Contents

1 Faire du calcul numérique (à une dimension)

1.1 Environnement de calcul : les modules python

Le module `numpy` regroupe `fonctions constantes` et `méthodes` utiles aux traitement numériques réalisés dans ce TP. Des fonctions avancées de traitement mathématiques et graphiques pourront être utilisés via les modules `matplotlib` et `scipy`

Calcul vectoriel : consiste à réaliser des opérations vectorielles ou matricielles plut que des boucles (`for`, `while`) classiquement utilisées pour les listes. Ces opérations ont été programmées dans des langages de programmations plus rapides (`C` pour `numpy` et `C` et `Fortran` pour `scipy`)

```
import numpy as np
L = []
for i in range(100000000):
```

```
    L.append(i)
a = np.array(L)
```

```
import numpy as np
a = np.arange(100000000)
```

Les `ndarray` et les fonctions `numpy` permettent d'accélérer les calculs et sont très utiles dans le contexte de traitement de gros volumes de données.

1.1.1 remarque timer

```
import timeit
timeit.timeit('"-".join(str(n) for n in range(100))', number=10000)
```

0.18152295495383441

1.2 Tracés de graphiques

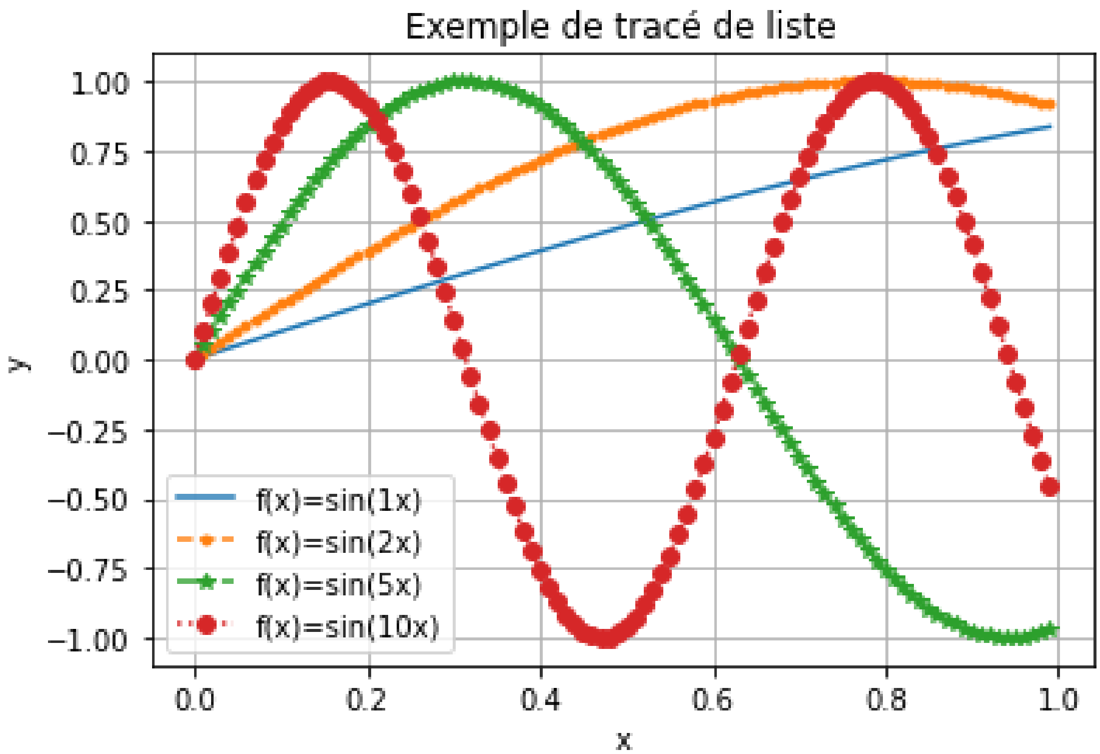
```
import matplotlib.pyplot as plt
```

Le sous module `pyplot` et sa fonction principale `plot` utilisent une syntaxe proche de celle utilisée par les logiciels de calcul numérique courant `MATLAB/Scilab`

Voici un petit exemple pour rappel :

```
style_lign = ['solid', 'dashed', 'dashdot', 'dotted']
style_mark = [ ' ', '.', '*', 'o']
```

```
k = [ 1,2,5,10 ]
for i in range(len(k)):
    x = []
    y = []
    for j in range(100):
        x.append(j/100)
        y.append(np.sin(k[i] * x[j]))
    plt.plot(x,y,linestyle = style_lign[i],
             marker=style_mark[i], label = 'f(x)=sin('+ str(k[i]) + 'x)')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Exemple de tracé de liste')
plt.legend()
plt.grid()
plt.show()
```



1.3 Résolution numérique d'équations

1.4 Résolution numérique d'équations différentielles

1.4.1 méthode d'Euler (ordre 1)

1.4.2 Méthode d'Euler (ordre 2)

1.5 Applications

1.5.1 tracé d'un courbe paramétrée

1.5.2 mise en oeuvre de la méthode d'Euler

2 Euler et RK

2.1 Mouvement d'une planète

```
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
```

2.2 implémentons une méthode de Runge Kutta

2.2.1 préalable : Euler explicite

```
def euler_explicit(f,x0,t):
    # initialisation du vecteur
    x = np.zeros((len(t),len(x0)))
    # données initiale
```

```
    x[0] = x0
    # boucle en temps
    for i in range(len(t)-1):
        x[i+1] = x[i] + ( t[i+1] - t[i] * f(t[i],x[i]) )
    return x
```

2.2.2 méthode de Runge-Kutta

Le code est pratiquement identique a celui écrit plut tôt pour la méthode d'Euler explicite

```
# M est une fonction Méthode
def integrate(f,x0,t,M):
    # initialisation du vecteur
    x = np.zeros( (len(t),) + x0.shape)
    # données initiale
    x[0] = x0
    # boucle en temps
    for i in range(len(t)-1):
        x[i+1] = x[i] + M( f, t[i] , x[i] , t[i+1]-t[i] )
    return x

def euler(g,t,x,h):
    return h * g(t,x)

def rk2(g,t,x,h):
    return h * g(t+h/2, x+h/2 * f(t,x))

def rk4(g,t,x,h):
    k1 = g(t,x)
    k2 = g(t+h/2, x+h/2*k1)
    k3 = g(t+h/2, x+h/2*k2)
    k4 = g(t+h, x+h*k3)
    return h/6*(k1+2*k2+2*k3+k4)
```

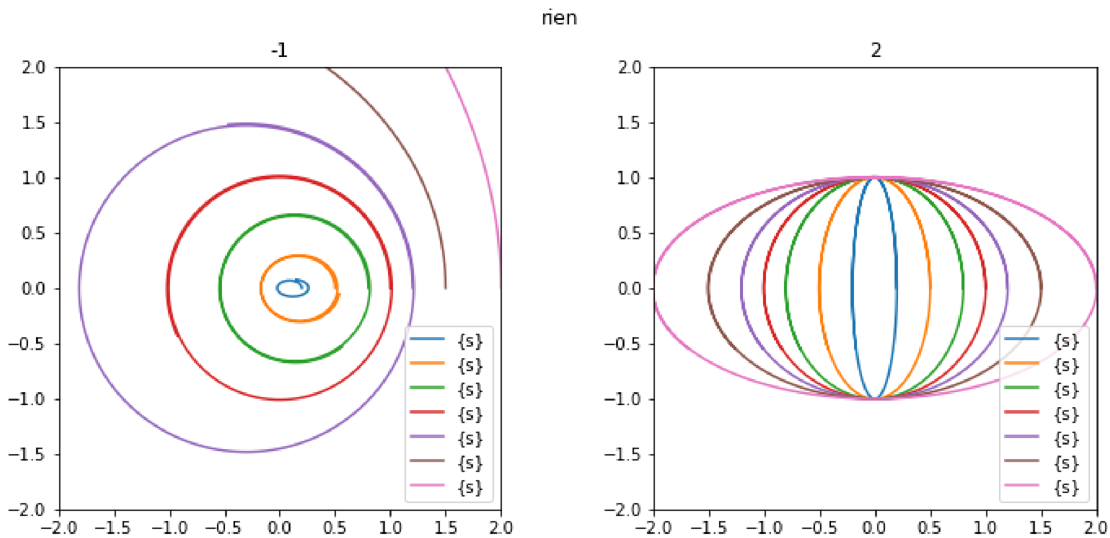
2.3 application à l'étude du mouvement d'une palanète

```
def f(t,xp):
    dxp = xp.copy()
    dxp[0:2] = xp[2:4]
    dxp[2:4] = -xp[0:2] * np.linalg.norm(xp[0:2])**(alpha-2)
    return dxp
```

2.3.1 Obtention de trajectoires fermées et bornées

```
fig = plt.figure(figsize=(12,5))
fig.suptitle(r'rien')

for i,alpha in enumerate([-1,2]):
    sub = fig.add_subplot(1,2,i+1)
    sub.set_title(f"{alpha}")
    for s in [0.2, 0.5, 0.8, 1, 1.2, 1.5, 2]:
        x0 = np.array([s,0,0,1])
        if alpha == -1:
            t = np.linspace(0,10*s**2, 10000)
        elif alpha == 2:
            t = np.linspace(0,10, 10000)
        sol = integrate(f,x0,t,euler)

        sub.plot(sol[:,0],sol[:,1], label="{s}")
    sub.set_xlim([-2,2])
    sub.set_ylim([-2,2])
```

```
    sub.set_aspect('equal')
    sub.legend()
    plt.savefig("test.png")
```

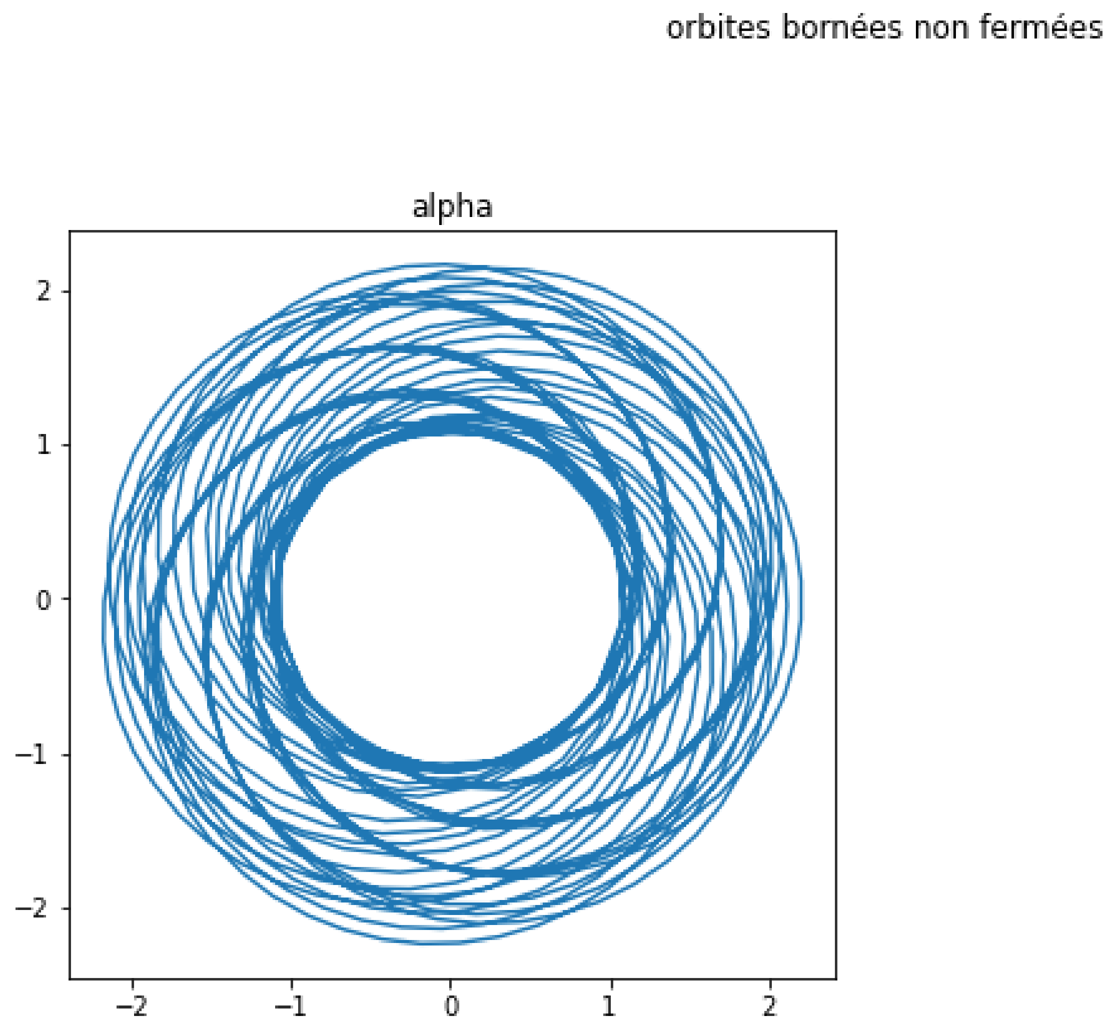



```
for value in enumerate([1,2,3,4]):
    print(value)
    # print(count, value)
```

(0, 1) (1, 2) (2, 3) (3, 4)

2.4 obtention de trajectoires bornées non fermées

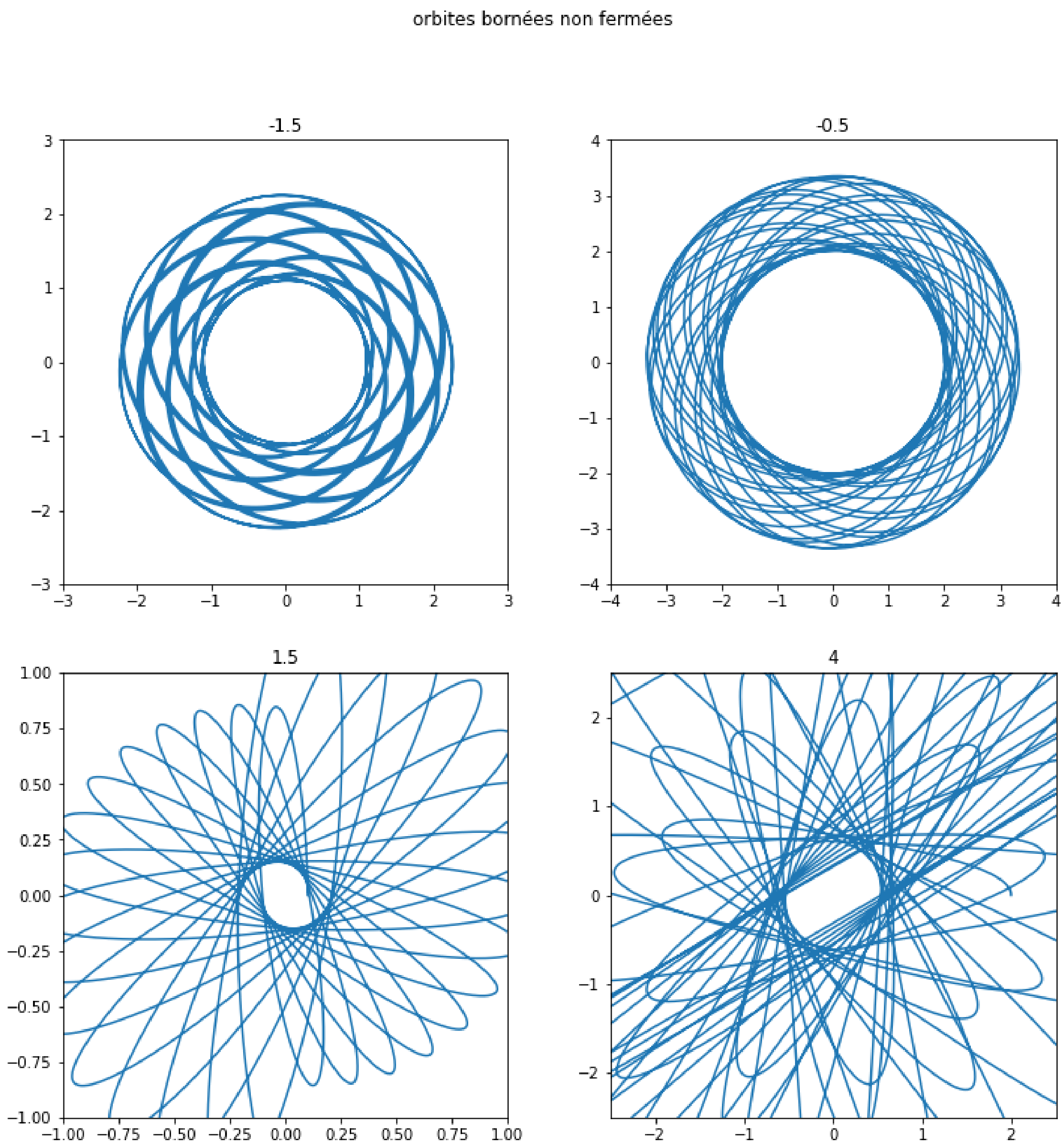
```
fig = plt.figure(figsize=(12,12))
fig.suptitle('orbites bornées non fermées')
alpha = -1.5
sub = fig.add_subplot(2,2,1)
sub.set_title("alpha")
x0 = np.array([1.1,0,0,1])
t = np.linspace(0,500,1000)

sol = integrate(f,x0,t,rk4)
sub.plot(sol[:,0], sol[:,1])
plt.savefig("test.png")
```




```
fig = plt.figure(figsize=(12,12))
fig.suptitle('orbites bornées non fermées')

alpha = -1.5
sub = fig.add_subplot(2,2,1)
sub.set_title(f"{alpha}")

x0 = np.array([1.1,0,0,1])
t = np.linspace(0,500,10000)
sol = integrate(f,x0,t,rk4)
sub.plot(sol[:,0], sol[:,1])
sub.set_xlim([-3,3])
sub.set_ylim([-3,3])
sub.set_aspect('equal')
#
alpha = -0.5
sub = fig.add_subplot(2,2,2)
sub.set_title(f"{alpha}")
x0 = np.array([2,0,0,1])
t = np.linspace(0,500,10000)
sol = integrate(f,x0,t,rk4)
sub.plot(sol[:,0], sol[:,1])
sub.set_xlim([-4,4])
sub.set_ylim([-4,4])
sub.set_aspect('equal')
```

```python
#
alpha = 1.5
sub = fig.add_subplot(2,2,3)
sub.set_title(f"{alpha}")
x0 = np.array([0.1,0,0,1])
t = np.linspace(0,95,10000)
sol = integrate(f,x0,t,euler)
sub.plot(sol[:,0], sol[:,1])
sub.set_xlim([-1,1])
sub.set_ylim([-1,1])
sub.set_aspect('equal')


#
alpha = 4
sub = fig.add_subplot(2,2,4)
sub.set_title(f"{alpha}")
x0 = np.array([2,0,0,1])
t = np.linspace(0,95,10000)
sol = integrate(f,x0,t,euler)
sub.plot(sol[:,0], sol[:,1])
sub.set_xlim([-2.5,2.5])
sub.set_ylim([-2.5,2.5])
sub.set_aspect('equal')

plt.savefig("test.png")
```

<ipython-input-5-4cdac2502d30>:4: RuntimeWarning: overflow encountered in multiply dxp[2:4] = -xp[0:2] * np.linalg.norm(xp[0:2])**(alpha-2) <ipython-input-4-d934a3694110>:9: RuntimeWarning: invalid value encountered in add x[i+1] = x[i] + M(f, t[i] , x[i] , t[i+1]-t[i])



3 Sondage

3.1 Imports Constantes et Données

```
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
#import json
#import csv

M = 29.0e-3
R = 8.31

P0 = 1.0e5
g0 = 9.8

```

```
RT = 6.4e6
pi = np.pi

zexp = np.array([0.0, 5.0, 10.0, 12.0, 20.0, 25.0, 30.0, 35.0, 40.0, 45.0, 48.0, 52.0, 55.0, 60.0, 65.0, 70.0, 75.0, 80.0, 84.0,

Texp = np.array([15.0, -18.0, -49.0, -56.0, -56.0, -51.0, -46.0, -37.0, -22.0, -8.0, -2.0, -2.0, -7.0, -17.0, -33.0, -54.0, -65.
```

3.2 Interpolation

```
def T(z,unite):
    z_km = z / 1000 #conversion
    alpha = 1 # facteur de conversion

    if unite == 'C':
        alpha = 0

    i = 0
    while z_km > zexp[i+1]: # recherche de l'indice i
        i = i + 1

    rate =  ( Texp[i+1] - Texp[i] ) / ( zexp[i+1] - zexp[i] )
    temperature = alpha*273 + Texp[i] + rate * (z_km - zexp[i])
    return temperature
```

3.3 Temperature

```
N = 10000
zmax = 100.0e3
dz = zmax / (N-1)
print(N, zmax, dz)
zatm = np.array([ k * dz for k in range(N) ])
Tatm = np.array([ T(zatm[k], 'C') for k in range(N) ])
TatmK = np.array([ T(zatm[k], 'K') for k in range(N) ])
gatm = np.array([ g(zatm[k]) for k in range(N)])
```

10000 100000.0 10.001000100010002

NameErrorTraceback (most recent call last) <ipython-input-12-2c84e93431ff> in <module> 6 Tatm = np.array([T(zatm[k], 'C') for k in range(N)]) 7 TatmK = np.array([T(zatm[k], 'K') for k in range(N)]) —-> 8 gatm = np.array([g(zatm[k]) for k in range(N)])

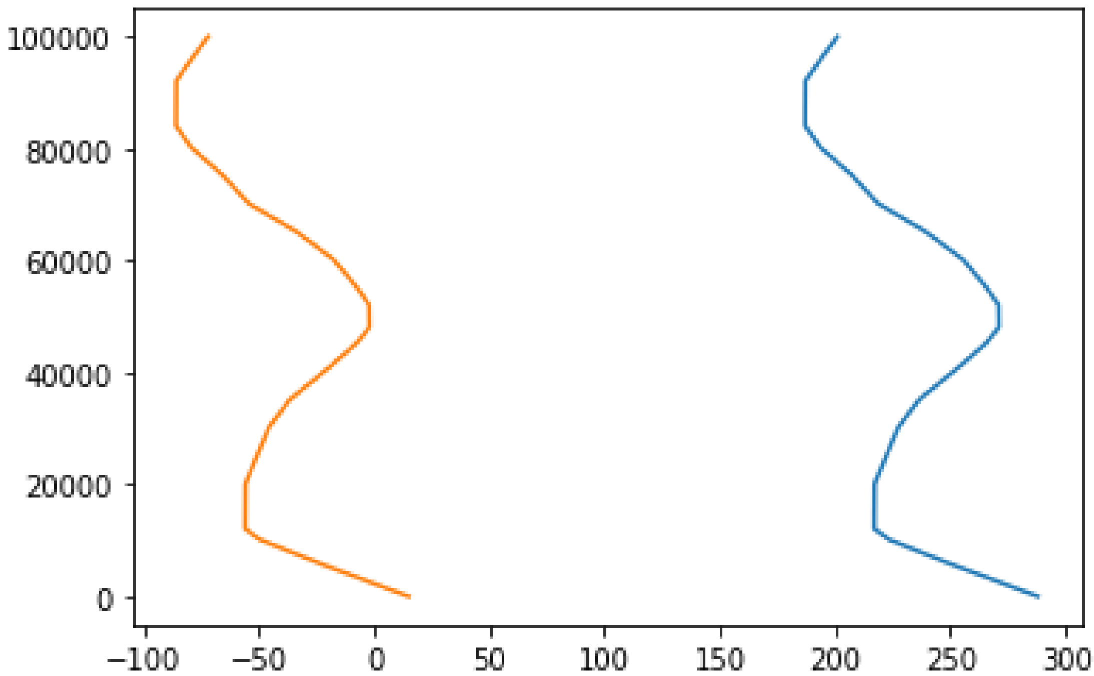
<ipython-input-12-2c84e93431ff> in <listcomp>(.0) 6 Tatm = np.array([T(zatm[k], 'C') for k in range(N)]) 7 TatmK = np.array([T(zatm[k], 'K') for k in range(N)]) —-> 8 gatm = np.array([g(zatm[k]) for k in range(N)])

NameError: name 'g' is not defined

3.4 Champ de pesanteur

```
def g(z):
    return g0 * RT**2 / (RT + z)**2
    return g0
```

```
fig, ax = plt.subplots()
ax.plot( TatmK,zatm)
ax.plot( Tatm,zatm)
plt.savefig("ffffffffff")
```

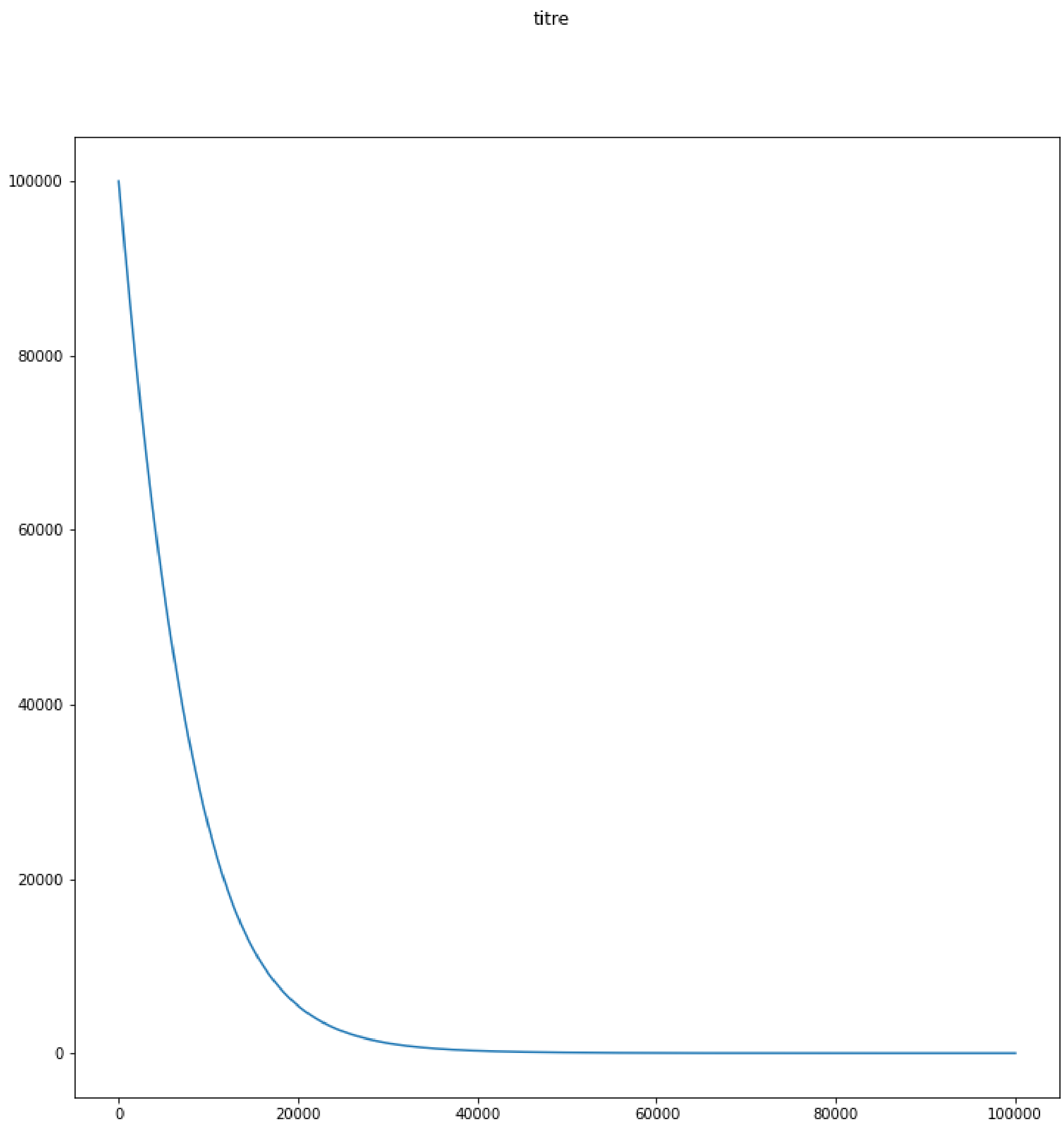


3.5 Pression

```python
Patm = [P0]
gatm = [g0]
deltap = 0
gradient = 0
for k in range(N-1):
    gradient = - M * g(zatm[k]) / (R * TatmK[k] )
    deltap = gradient * dz * Patm[k]

    Patm.append( Patm[k] + deltap )
#    gatm.append( gatm[k] )
Patm = np.array(Patm)
print(M,R,P0,g0,RT)
```

0.029 8.31 100000.0 9.8 6400000.0

```python
fig = plt.figure(figsize=(12,12))
fig.suptitle('titre')

plt.plot(zatm,Patm)
plt.savefig("test.png")
```




```python
def masse_atm(z):
    masse = 0
    k = 0

    Cte = 4*pi*M/R
    while zatm[k] < z:
        dm = Cte * (RT + z)**2 * Patm[k] / T(zatm[k],'K') * dz
        masse = masse + dm
        k = k+1
    return masse
```

```python
mtot = masse_atm(100e3)
print(mtot)
```

5.43005982435075e+18

```python
mtropo = masse_atm(15e3)
print(mtropo/mtot)
```

0.8571666627299532

```python
print(zatm)
```

[0.00000000e+00 1.00010001e+01 2.00020002e+01 … 9.99799980e+04 9.99899990e+04 1.00000000e+05]
test

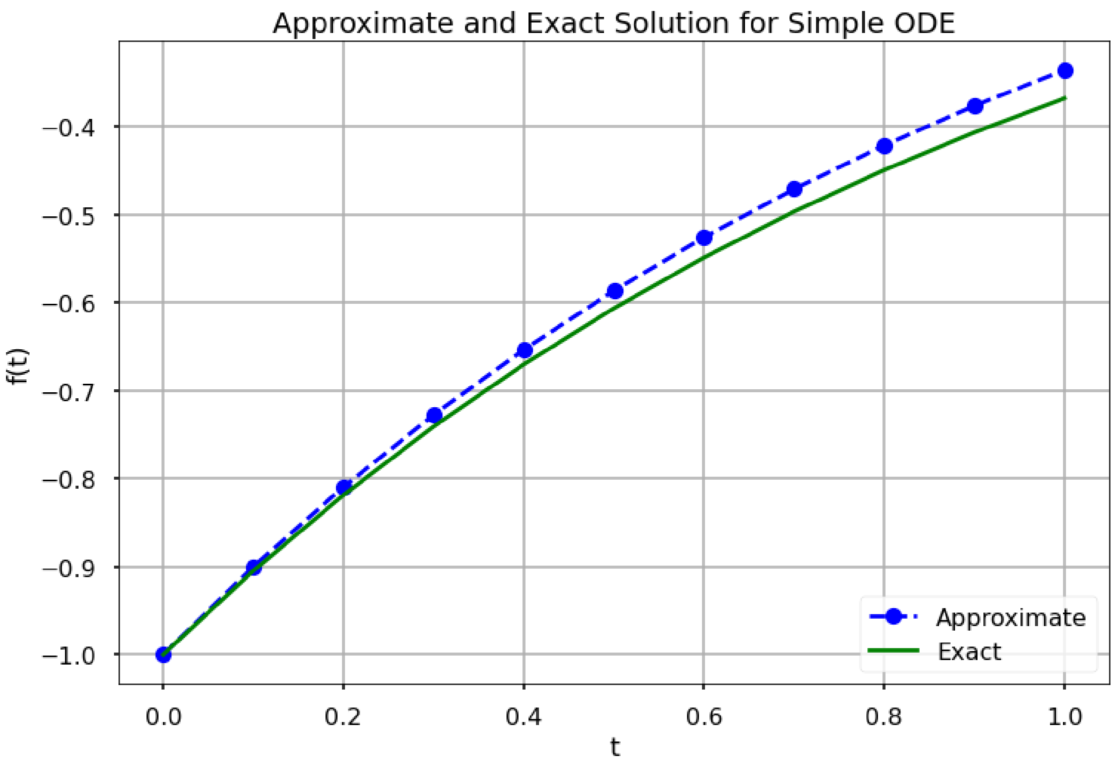
```python
vec = np.array([ masse_atm(zatm[k]) for k in range(10) ])
print(vec)
```

[0.00000000e+00 6.23762249e+15 1.24693043e+16 1.86950498e+16 2.49148633e+16 3.11287491e+16 3.73367115e+16 4.35387549e+16 4.97348836e+16 5.59251018e+16]

4 Python Numerical Methods (Berkeley)

```python
import numpy as np
import matplotlib.pyplot as plt

plt.style.use('seaborn-poster')
%matplotlib inline

# Define parameters
f = lambda t, s: np.exp(-t) # ODE
h = 0.1 # Step size
t = np.arange(0, 1 + h, h) # Numerical grid
s0 = -1 # Initial Condition

# Explicit Euler Method
s = np.zeros(len(t))
s[0] = s0

for i in range(0, len(t) - 1):
    s[i + 1] = s[i] + h*f(t[i], s[i])

plt.figure(figsize = (12, 8))
plt.plot(t, s, 'bo--', label='Approximate')
plt.plot(t, -np.exp(-t), 'g', label='Exact')
plt.title('Approximate and Exact Solution \
for Simple ODE')
plt.xlabel('t')
plt.ylabel('f(t)')
plt.grid()
plt.legend(loc='lower right')
plt.show()
```

<ipython-input-14-53e864c3c24f>:4: MatplotlibDeprecationWarning: The seaborn styles shipped by Matplotlib are deprecated since 3.6, as they no longer correspond to the styles shipped by seaborn. However, they will remain available as 'seaborn-v0_8-<style>'. Alternatively, directly use the seaborn API instead. plt.style.use('seaborn-poster')



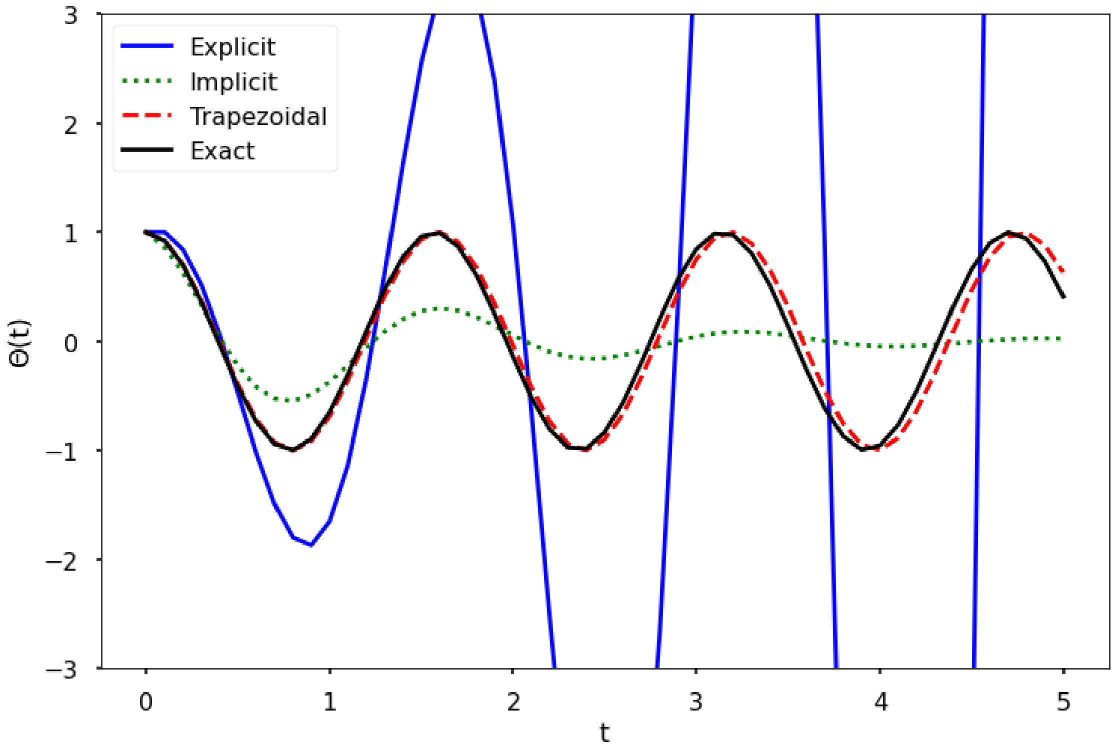

```python
import numpy as np
from numpy.linalg import inv
import matplotlib.pyplot as plt

plt.style.use('seaborn-poster')

%matplotlib inline
```

<ipython-input-15-bcef546ae31a>:5: MatplotlibDeprecationWarning: The seaborn styles shipped by Matplotlib are deprecated since 3.6, as they no longer correspond to the styles shipped by seaborn. However, they will remain available as 'seaborn-v0_8-<style>'. Alternatively, directly use the seaborn API instead. plt.style.use('seaborn-poster')

```python
# define step size
h = 0.1
# define numerical grid
t = np.arange(0, 5.1, h)
# oscillation freq. of pendulum
w = 4
s0 = np.array([[1], [0]])

m_e = np.array([[1, h],
               [-w**2*h, 1]])
m_i = inv(np.array([[1, -h],
               [w**2*h, 1]]))
m_t = np.dot(inv(np.array([[1, -h/2],
    [w**2*h/2,1]])), np.array(
      [[1,h/2], [-w**2*h/2, 1]]))

s_e = np.zeros((len(t), 2))
s_i = np.zeros((len(t), 2))
s_t = np.zeros((len(t), 2))
```

```python

# do integrations
s_e[0, :] = s0.T
s_i[0, :] = s0.T
s_t[0, :] = s0.T

for j in range(0, len(t)-1):
    s_e[j+1, :] = np.dot(m_e,s_e[j, :])
    s_i[j+1, :] = np.dot(m_i,s_i[j, :])
    s_t[j+1, :] = np.dot(m_t,s_t[j, :])

plt.figure(figsize = (12, 8))
plt.plot(t,s_e[:,0],'b-')
plt.plot(t,s_i[:,0],'g:')
plt.plot(t,s_t[:,0],'r--')
plt.plot(t, np.cos(w*t), 'k')
plt.ylim([-3, 3])
plt.xlabel('t')
plt.ylabel('$\Theta (t)$')
plt.legend(['Explicit', 'Implicit', \
            'Trapezoidal', 'Exact'])
plt.show()
```

findfont: Font family ['STIXGeneral'] not found. Falling back to DejaVu Sans. findfont: Font family ['STIXGeneral'] not found. Falling back to DejaVu Sans. findfont: Font family ['STIXGeneral'] not found. Falling back to DejaVu Sans. findfont: Font family ['STIXNonUnicode'] not found. Falling back to DejaVu Sans. findfont: Font family ['STIXNonUnicode'] not found. Falling back to DejaVu Sans. findfont: Font family ['STIXNonUnicode'] not found. Falling back to DejaVu Sans. findfont: Font family ['STIXSizeOneSym'] not found. Falling back to DejaVu Sans. findfont: Font family ['STIXSizeTwoSym'] not found. Falling back to DejaVu Sans. findfont: Font family ['STIXSizeThreeSym'] not found. Falling back to DejaVu Sans. findfont: Font family ['STIXSizeFourSym'] not found. Falling back to DejaVu Sans. findfont: Font family ['STIXSizeFiveSym'] not found. Falling back to DejaVu Sans. findfont: Font family ['cmsy10'] not found. Falling back to DejaVu Sans. findfont: Font family ['cmr10'] not found. Falling back to DejaVu Sans. findfont: Font family ['cmtt10'] not found. Falling back to DejaVu Sans. findfont: Font family ['cmmi10'] not found. Falling back to DejaVu Sans. findfont: Font family ['cmb10'] not found. Falling back to DejaVu Sans. findfont: Font family ['cmss10'] not found. Falling back to DejaVu Sans. findfont: Font family ['cmex10'] not found. Falling back to DejaVu Sans. findfont: Font family ['DejaVu Sans Display'] not found. Falling back to DejaVu Sans.

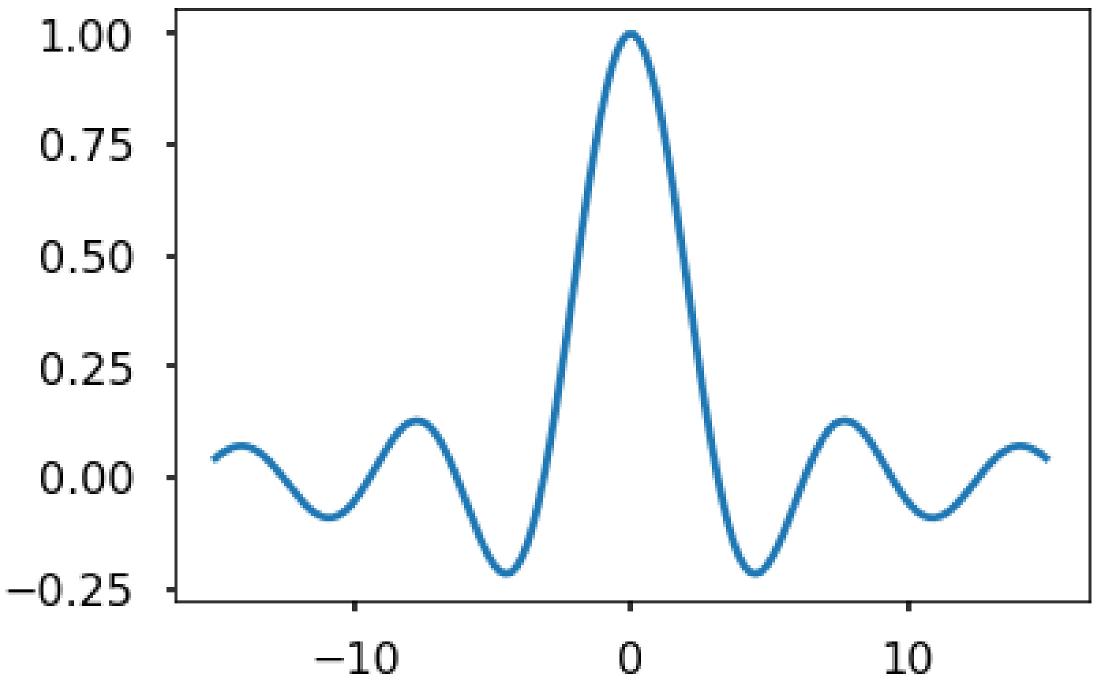


5 TEST Casamayou

```
import os

def nrange(a, b, numpoints):
    """Renvoie une subdivision de [a, b] à N+1 points."""
    pas = (b - a) / numpoints
    return (a + i * pas for i in range(numpoints + 1))

def srange(a, b, pas):
    """Renvoie une subdivision de [a, b] avec un pas donné."""
    numpoints = int((b - a) / pas)
    return (a + i * pas for i in range(numpoints + 1))

def preambule(nomFichier, boite, zoom, delta):
    """ Écrit le préambule du fichier EPS."""
    cadre = [x * zoom * delta for x in boite]
    s_debut = ("%!PS-Adobe-2.0 EPSF-2.0\n"
    "%%BoundingBox: {0[0]:.1f} {0[1]:.1f} {0[2]:.1f} {0[3]:.1f}\n"
    "{1} {1} scale\n").format(cadre, zoom)
    with open(nomFichier + ".eps", 'w') as f:
        f.write(s_debut)

def fin(nomFichier):
    """ Cloture le fichier EPS."""
    s_fin = "\nshowpage\n"
    with open(nomFichier + ".eps", 'a') as f:
        f.write(s_fin)

def ajouteCourbe(nomFichier, liste, boite, zoom, epaisseurTrait, rgb):
    """Ajoute une courbe donnée sous forme de liste."""
    with open(nomFichier + ".eps", 'a') as f:
        f.write("\nnewpath\n")
        for i, point in enumerate(liste):
```

```python
            if i == 0:
                f.write("    {0[0]: .4f}  {0[1]: .4f}   ".format(point))
                f.write("moveto\n")
            elif (boite[0] <= point[0] <= boite[2]
                    and boite[1] <= point[1] <= boite[3]):
                f.write("    {0[0]: .4f}  {0[1]: .4f}   ".format(point))
                f.write("lineto\n")
        f.write("{1} {0} div setlinewidth\n"
                "{2[0]} {2[1]} {2[2]} setrgbcolor\n"
                "stroke\n".format(zoom, epaisseurTrait, rgb))

def affiche(nomFichier):
    """Affiche le graphique via ghostview."""
    os.system("gv {0}.eps &".format(nomFichier))

if __name__ == "__main__":
    from math import pi, cos, sin, floor

    N, a, b = 1000, 0, 1 + floor(38 * pi)
    nomFichier, zoom, epaisseurTrait = "polar", 100, 0.4
    rgb = (0, 0, 1) # tracé en bleu
    # rgb = (0.2, 0.2, 0.2) # tracé en gris
    boite = [-1.5, -1.5, 1.5, 1.5] # xmin , ymin, xmax, ymax

    # Fonction définissant la courbe polaire
    def f(theta):
        return 1 + cos(theta*20/19) / 3

    # Liste de points de la courbe
    liste = ([f(theta) * cos(theta), f(theta) * sin(theta)]
            for theta in nrange(a, b, N))

    # Création du fichier EPS
    preambule(nomFichier, boite, zoom, 1.1)
    ajouteCourbe(nomFichier, liste, boite, zoom, epaisseurTrait, rgb)
    fin(nomFichier)
    affiche(nomFichier)
```

```python
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(-15, 15, 150)
y = np.sin(x) / x

plt.plot(x, y)
plt.show()
```



5.1 test

```python
import turtle as tt
# Set the background color as black,
# pensize as 2 and speed of drawing
# curve as 10(relative)
tt.bgcolor('grey')
tt.pensize(1)
tt.speed(60)

# Iterate six times in total
for i in range(6):
    # Choose your color combination
    for color in ('magenta','yellow'):
        tt.color(color)
        # Draw a circle of chosen size, 100 here
        for j in range(100):
            tt.circle(j*4)
            # Move 10 pixels left to draw another circle
            tt.left(10)
            # Hide the cursor(or turtle) which drew the circle
            tt.hideturtle()
```

AttributeErrorTraceback (most recent call last) <ipython-input-1-87aeeb7a6e43> in <module> 14 # Draw a circle of chosen size, 100 here 15 for j in range(100): —> 16 tt.square(j*4) 17 # Move 10 pixels left to draw another circle 18 tt.left(10)

AttributeError: module 'turtle' has no attribute 'square'

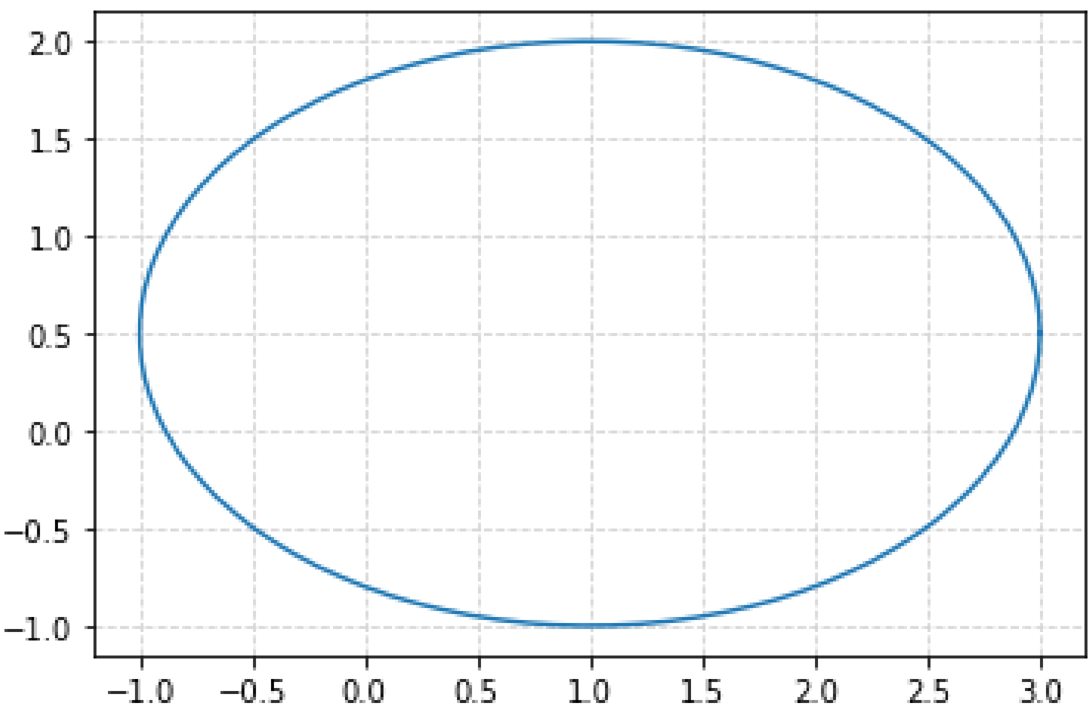
```python
import turtle as tt
tt.bgcolor('grey')
tt.pensize(1)
tt.speed(60)
shape("circle")
shapesize(5,4,1)
```

```
fillcolor("white")
tt.circle()
```

NameErrorTraceback (most recent call last) <ipython-input-1-743e3db4db16> in <module> 3 tt.pensize(1) 4 tt.speed(60) —-> 5 shape("circle") 6 shapesize(5,4,1) 7 fillcolor("white")

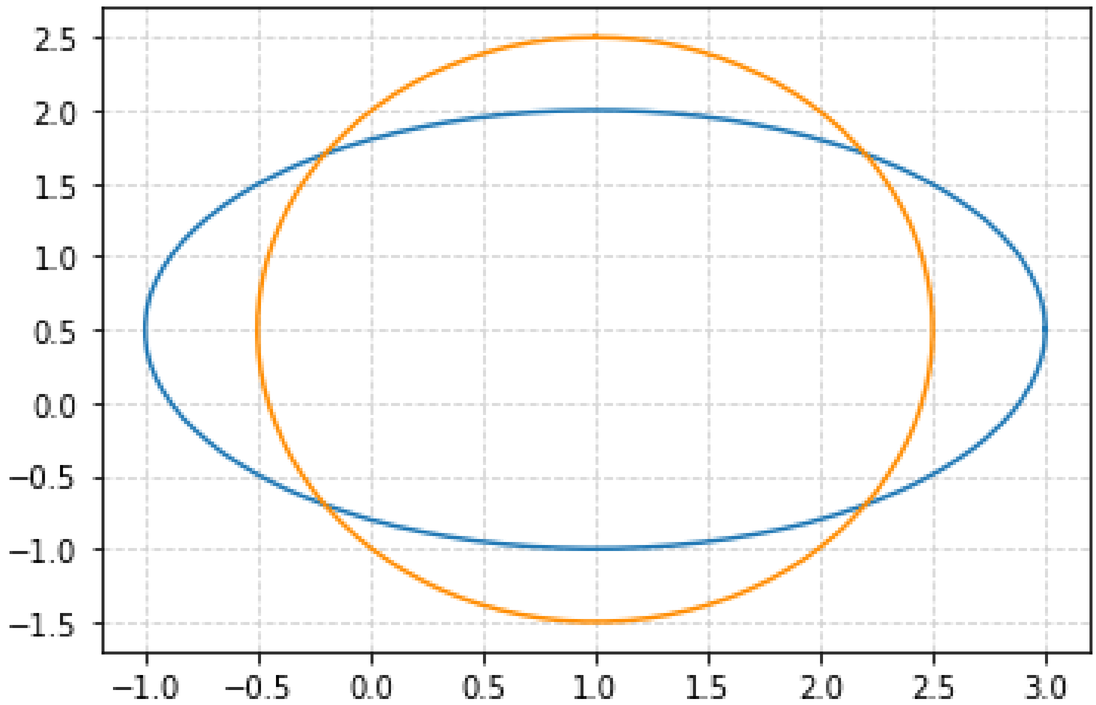
NameError: name 'shape' is not defined

```
import numpy as np
from matplotlib import pyplot as plt
from math import pi

u=1.     #x-position of the center
v=0.5    #y-position of the center
a=2.     #radius on the x-axis
b=1.5    #radius on the y-axis

t = np.linspace(0, 2*pi, 100)
plt.plot( u+a*np.cos(t) , v+b*np.sin(t) )
plt.grid(color='lightgray',linestyle='--')
plt.show()
```




```
import numpy as np
from matplotlib import pyplot as plt
from math import pi, cos, sin

u=1.       #x-position of the center
v=0.5      #y-position of the center
a=2.       #radius on the x-axis
b=1.5      #radius on the y-axis
t_rot=pi/2 #rotation angle

t = np.linspace(0, 2*pi, 100)
Ell = np.array([a*np.cos(t) , b*np.sin(t)])
```

```
    #u,v removed to keep the same center location
R_rot = np.array([[cos(t_rot) , -sin(t_rot)],[sin(t_rot) , cos(t_rot)]])
    #2-D rotation matrix

Ell_rot = np.zeros((2,Ell.shape[1]))
for i in range(Ell.shape[1]):
    Ell_rot[:,i] = np.dot(R_rot,Ell[:,i])

plt.plot( u+Ell[0,:] , v+Ell[1,:] )     #initial ellipse
plt.plot( u+Ell_rot[0,:] , v+Ell_rot[1,:],'darkorange' )    #rotated ellipse
plt.grid(color='lightgray',linestyle='--')
plt.show()
```

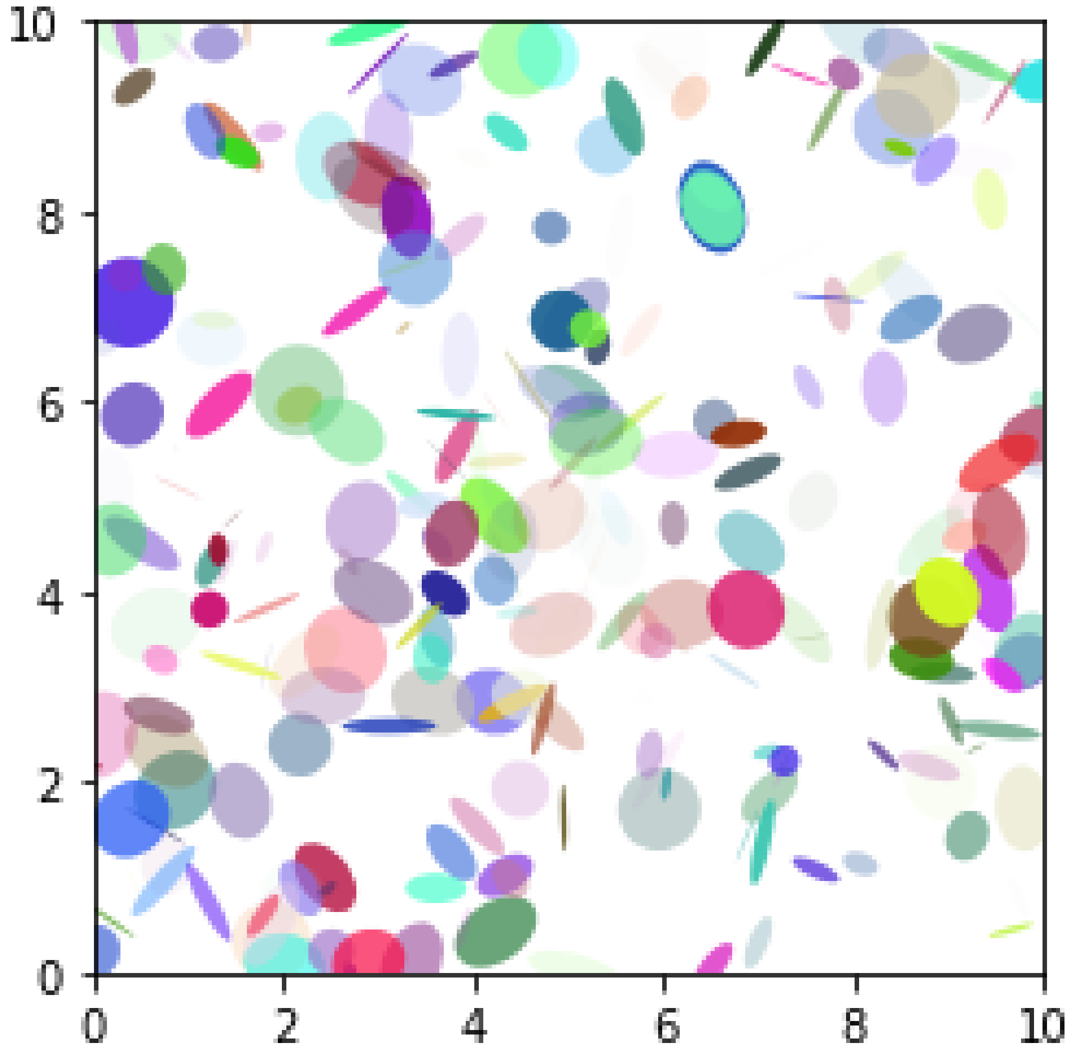



```
from matplotlib.patches import Ellipse

plt.figure()
ax = plt.gca()

ellipse = Ellipse(xy=(157.18, 68.4705), width=0.036, height=0.012,
                        edgecolor='r', fc='None', lw=2)
ax.add_patch(ellipse)
```

```
<matplotlib.patches.Ellipse at 0x7f288c6ad610>
```

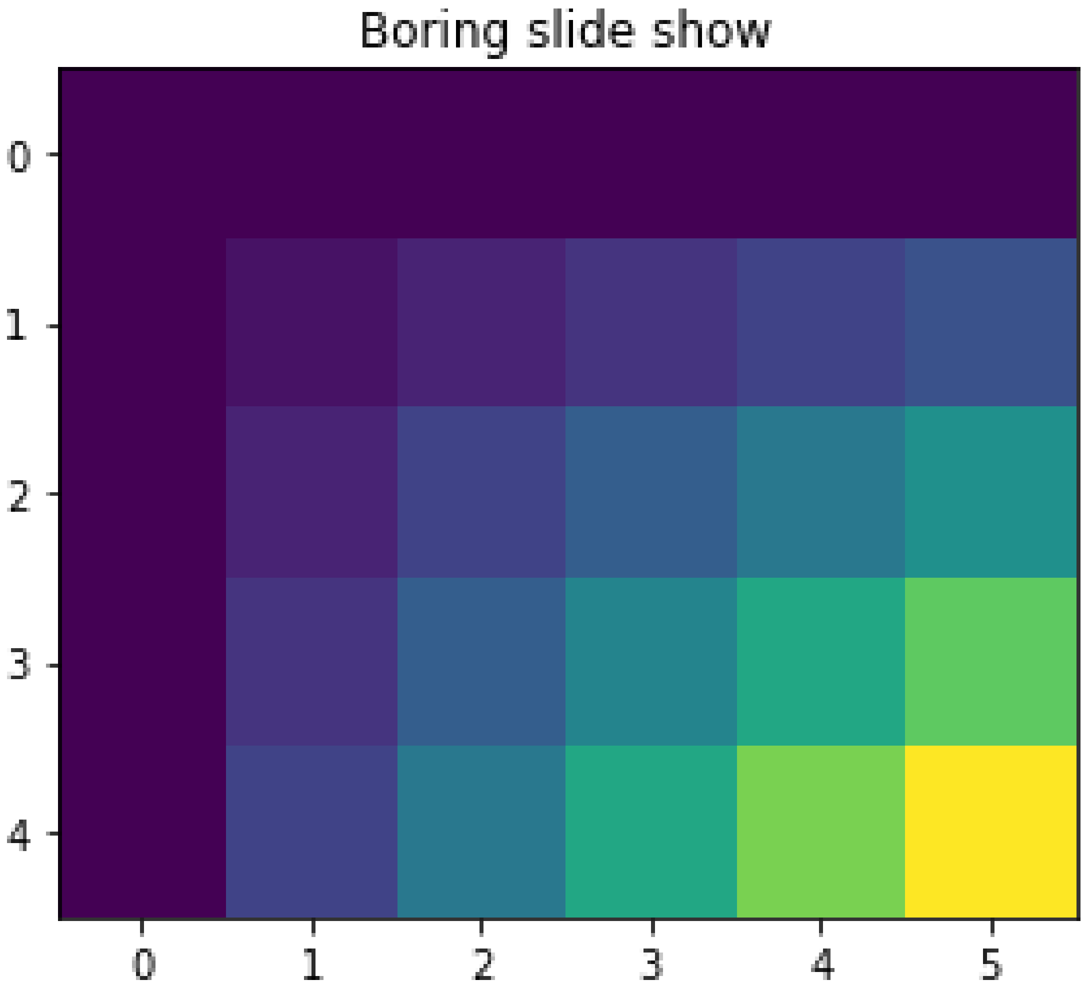
./.ob-jupyter/61d2780d035746e2a1d41c3696837d1141c39f65.png

```
import matplotlib.pyplot as plt
import numpy.random as rnd
from matplotlib.patches import Ellipse

NUM = 250

ells = [Ellipse(xy=rnd.rand(2)*10, width=rnd.rand(), height=rnd.rand(), angle=rnd.rand()*360)
        for i in range(NUM)]

fig = plt.figure(0)
ax = fig.add_subplot(111, aspect='equal')
for e in ells:
    ax.add_artist(e)
    e.set_clip_box(ax.bbox)
    e.set_alpha(rnd.rand())
    e.set_facecolor(rnd.rand(3))

ax.set_xlim(0, 10)
ax.set_ylim(0, 10)

plt.show()
```




```python
import matplotlib.pyplot as plt
import numpy as np

x = np.arange(6)
y = np.arange(5)
z = x * y[:, np.newaxis]

for i in range(5):
    if i == 0:
        p = plt.imshow(z)
        fig = plt.gcf()
        plt.clim()    # clamp the color limits
        plt.title("Boring slide show")
    else:
        z = z + 2
        p.set_data(z)

    print("step", i)
    plt.pause(0.5)
```

step 0



step 1 step 2 step 3 step 4

```python
import matplotlib.pyplot as plt
import numpy as np
from matplotlib.patches import Ellipse

delta = 45.0  # degrees

angles = np.arange(0, 360 + delta, delta)
ells = [Ellipse((1, 1), 4, 2, a) for a in angles]

a = plt.subplot(111, aspect='equal')

for e in ells:
    e.set_clip_box(a.bbox)
    e.set_alpha(0.1)
    a.add_artist(e)

plt.xlim(-2, 4)
plt.ylim(-1, 3)

plt.show()
```

<ipython-input-4-de3ee4e9bc8b>:8: MatplotlibDeprecationWarning: Passing the angle parameter of __init__() positionally is deprecated since Matplotlib 3.6; the parameter will become

keyword-only two minor releases later. ells = [Ellipse((1, 1), 4, 2, a) for a in angles]