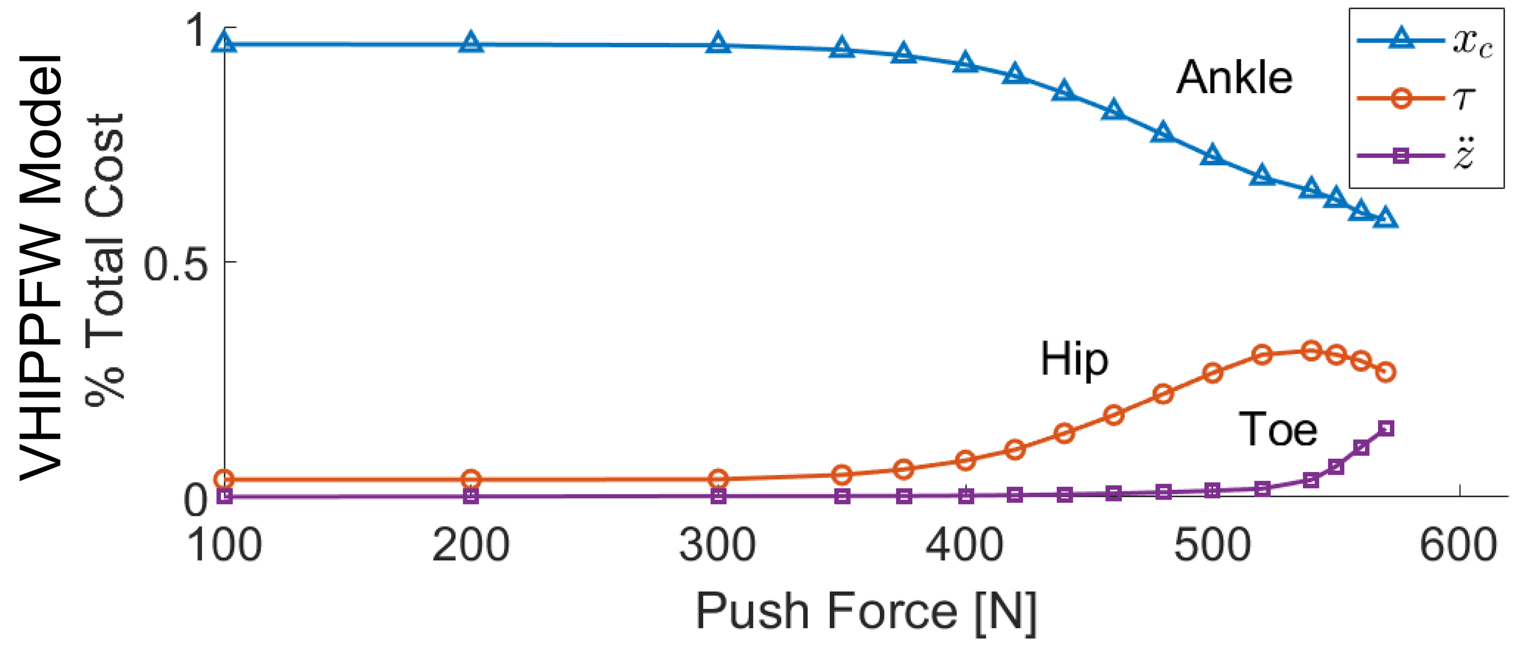
Results

Ankle was the dominant strategy for small pushes

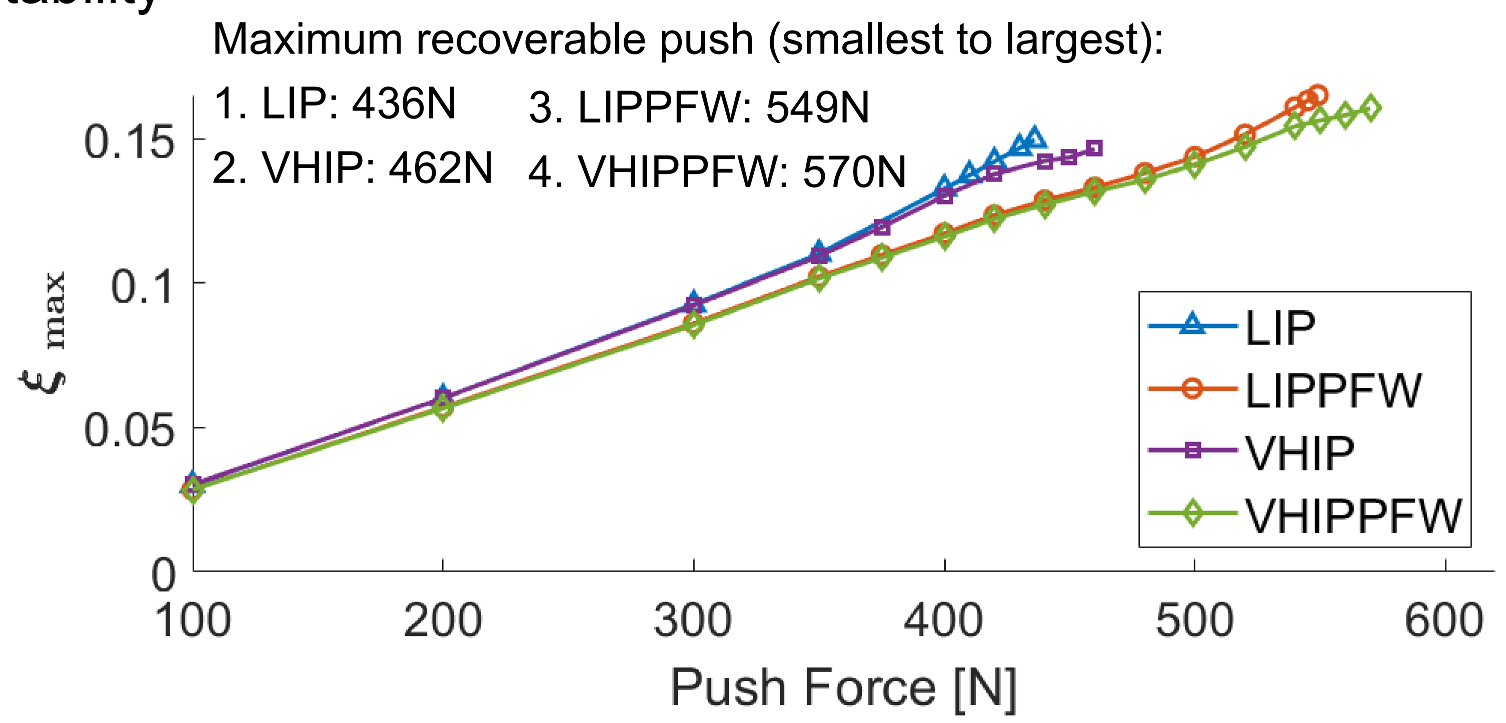
- Was 96% of the total cost of control for pushes < 300N
- Decreased in cost as pushes approached limits of ankle strategy (LIP model failure)

Hip and Toe strategies applied in hierarchical manner

- Hip strategy increased as ankle strategy failed
- Toe strategy increased as ankle & hip strategy failed



Model comparisons: Maximum capture point for a given disturbance was lowest for the VHIPPFW, indicating greater stability



Discussion

The unified balancing model VHIPPFW extends the stability range and improves recovery performance, and its transition from ankle to hip balancing parallels human preferences [5]. While we could not determine whether the model transitions would occur at similar push levels as for humans, our unified model demonstrates increased stability from the availability of multiple strategies when stepping is not an option.

References: [1] McGreavy et al. 2020, [2] Kajita et al. 2001, [3] Pratt et al. 2007, [4] van Hofslot et al. 2019, [5] Hof 2007

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