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Lab 2: Prelab

1) a) $I_s = 10^{-15}$ A

$I = 10^{-9}$ A

$U_T = 25\text{ mV} = 25\times10^{-3}$ V

$\frac{10^{-9}\text{A}}{10^{-15}\text{A}} = 10^{6}$

$I = I_s\left(e^{\frac{V}{U_T}} - 1\right)$

$\frac{I}{I_s} = \left(e^{\frac{V}{U_T}} - 1\right)$

$10^{6} = \underbrace{e^{\frac{V}{25\times10^{-3}}}} - 1$

This will be a very large number on the scale of 10^6. Therefore the -1 at the end doesn't make much difference. So this would be a good approximation

b) Increase current by e

$I = I_s\left(e^{V_1/U_T}\right)$

$I = I_s e^{V_1/U_T}$

$eI = I_s\left(e^{V_2/U_T}\right)$

$I = I_s\left(e^{V_2/U_T}\right)\cdot e^{-1}$

$I = I_s e^{V_2/U_T - 1}$

Set the I values equal and take natural log

$\frac{V_1}{U_T} = \frac{V_2}{U_T} - 1 \Rightarrow V_1 = V_2 - U_T \Rightarrow \boxed{V_2 = V_1 + U_T}$

with increased current

Increases by U_T

Increase current by 10 (one decade)

$I = I_s\left(e^{V_1/U_T}\right)$

$\frac{I}{I_s} = e^{V_1/U_T}$

$\ln\frac{I}{I_s} = \frac{V_1}{U_T}$

$10I = I_s\left(e^{V_2/U_T}\right)$

$10\cdot\frac{I}{I_s} = e^{V_2/U_T}$

$\ln 10 + \ln\frac{I}{I_s} = \frac{V_2}{U_T}$

$\ln\frac{I}{I_s} = \frac{V_2}{U_T} - \ln 10$

Set the equations equal

$\frac{V_1}{U_T} = \frac{V_2}{U_T} - \ln 10 \Rightarrow \boxed{V_2 = U_T\ln 10 + V_1}$

New voltage with increased current

Voltage increases by $U_T\ln 10$

1) c) Incremental diode resistance (dynamic or slope resistance)

$$r_d = \frac{\partial V}{\partial I} \qquad I = I_s\left(e^{V/u_T}\right)$$

the -1 would be a constant and would be zero with the derivative anyways.

$$\frac{I}{I_s} = e^{V/u_T}$$

$$\ln\frac{I}{I_s} = \frac{V}{u_T} \Rightarrow \ln I - \ln I_s = \frac{V}{u_T} \Rightarrow u_T(\ln I - \ln I_s) = V$$

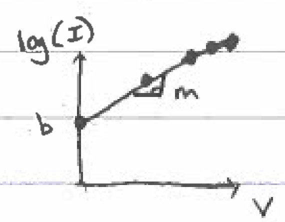
$$\frac{\partial V}{\partial I} = u_T \cdot \frac{1}{I} = \boxed{\frac{u_T}{I} = r_d = \frac{\partial V}{\partial I}}$$

d) No, the situation would not differ the relationship between I and V from the ideal diode equation remains the same if you apply a voltage or current.

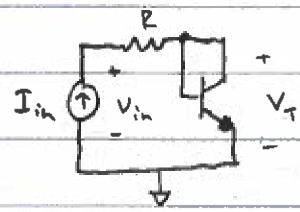
e) $I = I_s e^{\frac{V}{u_T}} \Rightarrow \ln I = \ln I_s + \frac{V}{u_T}$

$$\boxed{\underbrace{\ln I}_{y} = \underbrace{V}_{x} \cdot \underbrace{\frac{1}{u_T}}_{m} + \underbrace{\ln I_s}_{b}}$$

with the I-V characteristics, they can be plotted on a semilog scale (semilog for y) and fit with the equation above. The slope of the best fit line would be $\frac{1}{u_T}$ and by looking at the y-intercept we can find I_s.



2) a)



$I = I_s e^{V/u_T}$

$\ln \frac{I}{I_s} = \frac{V}{u_T}$

$V_D = u_D \ln \frac{I}{I_s}$

$V_{in} - V_D = IR$ ↙ Voltage across R

$V_{in} - u_D \ln \frac{I}{I_s} = IR$

$\boxed{V_{in} = IR + u_T \ln \frac{I}{I_s}}$ ↙ V_{in}

$\boxed{V_T = u_D \ln \frac{I}{I_s}}$ ↙ Voltage across diode

$\boxed{V_R = IR}$ ↙ Voltage across R

b) $I + \delta I$

Change in Voltage across resistor ↙

$\boxed{\delta V_R = \delta I R}$

Change in Voltage across diode ↙

$\boxed{\delta V_D = \delta I \cdot r_d}$

$V_{in} = \delta V_R + \delta V_D$

$\delta V_{in} = \delta I R + \delta I r_d \Rightarrow \boxed{\delta V_{in} = \delta I (R + r_d)}$

↑ Change in V_{in}

Where $r_d = \frac{u_T}{I}$

c) $\Delta I R = \Delta I r_d$

$R = \frac{u_T}{I} \Rightarrow \boxed{I_{on} = \frac{u_T}{R}}$

d) $I_{on} = I_s e^{V_{on}/u_T} \Rightarrow \ln\left(\frac{I_{on}}{I_s}\right) = \frac{V_{on}}{u_T} \Rightarrow \boxed{V_{on} = u_T \cdot \ln\left(\frac{I_{on}}{I_s}\right)}$

2) e) $$\frac{\delta V_R}{\delta V_{in}} = \frac{\cancel{\Delta I} R}{\cancel{\Delta I}(R+r_d)} = \frac{R}{R+r_d}$$

$$= \frac{\frac{U_T}{I_{on}}}{\frac{U_T}{I_{on}} + \frac{U_T}{I}} = \frac{\frac{U_T}{I_{on}}}{\frac{I U_T + I_{on} U_T}{I I_{on}}}$$

$$= \frac{\cancel{U_T}}{\cancel{I_{on}}} \cdot \frac{I\, I_{on}}{I \cancel{U_T} + I_{on} \cancel{U_T}} = \boxed{\frac{I}{I+I_{on}} = \frac{\delta V_R}{\delta V_{in}}}$$

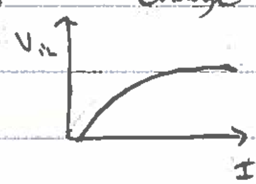
$$\frac{\delta V_T}{\delta V_{in}} = \frac{\cancel{\Delta I} r_d}{\cancel{\Delta I}(R+r_d)} = \frac{r_T}{R+r_T} \Rightarrow \frac{\frac{U_T}{I}}{\frac{U_T}{I_{on}} + \frac{U_T}{I}}$$

$$= \frac{\frac{U_T}{I}}{\frac{I U_T + I_{on} U_T}{I I_{on}}} = \frac{\cancel{U_T}}{\cancel{I}} \cdot \frac{\cancel{I}\, I_{on}}{I \cancel{U_T} + I_{on} \cancel{U_T}} = \boxed{\frac{I_{on}}{I+I_{on}} = \frac{\delta V_T}{\delta V_{in}}}$$

f) $I << I_{on}$: $\frac{I_{on}}{0 + I_{on}} = 1$ ∴ δV_{in} is largely contributed by δV_d

V_{in} will change with I following a logarithmic function

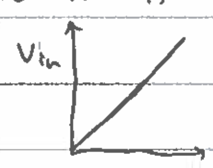
this shape



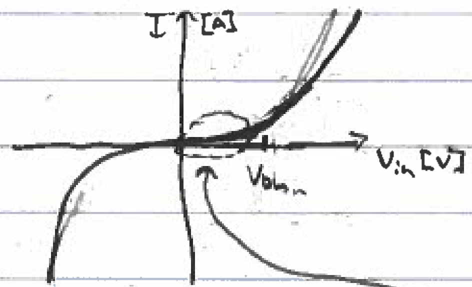
$I >> I_{on}$: $\frac{I_{on}}{\infty + I_{on}} \simeq 0$ ∴ δV_{in} is barely if at all contributed by δV_d

Thus, the voltage drop across the transistor at $I >> I_{on}$ because it is proportional to R with a linear relationship

$\delta V_{in} \propto R$



2) g) Ideal diode I - V plot

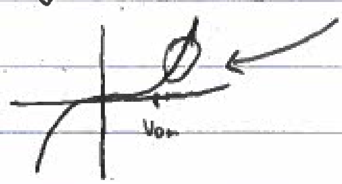


$V_{in} < V_{on}$ by a few U_T

This would be in this region. Therefore the current will remain close zero and depend very little on V_{in}. It presumably can be modelled with a linear fit.

$V_{in} > V_{on}$ by a few U_T

This regime would be in this region



Therefore, the current will depend largely on V_{in}. It will follow an exponential relationship in this region.