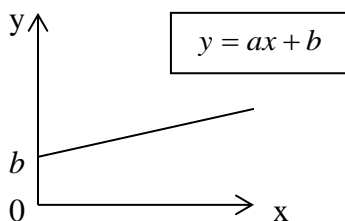


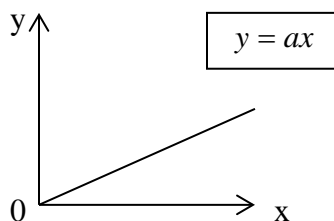
## Linear graphs and linearization of curved graphs

### 1. Straight-line graphs.

If the trend of the data is a straight line, then the graph is called *linear*.



Here, there is a linear relationship between y and x



This one is a special type of linear relationship between y and x, because  $b=0$  so the line passes through the origin (0,0).

Variable y is directly proportional to x.  
(Here, doubling x will double y.)

Data from experiments will never lie exactly on a line, but it is usually clear to the eye if the trend is linear or curved. If there is a linear relationship, the next step is usually to find (i) the slope  $a$  and (ii) the y intercept  $b$ . These two quantities fully describe the line.

#### Guidelines for plotting the points

- Each graph must have your name, and the date
- The graph must have a title
- “widgets versus gizmos” means widgets on the vertical axis and gizmos on the horizontal axis
- The axes must each be labeled with the name of the variable (eg. speed), the symbol for the variable (eg.  $v$ ), and the units of the variable (eg. m/s)
- The scale of each axis must be chosen so that it's easy to plot the points and to read off the coordinates of points. Use only multiples of 2 or 5 to label the bold grid lines on the graph paper. (eg. 8, 10, 12, 14,..., but not 8, 10.5, 13, 15.5,...)
- The axes don't have to start at zero; the range should start near the value of the first point
- Use one side of graph paper for each graph. The range of values for each axis must be selected so the data points use at least half of the extent of the axis.
- The points must be **tiny**. Draw a circle around them to clarify where they are.

### The best line:

To obtain the slope and y intercept, the *best line* must be drawn through the data. Although the slope can be calculated from the data points (using the method of least-squares), it can be done quite well by eye. Use a sufficiently long ruler and construct a *single* line that approximates the trend of the data points as accurately as possible. A transparent ruler is best. Don't pick up the pencil. Note that the line need not touch any of the actual data points. Sometimes, a point lies so far from the trend of the other points that it must be incorrect. This is called an *outlier* and should be ignored in finding the best line. Think: since you use all the acceptable points to interpolate the best line, it represents information "crystallized" from *all* the data.

### The slope:

- Select two new "slope points" on the best line. Indicate them by putting a square around each and number them 1 and 2. They must not be data points.
- The slope points must be near the ends of the best line – ie. far apart. Don't select them by looking for a point where the best line intersects with the grid lines
- On the graph, label the slope points with their coordinates and units
- The slope formula is:

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

- Show the slope calculation (on the graph if possible) by first writing the above equation, then substituting the slope-point coordinates with units, and only then giving the answer.
- The slope will be wrong if you leave out the unit. This is a common mistake!

### The y intercept:

After you have the calculated numerical value of the slope  $a$  (with its unit), you can calculate the y intercept  $b$  of the point by solving the straight-line equation for  $b$ :

$$b = y - ax$$

To evaluate the right-hand side, use the coordinates  $(x,y)$  of any point on the best line, and not a data point.

Why do we never use the data points after the best line is found? The idea is that all the information comes from the best line, which contains more information than any one data point.

## 2 Example: Linear plot

Height $h$ (meters)	Gravitational potential energy $U$ (Joules)
.20	.49
.40	1.04
.60	1.33
.80	1.96
1.00	2.48
1.20	3.21
1.40	3.43
1.60	3.93
1.80	4.54
2.00	1.73

We will plot a graph of gravitational potential energy versus height. Then, we will find the slope and the intercept.

## Exercise: a linear plot

The data tabulated here is hypothesized to have a linear trend. Make a graph of  $R$  versus  $v$  to verify this. Find the slope and the intercept. Follow the same methods as for the example above.

$v$ (m/s)	$R$ (kg/s)
2.5	32.75
3.5	28.25
4.5	23.75
5.5	19.25
6.5	14.75
7.5	10.25
8.5	5.75
9.5	1.25
10.5	-3.25
11.5	-7.75

# Gravitational Potential Energy versus height.

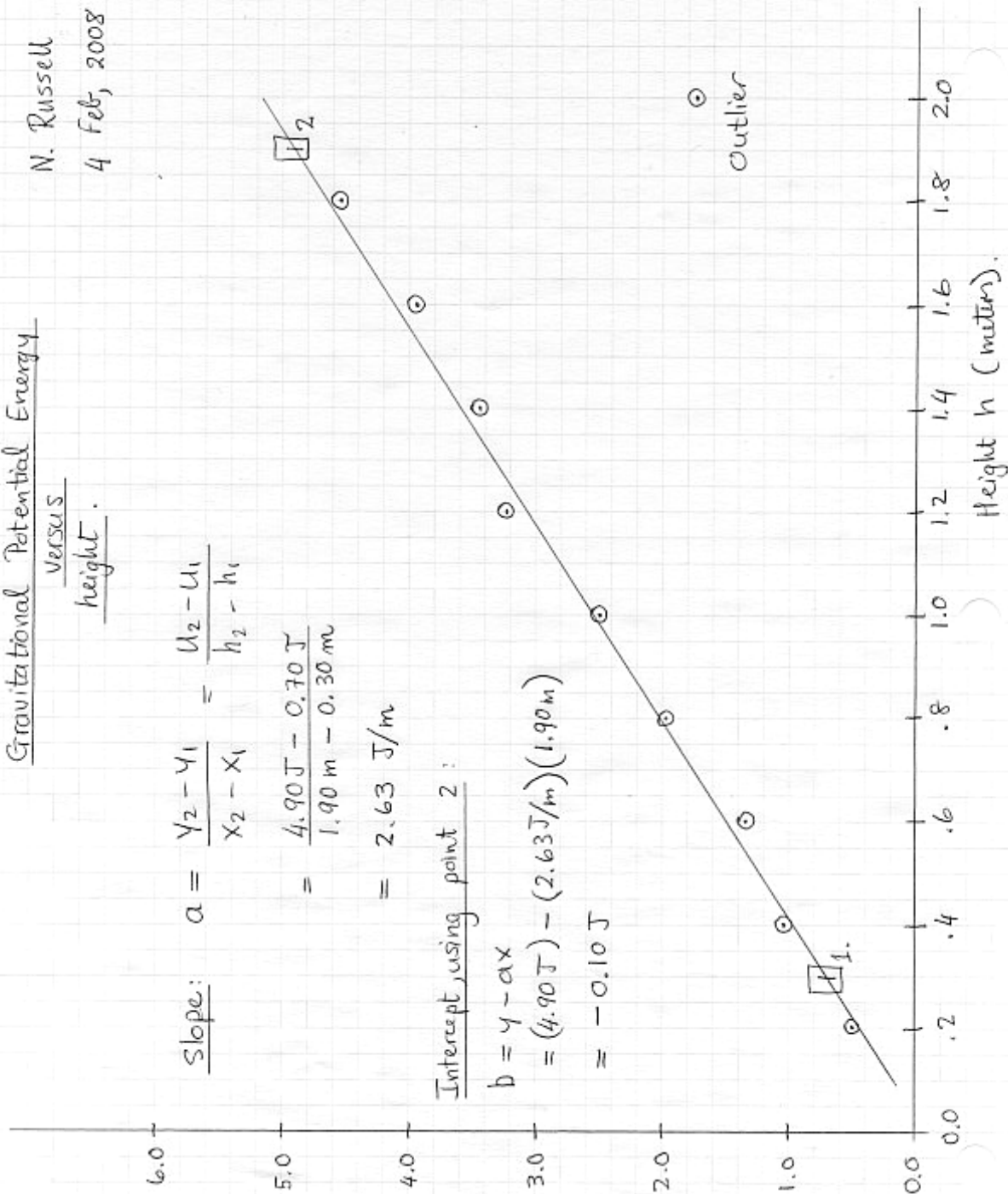
N. Russell  
4 Feb, 2008

(Gravitational Potential Energy,  $U$ , in Joules.

$$\begin{aligned}\text{slope: } a &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{U_2 - U_1}{h_2 - h_1} \\ &= \frac{4.90 \text{ J} - 0.70 \text{ J}}{1.90 \text{ m} - 0.30 \text{ m}} \\ &= 2.63 \text{ J/m}\end{aligned}$$

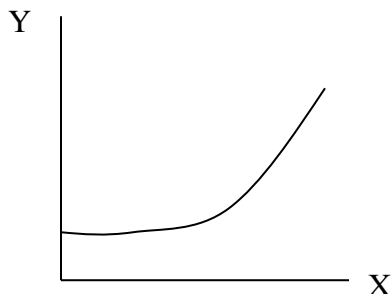
Intercept, using point 2:

$$\begin{aligned}b &= y - ax \\ &= (4.90 \text{ J}) - (2.63 \text{ J/m})(1.90 \text{ m}) \\ &= -0.10 \text{ J}\end{aligned}$$



### 3. Curved graphs

Often the plotted data will be curved. The eye can't tell one type of curve from another just by looking.



For example, it could be one of ...

$$Y = k\theta^n \quad (\text{power-law relationship})$$

$$Y = ke^{cX} \quad (\text{exponential relationship})$$

In the first equation, the plotted variables are  $Y$  and  $\theta$ , and the other two quantities,  $k$  and  $n$ , are constants. In the second equation, the plotted variables are  $Y$  and  $X$ , and the constants are  $k$  and  $c$ . Also, recall that  $e$  is the base of the natural logarithm and has value 2.71828...

When experimental data are plotted, the numerical values of the constants are often needed to find the results. There are many different approaches to extracting the constants from a graph. We do it in a couple of ways by plotting a new graph that converts the data into a straight line. The constants can then be found by calculating the slope and intercept of this line.

### 4. A parabola

A parabola is a power-law relationship where  $n = 2$ , so the one variable is proportional to the square of the other.

time (sec)	mass (kg)		
2.0	4		
3.0	21		
4.0	43		
5.0	43		
6.0	74		
7.0	90		
8.0	126		
9.0	163		
10.0	194		
11.0	230		

The data in this table is hypothesized to follow the equation  $m = \frac{1}{2} q t^2$ . Since measurements are always limited in their accuracy, you should not expect the plot to form a perfect parabola because of this randomness.

Here are two ways to find the value of the constant  $q$ . Both involve matching the plotted quantities with the  $x$  and  $y$  of the straight line,  $y = mx + b$ .

1. Plotting  $m$  versus  $t^2$ . Your instructor will discuss why we expect a graph of  $m$  versus  $t^2$  to follow the trend of a straight line through the origin. If you plot the graph, the slope should be the value of the constant multiplying  $t^2$ , in other words:  
 $slope = \frac{1}{2}q$ . From this you can find the value of the constant  $q$  in the experiment.
2. Plotting  $\sqrt{m}$  versus  $t$ . Your instructor will discuss why this is also expected to be a straight line if the hypothesis is correct. The constant  $q$  can be found again, but pay attention to the logic needed, since it is slightly different this time.

**What to hand in:**

1. Your graph of  $R$  versus  $v$  in the linear plots section. Include your calculation of the slope and the intercept, neatly written out, on a separate page.
2. One of the two plots for the parabola. Include your calculation of the value of  $q$ , neatly written out.

On the top right of your first page, give the Day and Time of the lab and write your name.

For example:

Tue 10am, Maggie Belenkins

Use a staple to keep your pages together in the correct order and avoid lost sheets. Points will be awarded for neatness and following the instructions.