

# Quantitative Engineering Analysis II

First Edition

Fall 2019

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## *Module I*

# *IT'S GETTING HOT IN HERE: CLIMATE CHANGE, THERMAL MODELING, AND ORDINARY DIFFERENTIAL EQUATIONS*

## *Module II*

### *SHAKE IT OFF: MODELING MOTION WITH DIFFERENTIAL EQUATIONS AND FREQUENCY DOMAIN ANALYSIS*

## Chapter 17

### Module 2 Project Description

As you reach the end of your QEA journey, we want to provide you the opportunity and space to choose your final grand adventure. To that end, the module two project will have two options, both with a significant level of flexibility. In many ways, these options reflect the Fitbit and Choose Your Own Adventure projects from previous iterations of the class, but with the scope modified to reflect the four-credit footprint of this year's course. In both cases, we want you to focus not only on the application of quantitative tools, but on creating value for an individual or group of individuals. The project options are as follows:

1. Measurement and Analysis of Motion
2. Deep Dive into a Topic of Your Choice

#### 17.1 Measurement and Analysis of Motion

##### Overview

The development and miniaturization of powerful sensors has led to an explosion in the application of wearable sensing to the analysis and characterization of human behavior. Perhaps the most popular application of these sensors is for fitness tracking (e.g., FitBit and smartphone activity trackers), however, these sensors have been utilized for a wide array of applications, including early detection of disorders (e.g., Parkinson's, Dementia, Chronic Obstructive Pulmonary Disease), monitoring home rehabilitation (e.g., for home exercise to alleviate low-back pain), navigation for people who are blind or visually-impaired, safety monitoring (e.g., fall detection, detection of epileptic seizures, etc.), and sleep monitoring (see the *Resources* section for links to more examples).

Each of these applications can be characterized using a framework of value generation. Specifically, each of the applications listed above has the following components.

1. **A person wearing the sensor** (e.g., someone going for a run, a person in an eldercare facility, someone at a rehab clinic).
2. **A motion model** that relates the application of interest to a meaningful understanding of the degrees of freedom of the motion, the time derivatives of those degrees of freedom, and the expected frequency of the motion. This model should also help inform the selection of appropriate sensor type and sensing range. Your motion model could be a set of kinematic equations or it could simply be a set of well-thought-out graphs that capture both qualitative and quantitative aspects of the motion.
3. **A sensor suite** (e.g., accelerometers, gyroscopes, GPS).
4. **Algorithms for analyzing raw sensor data** to extract meaningful information and relate the experimental data to the predictions from the motion model (e.g., falls, sleep quality).
5. **An end user** who derives value from the extracted information. The person wearing the sensor and the end user could be the same person (as in the case of someone using a fitness tracker), or they could be different (in the case of someone remotely monitoring a family member to detect whether their loved one has fallen and become injured).

In this project **you'll be developing a proof-of-concept system that uses wearable sensing to extract valuable information for a specific user group**. You will have the freedom to choose your specific user, application, and sensors subject to several requirements.

## *Project Requirements*

### *Motion Model*

For your chosen application, you must demonstrate an understanding of the dynamics of the underlying motion. You will need to identify the degrees of freedom of the motion and think about the time derivatives of each of those degrees of freedoms. It will be very useful to sketch out predictions of these quantities as a function of time, as it is important to know what you are expecting to see before collecting experimental data. Thinking hard about your motion early in the development of your project will also help in the selection of an appropriate sensor and inform where the sensor should be placed to maximize the usefulness of collected data.

We want you to show that you have thought carefully about the motion, have an informed prediction of what frequencies you expect to see in your experimental data, and understand what implications these insights have on sensor selection and choice of analysis techniques. Your understanding of the motion can be demonstrated through diagrams, sketches of your expectations for collected data, or an mathematical equation (potentially implemented on a computer).

Just as in the design process, iteration is important in developing a useful motion model. Your motion model provides an expectation for what experimental data should look like and helps identify and debug sensor issues. Once you have collected quality experimental data, the motion model can be updated and refined to better reflect the real world. Similarly, your data analysis will be developed based on the expectations from your motion model and will be refined based on experimentation.

**Example:** It might be useful to read through section 5 of [this solution](#) from last year's QEA II class that walks through the analysis of accelerometer data captured on a bicycle.

### *Sensor Selection*

We suggest that you use the sensors on a smartphone (iPhone or Android) for your project. The reasons for this are many. For one, you've already had experience processing data from a smartphone earlier in the module. Secondly, if you decide to pursue your project and develop it into an actual product you can distribute it to a mass audience through either the Google Play Store or the iOS App Store. Thirdly, the barriers to getting data from the sensors into an analysis platform, such as MATLAB, are minimal.

If you would like to use other sensing technology instead of (or in addition to) a smartphone we will allow it, but only if you run it by one of the instructors on the teaching team. Here are some issues to consider when choosing your own sensors.

1. The sensors must be easily and quickly obtainable.
2. The data from the sensors must be easy to acquire. If the sensors require a lot of work just to get data from them, then that will be taking away from doing meaningful analysis and design with the data.
3. You should be able to obtain synchronized data between all sensors easily. If you are using multiple devices to collect data (e.g., an Arduino and a smartphone), you will face the challenge of synchronizing



the two sensor streams. Again, if this takes a significant amount of time you will be missing out on other aspects of the project.

### *Signal Analysis Techniques*

We want you to maintain good alignment between the analysis techniques you have learned thus far in this module (most notably Fourier analysis) and those that you will select for your project. Given the open-ended nature of the project, it is of course likely that these two things will not be in complete alignment, however, we are requiring you to use frequency domain analysis of your sensor data in at least one part of your project. This analysis should consist of interpreting at least two frequencies related to the phenomenon you are characterizing.

### *Focus on Value Creation*

Your project should focus on extracting useful information from wearable sensors. You should define usefulness in terms of a specific group of end users (e.g., physical therapists, people who are blind, people with impaired sensing in their feet). You should be able to make a convincing argument as to how the system you will develop creates value for these users.

### *Focus on Proof-of-Concept*

This project is primarily about doing the experimentation, analytical, and design work to evaluate the feasibility of the core of your proposed application. Therefore, you should try to not spend too much time on the finish, polish, and packaging that you would need for a fully realized system (e.g., user interface, additional features). Remember, before you build something you should evaluate whether or not it is feasible. The focus here should be on making that determination.

### *(Some) Project Options*

There are limitless options, but to help get you going, here are a few!

1. Distinguish two or more different types of activities or conditions for diagnosis based on frequency spectrum data.
2. Create a monitoring system to help the user improve their general level of wellness (e.g., by exercising more, getting more sleep).
3. Create a system to facilitate navigation for people who are blind or visually-impaired.
4. Create a system to help train someone to be better at a sport.
5. Detect some sort of catastrophic event (e.g., a fall, a heart attack, etc.).

### *Deliverables*

#### *Project Consulting Days*

While this is not a deliverable in the traditional sense, we are planning two or three in-class consulting days for you to work on your projects. We expect you to have made significant progress on your project on

each of the two consulting days (i.e. you need to have done a bunch of work outside of class). We expect you to come to the consulting sessions with specific questions, graphs, analysis etc.

### *Website Pitch and Description*

In lieu of a final report, you will design a website to describe your project. This website should describe the overall idea of your project, the value it is intended to deliver and to whom, your system architecture, your motion model and expectations for the experimental data, a description of any experiments you did (or data you obtained), a detailed description of the analysis techniques you applied to your data, and the results of applying these techniques and how the results aligned (or didn't) with your predictions. Think of this as an amazing kickstarter page (because it would actually included evidence of the technical analysis that is at the core of the proof-of-concept) OR an awesome product description on a company website (see resources section for examples).

You will be asked to give a short pitch of your project and website on the final day of the module. We expect most of you to project the website on the screen and describe your project.

Feedback on your project and website will be based on the following criteria:

- *Proof-of-Concept:* The proof-of-concept should include a selection of sensors that are appropriate for the specific application. The experiment(s) completed should demonstrate some aspect of how the product would work in reality. Next steps for the proof-of concept (e.g., experiments, additional sensors) should be well articulated.
- *Value Creation:* The proof-of-concept supports specific user groups in clearly defined ways. The connection between the proof-of-concept and the user group is clear and built on existing research and/or a strong understanding of particular areas of opportunity.
- *Motion Model:* Your motion model should demonstrate a clear understanding of the dynamics of your motion, the degrees of freedom and their time derivatives, and the important frequencies. You should explain how this model informed your data collection, and what, if any, modifications were made to the model following experimentation.
- *Algorithm Development:* The algorithm(s) for data analysis should demonstrate a clear understanding and application of Fourier analysis, the frequency domain, and motion model dynamics. The project website should clearly explain the application of the algorithm to experimental data through the use of appropriate equations and graphics.
- *Overall Presentation and Delivery of Information:* The website is professional, well-organized, and the information is clear and easy to understand.

Some examples of nicely done projects and websites from the last couple years of class can be found below.

1. “**ConductAid: Wearable devices assisting visually-impaired musicians.**” (Nice description of the project and algorithm. Again, some equations to help explain the approach would be useful.)
2. “**Fidget Fighter**” (Again would benefit from more information on the data analysis.)
3. “**Dad Dancing**”

4. “Eigentricks”
5. “Art Visualizer”

### *Timeline*

- 11/14: Project ideation and group formation
- 11/18: Prepare project proposal in class; Night 7 (project ideation and resource finding) due
- 11/21: In-class consulting; Night 8 (project timeline and concept map) due
- 12/2: Project work day
- 12/5: Draft analysis section due
- 12/10: Final event- website due and in-class presentation

### *Resources*

There are many papers that cover prior work on wearable sensing. Here are some places to start.

1. “A review of wearable sensors and systems with application in rehabilitation”
2. “Human Activity Recognition with Smartphones” (a Kaggle data science challenge).
3. “Assessment of spatio-temporal gait parameters from trunk accelerations during human walking.”

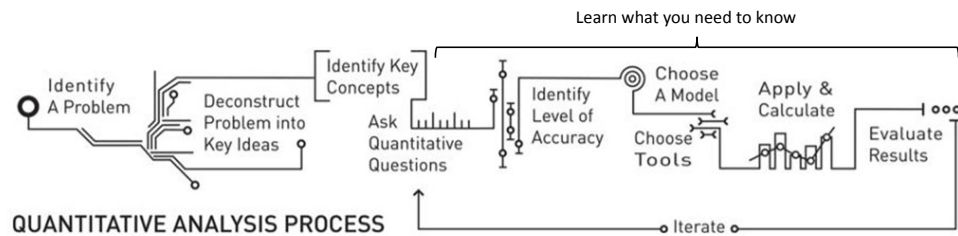
There are many more examples of product description websites and you should feel empowered to be as creative as you would like with this. Just be sure to included the information described earlier in the Website Pitch and Description section.

1. “imitone: mind to melody“
2. “OURA ring. Improve Sleep. Perform Better.“ (would have benefited from QEA level analysis on their campaign page)
3. “Product Embrace“ (Again would benefit from more information on the data analysis)

## *17.2 Deep Dive into a Topic of Your Choice*

### *Overview*

QEA has been one grand adventure – integrating theory across disciplines and learning through application and analysis in a variety of creative and challenging ways. In the very first module, you explored the stability of boats and the idea of buoyancy through the analysis AND the design (and implementation!) of your own boat. This semester you have analyzed thermal systems, human motion, and classical mechanics problems like the inverted pendulum. The Deep Dive project has been designed as an opportunity for you to **dive deeper, take a topic in a new direction, explore something new**, and/or **hone your skills** before setting off on your next pedagogical adventure as well as learning to explicitly navigate the Quantitative Analysis Process in a context of your own choosing.



Each of these adventures WILL look incredibly different, as they will be guided by the personal learning goals of the project team members. Yet, each will contain specific core components:

1. **A set of guiding learning goals.** We will coach your initial development of those goals during the Day 1 activities.
2. **A deep dive into particular concepts or approaches** to further your learning in QEA and hone your skills.
3. **A substantial analysis component.** It is QEA after all.
4. **A product** in this case, we value creation is around learning and the communication of the topic of your choice. You will create a QEA style overnight assignment with solutions, as well as an in-class demo.

### Timeline

- 11/14: Project ideation and group formation
- 11/18: Prepare project proposal in class; Night 7 (project ideation and resource finding) due
- 11/21: In-class consulting; Night 8 (project timeline and concept map) due
- 12/2: Project work day
- 12/5: Draft homework assignment due
- 12/10: Final event- homework assignment, demo, and adventure map due

### Deliverables:

#### *Detailed Project Proposal [Due End of Class, Monday 11/18]*

You and your partner will write up a detailed PROSE (not bulleted list) project proposal including

- A complete description of your project idea and goals.
- A discussion of how this project is connected to course material and other topics including the learning you will need to do to execute on this project.

- Identification of the key quantitative analysis you will perform towards these goals and the tools you will use for this analysis.
- A discussion of the learning objectives of the team members and how this project supports your learning objectives.

*Detailed and Annotated Concept Map and timeline [Due Beginning of Class, Thursday 11/21]*

As a first step after deciding on your project topic, you will map the space in which this project lives and the connections to material we have already covered and to new things you need to learn. You will find good, curated resources to support the learning you need to do over the course of this project. You will create a detailed project timeline.

*Create and Do a QEA-style Overnight Assignment [Draft due 12/5, Final due 12/10]*

This project truly puts your learning journey in the hands of you and your project team, as you all map out how you will achieve your project goals. As part of that learning journey, we would like you to share your resources and the critical concepts and skills you are learning with us and with future QEA cohorts. The mechanism for this sharing will be in the form of an *Overnight Assignment*, which should include:

- Learning Goals of This Assignment
- Readings and Resources
- Textbook-style review of Key Concepts - written or video.
- Exercises
- Comprehensive solutions

These exercises should span from relatively simple exercises aimed at learning a core mathematical execution skill to more detailed, project-directed numerical calculations. The aim is to create an assignment that would take a QEA student at this point in the course about 8 hours to complete.

*Create an in-class demo/interactive exercise [Due Final Event Period, Tuesday, Dec. 10]*

We would like you to create an in-class demo or interactive learning exercise. You don't need to write a full in-class assignment, just a component exercise that reinforces a concept of your choice. Some examples:

- The center of mass exercise of rulers/plates and masses from boats (hands on)
- Neato drives on Parametric Avenue (conceptual board work)
- A heat transfer coefficient experiment from the Solar House project (hands on)
- A portion of the spring-mass cart experiment (board work and hands on)

These exercises generally shouldn't take longer than 15-30 minutes in class. Please discuss your ideas with the teaching team to make sure they are scoped appropriately.

*Map your Adventure [Due Final Event Period, Tuesday, Dec. 10]*

Feedback on your project will be based on the following criteria:

- *Evidence of Learning:* How did the activities you performed to complete the project support your learning objectives?
- *Evidence of Analysis:* How did analysis support the completion of the project?
- *Evidence of Implementation:* How did implementation support your learning and your analysis?

Create a single-page document that highlights your learning, your approach to analysis, and your implementation within this project, by pointing to aspects of the overnight and demo.