Lab 1: Resistors and Resistive Networks

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1 Experiment 1

1.1 Background and Procedure

During this experiment, we found the resistance of a resistor in three ways. We first looked at the expected resistance value by the resistor color band code. Second, we measured the resistance using the Keithley 2400 SourceMeter in resistance mode. Third, we used the SMU to create an I-V characteristic plot. In doing this, we could fit a line to the measured current, with a given V_{in}, and the slope of the line would correspond to the resistance.

For the third method of measurement, we used the following circuit. In doing this, we measured the current through Ch1 while supplying various voltages.

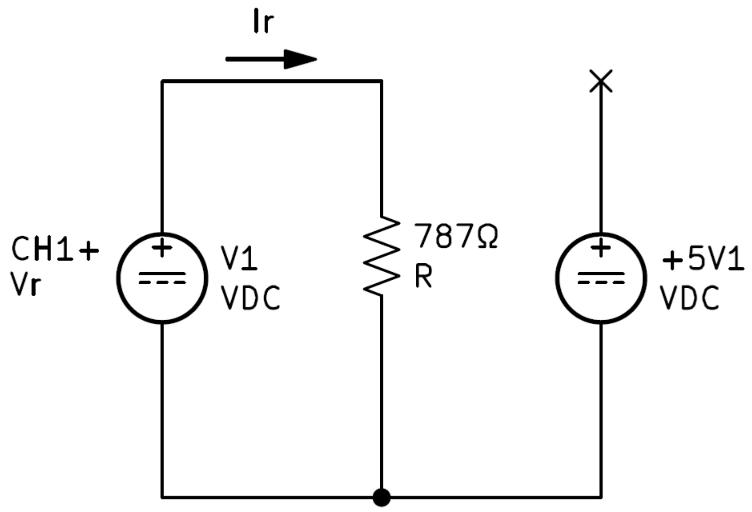

Figure 1: Schematic for Experiment 1

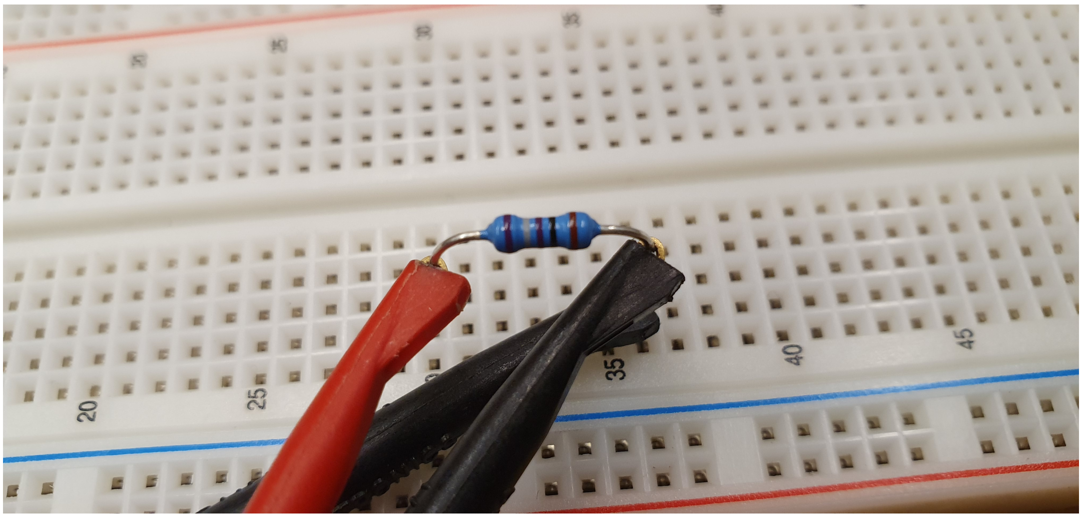

Figure 2: Picture for the Experiment 1 circuit

1.2 Expectations

We used a resistor value of 787 Ω with a tolerance of 1%. Therefore, when measuring the resistance in other ways, we expected to measure a resistance value between 779.13 Ω and 794.87 Ω. Furthermore, we expected the resistance measured by the Keithley 2400 SourceMeter to match the resistance extracted from the I-V characteristic plot.

1.3 Results

Using the resistor color code, we found the resistor was 787 Ω with a tolerance of 1%. Using the Keithley 2400 SourceMeter, we measured a resistance of 782.90 Ω. Finally, using the circuit shown in Figure 1, we extracted the resistance from the line of best fit on the measured I-V characteristic plot to be 781.86 Ω.

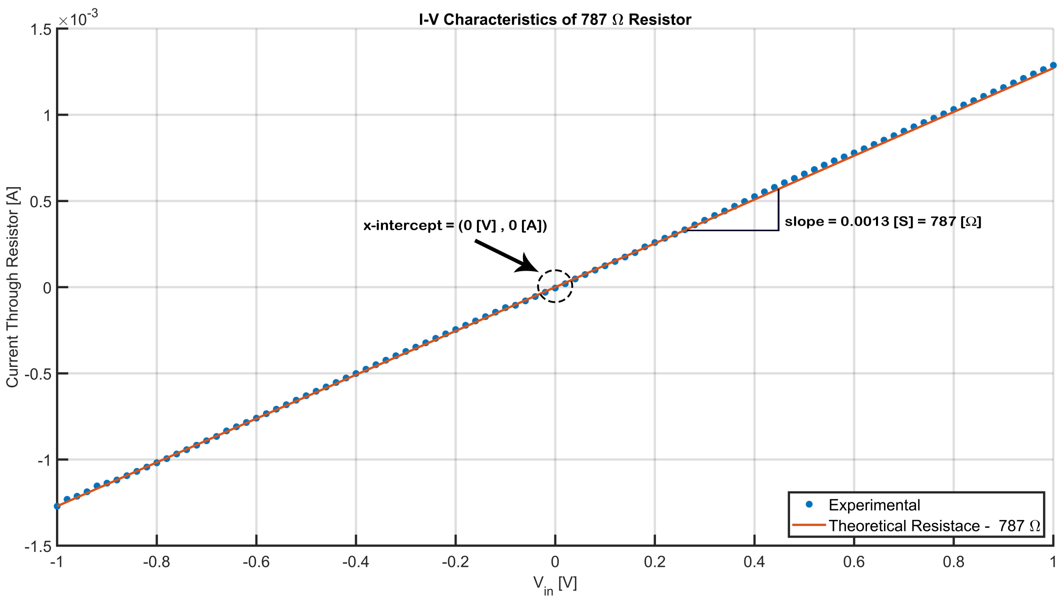

Figure 3: Theoretical I-V characteristics for a 787 Ω resistor and measured I-V values with SMU.

The I-V characteristics of the 787 Ω Resistor is shown in Figure 3. The x-axis represents the input voltage [V] while the y-axis represents the current through the resistor in amps [A]. The x-intercept for the line of best fit (which is represented in red) is (0[V] , 0[A]). The measured slope for the fit is 0.0013 [S] = 787[Ω], which according to Ohm's Law will be the theoretical value of the resistance. This observation matches our expectations, based off of the manufacture specified values.

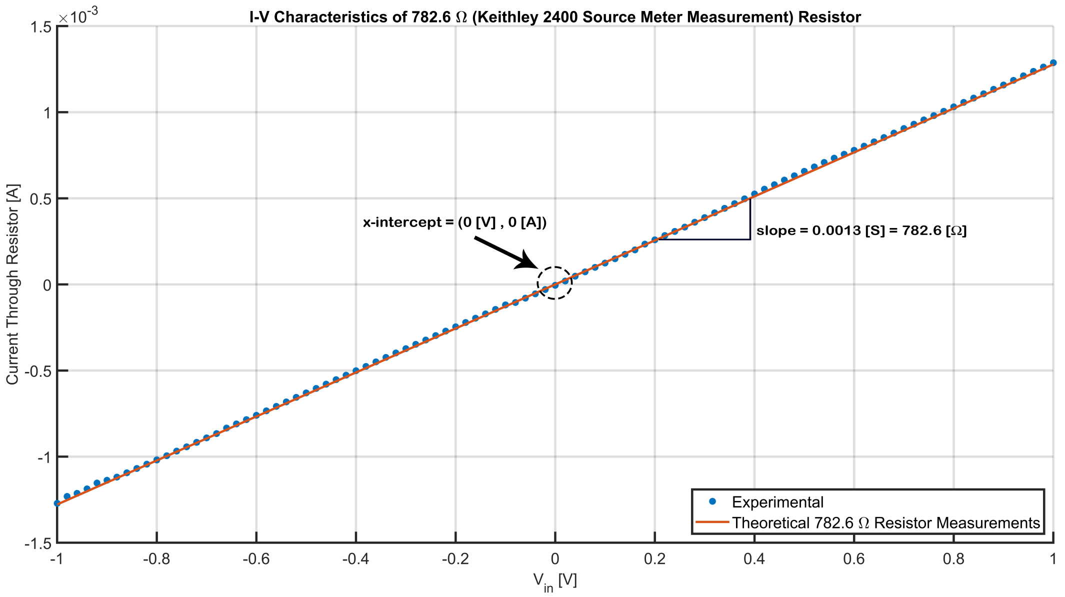

Figure 4: Theoretical I-V characteristics for a 782.6 Ω resistor compared to the measured values using the SMU. 782.6 Ω is the resistance computed with the Keithley 2400 SMU.

The I-V characteristics of the 782.6 Ω Resistor is shown in Figure 4. The x-axis represents the input voltage [V] while the y-axis represents the current through the resistor in amps [A]. The x-intercept for the line of best fit (which is represented in red) is (0[V] , 0[A]). The measured slope for the fit is 0.0013 [S] = 782.6[Ω], which according to Ohm's Law will be the experimental value of the resistance. This observation is within the manufacturer provided threshold for this particular component.

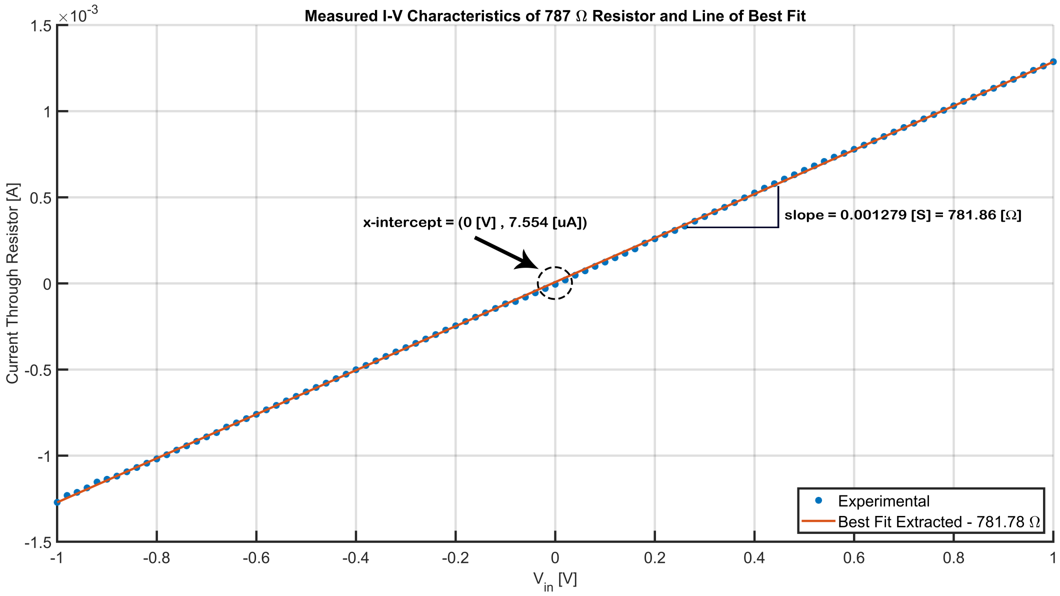

Figure 5: Measured values and line of best fit for I-V characteristics of a 787 Ω resistor.

The I-V characteristics of the 787 Ω Resistor is shown in Figure 5. The x-axis represents the input voltage [V] while the y-axis represents the current through the resistor in amps [A]. The x-intercept for the line of best fit (which is represented in red) is (0[V], 7.554[uA]). The measured slope for the fit is 0.001279 [S] = 781.86[Ω], which according to Ohm's Law will be the theoretical value of the resistance. This observation matches our expectations, based off of the manufacture specified values.

1.4 Discussion

The measurement for resistance using the Keithley 2400 Source-Meter unit was within the tolerance set by the manufacturer. Therefore, we believe the discrepancy between the bands (787 Ω) and the measured resistance is because of how it was manufactured. When extracting the resistance from the I-V characteristic plot, we found a resistance of 781.78 Ω. This was only 1 Ω different than the resistance computed with the Keithley SourceMeter (782.6 Ω). We believe this discrepancy is a result of the capacitance in the wires. A reason for this belief is because of the x-intercept on Figure 5. We find that the line of best fit when $V_{in} = 0$ $[V]$ is 7.554 $[\mu\Omega]$, which can be a result of residual current from the capacitance from when the V_{in} approaches $0[V]$. Although the SMU takes current calculations at a range of V_{in} values, we believe that the Keithley SourceMeter is the most accurate method because it will have the least and a more consistent capacitance in the wires. We believe it has the least amount of capacitance in the wires because the wires are the shortest for this measuring method. Furthermore, if we were to consider just the bands, the resistance could be any value within the tolerance; whereas, the Keithley SourceMeter and the SMU is able to provide a more precise resistance value.

2 Experiment 2

2.1 Background and Procedure

For this experiment, we created a resistive voltage divider. We used a Bourns resistor array chip (pinout shown in Figure 6) which had resistances that matched one another with a

±2% tolerance. After constructing the resistive voltage divider, we created a voltage transfer characteristic plot and extracted the voltage divider ratio.

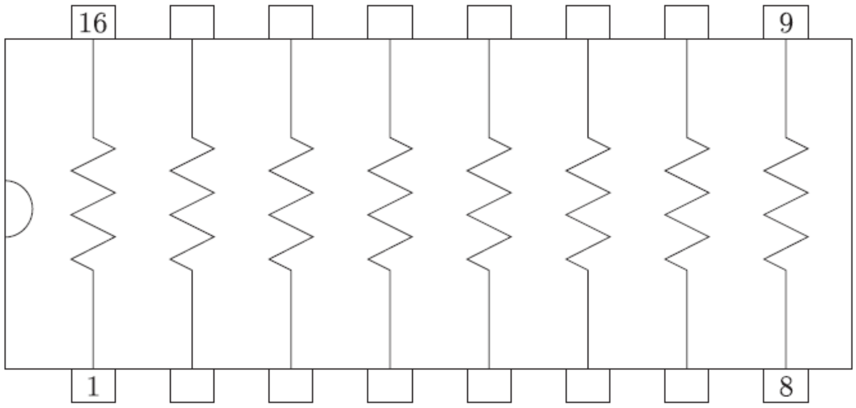

Figure 6: Pinout for Bourns resistor array chip

For this experiment, we used the following circuit. Ch1 supplied voltage to the circuit, and Ch2 was a set to be a current source of 0A so we could measure the voltage across the terminals.

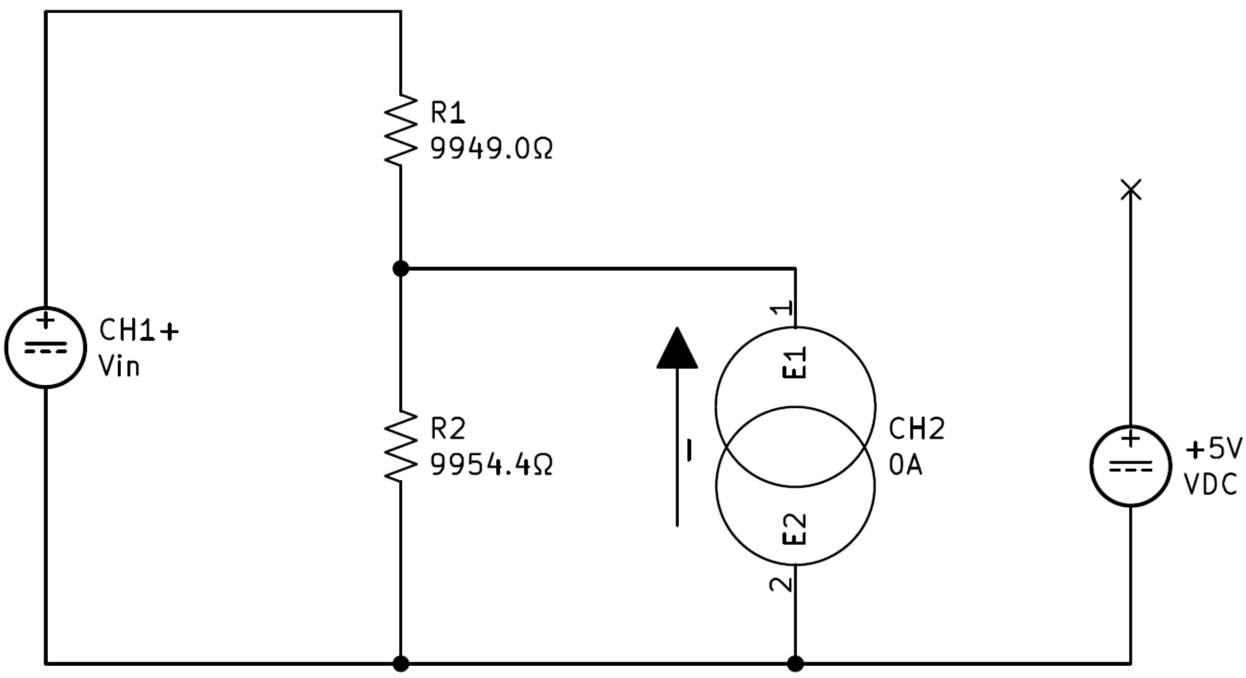

Figure 7: Schematic for Experiment 2

The following image shows the setup we used,

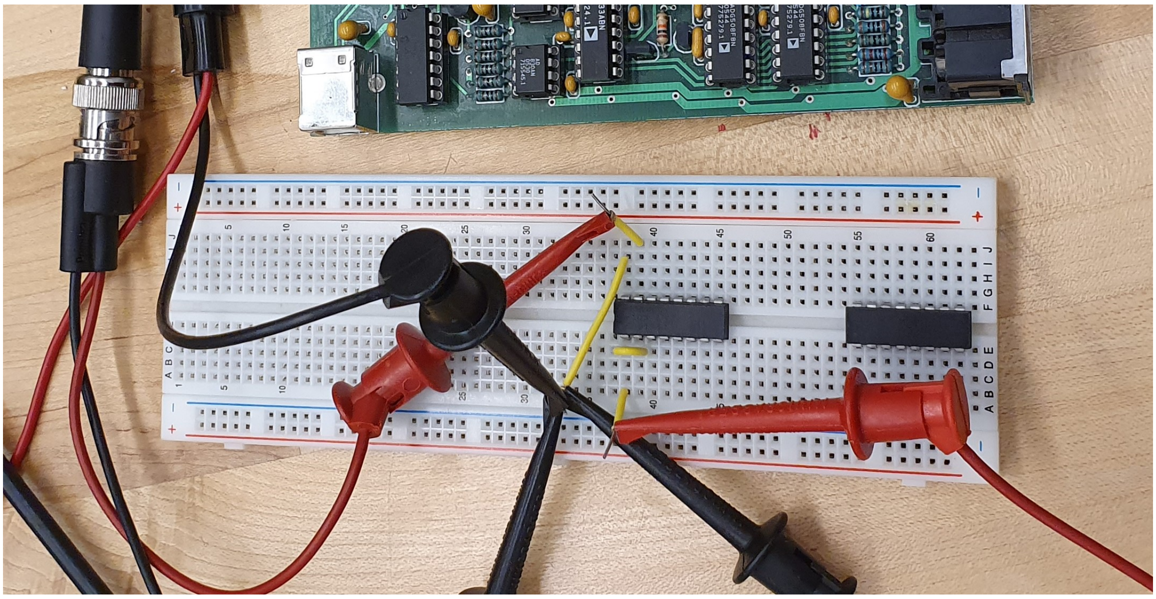

Figure 8: Picture for Experiment 2 Circuit

2.2 Expectations

After measuring the resistance of the Bourns resistor array chips, we calculated the voltage divider ratio using the voltage divider equation,

$$V_{out} = \frac{R_2}{R_1 + R_2} * V_{in} \tag{1}$$

From the voltage divider equation, we can extract the voltage division ratio as,

$$\frac{R_2}{R_1 + R_2} \tag{2}$$

Using the first Bourns resistor array, the resistor between pins 1 and 16 and the resistor between pins 2 and 15, we compute the expected voltage division ratio (refer to equation 2) to be,

$$\frac{9949.0[\Omega]}{9954.4[\Omega] + 9949.0[\Omega]} = 0.4999 \tag{3}$$

Please note, the resistor values used can be found in the "Resistor Values on Bourns Resistor Array" table below.

2.3 Results

The following table show the resistance values for the Bourns resistor array chips using the Keithley 2400 SourceMeter.

Resistor Values on Bourns Resistor Array		
Resistor Between Pin Number	Array 1 Resistance Value [Ω]	Array 2 Resistance Value [Ω]
1-16	9954.4	9949.0
2-15	9949.0	9947.8
3-14	9937.6	9951.0
4-13	9940.2	9940.6
5-12	9938.7	9943.0
6-11	9941.6	9948.0
7-10	9936.7	9941.0
8-9	9941.0	9950.8

Using the first (pins 1 and 16) and second (pins 2 and 15) resistors on the first resistor array chip for the resistive voltage divider, the following plots show the measured I-V characteristics.

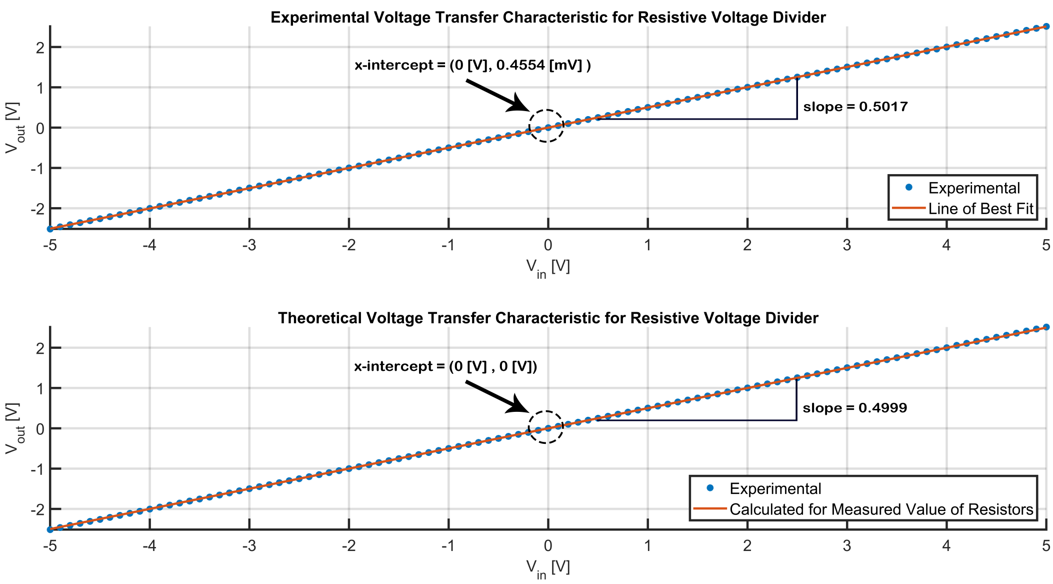

Figure 9: Voltage transfer characteristics for resistive voltage divider shown in Figure 7.

This figure shows the measured voltage transfer characteristics for the resistive voltage divider circuit. The first subplot shows the measured V_{out} $[V]$ for a given V_{in} $[V]$. Using a line of best fit, we find that the experimental voltage divider ratio is 0.5017. We also find that the line of best fit has an x-intercept of 0.4554 mV. The second subplot shows the measured resistance values compared to the theoretical voltage transfer characteristics.

2.4 Discussion

The average resistance of the first resistor array chip is 9942.4 Ω. With a tolerance of ±2%, there was a range of 198.85 Ω that would be considered within tolerance (between 9743.5 Ω and 10141.3 Ω). We can see that all of of the resistor values fall within this range. To consider the variation within a single chip, we computed the tolerance by calculating the percentage difference using the furthest from average resistance and the average resistance value within the chip. In doing this, we found the tolerance for chip 1 to be 0.12%.

The average resistance of the second resistor array chip is 9946.4 Ω. Similar to the last chip, the tolerance of the chip is ±2%, which is a range of 198.93 Ω (or between 9747.5 Ω and

10145.3 Ω). However, with consideration of the values in the table above, there is only a range of 11 Ω, which corresponds to a tolerance of 0.06% within the chip.

The average resistor values for the first and second resistor array chips are within 4 Ω of one another. This is within 0.04% of one another, and therefore is very close.

Using the circuit in Figure 7, we measured the voltage transfer characteristics of the voltage divider. The theoretical voltage divider ratio computed was 0.4999; however, the experimental divider ratio found through the line of best fit was 0.5017. This is a difference of 0.0018. From the prelab, we know that

$$T_\gamma = |S_{R1}{}^\gamma| T_{R1} + |S_{R2}{}^\gamma| T_{R2} \tag{4}$$

The tolerance for R_1 based on the average resistance of the first resistor array chip is 0.066%, and for R_2 is 0.068%.

The maximum tolerance that we calculated based on the resistor that varied the most from the mean resistance value for resistor array 1 was 0.12%.

$$T_\gamma = |0.4999| * 0.0012 + |-0.4999| * 0.0012 = 0.0012\% \tag{5}$$

With consideration of the difference we saw among the theoretical and measured voltage divider ratio is within the tolerance that is set forth by the overall tolerance equation (Equation 9).

3 Experiment 3

3.1 Background and Procedure

For this experiment, we created a resistive current divider. To do this, we used the same Bourns resistor array chips used for Experiment 2. After constructing the resistive current divider, we created a current transfer characteristic plot in order to determine the current division ratio. To do this, we used the following circuit,

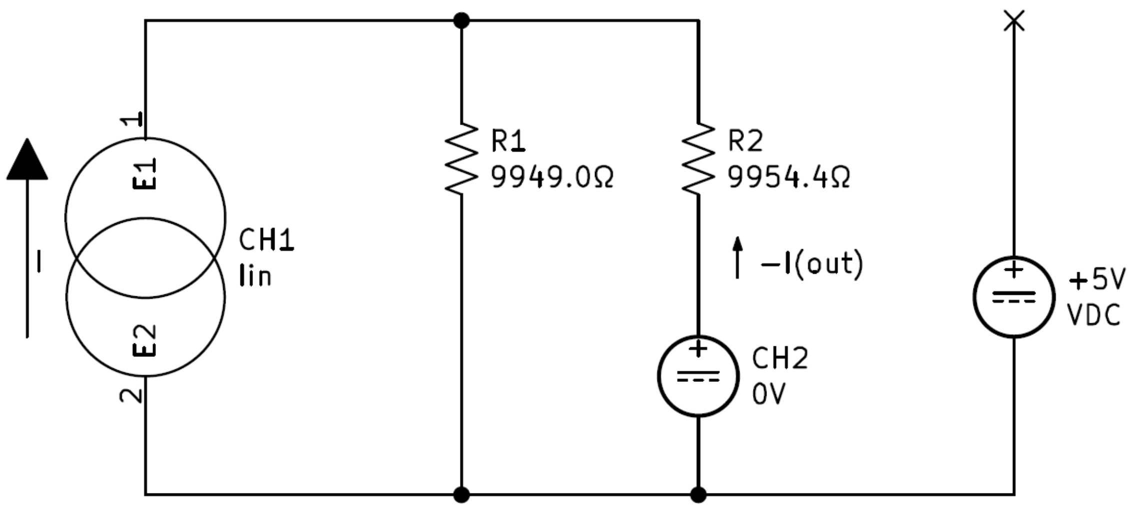

Figure 10: Schematic for Experiment 3

In this circuit, we supply a current through Ch1 and measure the current through Ch2 by creating a 0 V voltage source. The following image shows the setup we used.

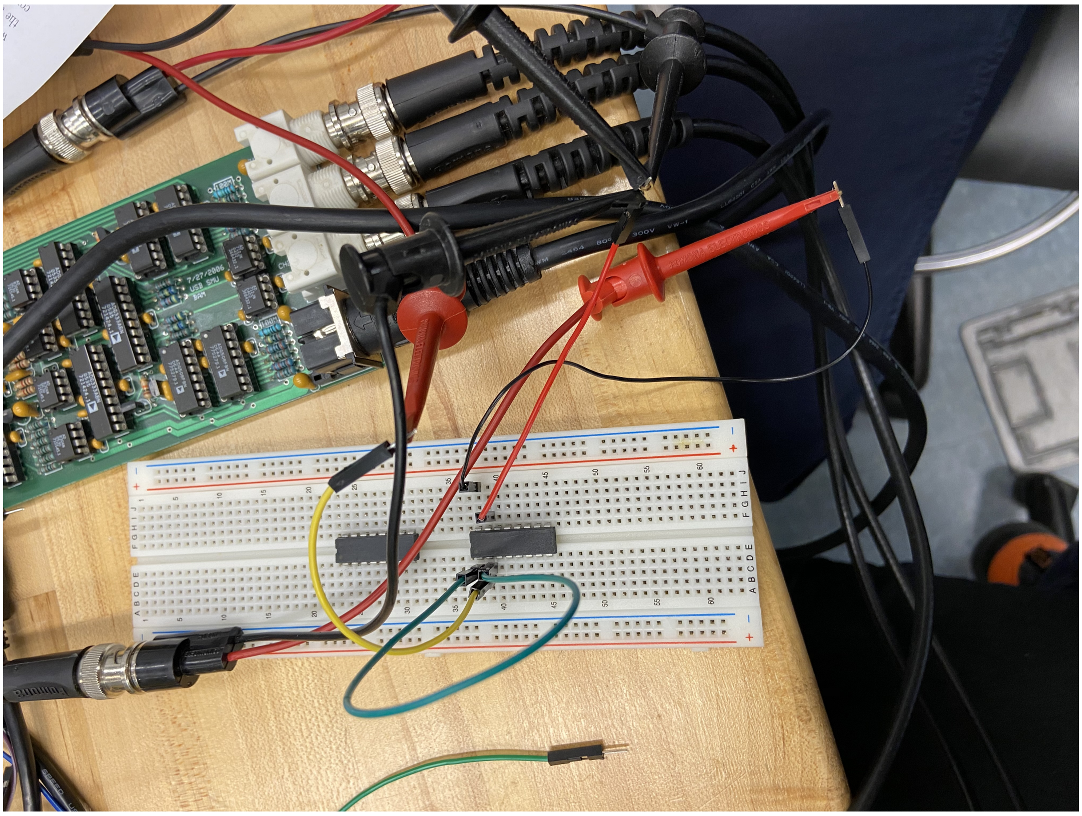

Figure 11: Picture for Experiment 3 Circuit

3.2 Expectations

After measuring the resistance of the Bourns resistor array chips, we were able to compute the current division ratio with the current divider eqaution,

$$I_{out} = \frac{R_1||R_2}{R_2} * I_{in} \tag{6}$$

From the current divider equation, we can extract the current division ratio as,

$$\frac{R_1||R_2}{R_2} \tag{7}$$

Using the first Bourns resistor array, the resistor between pins 1 and 16, and the resistor between pins 2 and 15 we compute the expected current division ratio (refer to Equation 7) to be,

$$\frac{9954.4[\Omega]\ ||\ 9949[\Omega]}{9949[\Omega]} = 0.5001 \tag{8}$$

Please note, the resistor values used can be found in the "Resistor Values on Bourns Resistor Array" table under Experiment 2 Results.

3.3 Results

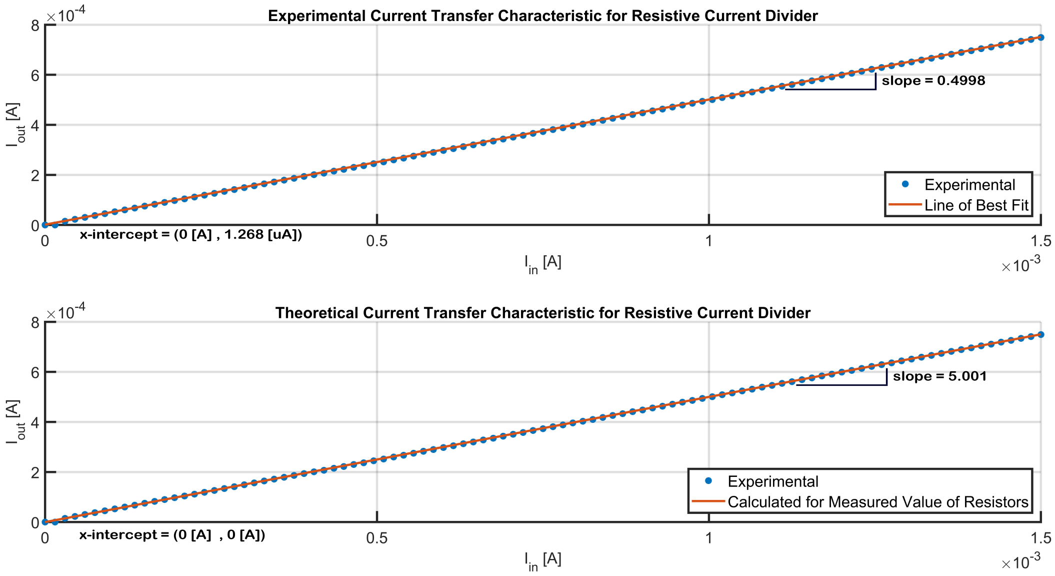

Figure 12: Current transfer characteristics for resistive current divider shown in Figure 10.

This plot shows the current transfer characteristics for the current divider schematic shown in Figure 10. The measured values are shown as points. The first subplot shows the line of best fit. Using the line of best fit, we can find the current divider ratio to be 0.4998. This line of best fit has a x-intercept of 1.268 μA. The second subplot shows the theoretical current divider ratio compared to the measured values for I_{out} for a given I_{in}.

3.4 Discussion

The theoretical computed current divider ratio was 0.5001, and the experimentally determined current divider ratio was 0.4998. This is a difference of 0.0003. From the prelab, we know that

$$T_\gamma = |S_{R1}{}^\gamma| T_{R1} + |S_{R2}{}^\gamma| T_{R2} \tag{9}$$

The tolerance for R_1 based on the average resistance of the first resistor array chip is 0.066%, and for R_2 is 0.068%.

The maximum tolerance that we calculated based on the resistor that varied the most from the mean resistance value for resistor array 1 was 0.12%.

$$T_\gamma = |0.5001| * 0.0012 + |-0.5001| * 0.0012 = 0.00120024 \tag{10}$$

With consideration of the difference we saw among the theoretical and measured voltage divider ratio is within the tolerance that is set forth by the overall tolerance equation (Equation 9).

4 Experiment 4

4.1 Background and Procedure

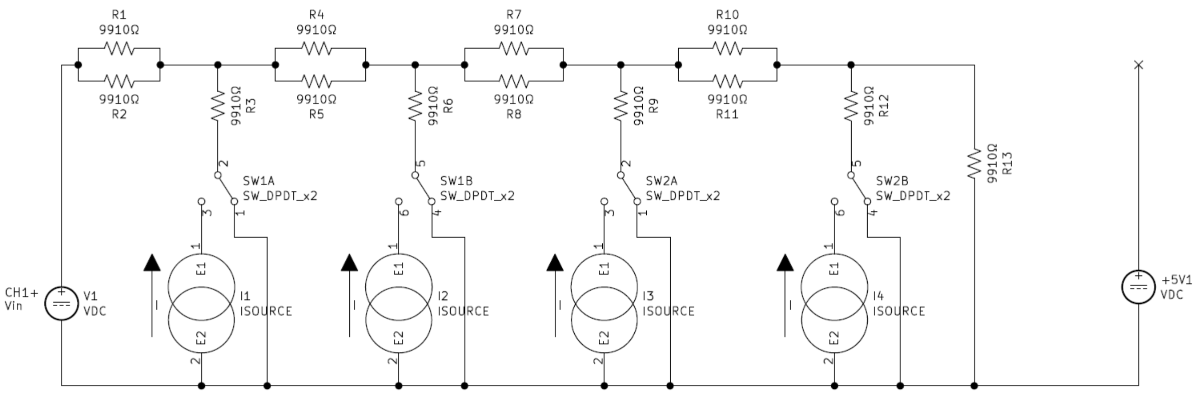

Figure 13: Schematic for Experiment 4

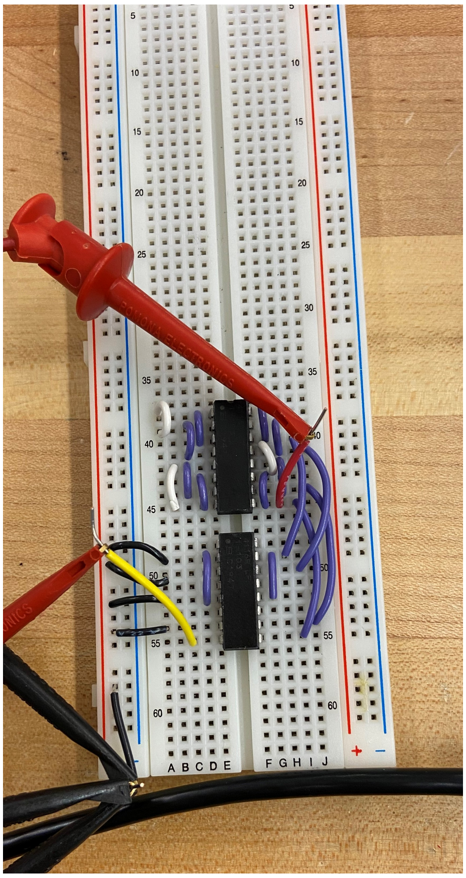

Figure 14: Picture for Experiment 4 Circuit

4.2 Expectations

We expect the current out through the n^{th} 2R branch can be given through the following equation,

$$I_n = \frac{1}{2^{n+1}} * \frac{V}{R} \tag{11}$$

4.3 Results

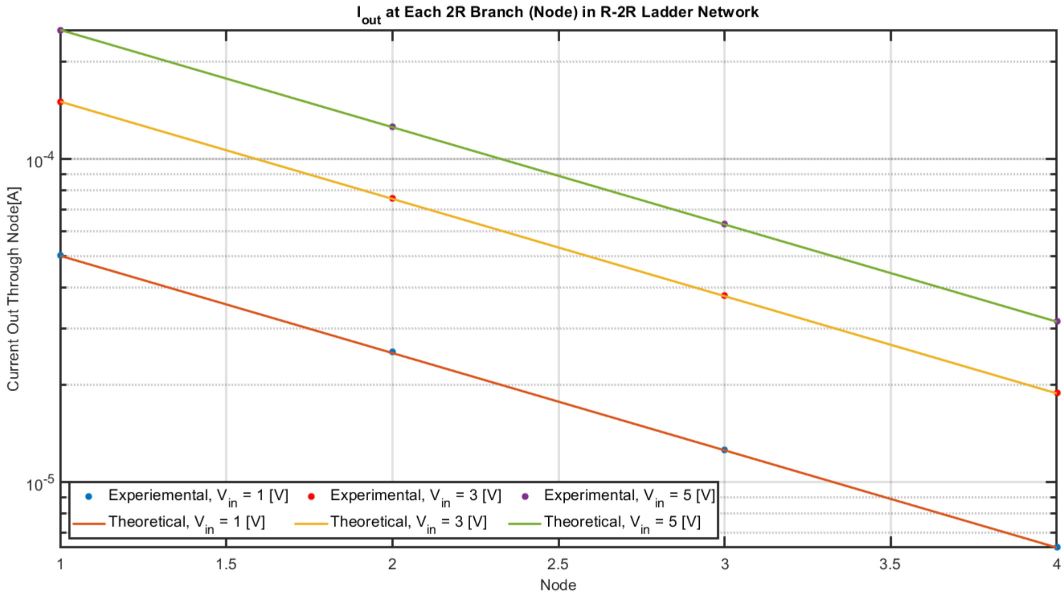

Figure 15: Measured and Theoretical I_{out} values for various input voltages. Each node refers to the 2R branch of the ladder.

Using the R-2R ladder network shown in Figure 13. The plot above shows the theoretical and measured I_{out} at various branches for various input voltages. To find the theoretical value, we used the average resistance value for both chips.

4.4 Discussion

Theoretical and Experimental I_{out} $[A]$ at Various Nodes and Voltages				
V_{in} $[V]$	Node	Theoretical I_{out} $[A]$	Experimental I_{out} $[A]$	Percent Difference between Theoretical and Experimental [%]
1	1	5.0280e-5	5.040e-5	6.0632e-10
	2	2.5140e-5	2.530e-5	4.0408e-10
	3	1.2570e-5	1.26e-5	3.7895e-11
	4	6.2849e-6	6.31e-6	1.5779e-11
3	1	1.5084e-4	1.509e-4	9.2539e-10
	2	7.5419e-5	7.570e-5	2.1207e-09
	3	3.7710e-5	3.79e-5	7.1956e-10
	4	1.8855e-5	1.888e-5	4.7484e-11
5	1	2.5140e-4	2.500e-4	3.5042e-08
	2	1.2570e-4	1.262e-4	6.3115e-09
	3	6.2849e-5	6.31e-5	1.5779e-09
	4	3.1425e-5	3.160e-5	5.5234e-10

As we can see in the table above, the expected and measured currents through each of the branches is very close. The percent difference is on the scale of 10^{-9}% (as a maximum percent difference). Thus, we conclude, the currents at each of the branches is what we expected them to be.