RUTGERS
School of Engineering

Department of Electrical and Computer Engineering

332:438:01 Capstone Design – Electronics Spring 2013

Electromagnetic Coil Gun

Final Report

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May 1st, 2013

Abstract

An electromagnetic coil gun with three stages of accelerating coils is designed and built. Conventional projectile propulsion mechanisms include the use of compressed air/spring or explosion which places theoretical limits on the maximum muzzle velocity governed by laws of thermodynamics. The electromagnetic coil gun, on the other hand, explores the use of electromagnetism in accelerating projectiles which offers a much higher theoretical limit on muzzle velocity. One attractive feature of an electromagnetic acceleration system that cannot be provided by conventional propulsion systems is that acceleration can be provided to a projectile in different stages as it moves along a barrel, where conventional propulsion system can only provide one burst of impulse at triggering. We place emphasis on the use of multi-stage accelerating coils in this project.

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1. Introduction

1.1 Overview .

Although there have been many iterations of an electromagnetic coil gun in novels, television, and video games there has not been any introduction of such an apparatus for public consumption outside of "something cool" for entertainment. Given the way electromagnetism is understood today, it is not inherently clear to the team why using the electromagnetic fields generated by coils hasn't seen much use as compared to mechanical counterparts. It is conceptually simple: an electromagnetic field generated by current surging through a coil would act on an object passing through said coil and accelerate it along a track. All applications of this concept would involve some kind of projectile launching that could be used in similar but differentiable ways: moving a subway car, creating an elevator system, high speed weapon launching, nail guns, etc. While information on coil gun design exists in many fragmented pieces in literature and on the world wide web, the team was unable to find a concentration of material that details the process from a first principles perspective to a practical application.

As such, the capstone team proposes the construction of material that fully details the design process of a coil gun through physical models derived by the team in an attempt to understand the governing physics behind coil gun creation. Through the models, we seek to address and explain why coil guns haven't seen widespread use and what needs to change in the environment to facilitate their use.

Using the developed concepts, we hope to generate high strength magnetic fields which will fire a nail through a series of coils and embed it into a material such as wood – a nail gun. Within the project we will explore how much energy is required to do this, what causes inefficiencies in the system, and what improvements can be made in comparison to what is readily available in current markets. We seek to construct the nail gun as a feasibility experiment; that is to say could an electromagnetic nail gun see practical use over other types of nail guns? Why or why not? And then demonstrate the answer with our product. As such, this project is a concert in two parts: one part to develop the theory and the other part to practically apply it and answer the questions posed above. At the end, we hope to have constructed a nail gun that fires with no moving parts other than a switch and a projectile and see whether or not it could compete with practical nail guns today.

1.2 Nail Gun Types & Comparisons.

Nail guns can be divided in two categories depending on their purpose and how they operate. Based on its purpose, a nail gun can be categorized in the following way:

1.2.1 Classification By Use

1.2.2.1 Framing Nailer

The most powerful nail guns available to the public belong in this category and is the most popular among small contractors. As its name implies, framing nailers are typically used to build house frames and any other kind of heavy work on thick/dense wood, therefore it usually works with three inch long nails.

1.2.1.2 Finish Nailers

This type of nailer uses finishing nails. The main difference between finishing nails and regular nails is that finishing nails lack the typical flat circular head that common nails have, allowing it to be driven farther and remain hidden or unseen when the project is done.

1.2.1.3 Brad Nailers

Brad Nailers are the smallest and the least powerful type of this family and use brad nails, which are thin nails with a very small head; they are suited for people who want to work on small projects at home.

1.2.1 Classification by Mode of Operation

In addition, nail guns can be classified according to their method of operation, i.e. how they drive the nail into the wood or any desired surface.

1.2.2.1 Pneumatic Nailer

The pneumatic nail gun uses compressed air to push the hammer that drives the nail. The relatively low cost, light weight and high power makes it the most popular choice among professionals and homeowners. The downside is that it requires an air compressor, which negates the low weight and price of the pneumatic nail gun itself. Furthermore, they also require a hose which reduces mobility and has to be inspected and replaced regularly due to cracks and/or leakages. This is the most popular type of nail gun.

1.2.2.2 Combustion-Drive Nailer

Similarly to how an actual gun works, this nailer uses a combustion engine to drive pistons that cock and release the hammer that drives the nail. Although they are cordless and offer the same power as pneumatic nailers, they are not popular because they are highly dangerous (the firing mechanism involves flammable gases). These guns have begun to fade out of the market due to a lack of popularity.

1.2.2.3 Electric Nail Gun

An electric nail gun uses an electric motor to pull a spring, thereby cocking the hammer and driving the nail out. Cordless versions are available at the expense of power and weight. This is the second most popular type of nail gun.

Based on the information shown above, the team proposes to introduce a fourth type of operating nail gun: the electromagnetic nail gun. The goal is to construct a prototype with similar 'firepower' to current brad nails gun in the market that could fire either brad nails or finishing nails (they are both nails of similar structure that lack the rounded cap of the prototypical 'nail'.

1.3 Why a nail gun over other applications?

The motivation behind this design is that the team feels that this application is the most feasible one that can be implemented in the time allotted and with given resources for the capstone as well as one that could see the most practical use in the real world. Other situations are largely impractical due to the scale of these projects or costly to prototype for, especially the subway system or the elevator where the object moving with magnetic force is a ferromagnetic exterior pulling non-magnetic loads in its interior. It is entirely possible that the nail gun can offer advantages in the marketplace over at least one of the other types of nail guns. While the pneumatic nail gun offers the highest firepower, it lacks mobility since it is constricted by a hose to a heavy air compressor. The opposite is true for electric nail guns; however, the addition of mobility for electric models makes the price very prohibitive for most homeowners who look for moderate firepower at a low price.

The coil nail gun establishes a middle ground between electrical and pneumatic nail guns; it combines some of the better features of the available nail guns in the market: low weight and high power from pneumatic models and the mobility/low maintenance of electric versions.

1.4 Establishing a Benchmark for the Nail Gun

Gun #	MODEL	Type	Nail Gauge	Nail Range	Nail Capacity
1	Hitachi NT50AE2 18-Gauge 5/8-Inch to 2-Inch Brad Nailer	Pneumatic	18	5/8"-2"	100 nails
2	Arrow Fastener 2 in. Electric Brad-Nail Gun	Electric (corded)	18	up to 2"	100 nails
3	Senco PC0947/FP18KIT 18-Gauge Brad Nailer Compressor Combo Kit *	Pneumatic	18	5/8"-2"	110 nails
4	DEWALT DC608K 18-Volt 18-Gauge 2-Inch Brad Nailer Kit	Electric (cordless)	18	5/8"-2"	110 nails
5	WEN 61720 3/4-Inch to 2-Inch 18-Gauge Brad Nailer	Pneumatic	18	3/4"-2"	100 nails

Table 1. Comparison of Best Selling Nail Guns

Gun #	Magazine Type	Operating Pressure	Weight	Dimensions (LxH)	Price	Distance
1	Side Loading	70-120 PSI	2.2 lbs	10"x9-1/4"	$59.00	n/a
2	Side Loading	n/a	10 lbs	15.3"x16.7"	$139.00	2”
3	Side Loading	70-95 PSI	(3 + 20) lbs	13.5"x15"	$179.99	n/a
4	Side Loading	n/a	7.4 lbs	11.5"x (???)	$267.99	2”
5	Side Loading	60-100 PSI	5.4 lbs	11"x13"	$32.24	n/a

Table 2. Continuation of Table 1

Type	Muzzle Velocities [fps]
Pneumatic	~1400
Electric	~1100

Table 3. Approximate muzzle velocities for different gun types.

The above tables shows a comparison of different types of nail guns that are able to handle nails up to 18 gauges and 2” long; this is the starting point for the comparison because this is the closest nail size that the team desires to fire with the coil gun.

In the current environment, the magazine type and its capacity are common across all models, so the only points of comparison are the other parameters. The difference in price of the pneumatic and electric nails guns is apparent; the electric model is significantly more expensive than its pneumatic counterpart. Furthermore, additional features such as cordless connection and /or lower weight drives the prices even higher (for example compare model #2 and #4). Although electric may weigh more, there must be taken into account for a given pneumatic gun the weight of the compressor (a 1 gallon compressor weighs around 20 pounds; not too light).

It is extremely difficult to compare different nail guns given the information available on the internet and from manufacturers since they do not seem to denominate or rate nail guns in any standardized fashion. Air guns like to use PSI as (somewhat) an indicator of the power of the gun but the full pressure is not translatable back to the pressure that the nail itself exerts on the surface; it is only operating pressure of the gun.

Electric guns like to use driving distance as an indicator of strength. This seems like the ideal quantity to compare when using a nail gun so we are not sure why manufacturers wouldn’t list this for a given type of material when discussing pneumatic nail guns. When designing our nail gun, we will design with driving distance as the comparing factor.

In summary:

- Coil nail guns are relatively easy to carry around comparison to other models because they don't need cords/air compressors.
- Potentially cheaper than a compressor/pneumatic gun combo.
- Light weight, no motors or compressors.
- More powerful than electric nail guns at the same price.
- Low maintenance cost, no need to replace the hose or oiling the guns.
- Ideally less kickback than other nail guns; it could be used more continuously over a longer span of time without exhausting the user.

As a benchmark for pneumatic nail guns and electrical nail guns, the team elects to use the most popular pneumatic nailer (Hitachi NT50AE2 18-Gauge 5/8-Inch to 2-Inch Brad Nailer) and the most popular electric nailers: Arrow Fastener 2 in. Electric Brad-Nail Gun(corded), and DEWALT DC608K 18-Volt 18-Gauge 2-Inch Brad Nailer Kit (cordless). Design considerations will be taken into account to try to match these guns by the end of the capstone. Using an amalgamation of the three tables above, the scenario we hope to achieve for our nail gun are the following parameters:

Price	Distance	Velocity	Weight
~$60.00	~2"	~1100 fps	~2.2 lbs

Table 4. Design Criteria

Based on these criteria, we can begin a first iteration for the design of the coil.

1.5 Overall Design of the Coil Gun

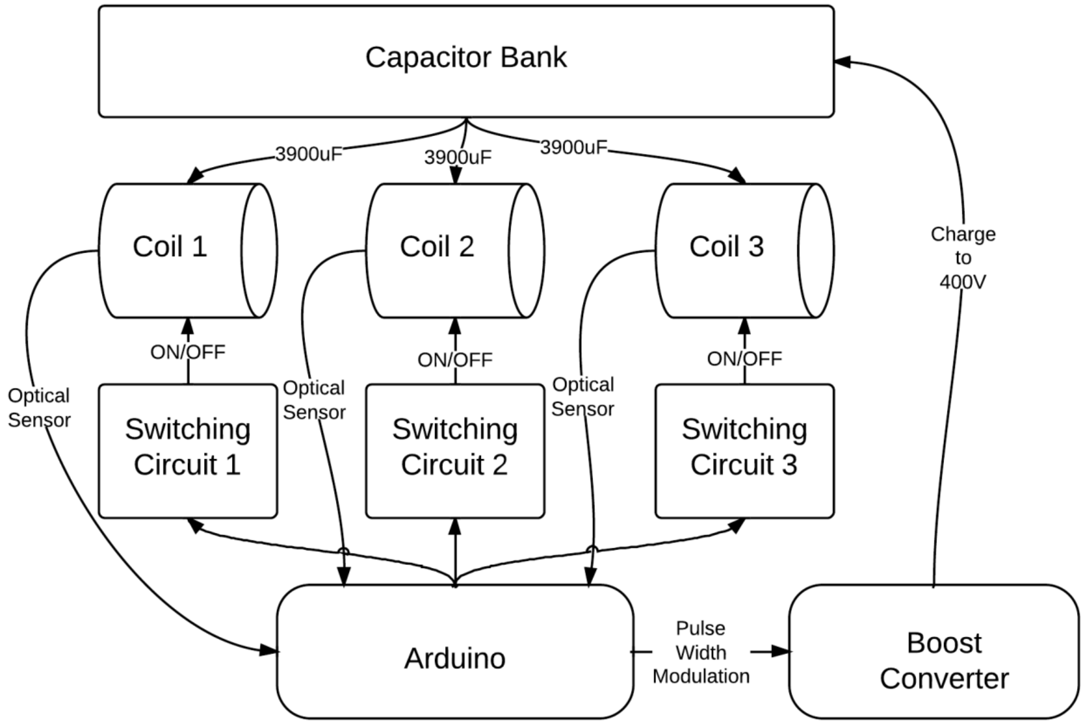


Figure 1. General Block Diagram

Our design begins with one or more capacitors which will be charged up to 200V - 500V by a charging circuit that is powered by a battery acting as a power supply. High voltage capacitors are necessary to get the large current spikes that can generate strong magnetic fields. The stronger the generated field, theoretically, the stronger the force exerted (see Section 2.6). The charging circuit is split into an oscillator circuit connected to a boost converter; a necessary evil to charge the high voltage capacitors in this project. The only alternative to the boost converter and oscillator would be a transformer, which the team did not want to build into the coil gun for safety reasons and a desire to use methods geared towards solid state circuit design for better scalability. Once the capacitors have been charged and a projectile has been loaded into the firing chamber, the device will be ready to fire. The triggering mechanism will be a switch which sends a logic high signal into the Arduino microcontroller. At this point, the Arduino is in control of guiding the projectile by turning the coils on and off sequentially through the use of transistors which will patch the coils to the capacitors. This is a key step in converting the electrical energy into kinetic energy in the projectile. Precise timing of when each coil will turn on and off will have to be experimentally determined to ensure optimized efficiency. Infrared sensors are employed by the Arduino to detect when the projectile's magnetic center has nearly reached the coil's center. If the coil is left on after the projectile has reached this point, it will apply a force opposite of that intended, thus slowing the projectile; the magnetic theory up to this point confirms this (See Section 2.6).

Creation of a viable coil gun will only be possible if there is successful application of the electromagnetic theory. We have determined qualitatively that a coil with a large enough burst of current into its turns of wire can apply a large enough force to a ferromagnetic object to accelerate it out of the coil. By varying the energy put into the system, we are optimistic that we can put enough energy into the projectile to match that used in current nail guns. As an exercise in optimization, we will also test the use of a ferrite rod as a projectile. Though ferrite would not be a suitable replacement for a nail (ferrite is soft and brittle, not ideal for use as a nail and very expensive too!), its high permeability and the fact that it has low conductivity will make it an ideal choice to see the limiting case for the gun.

2. Design

2.1 Projectile Analysis

The first step in the analysis is to analyze and select the proper material for the projectile. The best case design would require that the length of the coil be a minimum size equivalent to the length of the projectile to prevent too much of the "suckback" problem. What follows is a list of design considerations that govern a selection of a suitable material for the object being fired.

2.1.1. Projectile Magnetism

We have to consider that magnetism means memory (think hard disk); there is a hysteresis at play.

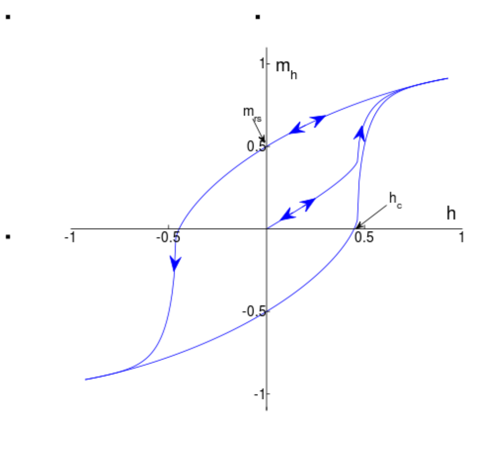


Figure 2.Hysteresis Diagram

Consider the hysteresis curve shown. The curve is of magnetization (y) versus applied magnetic field (x). What is desired for the projectile is to have a very thin hysteresis curve;

i.e. it is able to magnetize and de magnetize quickly under an applied field. Materials that are not capable of this are used in magnetic memory devices or permanent magnets; things we do not want for this project else the projectile become useless. A hysteresis implies a waste of energy; to demagnetize an object it requires (sometimes large) fields applied in the opposite direction to realign the poles inside the material. This doubles as a reason to why a purely magnetic gun may have problems accelerating an object: it would have to magnetize and demagnetize it. Although a magnetic gun would be conceivably more efficient, it is a hurdle to overcome and thus the coil gun remains our focus.

2.1.2 Projectile Weight/Mass/Density

Lower mass projectiles will require less energy to accelerate to a higher velocity (work kinetic energy theorem). But if the object is too small then it will saturate too quickly; meaning applied field cannot increase magnetization further so the flux density increases much more slowly. This is a trade off in design. If the mass is too heavy, though, it risks not being magnetized at all. So the projectile acts as if it is carrying additional weight since the left part of it may be magnetized and pulling its unmagnetized right side across some distance before stopping; it's the same as driving your car with only the front tires while the back ones are locked from rotating.

2.1.3 Projectile Dimensions

Have to be very close to the coil to maximize flux linkage, else the field energy be stored in the air around the projectile rather than in the projectile itself. For an air core design like the one we are considering, this means the diameter needs to be as close as possible to the diameter of the coil but not exactly equal to it (to reduce frictional forces slowing down the projectile). Recall from basic dynamics that a projectile, when fired, should have a length equal to three times its diameter or more. This is one constraint on the projectile length; the second constraint is that the length of the projectile should not be more than the length of the coil. If the projectile is longer than the coil then there are portions of the projectile not being magnetized at all or being pulled back towards the coil while the bulk of it is still passing through and thus leads back to the problem outlined above. One way to reduce a loss of flux linkage is to design the inside with a core, but that could raise costs so for now we will only consider the air core.

2.1.4 Projectile Material

The material of the projectile needs to be ferromagnetic. There's no debate on this issue; diamagnetic is useless due to negative magnetic susceptibility (negative susceptibility means the the magnetic field is weakened by the magnetization of the material $B = u_0(1+x)H$); paramagnetic has small magnetic susceptibility. Ferromagnetic has large magnetic susceptibility so it is ideal for the application we're undertaking. The force exerted on an object is proportional to the square of its magnetic permeability; taking this into consideration means that permeability may be the most important quality for selection of a projectile. Most nails are made from either iron or steel, both materials which have very high magnetic permeabilities so they are suitable to be fired by the coil gun.

2.1.5 Projectile Conductivity

The material <u>should not be conductive</u>. And if it is conductive it needs to have as low a conductivity as possible. Conductive materials are subject to eddy currents, which will act as a form of resistance when attempting to fire the projectile. The induced currents on the surface of the projectile will generate its own magnetic fields that tend to oppose the magnetic field acting on the projectile (Lenz's Law) induced by the coil , resulting in the transformation of kinetic energy to a much more wasteful energy: heat.

2.1.6 Summary

In summary then, we want a non conducting ferromagnetic material whose length is slightly less than the length of the coil and whose diameter is 3 times smaller than its length and yet very close to the diameter of the coil. In other words, the problem of selecting a projectile is the same as the problem for selecting the material for the core of a transformer or magnetic circuit!

The materials under consideration are thusly: Soft Iron, Laminated silicon steel, Carbonyl iron, ferrite, or a vitreous metal. The best options are likely soft iron (high concentrated magnetic field, low hysteresis loss, unfortunate high conduction and thus a loss to eddy currents) or ferrite material (low eddy current loss, low hysteresis, medium magnetic field permeability). The best option would be a soft ferrite rod since they are popular to use in transformers and Ferrite rods are already prepackaged and sold at some set diameters; also the low conductivity and decent permeability are the best bet for the design.

2.2 Selecting Capacitors

A Current VS. Time relation is plotted in the figures below for two different types of capacitors. Although the rational choice is to choose a capacitor with a higher voltage because of the Energy relationship ($W= \frac{1}{2} CV^2$), there is an additional reason to give more importance to the voltage rating rather than the capacitance for the purpose of this project.

While it is true that inductance limits how fast the current can rise to its maximum, the coil of the nail gun is a fixed component of the design and there is not much room to tweak with it. Resistances throughout the circuitry also limit the current output; this limits the final velocity of the projectile since the integral of the current is proportional to the force and acceleration inside the coil. In other words, one has to choose a capacitor that maximizes the area under this curve (up to a time cutoff when the projectile passes through the middle of the tube).

To reach ~35A of current at the peak, we would need V_{Cmax}/R ratio to be roughly 11.4.

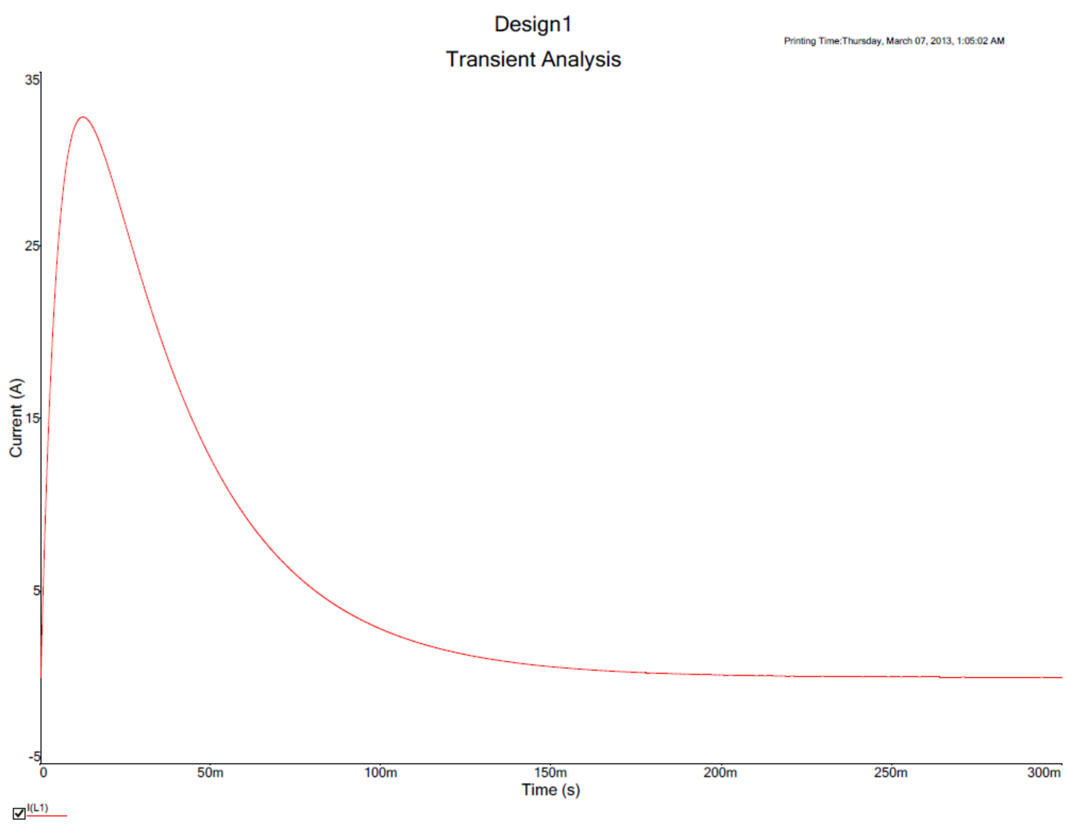


Figure 3: Transient Response of RLC Circuit with 35V, 3900uF Capacitor

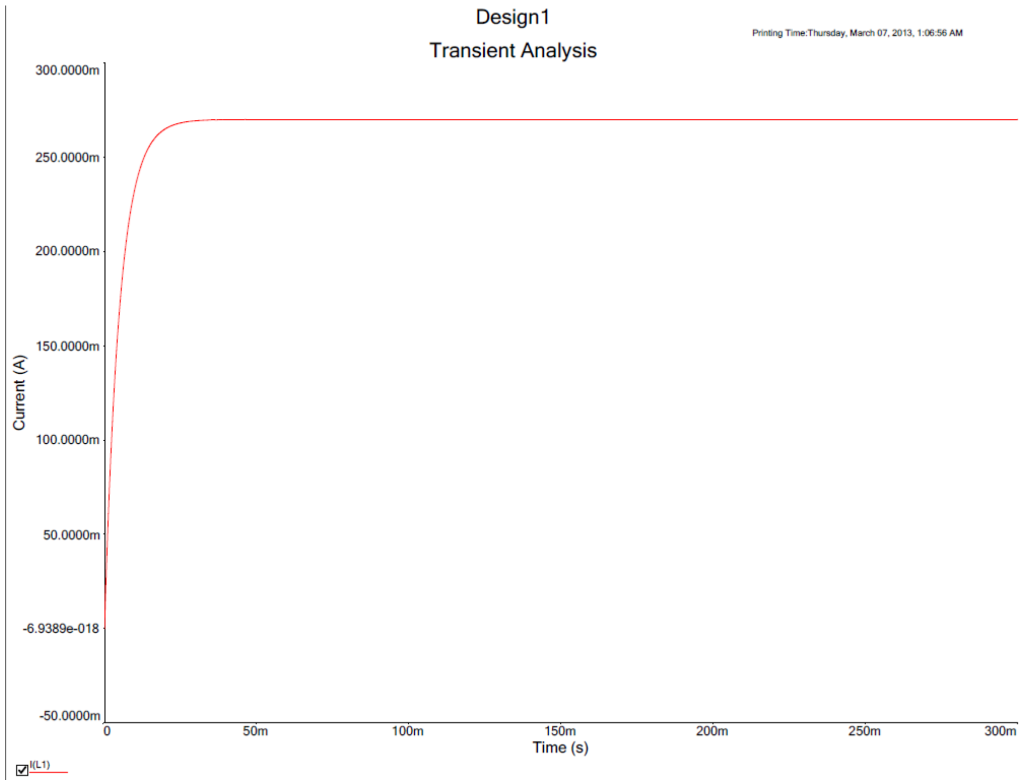


Figure 4: Transient Analaysis with 2.7V, 3000F Capacitor

In summary, although we considered an ultracapacitor for this apparatus for its high capacitance and low series resistance (fast charging cycle and long energy storage), they are not the best option because the low voltage rating limits the output current, so more conventional capacitors with high voltage ratings are better suited for this project. The constant output current would cause problems with suckback at any rate – the only advantage this capacitor offers is as an energy storage device; effectively an expensive battery.

So the design goal now shifts to maximizing current peaks for a given voltage. The RC time constant and current peaks are modifiable through the introduction of discrete components, so the best option is to pick a high voltage that can successfully drive a single stage. We want it ideally to be fast charging and fast discharging – so take the smallest series resistance for a high voltage rating and as high a capacitance as possible for good energy storage. After some running around, it was determined that the best capacitor to use would be a 3900uF, 400V capacitor supplied by Nippon Chemi-Con. It has the high voltage rating we desire with a appreciably high capacitance at a substantially low cost for the economy of scale; $30 for three capacitors averaging out to $10 per stage to energize the projectile. The description is as follows:

Capacitance	3900uF
Voltage	400V
ESR	39mOhm
Height	5.125"
Diameter	2.5"

Table 5: Capacitor Characteristics

An aluminum electrolytic capacitor is just fine for the application we're undertaking since electrolytic capacitors have the second highest capacitance rating below super/ultracapacitors and can achieve much higher voltages for the trade off. Ceramic and film capacitors can achieve higher voltages at lower capacitances, so it is a trade off in design. Aluminum electrolytic in this regard also averaged out to be more cost effective for the given ratings, so everything works out. If we were considering kV ratings, then we would need plastic film or ceramic capacitors but not so much here.

2.3 Selecting the Appropriate Size for the Coil

The length of the coil is fixed earlier by the length of the projectile. But there are still other parameters at play to consider here; i.e. the turns ratio of the coil and the number of layers. Each coil requires at least one layer, and there is no reason as of this writing to leave a layer unfinished. The gauge of the copper wire determines the number of turns we can fit on a single layer; the problem almost reduces to one of just selecting the number of layers.

2.3.1 Design Tradeoff

The coil's magnetic field is directly proportional to the numbers of turns and coil current. Using thinner wires allow us to make more turns, resulting in a higher magnetic field; on the other hand, the resistance is increased as we increase the number of loops, resulting in a lower current. Going further, the inductance is increased by more than the resistance is by increasing the number of turns, so there's another trade off there. A large inductance can mean a nastier voltage spike when the inductor can't hold it's charged energy any longer. But on the other hand, the larger inductance is analogous to the larger magnetic field. So, in effect, a small wire is very desirable in this design. But how small can we go?

Another concern for the design is that we have to use a wire thick enough so that it is able to handle the large amount of currents and/or power dissipation, in other words, the wire has to be capable of withstanding the high temperatures that comes with operating at relatively high current/voltage.

2.3.2 Preece's Equation

W.H Preece studied the melting current in wires and developed the following relationship:

$$\boldsymbol{I = A * D^{1.5}}$$

where A is a constant and D is the diameter of the wire in inches.

If the wire is made of copper, this equation simplifies to:

$$\boldsymbol{I = 12,277 * A\text{^}(0.75)}$$

where I is the fusing current in Amps and A is the cross sectional area of the wire in square inches.

2.3.3 Oderdonk's equation

I.M Onderdonk also developed an equation that predicts the time it takes to melt a wire given a certain current:

$$I_{Fuse} = A\sqrt{\frac{(\log(T_{melt} - T_{ambient}) + 1)}{Time * 33}}$$

Where:

- T_{melt} = Melting temperature of wire in Celsius
- $T_{ambient}$ = Ambient temperature in Celsius
- Time = Melting time in seconds
- I_{FUSE} = Fusing current in amperes
- Area = Wire area in circular mils
- Circular Mils = the diameter of the wire in thousandths of an inch (mils) squared. That is, it is the area of a circle 0.001" in diameter.

For Copper this equation reduces to:

$$I = 0.188 * \frac{A}{t^5}$$

where A is in square mils and t is in seconds.

2.3.4 Summary

Based on Oderdonk's Equation and Preece's Equation, the choice of gauge wire is entirely up to the the team. The current that will flow through the windings will flow for extremely short periods of time and therefore won't be long enough to melt the copper wire. A calculation isn't necessary; the time it would take to melt the wire varies with the fifth root of the current! The current could be 100 amperes; at the cross sectional area of a given wire for the time the pulse will be in the system (milliseconds), the wire will be just fine. Although the highest gauge wire would be desirable in this project since it is the thinnest, it is incredibly difficult to wind by hand. As such, the team elected to use 20 AWG wire for moderate to low thickness and improve the winding scheme.

Aside: An alternative to this analysis would be to apply more fundamental thermodynamic principles and use Stefan-Boltzmann Law to calculate the amount of heat radiated to the surroundings by the copper wire through the insulation. This also shows that the wire gauge is an arbitrary design factor; it only influences the resistance of the circuit.

2.4 Boost Converter Design

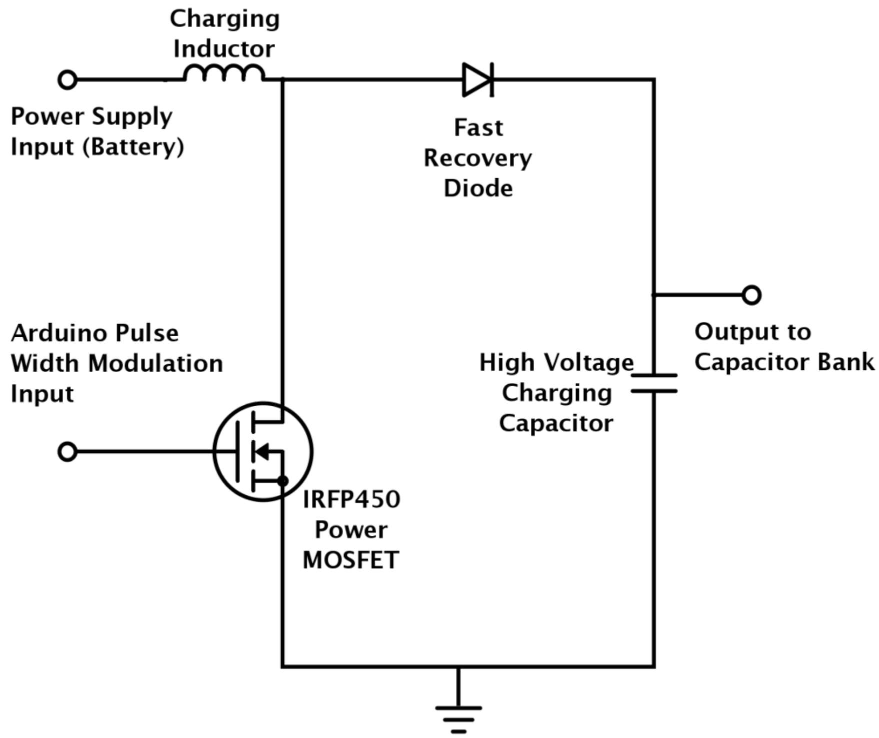


Figure 6: Simple Boost Converter

When the switch (the power MOSFET) is closed, the circuit will be fully completed with the supply->inductor->diode->capacitor forming a series circuit to ground. The DC current from the supply will store energy in the magnetic field of the inductor and be unable to pass it. When the switch is closed, the inductor will begin to oppose the change in the flow of current since the circuit resistance and voltage drops have begun to realign. The terminal of the inductor connected to the battery will become negative with respect to the terminal connected to the diode, effectively locking the two in series as a much larger voltage which will proceed to dump their combined energy into the capacitor as a transient waveform. The diode will block the capacitor from discharging itself through the inductor, thus absorbing all the energy and allowing its voltage to steadily climb.

Mathematically, this is straightforward to model as well.

The voltage across the inductor is given by:

$$v_L = L\frac{di}{dt}$$

Let's assume **continuous mode** of operation (meaning that the current through the inductor never falls to zero).

When the circuit is completed, the DC current supplied by the battery can't pass through the inductor. As such, all of the battery voltage must appear across it such that:

$$V_{DC} = L\frac{di_L}{dt} = L\frac{(I_2 - I_1)}{(\mathrm{t_{on}} - 0)} = L\frac{\Delta I}{t_{on}}$$

I_1 is the initial current through the inductor prior to closing the switch (if any); I_2 is the current the inductor gains after closing the switch. t_{on} is the amount of time the switch remains open.

The maximum amount of current that builds up through the inductor is proportional to the time t_{on} that the switch remains open and varies with the reciprocal of the inductance.

When the switch closes, the inductor dumps some of its energy into the capacitor. The change in voltage is given by:

$$V_{DC} - V_C = -L\frac{\Delta I}{t_{off}}$$

This should make sense; the inductor is only losing energy. We obtain $-\Delta I$ because we assume a linear fall from I_2 back to I_1. This may not always be the case.

These equations can be rewritten in various forms:

$$t_{on} = \frac{L\Delta I}{V_{DC}}$$

$$t_{off} = -\frac{L\Delta I}{V_{DC} - V_C}$$

$$\Delta I = \frac{V_{DC} t_{on}}{L} = \frac{(V_{DC} - V_C) t_{off}}{L}$$

At this point, the duty cycle D is typically introduced:

$$t_{on} = DT; t_{off} = (1 - D)T$$

And we get:

$$V_C = V_{DC} * \frac{t_{on}}{t_{off}} = \frac{V_{DC}}{1 - D}$$

The period can be obtained:

$$T = t_{on} + t_{off} = \frac{\Delta I L V_C}{V_{DC}(V_C - V_{DC})}$$

Back solving for the maximum ripple current:

$$\Delta I = \frac{V_{DC} D}{f * L}$$

Where $f = \frac{1}{T}$.

The change in capacitor voltage is similar to the change in inductor current:

$$\Delta V_C = \frac{I_0 T_{on}}{C}$$

I_0 is the current through the capacitor (on average… rough estimate here).

But we have enough from the previous page and a half to remove most of these terms:

$$\Delta V_c = \frac{I_0 D}{f * C}$$

2.4.1 Selecting Components for the Converter

It is possible that about 800mA~2A of current can be pulled from a battery to a simple boost converter connected to the capacitor bank. The frequency for the PWM is fixed by the Arduino at 15384 Hz. The duty cycle is also fixed; the Arduino is high for 50 μs and low for 15μs for a duty cycle of:

$$D = \frac{50}{65} = \frac{10}{13}$$

And it is known that the change in the capacitor voltage to be ~400V to match the capacitors selected for this project. We can pick a reasonable estimate for the change in the capacitor voltage for the short period, say, around 0.5V.

Calculating a value for C, then:

$$C = 0.8 * \frac{\frac{10}{13}}{15384 * 0.5} = 80\mu F$$

It was decided to instead to use a $100\mu F$ capacitor since these are of a more abundant type; this one would be rated at around 450V. Although the capacitor would be charged up to 400V, the extra 50V is a buffer to prevent damaging the capacitor from overvoltage. This means that:

$$\Delta V_C = 0.8m * \frac{\frac{10}{13}}{15384 * 100\mu} = 0.4V$$

2.4.2 Estimations for the Converter

The charging time can be guesstimated by saying:

$$P_{DC} = V_{DC} I_{DC} = 12 * 0.8 = 9.6W$$

$$E_C = \left(\frac{1}{2}\right) CV^2 = \left(\frac{1}{2}\right)(100\mu)(450)^2 = 10.125J$$

$$t = \frac{10.125}{9.6} = 1.05s$$

Although in actuality, this is far from the case. The coil gun will never charge this quickly. Why? Because this capacitor is not the same capacitor as the ones in the bank. The battery voltage weakens as it loses energy and the capacitor may not be able to change its voltage as quickly as the theory assumes.

With a strong voltage it is very easy to charge the capacitors up to 200, 300 volts in about one second. But the 400V charge takes much larger since we are charging the big caps. So:

$$t_{bank} = 39 * t = 40.95s$$

Where the 39 comes from the fact that the larger capacitors are 3900μF, so the charging time is just 3900/100 times the time it takes to charge the small capacitor. In practice, it was found that around 30 seconds is the charging time necessary for the larger caps; this could be because of larger current draws from the battery than the lower bound estimate used here.

2.5 Magnetic Field of a Finite Solenoid

Consider a solenoid oriented such that the axial direction is along the z.

The field at an arbitrary point 'r' from the center of solenoid is given most generally by:

$$d\vec{H} = \left(\frac{1}{4\pi}\right)\frac{(Id\vec{s}x\vec{r})}{|\vec{r}|^3}$$

- $d\vec{H}$ is the differential magnetic field intensity
- I is the magnitude of current passing through a cross sectional area of wire
- $d\vec{s}$ is the differential vector in the direction of the current element through which I is flowing
- $Id\vec{s}$ taken together is referred to as the "current element"
- $\vec{r}$ is the vector directed from the center of the current element to the field point where $d\vec{H}$ is to be measured

We can establish $d\vec{s}$:

$$d\vec{s} = ds(-\sin(\theta)\,x + \cos(\theta)\,y)$$

This is a segment of the wire that is perpendicular to some vector that is directed from the origin to a point on the wire (in the first octant, first quadrant). $d\vec{s}$ is sufficiently short such that it can be assumed to be diagonally straight.

The angle θ is the angle with the x axis relative to the y axis.

Moving on to $\vec{r}$. This vector is the direction from the current element $d\vec{s}$ to the arbitrary field point. The arbitrary field point vector $\vec{r}$ can be written most generally as:

$$\vec{r} = \mathrm{r_x x + r_y y + r_z z}$$

Dealing with the cross product:

$$d\vec{s}x\vec{r} = \begin{vmatrix} \mathrm{x} & \mathrm{y} & \mathrm{z} \\ -\,\mathrm{dssin}(\theta) & \mathrm{dscos}(\theta) & 0 \\ \mathrm{r_x} & \mathrm{r_y} & \mathrm{r_z} \end{vmatrix}$$

$$= \mathrm{dssin}(\theta)\begin{vmatrix} \mathrm{y} & \mathrm{z} \\ \mathrm{r_y} & \mathrm{r_z} \end{vmatrix} + \mathrm{dscos}(\theta)\begin{vmatrix} \mathrm{x} & \mathrm{z} \\ \mathrm{r_x} & \mathrm{r_z} \end{vmatrix}$$

$$= \mathrm{dssin}(\theta)\left[\mathrm{r_z y - r_y z}\right] + \mathrm{dscos}(\theta)\,[\mathrm{r_z x - r_x z}]$$

$$= \left(\mathrm{r_z dscos}(\theta)\right)\mathrm{x} + \left(\mathrm{r_z dssin}(\theta)\right)\mathrm{y} - \mathrm{ds}\left(\mathrm{r_y}\sin(\theta) + \mathrm{r_x}\cos(\theta)\right)\mathrm{z}$$

The next step is how to quantify the vector $\vec{r}$ and the expression ds in terms of more definable quantities.

The vector $\vec{r}$ is the distance from the current element to the field point. This is not a convenient distance to be performing a calculation from given that we have multiple coils of wire wrapping around; we would need to move. It is instead easier to quantify the vector as it relates to the geometric center of the coil by writing it as the difference between a vector, $\vec{A}$ extending from the center of the coil to the field point, with the vector extending from the geometric center of the coil to the current element,$\vec{B}$.

$$\vec{r} = \vec{A} - \vec{B}$$

$$\vec{A} = A_{radial}[\cos(\theta_2)\,\mathrm{x} + \sin(\theta_2)\,\mathrm{y}] + \mathrm{A_z z}$$

This is an arbitrary field point. To most generally define the field point, we choose an arbitrary angle θ_2 with radial arm A_{radial} and axial height A_z.

$$\vec{B} = R_B(\cos(\theta)\,x + \sin(\theta)\,y) + z_B z$$

This is the distance from the center of the coil to our current element a height z_B above or below the center.

$$\vec{r} = [A_{radial}\cos(\theta_2) - R_B cos(\theta)]x + [A_{radial}\sin(\theta_2) - R_B sin(\theta)]y + [A_z - z_B]z$$

And this means that the cross product is now:

$$\begin{aligned} d\vec{s}x\vec{r} = {} & [(A_z - z_B)\mathrm{ds}\cos(\theta)]\mathrm{x} + [(A_z - z_B)\mathrm{ds}\sin(\theta)]\mathrm{y} \\ & - \mathrm{ds}\big[\big(A_{radial}\sin(\theta_2) - R_B sin(\theta)\big)\sin(\theta) + (A_{radial}\cos(\theta_2) \\ & - R_B cos(\theta))\cos(\theta)\big]\mathrm{z} \end{aligned}$$

In general, ds is angularly dependent. It is not satisfactory to leave it written in such a way; if we consider the geometry we used to derive $d\vec{s}$ then we can replace ds in terms of a differential angle.

A small change in the angle, dθ, is presented by:

$$\tan(\mathrm{d}\theta) \approx \mathrm{d}\theta \approx \frac{\mathrm{ds}}{\mathrm{R_{Lth}}}$$

$$ds = R_{Lth} d\theta$$

Where R_{Lth} is the radial distance from the center of the coil to the center of the Lth layer of wire windings. What is meant by "Layer" in this regard is that the coil can be covered with a finite number of turns before overlapping. So if a coil can fit 50 turns without overlapping, we call these first 50 turns the first "Layer". To add an additional 50 turns (for a total of 100) we would need to wrap the wire over itself; these 100 turns of wire constitute two "Layers" on the coil.

The distance from the center of the coil to the center of the wire in the Lth layer is:

$$R_{Lth} = (Coil\ Radius) + \ (L\ layers * Wire\ diameter) - \frac{Wire\ Diameter}{2}$$

$$= R + Ld - \frac{d}{2}$$

Where we subtract half of the wire diameter because we assume L layers of wire but to get to the center of the Lth wire we have to go backwards by d/2.

It is now convenient to realize at this stage that $R_B = R_{L_{th}}$; they are defined in exactly the same way.

After this digression, $\vec{r}$ is now expressable as:

$$\vec{r} = [A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]x + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]y + [A_z - z_B]z$$

And the cross product becomes:

$$d\vec{s}x\vec{r} = [(A_z - z_B)R_{Lth}\cos(\theta)d\theta]\mathrm{x} + [(A_z - z_B)R_{Lth}\sin(\theta)d\theta]\mathrm{y}$$
$$- R_{Lth}d\theta\big[\big(A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)\big)\sin(\theta) + (A_{radial}\cos(\theta_2)$$
$$- R_{Lth}cos(\theta))\cos(\theta)\big]\mathrm{z}$$

And so the expression complicates itself further. The next problem is quantifying z_B. z_B came from the vector that is directed from the center of the coil to the current element, so z_B is height above or below the center of the coil to a given turn of a coil at a given layer. We can manipulate this.

We can relate the height of the coil, h, to the number of turns by saying:

$$\mathrm{h} = \mathrm{T_{turns}} * \mathrm{d}$$

How does this work? If we assume the turns of the wire are sufficiently close such that there is no gap and also assume that the turns wrap around the entire solenoid then a scalar multiple of the wire diameter = the height of the object.

To find the coordinate of the center of a given turn of wire then we say:

$$\mathrm{z_B} = (\text{Height of Turn}) - \frac{\text{Coil Height}}{2} - \left(\frac{\text{Wire Diameter}}{2}\right)$$

$$z_B = Td - \frac{d}{2} - \frac{h}{2}$$

Why does this work? We are measuring the height from the center of the coil. So 'Td' is the height of the Tth turn measured from **the bottom of the coil**. To shift the origin upwards to the center, it is necessary to subtract half of the coil height. So if the coil has 50 turns, the z coordinate of the 30th turn relative to the center of the coil is 30 – 50/2 = 30-25 = 5 turns above the center.

We subtract half the wire diameter to shift the z coordinate to the center of the wire.

So now we have quantified all the terms in the expression in terms of measurable physical quantities and design parameters that are meaningful to us.

Evaluating the magnitude of $\vec{r}$:

$$|\vec{r}| = \sqrt{[A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2}$$

$$|\vec{r}|^3 = ([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}$$

The expression is too long to write out on a single numerator, so splitting the components nets us:

$$dH_x = \frac{I}{4\pi}\frac{[(A_z - z_B)R_{Lth}\cos(\theta)d\theta]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

$$dH_y = \frac{I}{4\pi}\frac{[(A_z - z_B)R_{Lth}\sin(\theta)d\theta]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

$$dH_z = \frac{I}{4\pi}\frac{R_{Lth}d\theta\left[\left(A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)\right)\sin(\theta) + (A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta))\cos(\theta)\right]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

These expressions contain terms that include layers and turns, so they need to be repeatedly summed:

$$dH_x = \sum_{i=1}^{L_{layers}} \sum_{j=1}^{T_{turns}} \frac{I}{4\pi} \frac{[(A_z - z_B)R_{Lth}\cos(\theta)d\theta]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

$$dH_y = \sum_{i=1}^{L_{layers}} \sum_{j=1}^{T_{turns}} \frac{I}{4\pi} \frac{[(A_z - z_B)R_{Lth}\sin(\theta)d\theta]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

$$dH_z = \sum_{i=1}^{L_{layers}} \sum_{j=1}^{T_{turns}} \frac{I}{4\pi} \frac{R_{Lth}d\theta\left[\left(A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)\right)\sin(\theta) + (A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta))\cos(\theta)\right]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

Integration is likely impossible; if anyone thinks they can do this then they should stop eating dog food.

The expression is assuredly complicated. Obtaining a picture of the field would be sufficient verification for whether or not it is correct. We'll consider a case where we sweep the variable A_z through the center of the coil to some height above it and some height below it. We'll fix the field at the center and then move outward. By this approximation, the field equations reduce down to a simple case where dH_x is now the radial component of the magnetic field intensity and dH_y is zero. Obtaining plots for these expressions using MATLAB yields the following:

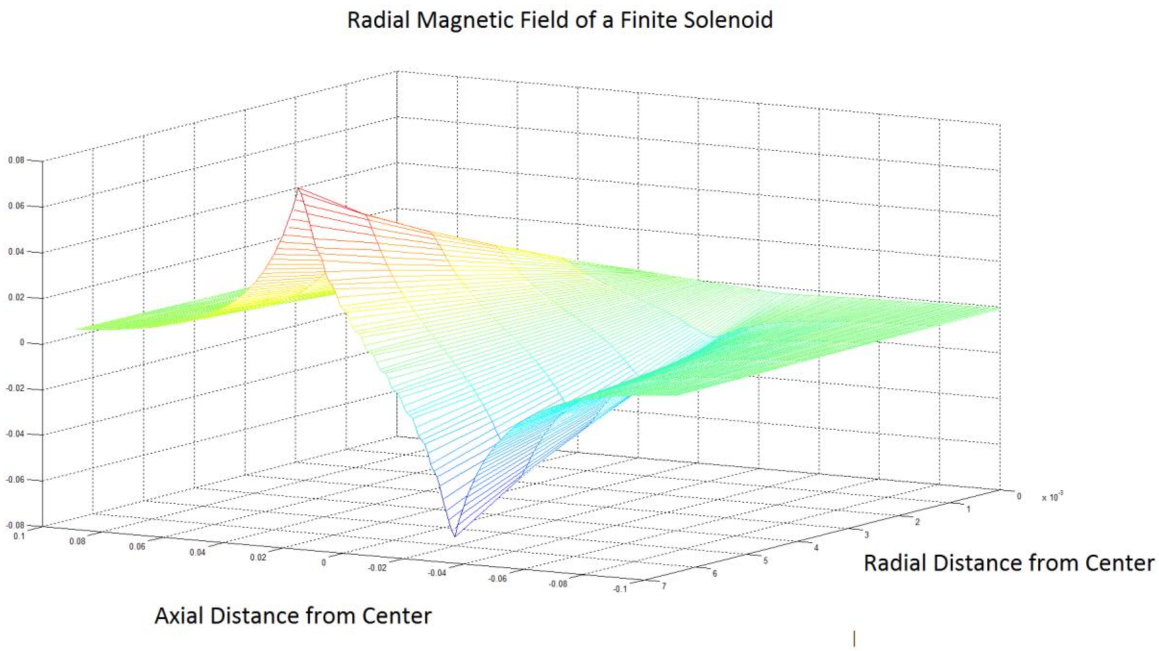


Figure 6: $H_r(A_z, A_{Radial})$

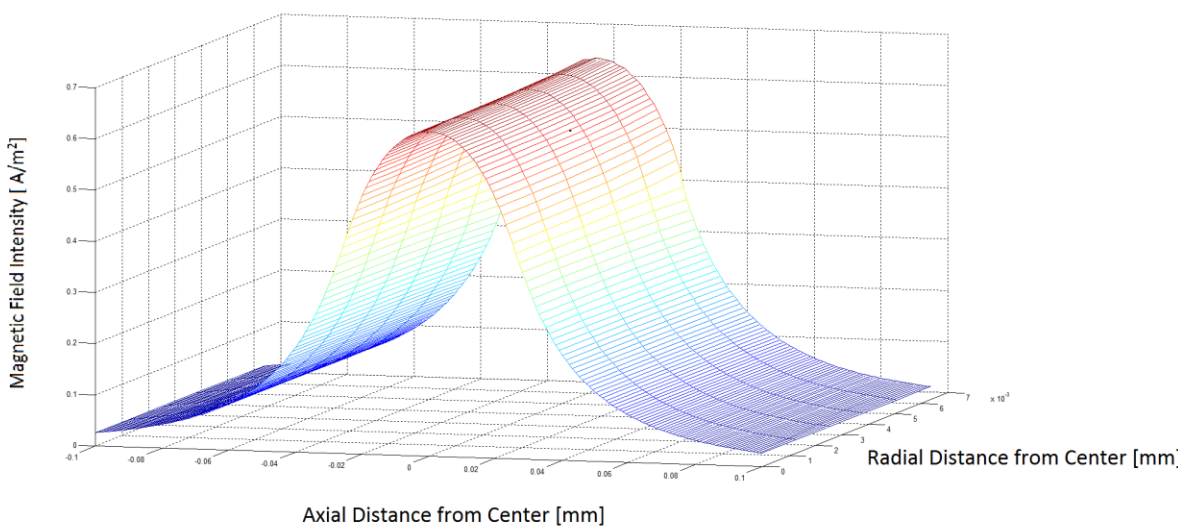


Figure 7: $H_z(A_z, A_{radial})$

The radial field plot looks like what one would expect; a strong spike at the two faces of the coil and a tendency towards zero inside the coil. There is nasty asymmetric behavior at $A_{Radial} \approx R$; this is because the approximation is getting closer to the inside of the windings. It is interesting to note that the radial field becomes more erratic the closer the calculation moves towards the edge. This could imply that there are stronger forces exerted on an object near the walls of the center than there are forces that act on the center of the object; strange indeed.

The axial field plot looks somewhat strange; the field should be approximately linear inside the coil but instead the field shape appears almost Gaussian in nature. Of course, this actually is exactly what should happen. If the coil were infinitely long, this shape would come closer and closer to approximating a square wave; exactly 0 outside of the coil but a near constant value inside. But if the length is not much much greater than the radius of the coil, the shape spreads out as seen in the above plot.

From the magnetic field equation, it is possible to move on to consider the Lorentz force acting on a particle moving through it and then consider the effects on velocity. The Lorentz force equation will take time to obtain and the force on an object moving in the axial direction will likely depend on both the axial field and the radial field so neglecting one in favor of the other is not the way to move forward.

2.6 Magnetic Force due to a Finite Solenoid

Now that we have sufficiently derived the magnetic field expression for the solenoid in a previous section, it is time to focus on obtaining the force expression over the coil.

Starting from the Lorentz Force equation:

$$\vec{F} = q\vec{v} \times \vec{B}$$

And Maxwell's Equations:

$$\nabla x\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla x\vec{B} = \mu \left[\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}\right]$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

$$\nabla \cdot \vec{B} = 0$$

It is not immediately apparent how to translate Maxwell's equations over to the Lorentz force equation. But this is because the Lorentz force as written is describing the force acting at a point; if we consider the force acting per unit volume then it would be possible to introduce the current density term:

$$\vec{F}_{vol} = \vec{J} \times \vec{B}$$

Substituting in for the density:

$$\vec{F}_{vol} = \left[\frac{1}{\mu}\nabla x\vec{B} - \epsilon\frac{\partial\vec{E}}{\partial t}\right] \times \vec{B}$$

We can consider the electric field to be negligible in pursuit of the solution (at least as far as the force is concerned) so that leaves us with:

$$\vec{F}_{vol} = \frac{1}{\mu_0}\left[\nabla x\vec{B}\right] \times \vec{B}$$

Applying the vector identity:

$$\vec{V} \times \left(\nabla x\vec{F}\right) = \nabla_F\left(\vec{V} \cdot \vec{F}\right) - (\nabla \cdot \vec{V})\vec{F}$$

The equation simplified to:

$$\frac{1}{\mu}\left[\nabla x\vec{B}\right] \times \vec{B} = \frac{1}{\mu_0}\left[\nabla_B\left(\vec{B} \cdot \vec{B}\right) - \left(\nabla \cdot \vec{B}\right)\vec{B}\right]$$

$$= \frac{1}{\mu_0}\left[\nabla_B\left(\vec{B} \cdot \vec{B}\right) - \left(\nabla \cdot \vec{B}\right)\vec{B}\right]$$

$$= \frac{1}{\mu_0}\left[\nabla\left|\vec{B}\right|^2 - \left(\nabla \cdot \vec{B}\right)\vec{B}\right]$$

The magnetic flux density is expressable as:

$$\vec{B} = B_x\boldsymbol{x} + B_y\boldsymbol{y} + B_z\boldsymbol{z}$$

Its' magnitude is:

$$\left|\vec{B}\right| = \sqrt{B_x^2 + B_y^2 + B_z^2} \rightarrow \left|\vec{B}\right|^2 = B_x^2 + B_y^2 + B_z^2$$

The gradient of the above is:

$$\nabla\left|\vec{B}\right|^2 = \frac{\partial\nabla\left|\vec{B}\right|^2}{\partial x}\boldsymbol{x} + \frac{\partial\nabla\left|\vec{B}\right|^2}{\partial y}\boldsymbol{y} + \frac{\partial\nabla\left|\vec{B}\right|^2}{\partial z}\boldsymbol{z}$$

Expand it:

$$\nabla\left|\vec{B}\right|^2 = \frac{\partial}{\partial x}[B_x^2 + B_y^2 + B_z^2]\boldsymbol{x} + \frac{\partial}{\partial y}[B_x^2 + B_y^2 + B_z^2]\boldsymbol{y} + \frac{\partial}{\partial z}[B_x^2 + B_y^2 + B_z^2]\boldsymbol{z}$$

The divergence of $\vec{B}$ is:

$$\nabla \cdot \vec{B} = 0$$

This is shown by Gauss's law for magnetism.

Expanding the Lorentz force expression with the above:

$$= \frac{1}{\mu_0}\left[\frac{\partial}{\partial x}[B_x^2 + B_y^2 + B_z^2]\boldsymbol{x} + \frac{\partial}{\partial y}[B_x^2 + B_y^2 + B_z^2]\boldsymbol{y} + \frac{\partial}{\partial z}[B_x^2 + B_y^2 + B_z^2]\boldsymbol{z}\right]$$

Previously, the field components were discovered to be:

$$dH_x = \frac{I}{4\pi}\frac{[(A_z - z_B)R_{Lth}\cos(\theta)d\theta]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

$$dH_y = \frac{I}{4\pi}\frac{[(A_z - z_B)R_{Lth}\sin(\theta)d\theta]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

$$dH_z = \frac{I}{4\pi}\frac{R_{Lth}d\theta\left[\left(A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)\right)\sin(\theta) + (A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta))\cos(\theta)\right]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

Where these are meant to be summed repeatedly over many turns and many layers.

Transforming to B field:

$$dB_x = \frac{\mu I}{4\pi} \frac{[(A_z - z_B)R_{Lth}\cos(\theta)d\theta]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

$$dB_y = \frac{\mu I}{4\pi} \frac{[(A_z - z_B)R_{Lth}\sin(\theta)d\theta]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

$$dB_z = \frac{\mu I}{4\pi} \frac{R_{Lth}d\theta\left[\left(A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)\right)\sin(\theta) + (A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta))\cos(\theta)\right]}{([A_{radial}\cos(\theta_2) - R_{Lth}cos(\theta)]^2 + [A_{radial}\sin(\theta_2) - R_{Lth}sin(\theta)]^2 + [A_z - z_B]^2)^{\frac{3}{2}}}$$

It is now necessary to recognize the appropriate x,y,z variables in the magnetic field equations in order to affect a solution. However, a simplification be made. We can assume that we are constrained to a line through the center of the coil such that $y = 0$ and that $x = r$. To make $y = 0$, we set $\theta_2 = 0$. Summing over the entire circle defined by θ would mean there is always a way to cancel competing B_y contributions. In addition, the variable A_z is defined as the distance above the origin in the axial direction; this is the same as 'z'. A_{radial} is defined as the radial arm's distance from the center; this is 'r'. 'x' becomes 'r' in this configuration, then.

$$dB_r = \frac{\mu I}{4\pi} \frac{[(z - z_B)R_{Lth}\cos(\theta)d\theta]}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{3}{2}}}$$

$$dB_z = \frac{\mu I}{4\pi}\frac{-R_{Lth}d\theta\left[-\left(R_{Lth}sin(\theta)\right)\sin(\theta) + (r - R_{Lth}cos(\theta))\cos(\theta)\right]}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{3}{2}}}$$

Distribute terms on B_z and simplify to:

$$dB_z = \frac{\mu I}{4\pi}\frac{R_{Lth}d\theta[\mathrm{R_{Lth}} - \mathrm{r\cos\theta}]}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{3}{2}}}$$

Applying the same simplifications to the force equation:

$$\vec{F}_{vol} = \frac{1}{\mu_0}\left[\frac{\partial}{\partial r}[B_r^2 + B_z^2]\boldsymbol{r} + \frac{\partial}{\partial z}[B_r^2 + B_z^2]\boldsymbol{z}\right]$$

$$\vec{F}_{vol} = \frac{1}{\mu_0}\left[\left(2B_r\frac{\partial B_r}{\partial r} + 2B_z\frac{\partial B_z}{\partial r}\right)\boldsymbol{r} + \left(2B_r\frac{\partial B_r}{\partial z} + 2B_z\frac{\partial B_z}{\partial z}\right)\boldsymbol{z}\right]$$

Get the derivatives of the components. We will treat dBr/dtheta the same as Br for now since an integral of that with respect to theta will not affect the variables r & z.

$$\frac{\partial B_r}{\partial r} = -\frac{\mu I}{4\pi}\frac{3\left(r - R_{Lth}cos(\theta)\right)[(z - z_B)R_{Lth}\cos(\theta)d\theta]}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{5}{2}}}$$

$$\frac{\partial B_r}{\partial z} = \frac{\mu I R_{Lth}cos\theta\ d\theta}{4\pi}\frac{[[r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 - 2[z - z_B]^2]}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{5}{2}}}$$

$$\frac{\partial B_z}{\partial r} = \frac{\mu I R_{Lth}d\theta}{4\pi}\frac{-3[\mathrm{R_{Lth}} - \mathrm{r\cos\theta}][\left(r - R_{Lth}cos(\theta)\right) - \cos\theta([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2))]}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{5}{2}}}$$

And the derivatives of the z components are:

$$\frac{\partial B_z}{\partial r} = \frac{\mu I R_{Lth} d\theta}{4\pi}\left\{-\frac{cos\theta[\mathrm{R_{Lth}} - \mathrm{rcos\theta}]}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{3}{2}}} - \frac{3[r - R_{Lth}cos(\theta)][\mathrm{R_{Lth}} - \mathrm{rcos\theta}]}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{5}{2}}}\right\}$$

$$\frac{\partial B_z}{\partial z} = -\frac{\mu I}{4\pi}\frac{(3R_{Lth}d\theta[\mathrm{R_{Lth}} - \mathrm{rcos\theta}])(z - z_B)}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{5}{2}}}$$

We're not concerned wit h the force exerted in the radial direction at this time since the projectile is constrained to slide along the axial direction so only the force in the axial direction (z) need be considered. Rewrite the force equation:

$$\vec{F}_{vol} = \frac{1}{\mu_0}\left[\left(2B_r\frac{\partial B_r}{\partial z} + 2B_z\frac{\partial B_z}{\partial z}\right)\boldsymbol{z}\right]$$

Substitute: (Note! We can substitute dBr as Br in these expressions just like before)

$$\vec{F}_{vol} = \frac{1}{\mu_0}\left[\begin{array}{c} 2\left(\frac{\mu I}{4\pi}\frac{[(z - z_B)R_{Lth}\cos(\theta)d\theta]}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{3}{2}}}\right) \\ \left(\frac{\mu I R_{Lth}cos\theta\ d\theta}{4\pi}\left[\frac{\mu I R_{Lth}cos\theta\ d\theta}{4\pi}\frac{[[r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 - 2[z - z_B]^2]}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{5}{2}}}\right]\right) + \\ 2\left(\frac{\mu I}{4\pi}\frac{R_{Lth}d\theta[\mathrm{R_{Lth}} - \mathrm{rcos\theta}]}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{3}{2}}}\right) \\ \left(\frac{\mu I}{4\pi}\frac{-(3R_{Lth}d\theta[\mathrm{R_{Lth}} - \mathrm{rcos\theta}])(z - z_B)}{([r - R_{Lth}cos(\theta)]^2 + [R_{Lth}sin(\theta)]^2 + [z - z_B]^2)^{\frac{5}{2}}}\right) \end{array}\right]$$

This expression is wholly useless by itself. It is more useful to consider the individual components and their derivatives by themselves rather than as the product. Plots of the derivatives along with the force term will give us a better understanding:

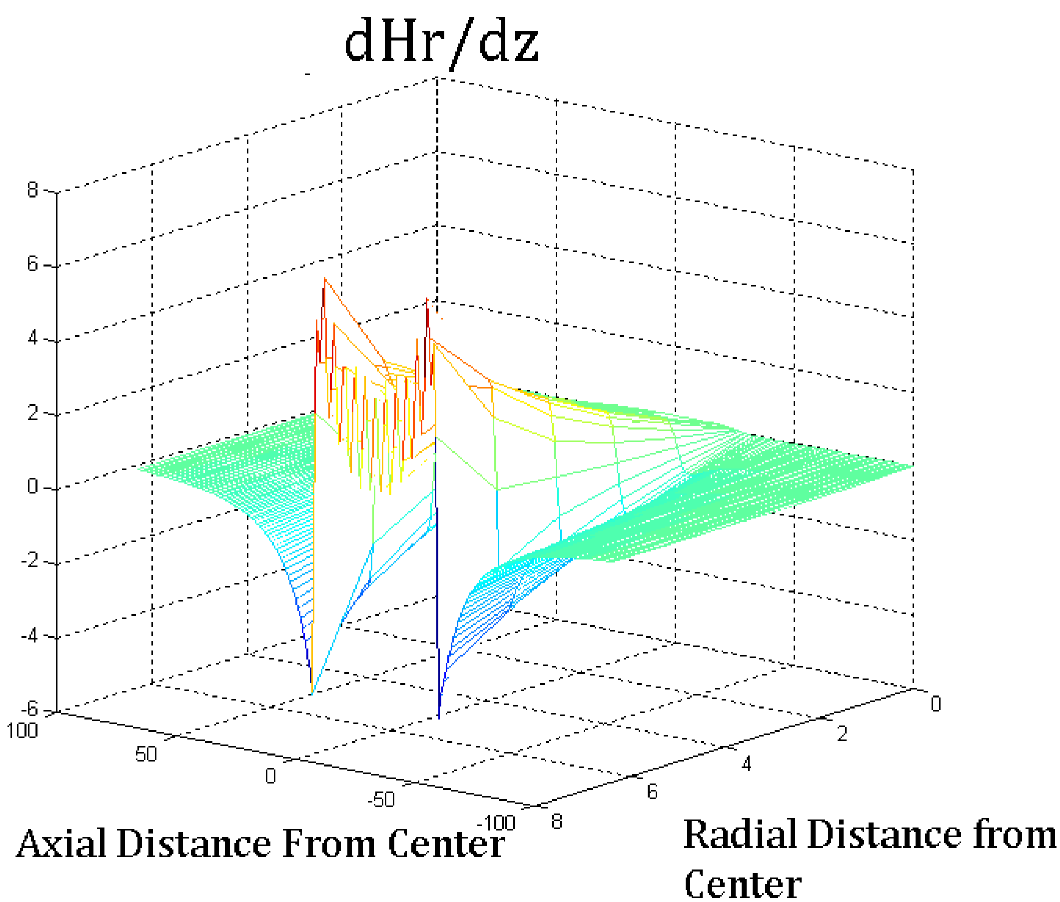


Figure 8: dHr/dz

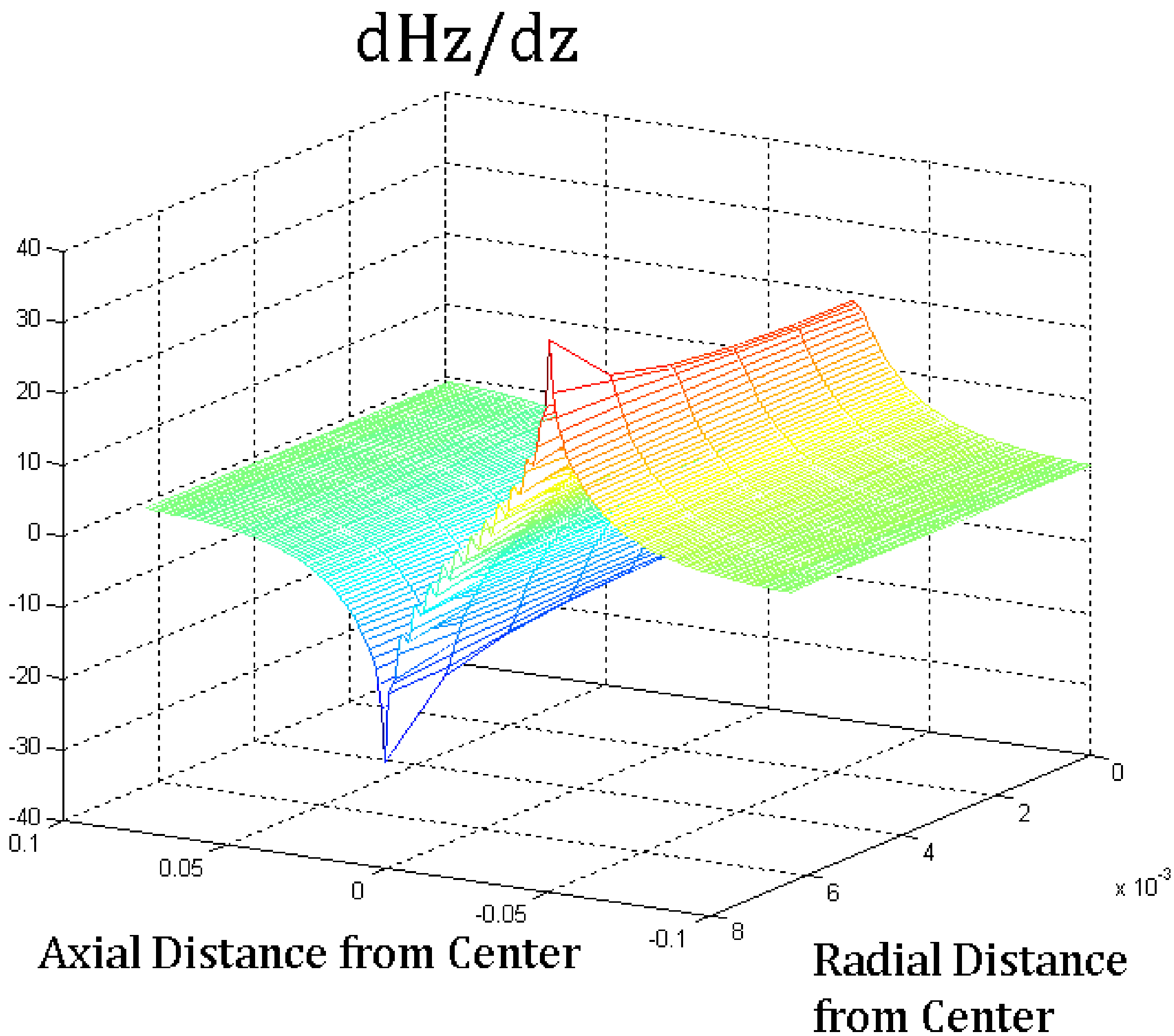


Figure 9: dHz/dz

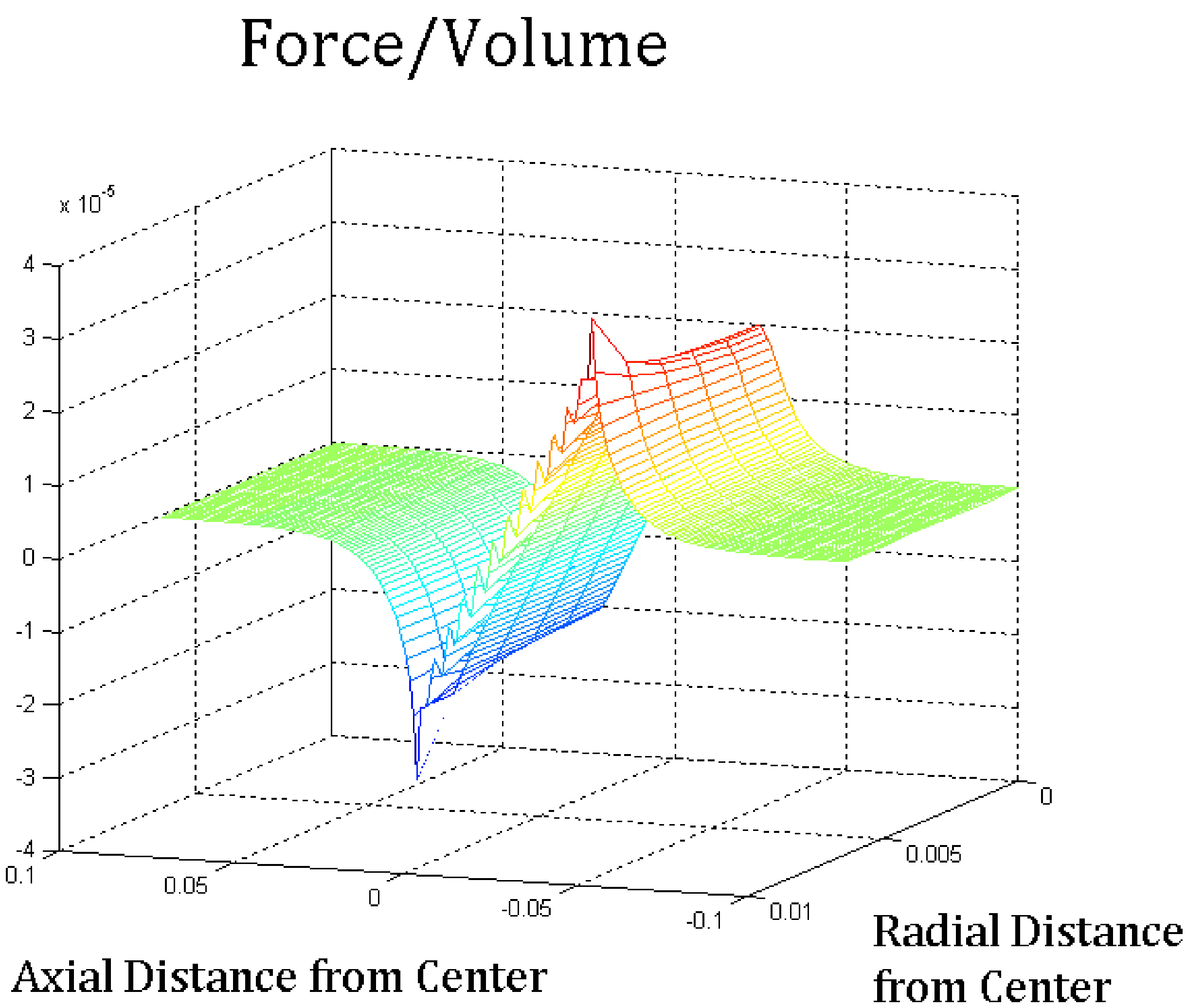


Figure 10: dHz/dz

Although Figure 9 and Figure 10 look similar, they are not the same plot. It is just that the force distribution mimics the radial field distribution since the derivative of the axial field was such a strange shape in and of itself that the radial field times its derivative tended to dominate the expression.

Examining the force equation again:

$$\vec{F}_{vol} = \frac{1}{\mu_0}\left[\left(2B_r\frac{\partial B_r}{\partial z} + 2B_z\frac{\partial B_z}{\partial z}\right)\boldsymbol{z}\right]$$

In conjunction with the plot basically tells us all we need to know. The force exerted per unit volume on a slug at the center of the coil is strongest at the face; it decreases radically at the center of the coil. So the force that acts on the coil effectively acts only as far as the face; once the object is a few lengths inside of the coil it no longer experiences a force until the tip begins to reach the other end, where the force will act to slam it back towards the center.

2.7 To Switch Or Not To Switch?

Most coil gun creators on the internet tend to design coil guns for a single stage. The reason is because they are trying to avoid having to pick a switching mechanism for their coil gun and to do timing calculations, which exacerbate many of the problems that exist in tuning a coil gun. Even multi-stage coil gun designers that do use a switch, do so reluctantly. Why?

2.7.1 Silicon Controlled Rectifier

They bypass the switching entirely and employ the use of solid state relays, also known as silicon controlled rectifiers. An SCR is basically a diode with a gate terminal added. One the voltage applied at the gate is set high, the anode-cathode portion becomes conducting and current is allowed to flow. SCRs can handle huge amounts of current without being fried. The down-side to this is that the SCR is a latch; once the gate voltage is set high it will not turn off unless the current flowing through it goes to zero. Setting the gate voltage to low does nothing to stop the current flow; the gate can only turn the device on, not off. This means that if a projectile moving through the coil passes the center while there is still current surging in the coil, the projectile will experience a force at the other face that will slow it down, reducing the overall efficiency. Not ideal. The only advantage to the SCR is that it is not damaged by inductive voltage spikes; otherwise it's just a voltage controlled current sink.

2.7.2 Solid State Switching: IGBTs or Power MOSFETs?

To switch the coil current on and off then, it is necessary to employ a device that can shut itself off AND handle huge currents and voltages. This has led the team to consider one of two devices: IGBTs or Power MOSFETs.

The result is that the team selected an IGBT; it was no contest. The Power MOSFET is good for high voltage ratings but they are not suitably rated for high currents that surge in the coils. While it is nice that the MOSFET can protect itself from voltage spikes, it is useless if the current fries it on a single shot. On the other hand, the IGBT (insulated gate bipolar transistor) is similar to the SCR: it is a BJT with a gate terminal replacing its base. The IGBT is capable of surviving large voltage spikes while, at the same time, employing the use of the BJT's current carrying capability to surge large currents. So for a solid state switch, the clear winner is the IGBT. The team went on to select very high rated IGBTs for the coil gun: IXXK160N65B4 from IXYS rated at a breakdown voltage of 650V and 310A continuous current; power dissipation of 940W and pulse currents of 860W.

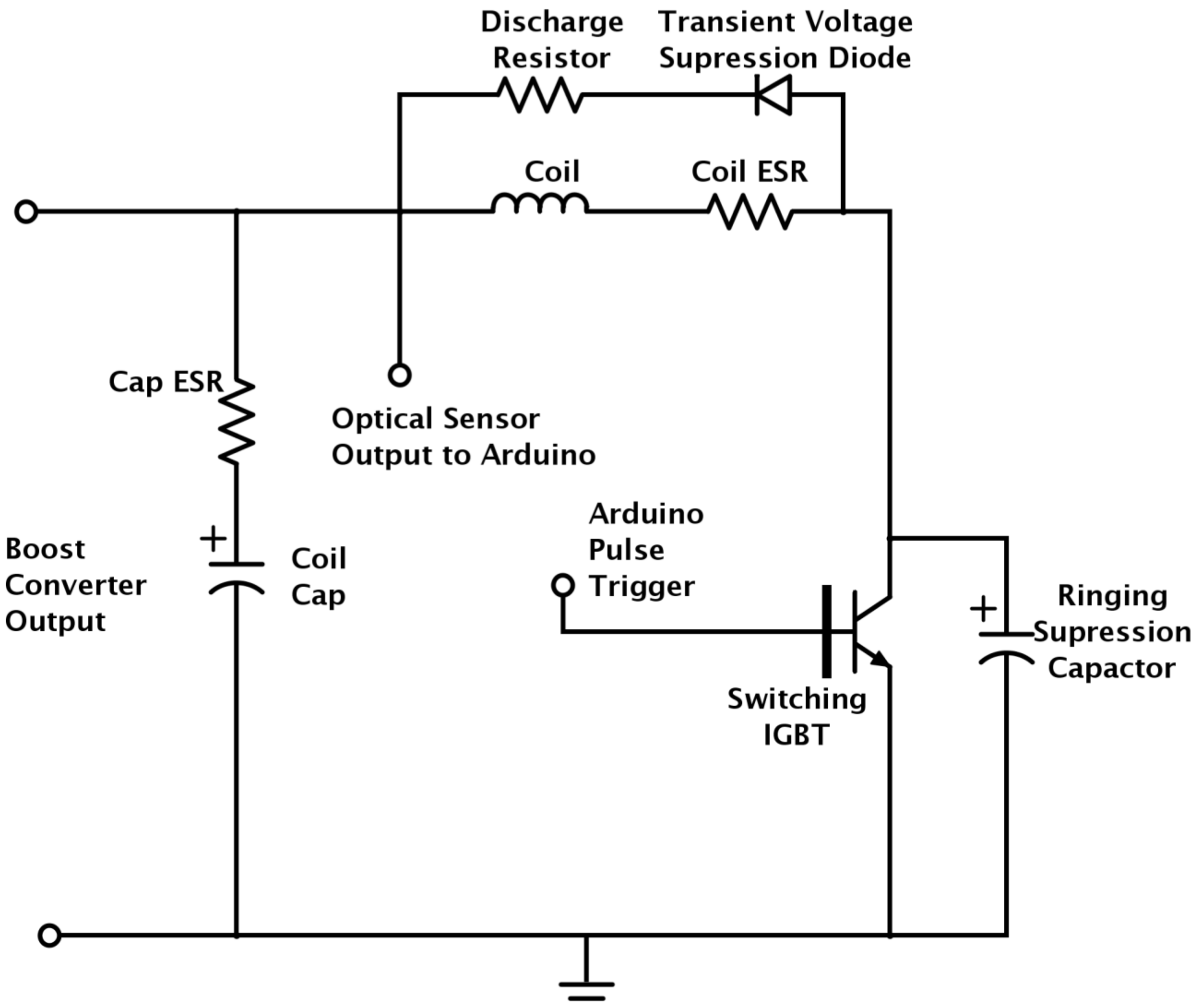
2.7.2 The Problem with Solid State Switching and Inductive Loads

The problem with switching in coil guns is that the switch needs to be rated for very high peak inverse voltages. The reason for that is because the coil is a large inductor. Inductors resist any change in current, like so:

$$v_L = L\frac{di}{dt}$$

While the current may be constant, the voltage drop across the inductor can change enormously. A large forward voltage change may be okay, but that's not what is occurring when the switch closes the current off. Lenz's Law explains this; a magnetic field that is fighting to stay alive will induce a current to oppose the change in flux. This means the polarity of the coil is switched when the switch turns off; it becomes negative and VERY largely negative since the current is falling to zero very rapidly after the switch is off. This can lead to PIVs of nearly 1kV at one terminal of the switch! It takes only a few pico to nanoseconds for this voltage to destroy the oxide layer of a switch, rendering it useless. Overvoltage is a serious problem in switching inductive loads; since the coil gun requires a solid state switch provisions must be made to protect the switch. Even though it may have high ratings the higher the current the higher the PIV.

2.8 Circuit Schematic for a Single Stage



Pictured above is the circuit schematic for a single stage of the coil. This is the same schematic for all three stages. The Boost Converter output is the output from Figure 6. This is fairly straightforward to understand; the Coil Cap is the 400V, 3900uF capacitor that gives the current spike through the coil. The coil is modeled by its inductance and its ESR; same as the capacitor. On one side of the coil is the optical sensor output to the Arduino;

once the object has finished moving through the sensor the Arduino will hit the brakes and switching off the IGBT.

There are two extra components here. One is a TVS diode connected across the coil; this helps to clamp the negative voltage spike described in Section 2.7.2. It is in series with a resistor because the resistor will help to burn off the energy of the coil very quickly; otherwise the coil may not switch off for a long time after the switch has closed.

The ringing suppression capacitor connected across the IGBT is another component added to help with the switching problems. The capacitor, like the inductor, has inertia like so:

$$i_C = C\frac{dv}{dt}$$

The capacitor will resist the change in voltage across it. If the capacitor resists the change in voltage from the inductor while it is connected in parallel with the coil, then the coil too will resist the change in voltage and give the TVS diode time to turn on and prevent overvoltage from killing the switch.

3. Economic Considerations

3.1 Cost

3.1.1 Summary of Total Cost of the Project

Table 5. Summary Cost of the Project				
Description	**Manufacture's Part #**	**Quantity**	**Unit Price**	**Subtotal**
Arduino	A000066	1	0	0
Photosensors	GPA157HRJ00F	3	1.95000	5.85
Capacitors 3900 uF 400 V	U32D 400 LG 392 M 63X130 HP	3	10	30
IFRP450	IRFP450APBF	3	2.92	8.76
Capacitors 100 uF 450 V	450TXW100MEFC18X35	3	4.24	12.72
Fast Recovery Diode	MR856RLG	3	0.45	1.35
Inductor	CTX500-1--52LP-R	3	0	0
IGBT	IXXK160N65B4	3	11.22	33.66
Photosensor Breakout Board	GPA157HRJ00F	3	1.5	4.5
Polycarbonate Tubing	n/a	1	17.14	17.14
Polycarbonate Sheet	n/a	1	7.86	7.86
Magnet Wire 20 AWG	MW0168	1	0	0
Black Grommets	n/a	1	3.29	3.29
			Grand Total	125.13

Table 5. Total Cost of the Project

Note: Item with a price shown as 0 were either provided by the school or were salvaged from old projects done by students. Additional components such as resistors/wires are not listed in this table because their prices are negligible.

3.1.2. Prototype Price with Bulk Order

				PROTOTYPE			ADDITIONAL STAGE	
Description	**Manufacture's Part #**	**Bulk**	**Unit Price**	**Quantity**	**Subtotal**		**Quantity**	**Subtotal**
Arduino	A000066	10	24.95	1	24.950		0	0.000
Photosensors	GPA157HRJ00F	5000	0.96380	3	2.891		1	0.964
Capacitors 3900 uF 400 V	U32D 400 LG 392 M 63X130 HP	3	10	3	30.000		1	10.000
IFRP450	IRFP450APBF	5000	1.32855	3	3.986		1	1.329
Capacitors 100 uF 450 V	450TXW100MEFC18X35	2500	1.77684	3	5.331		1	1.777
Fast Recovery Diode	MR856RLG	120000	0.059	3	0.177		1	0.059
Inductor	CTX500-1--52LP-R	2000	1.86	3	5.580		1	1.860
IGBT	IXXK160N65B4	10000	6.12	3	18.360		1	6.120
Photosensor Breakout Board	GPA157HRJ00F	100	1.2	3	3.600		1	1.200
Polycarbonate Tubing	5706 6B	240	0.6715	2	1.343		0.25	0.168
Polycarbonate Sheet	CUT	1	4.6	1	4.600		0.25	1.150
Magnet Wire 20 AWG	MW0168	10	13.063	1	13.063		0.333	4.354
			Grand Total		113.88			28.98

Table 6. Prototype Price with Bulk Order

3.1.3 Cost Per Additional Stage

As shown in Table 6, the cost per additional stage is approximately $28.98. However, we have to take into consideration that this adds 1.5 pounds of extra weight to the device (~1 lb per capacitor, 1/3 lb per coil and ~0.2 lbs for sensors and other small components). As mentioned in the abstract, the coil gun is theoretically the best option for the nail gun. No matter how good the other guns are, it is only necessary to add an additional stage to the coil gun and it will surpass the others. However, the gun itself becomes infeasible to use if it is too long and too heavy—thus the weight and the length is the true "cost" per stage.

3.2 Scaling

One design goal when dealing with any product is how to scale down the prototype. The prototype as it is now is in excess of twenty pounds, weighed down by three capacitors, heavy batteries, and copper wiring for the stages. So what can we do in this instance?

And then, that's only one type of scaling – in reference to its size. But what about scaling up in reference to making it a more powerful device?

3.2.1 Scaling Down the Product.

The prototype can only be scaled down in size at the expense of power and money. The obvious choice is to pick a smaller capacitor because it is the largest component in the design; changing other components is not an option because they don't offer as much size reduction as the big capacitors (i.e. IGBTs, diodes, MOSFETs, etc.) or they are entirely dependent on the size of the projectile (barrel's length and coil size).

The capacitor used in the prototype has a nominal case size of 2.5 in x 5.125 in (DxL), which yields a cylindrical volume of 25.157 in^3 and a total weight of 3 lbs.

Let's assume that the goal is to reduce the volume to 8 in^3, then one of the biggest capacitor available at this size is approximately of 1800 μF 400. In terms of energy this is equivalent to:

$$E = \tfrac{1}{2}\,(1800\ \mu F)(400\ V)^2 = 144\ \text{Joules/capacitor}$$

For 3 stages at 5% efficiency and a 6.5 gram projectile, the maximum attainable velocity is estimated by:

$$v = (2^*E/m)^{1/2} = (2^*(3^*144)^*0.05/0.0065)^{1/2} = 81.52\ m/s$$

For 3900 μF 400 V, the energy stored is 312 Joules and the expected speed is 120 m/s. So the design tradeoff in shrinking the size is a loss of ~40m/s, that's a 33.3% reduction in speed! Some will not want to use it if it's too heavy, but even less people will want to use it if it doesn't work as expected.

Another way to increase the comfort of the device would be to reduce the size of the wire down to the thinnest available. =

It is difficult to predict a well-defined relationship between sizing and speed because of the vast diversity of capacitors in terms of size, voltage rating and capacitance and the heavy dependence on the size and weight of projectile, so at the end of the day it all comes down to the quality of the capacitor and how much energy it can store for a given size. Another concern is that changing the capacitor for another with a voltage rating higher than 400 V will make the price go up because the rest of the circuitry is only tuned to operate at 400 V or under.

3.2.2 Scaling Up the Product.

The same situation occurs when scaling up; there is no clear relationship between size and power. If the goal is to keep the three stages while the firepower is increased, the designer has two choices: either increase the efficiency or use different capacitors. Again, there is no quantitative way to relative size to firepower given these conditions.

So the most logical thing to do is to compare the same coil gun with more stages. The following graph shows the relationship between number of stages and muzzle velocity.

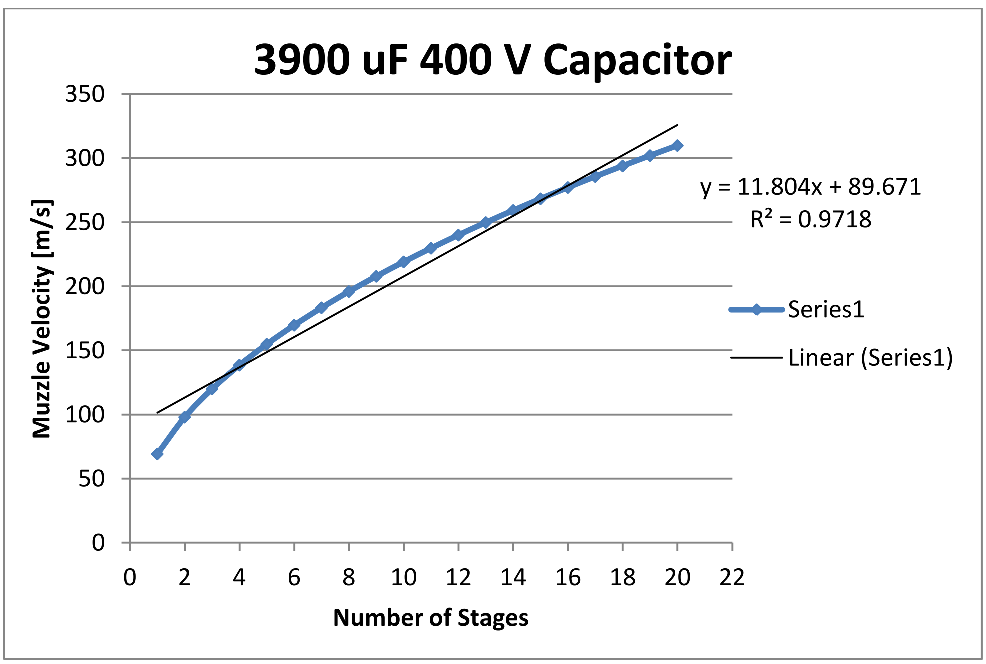


Figure 11. Number of Stages vs Muzzle Velocity.

As shown in the graph, the relationship between number of stages and muzzle velocity is not exactly linear. Adding one more stage to the coil gun approximately yields an additional 11.80.4 m/s at a price of $28.98 and extra weight.

3.2.2.1 Scaling Limitations

Is there a limitation regarding the number of stages we can put on a coil gun? Theoretically, if more stages are added the power of the coil gun increases. However, as more stage is added the extra benefits diminish and the device becomes very heavy and large, defeating the whole purpose of having a portable nail gun.

Stages	Energy [J]	Muzzle Velocity [m/s]	% increase
1	312	69.28	n/a
2	624	97.98	41.42
3	936	120.00	22.47
4	1248	138.56	15.47
5	1560	154.92	11.80
6	1872	169.71	9.54
7	2184	183.30	8.01
8	2496	195.96	6.90
9	2808	207.85	6.07
10	3120	219.09	5.41
11	3432	229.78	4.88
12	3744	240.00	4.45
13	4056	249.80	4.08
14	4368	259.23	3.77
15	4680	268.33	3.51
16	4992	277.13	3.28
...	...	...	...
98	30576	685.86	n/a
99	30888	689.35	0.51
100	31200	692.82	0.50

Table 7. Benefits of increasing the number of stages.

From the table, it can be observed that there is a diminishing return effect in the muzzle velocity, adding the 100^{th} stage only yields a 0.5 % increase in speed! This is the reason the team chose to do 3 stages; it yields the best benefits for the investment.

4. Sustainability Analysis

It is necessary to consider how the coil gun as designed will function in the event of wear and tear on the device. How does it need to be handled? If something happens to a component, will it continue to function? How can replacing parts affect the operation in anyway?

4.1 Large Scale Parts

What is meant by large scale in the context of this writing are the largest components in the nail gun: the capacitors, the coil, and the barrel.

In the event that a capacitor is damaged, there are few options here. Of course, this depends on which capacitor has failed. If the first stage capacitor fails, then the game is over. Capacitors from one of the other two stages can be disconnected and connected to the first stage, but then the coil gun is acting at reduced efficiency; only two stages (see Table 7).

4.1.1 Capacitors

Capacitors at any other stage can be treated in much the same way. It is left up to the owner to purchase additional capacitors to replace it; at some cost to themselves. It will be ~$10. On the bright side, capacitor failure will not be likely in this configuration since the capacitor won't be damaged by any current transients set up by the circuit. If the capacitor is damaged it would be because it is dropped or the gun is manhandled in some way such that it is punctured; in which case the housing on the gun is meant to protect it. If not and the capacitor just dies a natural death then it needs to be replaced; this could be covered by some sort of warranty (within a year) so the person who purchases the gun doesn't feel tricked. Otherwise, $10 again.

4.1.2 Coils

Now, the coils are very sturdy. Damaging the coil doesn't really do too much and that would be difficult to do as well. If someone was able to dent it in some way, it would only slightly change the distribution of the magnetic force; otherwise, it will work as expected. The coil rests on top of the barrel, so damaging it is a very difficult thing to do. Not to mention it is 8 layers of 20 AWG copper wire protected by a PVC housing; if someone did damage it then that was likely intentional.

4.1.3 The Barrel

The barrel is a solid plastic tube that is equal to the full length of all three stages. This portion of the large scale parts may be more susceptible to damage than the others. Because the barrel is long, a sharp and strong applied force that cracks the barrel in some way would ruin the gun irreparably. Of course, in standard operation this shouldn't happen at all; dropping this gun wouldn't cause this kind of damage so it would have to be intentional given how hard the barrel is. But if the barrel is cracked and broken between coils, then it could be used at reduced efficiency (2 or less stages) before needing to get it fixed. It would be a simple fix for the owner at any rate; just so long as they can fit the coils on top of another tube of similar length then the gun will still work as anticipated. The barrel, in the grand scheme of things, is really just there to restrict the motion of the nail in one direction. Replacing the barrel would only cost around \$1~\$2, so it's not a heavy cost.

4.2 Small Scale Parts

Small scale parts in this context refer to any of the small electronics that the coil gun needs to function properly. This ranges from the discrete components (resistors, capacitors, etc.) to the solid states (IGBT, Arduino, etc.). These components are where the sustainability analysis really matters and they need to be protected accordingly. Consideration needs to be taken for the two circuit schematics here: the boost converter and the coil circuit schematic.

4.2.1 Boost Converter

The boost converter consists of four components: the diode, the capacitor, the power MOSFET, and the inductor. There is an external output to a power supply which is just a 12V battery; if that fails then the consumer needs to buy another one. This is a fact of their life and one they should be well versed in at this point in purchasing a product. Simply put, if any one of the components on the boost converter fails then the boost converter itself fails; this means that the capacitors cannot charge up to the hundreds of volts at which the gun becomes feasible to fire. If the diode fails, there won't be any blocking so the capacitor will discharge its energy into the inductor when the switch toggles. If the inductor fails, there won't be any energy to dump in the capacitor when the switch toggles. If the capacitor fails, there won't be any build of energy which means no large voltage outputs (probably a DC open at this point). If the MOSFET fails then there's no switching and no pulse width modulation from the Arduino, rendering this a useless LC circuit with a diode in between.

There's not too much reason to suspect these components will fail on their own. The inductor and the capacitor are both passives; their lifetime is very long (especially in the case of the inductor). The same holds true for the diode and the MOSFET; these components aren't under any real strain when the boost converter is operational. The only component that could fail naturally is the capacitor, and even then it is an inexpensive replacement and won't be noticed at all! Why? The capacitor is only there in the boost converter in case the capacitor on the coil gun (the big one) fails (or is disconnected while the device is on); this little capacitor will pick up the slack in this instance. If there is no output load, the inductor will destroy the diode or the MOSFET, so the capacitor needs to take that energy before it builds up a nasty voltage spike. But! If the capacitor fails on the boost converter and the output is solidly connected, the capacitor on the coil will charge as if nothing happened. An owner may never know that capacitor has failed until the other capacitor on the coil fails too.

The best provision here is to just protect the boost converter from physical wear and tear and this is done with a metal housing on the outside solidly anchored to the gun.

Unless the gun is abused in some way, dropping it won't cause the damage necessary to break these components.

4.2.2 Coil Switching Schematic

The components that need to be considered for failure on this schematic are the IGBT, the optical sensor to the Arduino, and the TVS diode connected across the coil.

If the IGBT fails, then the coil gun won't work. Period. When the IGBT fails, it acts as a short circuit which means the coil gun is always on. If the gun is always on, then the boost converter can't charge the capacitor without the capacitor just dumping its energy into the coil. All provisions must be taken to protect this switch from damage; replacing it is costly at $11 – that's more than the capacitors! The IGBT could fail from repeated firing; charge may build up in the junctions of the IGBT and after some point enough voltage could be present there to cause it to fail. But the biggest reason for failure would just be charging the capacitor up to beyond what is considered safe for the switch and discharging through it.

If the optical sensor to the Arduino fails, the coil gun will stop working. The optical sensors act as triggers; there are no inputs sent to the gate of the IGBT if the optical sensors do not detect a nail in the barrel. If these fail, they need to be replaced **immediately**. They are relatively inexpensive at $3 and replacing them is simple; disconnect the one that's on and glue on the one that was bought and keep the wire connections the same. The optical sensors are sturdy however; if they fail it will likely be because of physical damage. They are insulated away from most of that stuff since dropping the gun would put strain on the PVC pipe and washers without touching the optical sensors; only a direct blow could really do that.

If the TVS diode fails, the coil gun will still work. At least, it will work for one more shot normally before the IGBT gives up the ghost. The TVS diode is helping to protect the IGBT from overvoltage a great deal; if it fails then the operating voltage of the coil gun must be reduced by around 100V to ensure protection of the IGBT, unless the owner wants to replace that one too. However, there's no reason the TVS diode would fail electrically so this scenario isn't feasible. It's also insulated from harm inside the metal box anchored to the outside of the gun so dropping it shouldn't hurt it. Very sustainable.

If the ringing suppression capacitor fails, thee coil gun will still work. It is recommended that the operating voltage be reduced anywhere from 20~100V to ensure that the IGBT is protected from overvoltage. Otherwise, replacing the capacitor will have a negligible cost and a small effect on efficiency.

4.2.3 Arduino

If the Arduino fails, the coil gun goes belly up. Without the Arduino, there's no way to switch the circuit for two reasons: it won't be sending a high voltage to the gate of the IGBT and it won't be detecting the nail moving through the coil at any given moment with the optical sensors. The boost converter won't work at all either since the Arduino controls the pulse width modulation at the input of the power MOSFET on that schematic. So the Arduino, much like the IGBT, acts as a gatekeeper: if it fails, the coil gun fails. This is absolute. Replacing an Arduino is the most expensive part since it is a small computer; $30 or so. This is not ideal. The only way to really break this is if a pin just happens to go bad, at which case then the Arduino needs to be reprogrammed (at an inconvenience to the owner; could be done remotely) and one wire needs to be moved to another pin. The

Arduino is insulated from circuit damage by being inside the aforementioned metal box. If the Arduino fails, the boost converter could still work b y employed a 555 digital timer IC to do the pulse width modulation for it. Even so, there is no way to switch the circuit so it is a moot point.

4.3 Summary

From the performed sustainability analysis, it is clear that three components need the most care and attention out of any of the others: the Arduino, the IGBT, and the optical sensor. All of the other parts are somewhat easily replaceable and the coil gun can continue trucking on in light of those failures (capacitors are everywhere, so fixing that is simple). Typically if one of these components fail then the gun has serious trouble. The IGBT is killed by overvoltage and the Arduino can just wear out pins until it doesn't work anymore. Physical damage is a big no-no to anything on this, but the gun is pretty will constructed so anything short of intentional damage would protect it. There is unfortunately no way to stop these problems; at most the cost of a single repair could run up to $30 and that would be the cost of the Arduino to the consumer. The team can only recommend the implementation of a warranty for these objects to help prevent against early failures when using the gun.

5 Problems, Results, Questions

There were an enormous amount of problems when designing this coil gun, to the point where collecting any type of data became more of a hindrance since it meant having to solve said problem first.

5.1 Problems

The biggest problem that slowed down the development of the coil gun, more than anything else, was the repeated failure of the solid state switching device. Every time the switch failed, the team would have to order better switches at a cost of waiting three days to a week to receive the part. As such, nothing is worse in the coil gun than having the solid state switch fails; it turns the gun into a paperweight.

For a while, the team operated under the belief that the solid state switch was frying under the large current transients flowing through the switch. This is what precipitated the switch from power MOSFETs to IGBTs; they could sink more current at a better rate and not die. But the IGBTs tended to die at a slightly higher voltage, and that was it.

Later, it became apparent that what was causing death of the switch was the overvoltage problem laid out in Section 2. Attempts were made to see the transient behavior on the oscilloscope, but the discharge time was too fast to capture anything meaningful. Instead, the team attempted a simulation to see what the voltage spike might have been. Multisim, PSPICE, and Virtuoso all failed in different ways to simulate the behavior of the circuit. Multisim would claim that the switch never turned on, so that was useless. PSPICE claimed that the inductor would be charged up by the capacitor, sink a healthy current into the switch, and turn off without so much as a hint of ringing. Virtuoso was the closest; it would claim that the voltage applied to the oxide layer exceeded breakdown voltage and proceed to plot nothing since t he simulation failed. So the team was unable to use simulation to determine the problems with the coil.

It was only later that the TVS diodes and RC snubbers as well as other diodes were clamped across the switch and coil to protect them. The TVS diode across the coil worked the best; the IGBT would fire from 100 up to 200V once that was placed on. A diode in parallel with an RC snubber in parallel with the switch was thought to be able to completely eliminate the overvoltage, but the problem was that it weakened the current waveform to the point where the coil gun was useless. Finally, the team settled on using a ringing suppression capacitor at the IGBT but this only marginally improved performance up to 230V for the highly rated IGBT. Any attempts to fire at voltages higher than this fries the solid state switch. As such, the team has had to make the choice of operating the coil gun at a reduced efficiency. The theoretical speed has become:

$$v = \sqrt{\frac{0.05CV^2}{m}} = \sqrt{0.05 * 3 * 3900\mu * \frac{230^2}{0.0065}} = 69m/s$$

Where this is an upper bound for the speed.

The solid state switching is still a major problem; it is now easy to see why coil gun designers avoid this type of switching and defer to latching instead: at least they are able to use all of their capacitor energy! It isn't even possible to charge the capacitors up to 400V and use their energy in stages; the switch will fry.

Another big problem was the Arduino itself; it would constantly act up and keep pins registered at a semi-high level (~2V), causing endless headaches. The reason why it would do this is still unknown; restarting the computer and re-uploading the sketch will sometimes fix the problem.

5.2 Results

The coil designed for one stage and replicated for all three is an 8 layer, 400 turn coil with 20 AWG magnet wire. The team selected 8 stages since a single $20 spool of magnet wire was capable of winding 1600 turns before running out. There shouldn't be any retarding potential for increasing the layers except for increasing the total resistance of the coil, weakening the overall current.

Firing some carpenter and finishing nails through a single stage coil at the 230V limit results in output velocities of 15~20 m/s. Firing through all three stages results in 40~50 m/s; slightly less than the upper bound of 69 m/s. This might be because the photo detectors aren't fast enough so they don't energize and de-energize the gun as quickly as they should be doing.

The boost converter circuit works well, charging the caps up to 400V anywhere from 30sec~1 minute (strangely enough, this does vary!).

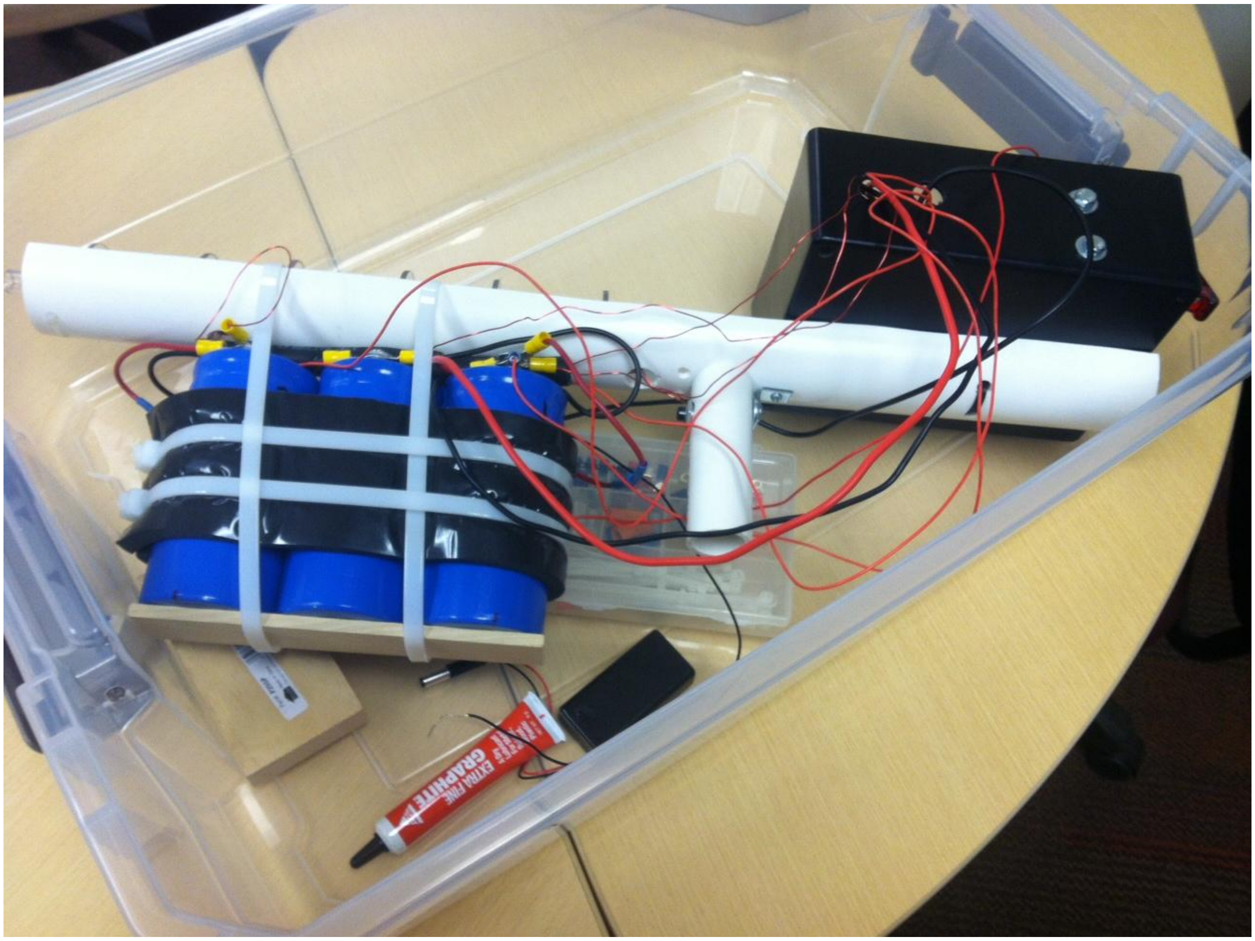


Figure 12: Side View of the Nail Gun

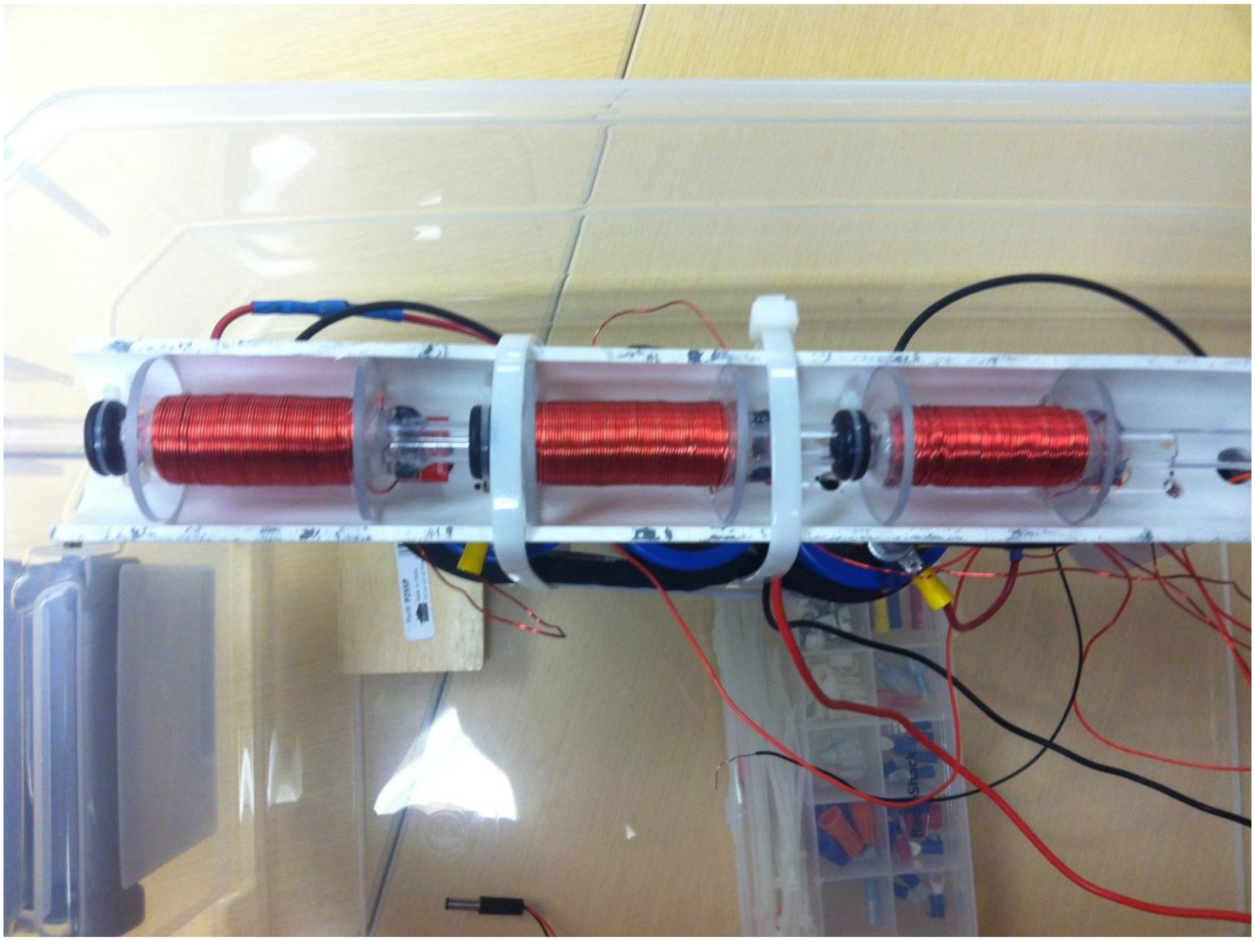


Figure 13: Top View of the Nail Gun Showing Coils

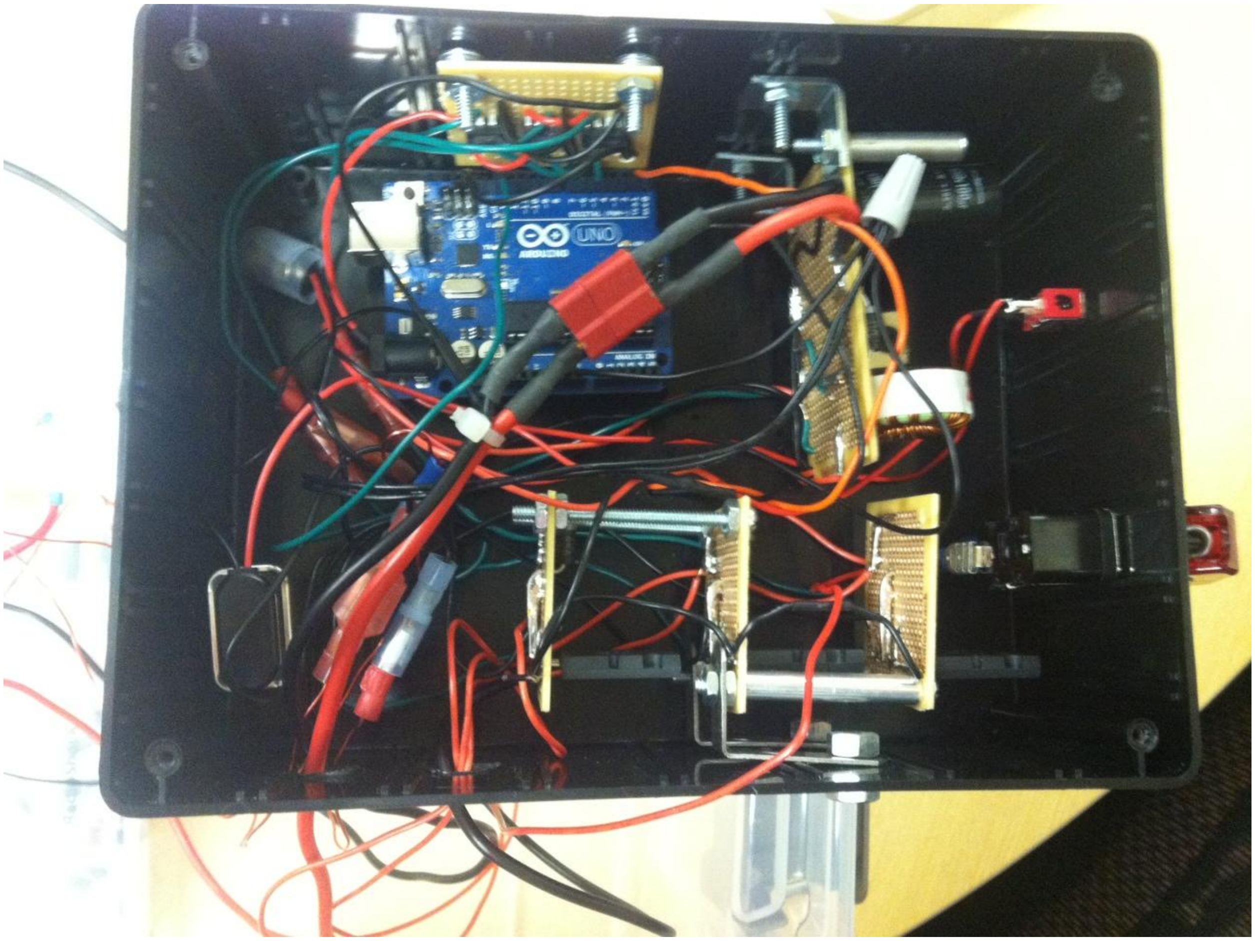


Figure 14: Top View of the Nail Gun Showing Circuitry

5.3 Questions

The following are the team's answers to a cluster of questions presented in the Midterm Report:

- How many shots will be possible on a single charge of the capacitor(s) by either method?

Only one shot since 90 % of the voltage is discharged when the projectile is fired.

- What will the kick-back be?

There is little if any kick back when firing this coil gun.

- Will the device be light enough for extended periods of use?

As of right now, the prototype weights approximately 15 pounds. The capacitors, the coil, and the batteries are all the main reasons why the weight is so high. One way to get the weight down would be to employ the mechanism that pneumatic nail guns use; i.e. use a hose connected to the gun and carry around the heavy capacitors and or batteries in a separate container.

- How will the projectile be loaded?

The nails have to be loaded into the gun manually; in other words, the nail is directly inserted into the barrel and the tip goes into the first coil. However the trigger mechanism is complete, meaning that the nail will fire with the press of a button.

How fast can this device fire nails back to back?

Currently, the only thing that doesn't allow the coil gun to fire back to back is the time needed to charge the capacitor, which is about 30 seconds. Because of time constraints and major problems with the main switch, the team was not able to come up with a mechanism that allows to fire nails continuously, but it is believed that it is possible to implement it by either increasing the input voltage of the boost converter or have a another bank of capacitors that will charge while the other capacitors are used to fire the nail.

-How safe is the device? Is a safety mechanism needed to prevent accidental firing?

The final prototype will have a master switch that will turn off/on all the components in the circuit for precaution, so two switches have to been flipped before firing the gun.

During experimentation, several sparks and electric were created due to the high voltages involved in this project. This problem was solved by moving all the components into a circuit board, since then no major catastrophes involving sparks or electric arcs has happened.

- How will the device function or react if a component fails?

Since all the three stages are independent, the device will be able to fire the nail as long as there is one working stage. But since the final prototype lacks an initial launching mechanism, the nail has to be moved close to the working coil. (See Section 4).

- Is anything on it easily replaceable?

The only easily replaceable component in the coil gun is the capacitor; the rest is wired and soldered into a board. (See Section 4).

- Can the device be made adjustable for different applications?

The force or penetration of the nail can be adjusted by varying the voltage of the capacitors. Changing the size of the nail could result in lower speeds depending on how permeable the nail is and its weight and length.

- Can critical timing to turn on and off each coil be achieved using a microcontroller?

Yes, using optical sensors.

- Do we need to install a discharging device to safely remove the charge on the capacitors in the event of a malfunction or if use is to be suspended?

This could be done but it isn't necessary. One idea the team had was to have a separate connection to a light bulb for discharging the capacitors, but in general just pushing the fire switch will discharge the bank.

- What are the effects of an initial position of the projectile relative to the coil? How will it affect efficiency?

Initial position changes the efficiency by a bit, at least for the first stage. If it is too deep into the coil, it won't be pushed hard enough and may remain in the center. This question was asked before the team understood the spatial distribution of the force; now that it is known that most of the force is at the face an initial position should only have the tip somewhat on the inside of the coil. This is the best position.

-What are the tradeoffs in design for all the variables? Are more layers preferable to more turns? Do we need to worry about resistance versus inductance tradeoffs for the coil?

Still a good question, but far too difficult to answer given the material to work with. Focusing on the second and third question only: more layers versus more turns was a naïve question since more layers = more turns. Resistance versus inductance tradeoffs do not need to be worried about as far as the coil is concerned; the circuit response is usually underdamped.

- Is there an optimal shape for the coil? What this means is, given the field expressions, can a different shape be designed for the coil that optimizes energy transmitted to the projectile? Is circular the best one?

The optimal “shape” for the coil is still circular windings around the tube. However, there was a result (not given in this report) that indicates that a triangular distribution of layers can result in a better distribution of the force, allowing more useful force to act on the left side of the face versus the right. This could not be tested as winding the coils is a very difficult process; 400 turns with this distribution would have taken about 4 hours for two people to do. 400 turns is necessary for comparing the performance to the other two coils.

- How will the chassis that holds the gun look like?

See pictures.

6. Conclusions

The main goals outlined in the beginning were to design a working prototype of a coil gun, use it to fire nails, and see if it would serve as a suitable replacement for pneumatic or electric nail gun. A second part to this was to fully understand the operation and physics of a nail gun.

With the current technologies, the answer is a soft turn towards ‘no’. The nail gun as designed is already heavier than all the ones we set out to compare to and consumes more energy. The biggest problem with this gun is its weight; this thing is much heavier than other nail guns and has a much longer barrel – so it is very unwieldy. A big kicker for the coil gun is the ultra low efficiency (5%! So much electrical energy is wasted) and the fact that the coil gun is its own worst enemy; the force that acts to push the nail towards the center of the coil at all times is a big hindrance.

Potentially, however, the coil gun can fire a nail pretty fast and likely embed it in the wood better than the other two guns. But so what? It can fire one nail at a higher velocity than these other two, but it doesn't do so efficiently. But then again, this is in the prototype stage. If there was a mechanism to charge two banks of capacitors at the same time then this nail gun would be capable of achieving the rapid fire capabilities of the other guns. However, this brings added cost of weight to the picture for the extra bank of the capacitors!

There are some possibilities here. Smaller capacitors would reduce the weight of the object considerably; if the switch can only handle 230V capacitance, then it wouldn't be wrong to use lighter capacitors rated for a lower voltage and a higher capacitance rating. Another thing would be the use of a machine to turn very thin copper wire when making the coils; this gives the smallest possible resistance for a given design and allows for the same inductance at a much lower weight. A smaller wire is best but too difficult to wind by hand for a prototype. With these changes, the gun might be adjustable to a manageable size.

The other big problem is the solid state switch. For efficient nail guns, the switching is necessary; the force exerted when the nail leaves the coil needs to be reduced to zero by collapsing the current through the coil. So a coil nail gun designer would need to remain vigilant of improved ICs, either that or try to use a latch to dump all the current through the coil as fast as possible. A latching approach was considered by the team but rejected because switching was (thought) to be the better option; repeated failure of the switch has shown this not to be the case. A firm, reliable switch is necessary for the success of the nail coil gun.

The prototype ended up being more expensive than anticipated; the cost target was meant to be in the $60.00 range, but ended up at best for $113.88 for bulk orders. However, a smaller wire can be purchased for +$10 than the AWG we have now but subtracted from future units because more than 3 coils can be wound off the thin wire. This would pull costs down to $100. One other thing would be to use less expensive iterations of the microprocessor; it isn't doing anything complicated since its only sending high signals and doing basic PWM. The PWM function can be replaced with a 555 timer IC anyway, so costs could be brought down about another $10. The capacitors, however, are always going to be $30 per unit unless cheaper and smaller capacitors are used as recommended above. Together, the capacitors and the Arduino together represented the target cost alone, ignoring the fact that switches were still needed along with wires, sensors, etc.

The coil nail gun is more complicated than either of the two guns and probably more prone to error as well. The sustainability analysis indicated that the gun can fail to work in a number of ways if the ICs aren't properly protected; these problems aren't as apparent in other nail guns that rely on pneumatics or use an electric spring.

The team concludes that the coil nail gun is not efficient enough to survive as an alternative to current technologies, although it presents an interesting alternative for future work.

7. Future Work

The future work that needs to be done on a project like this would have to begin with finding a better way to switch the current. It may be possible with SCRs or multiple SCRs in conjunction with optical sensors to fully dump all of the capacitors energy before the nail passes through the center of the coil; this would need to be examined to see if its feasible. An SCR is a much better alternative to the IGBT in terms of surviving the operation of the coil gun but cannot be switched off, leaving open the possibility of lower efficiency.

The gun needs to be made lighter and more cost effective, following the suggestions from the conclusions in Section 6.

The gun needs to be adapted for rapid fire so it can compete with other nail guns on that level. One way to do this is, as it says in Section 6, to use more capacitors with another boost converter set. But this will increase the weight and the cost of the product by another $40 or so.

Increasing the conversion efficiency above 5%. This may be possible by implementing a new winding scheme and trying shorter coils.

The coil gun is currently designed for a single type of nail and ferrite at a certain voltage; it would need to be adjusted to take multiple nail sizes and operate at different voltages. The firing speed should be adjustable as well.

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