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BACHELOR'S THESIS

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# Study of Domain Wall Dynamics under Nonlocal Spin-Transfer Torque using heterogeneous computing

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## *Abstract*

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### **Study of Domain Wall Dynamics under Nonlocal Spin-Transfer Torque using heterogeneous computing**

by Thomas Sanchez Lengeling

This work is an exploration of the role that Graphical Processing Units, also known as GPUs, can play in the acceleration of physical simulations. In particular, in the research of spintronic effects such as the dynamics of domain walls under nonlocal spin-transfer torque. Our study is relevant because it allows researchers to quantitatively test some of the effects of a phenomenon known as spin-diffusion on magnetic configurations at the nanoscale. Some of such configurations are known as domain walls. These magnetic configurations can be observed experimentally in NiFe soft nanostripes but they are really complicated to produce and image experimentally. Due to this, we use the massively parallel capabilities of a single GPU to numerically solve a mathematical equation, known as the Zhang-Li model. As a consequence of our implementation, we have observed a 13x speed-up in the solution of the Zhang-Li equation. This speed-up is obtained when we compare the time needed to obtain the result of a simulation in a GPU with that of a simulation with the same input parameters in a conventional processor e.g. Intel Xeon. The numerical method used for the solution is the method known as Finite Differences in the Time Domain (FDTD) whose integration is done using a 4th order Runge - Kutta integration

# *Acknowledgements*

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# Introduction

Commodity graphics processing units (GPUs) are becoming increasingly popular to accelerate scientific applications due to their low cost and potential for high performance when compared with central processing units (CPUs). A large number of contemporary problems and scientific research are being benefit from this new technology .There has been considerable progress in implementing the hardware and the supporting infrastructure for GPUs programming and streaming architectures. This thesis is a exploration and study of the role of accelerator hardware like the use of the GPUs on physical computing, more specific in the area of spin-diffusion effects within a continuously variable magnetization distribution.

The work begins with a overview of the current trends in computing, focusing our attention specifically on GPUs, on how they differ from the CPUs and common programming practices that uses heterogeneous computing. The second chapter focus on the use of techniques of heterogeneous computing to gain more performance out the GPUs when applying to a specific task. Also the necessary means how to test the speed-up against the CPU. The next chapter is overview of the CUDA code implementation of Dr. Claudio's work "Domain Wall Dynamics under Non-local Spin-Transfer Torque". The forth chapter are the results collected by applying optimization techniques to the initial CUDA code implementation. The outcome is compared by launching the code in-to several GPUs nodes. Finally the last chapter of the thesis is a conclusion of the work and future research.



# Chapter 1

## Heterogeneous Computing

Heterogeneous computing refers a system that combines several processor types to gain more performance. Typically using a single or multi-core computer processing units (CPUs) and a graphics processing units (GPUs). Frequently GPUs are know for 3D graphics rendering, video games and video editing, but GPUs are becoming increasingly popular for accelerating computing applications and scientific research due to their low price, high performance and relatively low energy consumption per FLOPS (floating point operations per second) when compared with the CPUs. This chapter provides an overview of GPUs within the High Performance Computing (HPC) context, their advantages and disadvantages and how they can be integrated in to a scientific software and research.

### 1.1 Motivation

The GPU has been essential part of personal computer since the early use. Over the course of 30 years the graphics architecture has evolve form drawing a simple 3D scene to be able to program each part of the GPU graphics pipeline. Their role became more important in the 90s with the first-person shooting video game DOOM by id Software. The demanding video game industry has brought year by year more realistic 3D graphics. Consequently new innovated hardware capabilities has been developed to increase the graphics pipeline and the render output. This lead to a more sophisticated programming environment with a massive parallel capabilities.

The fixed graphics pipeline (fixed functions on the GPU) was introduced in the early 90s, allowed various customization of the rendering process. However only allowed some modifications of the GPU output. Specific adjustment were extremely complicated did

not allow custom algorithms. In 2001 NVIDIA and ATI (AMD) introduced the first programmability to the graphics pipeline. Which could control millions pixels and vertex output in a single frame, moreover it out-performed the CPU in rendering video. In addition graphics shift from the CPU to the GPU. This was the beginning of GPU parallel capabilities.

At first the GPUs where only used for general-purpose computing like computer graphics, but in-till resent years the GPU has been used to accelerate scientific research, analytics, engineering, robotics and consumer applications.(GPGPU)[10].

GPUs are attractive for certain type of scientific computation as they offer potential seed-up of multi-processors devices with the added advantages of being low cost, low maintenance, energy efficient, and relative simple to program. Many algorithms in applied physics are using GPUs to improve their performance over the CPU. Some area of scientific research that obtain the benefit of heterogenous computing are: Molecular Dynamics, Quantum Chemistry, Computational Structural Mechanics and Computational Physics [15].

In any case, for a given simulation a compromise between speed and accuracy is always made. The current tendency of the CPU relies on increases the clock seeped, decreasing the size of transistor and finally adding more cores per unit and be able to work and a parallel manner, because of the there are some limitations[20]

### Power Wall

The CPUs single core has not gone beyond the 4GHz barrier, a paradigm shift from a single core to a multi-core CPUs, also the power use of CPUs is very high per Watt. The figure 1.1 shows the comparison of performance between the GPU and CPU.

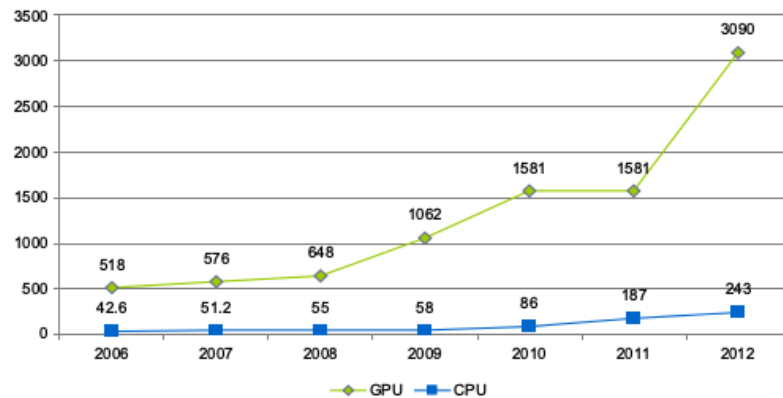


FIGURE 1.1: GPU and CPU peak performance in gigaflops

**Memory Wall**

This refers to the growing disparity of speed between CPU and the memory outside the CPU chip. Some applications have become memory bound, that is to say computing time is bounded by the transfer memory between the CPU and all the hardware devices connected to the CPU, commonly to the Peripheral Component Interconnect (PCI) chip. In conclusion the computing time is bounded by the memory not by the time calculations done on the CPU.

**Parallelism Wall**

This indicates a law that indicates the number of parallel processes. The number  $N$  parallel processes is never ideal and always depends on the problem. The seed-up can be described by Amdahl's Law in terms of the fraction of parallelized work ( $f$ ). [20].

$$speedup \leq \frac{N}{f + N(1 - f)}$$

The current paradigm of using CPUs for computing is growth unsustainable. In 2012, Japan among the countries with elite supercomputers, build the machine "K Computer", with 705,024 multi-core CPUs, it can achieve 11.3 petaflops ( $10^5$  flops). Furthermore the computer is one of the most power efficient supercomputer in the world with a total of 12.66 megawatts (MW), in other words 830 Mflops/watt. This is this is enough to power a small town of 10,000 homes. If the current trend of power use continues, the next supercomputer would require 200 MW of power, this would require a nuclear power reactor to run it.[26]. However in 2013 Oak Ridge National Laboratory (U.S) built a supercomputer that combines CPUs and GPUs, the Titan. It can archive an astonish 24 petaflops theoretical peak. Moreover with a power consumption of 8.2 MW. As showed is possible to built a supercomputers that combines CPUs and GPUs, which has a higher performance and lower power consumption compared to a CPU based supercomputer. [18]

As said the GPU exceeds the CPU in calculations per second FLOPS with a low energy consumption. However the GPU is designed to launch small amounts of data in parallel with only several instructions, in other words the GPU swap, switch threads very fast and they are extremely lightweight. In a typical system, thousands of threads are waiting to work. While the CPU only run up-to 24 threads on a hex-core processor. They can execute a single operation on comparatively large set of data with only one instruction. Although this can be extremely cost-wise operation on the GPU.

## 1.2 GPUs as computing units

A insight of the architecture of GPU can give a idea of why it outperforms the CPU on various benchmarking.

The GPU, unlike its CPU cousin, has thousands for registers per SM (streaming multiprocessor), this are arithmetic processing units. An SM can thought of like a multi-thread CPU core. On a typical CPU has two, four, six or eight cores. On a GPU as many as N SM core. We can see this in the figure 1.2. For a particular calculation, all the stream processors within a group execute exactly the same instruction on a particular data stream, then the data is sent to the upper level, the host (CPU). [5]

As commonly named CUDA cores are the number of processors in a single NVIDIA GPU chip. For example one of the first GPU capable of running CUDA code was the NVIDIA 9800 GT, which had 112 cores, while the latest high-end GPU GTX 980 has 2048 cores.



FIGURE 1.2: Architecture of a NVIDIA GeForce GTX 580

Each CUDA core can execute a sequential thread, just like a CPU thread, which NVIDIA calls it Single Instruction, Multiple Thread (SIMT). In addition all cores in the same group execute the same instruction at the same time, much like classical SIMD (Single instruction, multiple data) processors. SIMT handles conditionals somewhat differently than SIMD, though the effect is much the same, where some cores are disabled for conditional operations, in other word a single instruction is executed throughout the device.

Being able to efficiently use a GPU for an application requires to expose the inherent data-parallelism Optimized for low-latency, serial computation. This can be seen in contrast with a CPU, which is optimized for sequential code performance, fast switching registers and sophisticated control logic allowing to run single complex programs as fast as possible, which is not possible on the GPU. Memory management is very important

for GPUs. this refers how to allocate memory space and transfer data between host (CPU) and device (GPU). While the CPU memory hierarchy is almost non-existent, on the GPU inherent data is important. In figure 1.3 different levels of memory can be observer between the host and the device, which differs form the CPU [11].

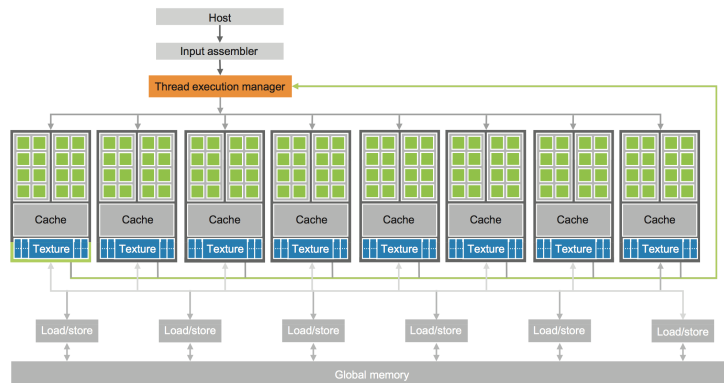


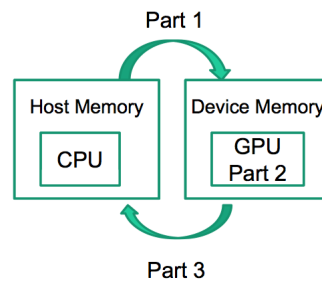
FIGURE 1.3: Memory transfer between the host and device

On the GPU precision and optimization are very important but there is a penalty for choosing performance or precession. All the GPUs are optimized for single precision floating operations, 24 bit size, Also provides double precision point, size of 53 bits. This is using the standard notation IEEE 754. Normally the GPU uses single precession(SP) by default, if chosen double precision (DP), normally there is a penalty of 2x - 4x seed-up. [28] Libraries such as CUBLAS and CUFFT provides useful information how NVIDIA handles floating point operations under the hood.

### 1.3 Programming on GPUs

There exist, among many, two main computing platforms, NVIDIA's Compute Unified Device Architecture (CUDA), and Khronos's Open Computing Language (OpenCL). NVIDIA's CUDA provides the necessary tools, frameworks and library to programs parallel computing, but for there GPUs. While OpenCL is a open standard framework meaning that is possible to do parallel computing on other GPUs, like on AMD cards. Programmers can easily port their code to others graphics cars. However CUDA has more robust debugging and profiling for GPGPU computing. This two frameworks are developed to be close to the hardware layer, using C programming language. CUDA provides both a low level API and a higher level API. Those who are familiar to OpenCL and CUDa, can easily modify their code to work on either platform.[11]

The CUDA programming model views the GPU as an accelerator processor which calls parallel programs throughout all the SMI. This programs are only called on the device and are called kernels, which launch a large amounts of threads to execute CUDA code. The basic idea of programming on a GPU is simple. We can observer this in the figure 1.4




---

FIGURE 1.4: Programming Cycle between the CPU and GPU

- Create memory(data) for the host (CPU) and devices (GPUs)
- Send the data host memory to the highly parallel device.
- Do something with data on the device, e.g. matrix multiplication, calculation, parallel algorithm.
- Return the data from the device to the host.

The structure of CUDA reflects the coexistence of CPU and GPUs. The CUDA code is a mixture of both host code and device code. The CUDA C compiler is called NVCC. The host code is the standard low level ANSI C language. The device code is marked is CUDA keywords for identifying data-parallel functions and has a extension file .cu.

When a kernel is launched, executed by a large amount of threads, where they are organized as a one, two or three dimensional grid of thread blocks. A thread is the simplest executing process. It consists of the code of the program, the particular point where the code is being executed. [11]. Many threads form a block, and many blocks form a grid. CUDA handles the execution of the random-access threads, which take up-to very few clock cycles in comparison to CPU threads. The threads per block can be observer in figure 1.5. All the threads in a kernel can access the global memory, figure 1.3.

Each of the threads can be access by implicit variable that identifies its position within the thread block and its grid. In a case of 1-D block. [24]

$$blockIdx.x \times blockDim.x + threadIdx.x$$

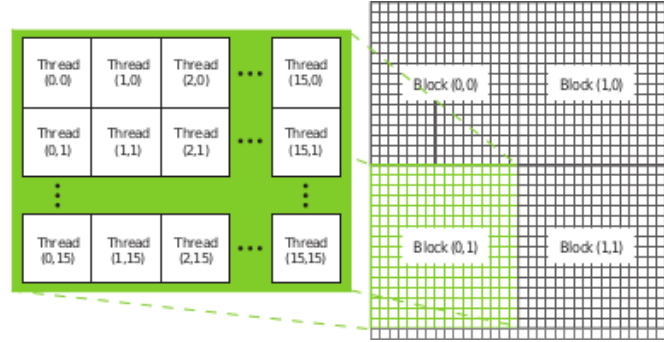


FIGURE 1.5: Part of a 2D CUDA's thread grid, divided in blocks, each block with its own respective threads.

In CUDA, host and device have separate memory spaces. This can be seen on the host and device with the DRAM(Dynamic random-access memory) data. For example a NVIDIA GTX 660m comes with 2GB of memory, which is the global memory for the device. As told the host and device allocates data. The programmer needs to send data from the host memory to the device's global memory. We can see this in the figure ???. Once the memory is transfer back to the host, is completely necessary to free the memory from the device and host. This is typically done with free or delete on C/C++. The CUDA's Application Programming Interface (API) functions performs this activities on behalf of the programmer. [11]

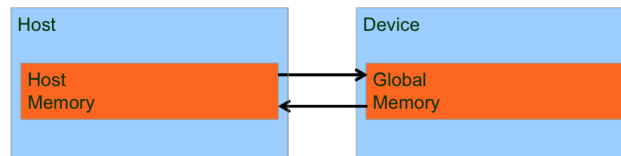


FIGURE 1.6: Separate memory spaces for the CPU and GPU

### 1.3.1 Vector Addition Example

A simple example of a vector addition to show the comparison between the GPU and CPU, input, two list of number which is sum up each corresponded element to produce a final output with the addition of both list. Figure 1.7 shows this process. [24]

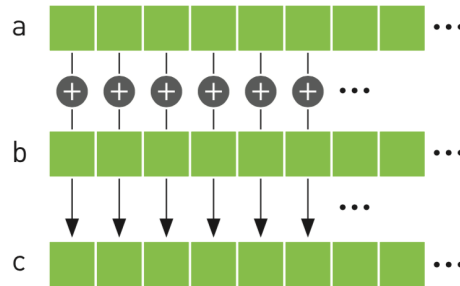


FIGURE 1.7: Simple Vector Addition Example

### 1.3.1.1 CPU Code

This first example illustrates the CPU code executed in a single thread. The code is straight forward to understand. First create the memory for each array, A, B and C with size N. Then calculate the sum of the two vectors with the function *add*. As we can see in the function *add*, we use the while loop to go through each element of the arrays A and B, which are added into a single array C.

---

```
#include <iostream>
#define N 100

void add( int *a, int *b, int *c );

int main()
{
    int A[N], B[N], C[N];

    //fill the arrays with values
    for(int i = 0; i < N; i++){
        A[i] = 1; B[i] = i;
    }

    add(A, B, C);

    //Display the results
    for (int i = 0; i < N; i++)
        std::cout << A[i] << ", " << B[i] << ", " << C[i] << std::endl;

    return 0;
}

void add( int * A, int * B, int * C )
{
    int index = 0;

    //go through each index of the arrays and make the operation
    while(index < N){
        C[index] = A[index] + B[index];
        index++;
    }
}
```



```

    }
}

```

---

LISTING 1.1: CPU Vector Addition

We can notice if we set *N* to be a large number, the function *add* could take a large amount of time to execute. But the example only illustrates the use of the CPU as a single core, however nowadays CPUs commonly have around 4-8 cores. To be able to execute the previous code on all the cores available in the CPU, threads are needed to be implemented. But you would need a reasonable amount of code and debugging to make that happen. Also is a complicated task to schedule all the threads in the CPU.

### 1.3.1.2 GPU Code

We can accomplish the same operation very similar in the GPU with CUDA. First create CPU and GPU memory, with their corresponding code. Send the CPU memory to the device, make calculations on the highly parallel GPU, finally return the results to the CPU.

---

```

#include <iostream>
#define N 100

// CUDA KERNEL
__global__ void add( int *a, int *b, int *c );

int main()
{
    int a[N], b[N], c[N];
    int *dev_a, *dev_b, *dev_c;

    // allocate the memory on the GPU
    cudaMalloc( (void**)&dev_a, N * sizeof(int) );
    cudaMalloc( (void**)&dev_b, N * sizeof(int) );
    cudaMalloc( (void**)&dev_c, N * sizeof(int) );

    //allocate the memory on the CPU
    for(int i = 0; i < N; i++){
        A[i] = 1; B[i] = i;
    }

    //calculate the vector addition in the GPU
    add<<<N,1>>>>( dev_a, dev_b, dev_c );

    //copy back the result from the GPU to the CPU
    cudaMemcpy( c, dev_c, N * sizeof(int), cudaMemcpyDeviceToHost );

    //Display the results
    for (int i = 0; i < N; i++)
        std::cout << A[i] << ", " << B[i] << ", " << C[i] << std::endl;

    cudaFree( dev_a );
}

```

```
    cudaFree( dev_b );
    cudaFree( dev_c );

    return 0;
}
```

---

LISTING 1.2: GPU Vector Addition

The function *cudaMalloc* and *cudaFree* are very similar to the C functions *malloc* and *free*, which allocating memory and deleting memory respectively.

CUDA automatically spams the threads to it correspondent block, so we only need to access the index of the block and pass it to the index arrays. To parallel code will stop in-till the block index reaches the number of elements of the arrays, N.

---

```
void add( int * A, int * B, int * C )
{
    // handle the data at this index if (tid < N)
    int index = blockIdx.x;
    if(index < N)
        c[index] = a[index] + b[index];
}
```

---

LISTING 1.3: GPU Vector Addition

The biggest difference between the CPU code and the GPU code is how threads are managed within the process. The CPU code has only one thread, thrust one loop, however if we want to expand the CPU code to multiples threads, extra loops are required for each additional thread. Based on this idea, the GPU code is launched to every threads accessible by the device chip.

Finally, this chapter provided a overview of heterogeneous programming in a modern context. CUDA enhance the C language with parallel computing support. Which is possible to launch enormous amounts of parallel threads, oppose of few threads on the CPU. The number of GPU cores will continue to increase in proportion to increase in available transistors as silicon process improve. In addition, GPUs will continue to go through vigorous architectural evolution. Despite their demonstration high performance on data-parallel applications. [11]

## Chapter 2

# Heterogeneous Performance Analysis and Practices

When working with GPUs hardware challenges emerges. How can we make the best usage of the GPU hardware. In the conventional CPU model we have what is called linear or flat memory model. This appears to the programmer as a single contiguous address space. The CPU can directly address all the available memory, in other words there is almost no efficiency penalty in creating global data, local data, or even access data that is located on a opposite memory location, all of this can be access as a contiguous block. [5] Meanwhile on the GPU there are expectations, their exists different memory hierarchies which dramatically change the performance output. By allocation the optimal memory types, seed-up and increase throughput can be accomplished, but also analyzing. To ensure optimization, some analysis should be done, like comparing latency, memory hierarchies and data bandwidth between CUDA kernels, The study of the performance of the CUDA code can be done by using NVIDIA's Visual Profiler.

### 2.1 Practices

There are three rules based on NVIDIAs standers to follow for creating a high performance GPGPU (General-purpose on the GPU) program.[7]

1. Get the data on the GPU device and keep it there
2. Process all the data en the GPU, give it enough work to do.
3. Focus on data reuse within the GPU context, to avoid memory bandwidth limitations

GPUs are plugged into the PCI Express bus of the host computer. The PCIe bus has extremely slow bandwidth compared with the GPU. This is why it is important to store the data on the GPU and keep it busy. And minimize the data transfer to the host and back to the device. We can see this in the following table 2.1. Because CUDA-enabled GPUs can carry out petaFLOP performance, they are fast enough to compute large amount of data. So each Kernel launch needs to use all the available resources of the GPU and avoid wasting compute cycles. If a single Kernel doesn't use all of the available bandwidth, multiple kernels can be launched at the same time on a single GPU. For example a DP vectors require 8 bytes of storage per vector element this will double the bandwidth requirement. So is important to take advantage of the memory usage, take advantage of the memory types, use less memory copies between the GPU. [7]

	Bandwidth (GB/s)	Speedup over PCIe Bus
PCIe x16 v2.0 bus (one-way)	8	1
GPU global memory	160 to 200	20x to 28x

FIGURE 2.1: PCIe bus and GPU bandwidth comparison

Some practices should keep in mind to rapidly identify the portions of code where it would be beneficial for GPU acceleration.[16]

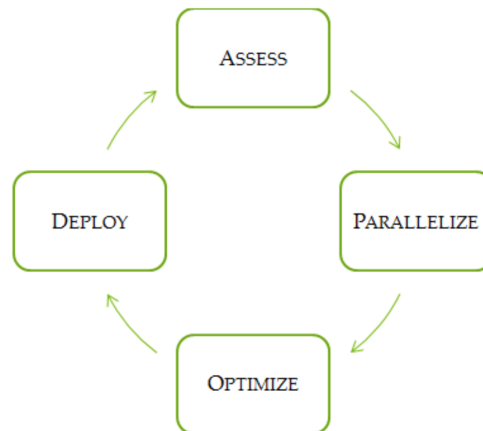


FIGURE 2.2: Different memory type and penalties usage

### Asses

The first step is to locate the part of the code where the majority of the execution time occurs. The programmer can evaluate memory bottlenecks for GPU parallelization.

### Parallelize

Increase parallelization from the original code, could be either adding GPU-optimized

libraries such as cuBLAS, cuFFT, or including more amount of parallelism exposure through the use of CUDA code.

#### Optimize

The developer can optimize the implementation performance through a number of considerations, overlapping kernel executing, kernel profiling, memory handling and fine-tuning floating-point operations.

#### Deploy

Compare the outcome with the original expectation. Determine the potential speedup by accelerating a given section. First a partial parallelization should be implemented before carrying out a complete change.

## 2.2 Performance Metrics

Before trying to make your simulation run faster, you should understand how it currently performs and where the bottlenecks are. There are many possible approaches to profiling the code, but in all cases the objective is the same: to identify the function or functions in which the application is spending most of its execution time and increase the throughput by a given kernel. Throughput is how many operations completed per cycle.

### 2.2.1 Timing

Timing a launched kernel can be done on either the GPU or the CPU. It is important to remember that the CPU and GPU are not synchronized. So it is necessary to synchronize the CPU thread with the GPU kernels launches. CUDA provides the required functions to synchronize the CPU with the GPU calling immediately before starting the timer.[16]. CUDA also can handle timers within the GPU, and record times in a floating-point value in milliseconds. This is done with *cudaEventRecord()*, just by including *start* and *stop* in the function inputs. Note that the timing are measured on the GPU clock, so the timing is independent from the OS. [5].

### 2.2.2 Bandwidth

The bandwidth refers to the rate at which data can be transferred between host and device and vice-versa. The bandwidth is one of the most important factors for testing performance of the GPUs. Choosing the right type of memory could dramatically increase performance and bandwidth. There are two main memory to indicate performance,

theoretical bandwidth and effective bandwidth. The theoretical bandwidth is base on the hardware specifications that is available by NVIDIA. This is calculated using the following formula:

$$theoreticalbandwidth = (clockrate * (bit - wide - memory - interface/8) * 2)/10^9$$

For example the NVIDIA GeForce GTX 280 uses DDR RAM with a memory clock rate of 1,105 MhZ and a 512-bit-wide memory interface

$$(1107 * 10^6 * (512/8.0) * 2)/10^9 = 141.6Gb/sec$$

The GTX 280 has a theoretical bandwidth of 141.6Gb/sec. The effective bandwidth is calculated by timing specific program activities and by knowing how data is accessed by the application. [16]

$$effective - bandwidth = ((Br - Bw)/109)/time$$

Where Br is the number of bytes read per kernel, Bw is the number of bytes written per kernel and t is the elapsed time given in seconds. [23]

In practice the difference between theoretical bandwidth and effective bandwidth indicated how much bandwidth is wasted on accessing memory and calculations. If the effective bandwidth is low compared to the theoretical bandwidth is one indication that there is not enough work being done in the GPUs. There a several solutions, analyze the code to make more parallelize instructions, execute more computational instructions on the GPUs, finally analyze the number of threads per block that are executing on execute kernels .

## 2.3 Memory Handling with CUDA

In this section four types of memory handling are going to be explained, shared memory, global memory (device memory) and finally host memory. As the figure 2.3 illustrates the position of the different types of memory inside the device chip. The global memory is faraway the registers and CUDA core locations, however is very large in comparison to the shared memory. [5].

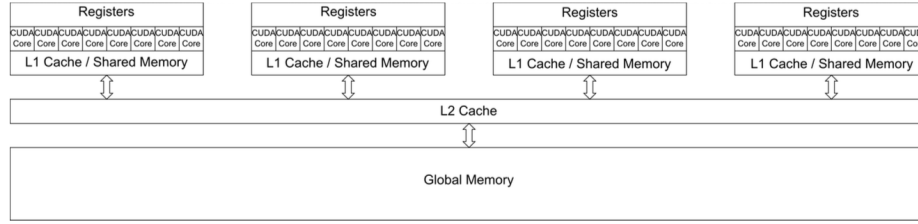


FIGURE 2.3: The schematic cache hierarchy of a CUDA GPU with 4 Streaming Multiprocessors and 8 CUDA Cores each.

In figure 2.4 each memory type has it's bandwidth penalty of used and latency in cycles. Each one can be used in different applications to maximize the memory used. The shared Memory is very limited so it cannot be handler for all the kernels, when performed wrong on the device there is a huge latency and bandwidth penalty, instead having a gain in performance

Storage Type	Registers	Shared Memory	Texture Memory	Constant Memory	Global Memory
Bandwidth	~8 TB/s	~1.5 TB/s	~200 MB/s	~200 MB/s	~200 MB/s
Latency	1 cycle	1 to 32 cycles	~400 to 600	~400 to 600	~400 to 600

FIGURE 2.4: Different memory type and penalties usage

### 2.3.1 Global Memory

Understanding how to efficiently use global memory is essential in CUDA memory management. Focusing on data reuse within the SM and caches avoids memory bandwidth limitations. Global memory on the GPU is designed to quickly stream memory blocks of data into the SM.

- Get the data on to the Device, keep it there.
- Give the GPU enough workload, this using all the resources available from the GPU.
- Focus on data reuse within the GPGPU to avoid memory bandwidth limitations.

In other words the global memory resides on the device, and it can be anything from 1 byte to 8GB, depending on the GPU. Also the memory is visible to all the threads of the grid. Any thread can read and write to any location of the global memory, The memory is always allocated with *cudaMalloc*. And only global memory can be passed to the kernels and are called with `__global__`. [7]

### 2.3.2 Shared Memory

CUDA C compiler treats variables differently than a typical variable, it creates a copy of the variable for each block that is launched on the GPU, now every thread in that block can access the memory, this is why is called shared memory. This memory reside physically on the GPU, because the memory is very close the cache, the latency is typical very low.[24]. One thing comes to mind, if the threads can communicate with others threads, so there should be way to synchronize all the threads. A simple case should be if thread A writes a value into the shared memory, and Thread B wants to access we need to synchronize, when thread A finish writing then Thread B can access it. This is typical case when shared memory with synchronize thread is needed. [5] Shared memory is magnitudes faster to access than global memory, essentially is like a local cache for each threads of a block. While the shared memory is limited to 48K a block, the global memory is the amount of DRAM on the device. The duration of the shared memory on the device is the lifetime of the thread block. Using `__shared__` in-front of the kernel call will invoke shared memory.

### 2.3.3 Constant Memory

Is an excellent way to store and broadcast read-only data to all the threads on the GPU. One thing to keep in mind is that the constant memory is limited to 64KB. [7]. A simple analogue is the `#define` or `const` attribute in the c++ programming language, the variable performed like a variable that cannot be modified. On CUDA is excitability the same, the value can only be read and not written, the value will not change over the course of a kernel execution and only the host can write the constant memory.[24]

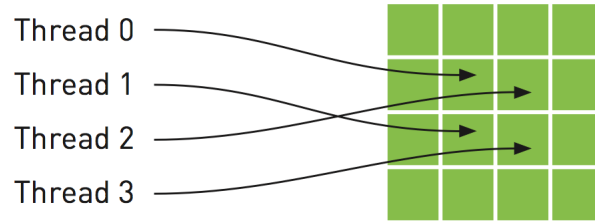
### 2.3.4 Texture Memory

Like constant memory, texture memory is another variety of read-only memory that can improve performance and reduce memory traffic when reads have certain access patterns. . Traditionally Texture memory id used for computer graphics applications, but it can also be use for HPC. The main idea of this read-only memory is that threads are likely to read from address "near' the address they nearby threads.[24]

The texture Memory in a form works like the GPU graphics Texture, when you want to use the texture bind with some sort of data is necessary and when you finish using it unbind the texture from the data. The usage can be summarized in the following table:

- Allocate global memory in the Host.






---

FIGURE 2.5: Mapping of threads into a two dimensional array of texture memory

- Create Texture reference and bind it to memory object.
- On the device obtain the reference from the texture.
- Use Texture memory operations on the device
- When the work is done on the Texture, unbind the texture reference on the host.

### 2.3.5 Thread Synchronization

This refers to synchronizing threads operations. For efficiency, a pipeline can be created by queuing a number of kernels to keep the GPGPU busy for as long as possible. Further, some form of synchronization is required so that the host can determine when the kernel or pipeline has completed. [7] Commonly used synchronization mechanisms are:

- Explicitly calling `cudaThreadSynchronize()`, which acts as a barrier causing the host to stop and wait for all queued kernels to complete.
- Performing a blocking data transfer with `cudaMemcpy()` as `cudaThreadSynchronize()` is called inside `cudaMemcpy()`.

The basic unit of work on the GPU is a thread. It is important to understand from a software point of view that each thread is separate from every other thread. Every thread acts as if it has its own processor with separate registers and identity. Will wait for all threads to finish there job. [7]

## 2.4 Concurrent Kernels

As we seen kernels are executed in a sequential form with parallel instructions. With CUDA's streams is possible to launch several kernels in parallel, overlap kernel in the same launch sequence. As illustrated in Figure 5.3.

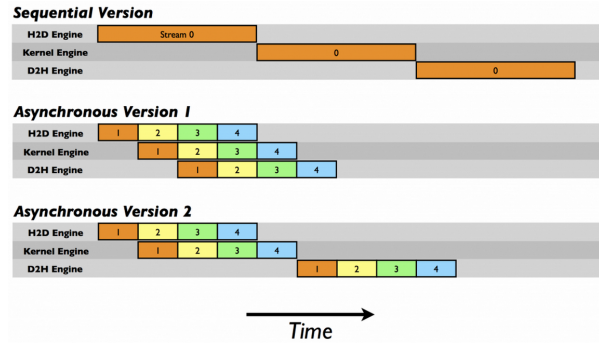


FIGURE 2.6: Overlapping kernel execution using CUDA streams

To overlap kernel execution and data transfers, in addition to pinned host memory, the data transfer and kernel must use different, non-default streams. Non-default streams are required for this overlap because memory copy, memory set functions, and kernel calls that use the default stream begin only after all preceding calls on the device (in any stream) have completed, and no operation on the device (in any stream) commences until they are finished. The following is an example of overlapping kernel execution and data transfer. As the 5.3 shows. [11]

By using two or more CUDA streams, we can allow the GPU to simultaneously execute a kernel while performing a copy between the host and the GPU. We need to be careful about two things. First, the host memory involved needs to be allocated, since we will queue our memory copies, we need to synchronize those copies. Second, we need to be aware that the order in which we add operations to our streams will affect our capacity to achieve overlapping of copies and kernel execution. The general guideline involves a breadth-first, or round robin, to assign work and queue work to the kernels. [24]

## 2.5 Kernel Analysis

As said before, kernels are the essential part of CUDA programming, threads are launched automatically throughout each thread blocks of the device. Furthermore it millions of threads execute the same code parallel, which is When execution the

### Memory Bandwidth Bound

This refers when the code/application is limited by memory access. Most GPUs card have 1GB- 6GB of memory, this is used to process the data on the GPU, while the CPU has massively amount of memory available for use. A solution to

this is to reuse the data, change the type of memory used in the GPU. A multi-GPU approach, launching kernels in several GPUs at once. This will dramatically increase the amount of memory in the application.

#### Compute Bound

Refers to the computation time execution, in other words calculations done in the device, under the assumption that there is enough memory for the calculations. What is actually the analysis time operations on the kernels. Theoretical bandwidth vs effective Bandwidth can measure performance for a compute-bound Kernel. Therefore it's possible to increase the FLOPS per device.

#### Latency Bound

Is one whose predominate stall reason is due to memory fetches. This is actually saturating the global memory, or any type, but still have to wait to get the data into the kernel. Physically can be the data being sent from one part of the Device to the other. Also depends the time required to perform an operation, and are counted in cycles of operations. A way to reduce the latency is to increase the number of parallel instructions (more calls per thread), in other words more work per thread and fewer threads.

The performance of relatively simple kernels, which perform computations across a large number of data elements, is more a function of the GPU's memory system performance than the processing performance. It can be beneficial for such memory-bound kernels to decrease the amount of memory access required by increasing the complexity of the computation. [5]

## 2.6 Hardware constraints

This refers to the limit how many threads per block a kernel launch can have. If exceed this values they kernel will never run. The threads per block really depends of the hardware capabilities. The compute capabilitiesThe compute capability of a device is represented by a version number, also sometimes called its "SM version". This version number identifies the features supported by the GPU hardware and is used by applications at runtime to determine which hardware features and/or instructions are available on the present GPU.[17] In a roughly summarized as:

- Each block cannot have more than 512/1024 threads in total. (Capability 1.x or 2.x-3.x)

- The Maximum dimensions of each block are limited to  $[512, 512, 64]/[1024, 1024, 64]$ (compute 1,1.2)
- Each block cannot consume more than to 8k, 16k, 32K registers total
- Each block cannot consume more than 16kb/48kb of shared memory

SM Resources, improve performance of an application by trading one resource usage for another. [16]

Another inefficiency that can cause low performance to the applications is the number transfers memory calls between the CPU and GPU. The GPU communicates with the CPU via a *PCIe* bus, in addition all of the massive FLOPS per second that can be achieve cannot actually be sent to CPU. The GPU should be filled with the enough workload at the beginning of the application and at the end only return the memory to the CPU.

### 2.6.1 Thread Division

The hardware has its limits, how much thread per block a kernel can handle. Launching a kernel with the hardware constrains for above can only ensure us that the kernel will actually be executed in the device, nonetheless not 100% optimal. Furthermore is necessary to launch kernels with the amount of threads per block base on the hardware contains and the problem. The block size that is chosen will determine how faster the code will run. By Benchmarking, is possible to find what configuration is the best for the problem. One thing to notice is that thread blocks should be a multiple number of SMs, with this idea is possible to obtain optimal thread block configuration.

## 2.7 Visual Profiler

Is a hard task to keep track of each individual thread. This becomes difficult for debugging highly parallel applications. The NVIDIA's Visual Profiler is a profiling tool that can be used to measure performance and find potential opportunities for optimization in order to archive maximum performance on the GPUs. The Profiler provides metrics in the form of plots and graphs, which describes opportunities to fully utilize the compute and data movements capabilities of the GPU, as well of each kernel launch in the application. See Figure 2.7.

NVIDIA's profiling tools comes in various flavors; a standalone profiler through the visual profiler compiler *nvvp*, integrated in a GUI *Nsight Eclipse Edition* as *NSight*

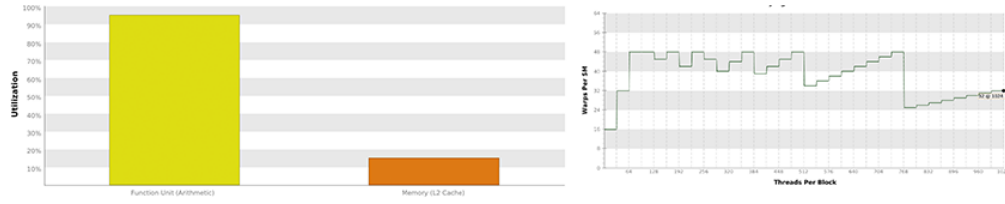


FIGURE 2.7: Profiler provides optimization metrics

command (Visual Profiler), and as a command-line profiler through `nvprof` command. Each one has its disadvantages and advantages. The command-line profiler is useful for remotely access, where a GUI is not available, while the NSight can show graphs, plots and timeline of the application. The Profiler support CUDA applications as well as openCL applications, however both have some exceptions.

The Visual Profiler, by default, will execute the entire application, nonetheless typically only some parts of application only need performance optimization. This enables to determine kernels, code where critical performances is needed. The common situations where profiling a region of the application is helpful.[17]

- Analyze data initialization and movement in the CPU and GPU, as well as evaluating CUDA calls.
- The application operates in phases, where a algorithm operates throughout each region. The application can be optimized independently from other phases of the code.
- The application contains algorithms that operate through a large number of iterations. In this case is possible to collect data from a portion of the iterations.

The Visual Profiler provides a step-by-step optimization guidance, where is possible to evaluate the GPU usage, examine individual kernels and analyze timeline of the application which the profiler shows memory movements and usage, CUDA calls, number of threads and performance. The figure 2.8 shows, each Kernel has its own percentage of execution time of the overall application.[16]

### 2.7.1 Profiler Kernel Report

The profiler will execute several times the application for it to collect data from each kernels. This enables to precisely optimize phases of the application[24]. The profiling tools can verify how long the application spends executing each kernel as well the

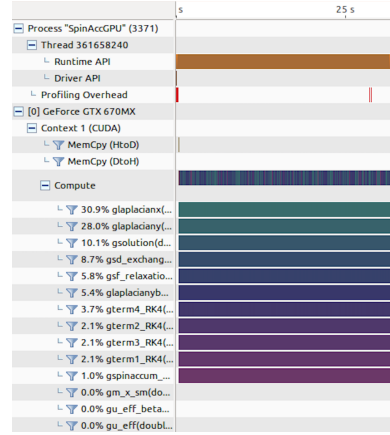


FIGURE 2.8: Visual Profiler kernel execution

number of used blocks and threads. Through this is possible to obtain various memory throughput measures, like global load throughput and global store throughput, indicate the global memory throughput requested by the kernel and therefore corresponding to the effective bandwidth mentioned in the last section.

As we know the profiler executes the application several time to collect data about each kernel. The information obtained by each kernel can be sum-up in-to a report that can be exported in a pdf file, which has the following information.

### 1. Compute, Bandwidth, or Latency Bound

The performance determines if the kernel is bounded by computation, memory bandwidth, or instructions/memory latency. It shows how is limiting the performance respectively.

### 2. Instructions and Memory Latency

Instruction and memory latency limit the performance of a kernel when the GPU does not have enough work to keep busy. The performance of latency-limited kernels can often be improved by increasing occupancy. Occupancy is a measure of how many warps the kernel has active on the GPU, relative to the maximum number of warps supported by the GPU.

### 3. Compute Resources

GPU compute resources limit the performance of a kernel when those resources are insufficient or poorly utilized. Compute resources are used most efficiently when instructions do not overuse a function unit.

### 4. Floating-Point Operation Counts

floating-point operations executed by the kernel, can be either single precision or double precision.

## 5. Memory Bandwidth

Memory bandwidth limits the performance of a kernel when one or more memories in the GPU cannot provide data at the rate requested by the kernel.

### 2.7.2 Collect Data On Remote System

As mention before, is possible to collect data from a remote system where a GUI is not available, using the command-line `nvprof`. Remote profiling is the process of collecting profile data from a remote system that is different than the host system at which that profile data will be viewed and analyzed. Once the data is collected is possible to access the data using the Visual profiler, which enables a GUI and more compressive information about the application. There are two ways to perform remote profiling. To use `nvvp` remote profiling you must install the same version of the CUDA Toolkit on both the host and remote systems. It is not necessary for the host system to have an NVIDIA GPU. [17]

Finally, this chapter gives a overview of practices and performance studies for GPGPU. Also a better understanding of the hardware and memory management, as well as hardware limitation, which determinate the best usage of the GPUs. As we can see NVIDIA's profiling tools is useful to analyze different stages of our application, moreover to determined which parts of the CUDA code is better to optimize from others to gain more performance.

## Chapter 3

# Introduction to Domain Wall Dynamics and a Implementation with CUDA

This chapter is a overview of the theory and experiments behind the study of Domain Wall Dynamics under Nonlocal Spin-Transfer Torque. This is a quantitatively test the effects of spin-diffusion, on real Domain Wall (DW) structures, by numerically implementing the Zhang-LI model into a NiFe soft nanostrip. The numerical implementation takes advantage of the highly parallel process capabilities of the GPU. The numerical method used for the solution is a the method known as Finite Differences in the Time Domain (FDTD) whose integration is done using a 4th order Runge - Kutta integration.

### 3.1 Theory

[4]

Contrarily to charge, spin accumulate in metals, The associated diffusion current flows in all directions, giving rise to nonlocal effects, Beyond transport properties, conduction electrons spin resonance and spin pumping provide further testimonies for non-locality in spin transport. These works all refer to samples consisting in piecewise uniform layers or blocks, magnetic or not. Of special significance to the present work in the non-collinear geometry where a spin current with polarization transverse to the magnetization exists, whose absorption in the vicinity of the surface of a magnetic layer creates a torque on the magnetization, known as spin transfer torque (SFT), [4]

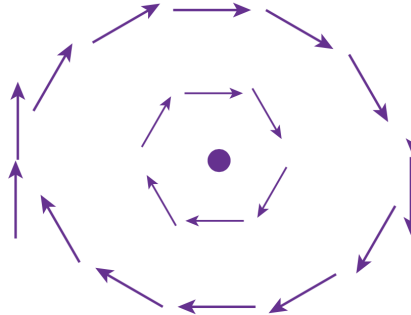


Spintronics is a new type of electronics that exploits the spin degree of freedom of an electron in addition to its charge. [30]

[22]

### 3.1.1 Domain Wall

An abrupt in magnetization at the boundary of two anti-aligned domains is not a favorable condition. Domain walls form between such domains as means of minimizing the energy of the two anti-aligned domains. Domain walls are transition layers in which the magnetization changes gradually from one magnetization to another. In other words the boundaries between regions of uniform magnetization. The gradual change prevents the large increase in exchange energy that would accompany an abrupt change in the magnetization angle. Common domain wall geometries include Bloch walls, Néel walls and vortex walls. In the case of Vortex wall the magnetization rotates in the plane perpendicular to the domain wall, but the local magnetization is wrapped around a single vortex point [8]. This can be seen in figure .



Vortex wall

---

FIGURE 3.1: Domain Wall - Vortex

Domain walls are the basis for various spintronics devices that use magnetic moments, in other words spin of electrons. The use of the spin degree of freedom. With this it is expected that electronics technology and devices will be faster, compact and more energy-saving. An interesting application using this idea is a new design for a different type of memory disk drive called racetrack memory by Parkin in 2008[19]

Spin-transfer torque is a torque that exerts on a magnetization by conduction electron spins, in other words the angular momentum transferred from spins to magnetic moment [31]. This has stimulated research into domain wall (DW) dynamics, particularly those

resulting from interactions with current passing through the DW via the phenomenon of spin momentum transfer (SMT) [27]

The study spin-diffuse effect within a continuously variable magnetization distribution, integrating with micromagenectis with diffuse model of Zhang and LI [4]

We Quantitatively test the effects of spin diffusion, on real Domain walls structures, this is done by numerically solve the Zhang-Li model [31] into micro-magnetics. The Zhang Li model refers to:

which is the following equation.

## 3.2 Experiment

Base on the work of Dr. Claudio [4]

At first we investigate the steady-sate velocity regime of DWs in NiFe soft nanostrips. applying current densities similar to those reported in experiments. The results that we are going to obtain

Experimentally measured spin-diffusion parameters are used, we want to the solution of.

$$\frac{\partial \delta \vec{m}}{\partial t} = D \triangle \delta \vec{m} + \frac{1}{\tau_{sd}} \vec{m} \times \delta \vec{m} - \frac{1}{\tau_{sf}} \delta \vec{m} - u \partial_x \vec{m} \quad (3.1)$$

The sample that is considerate is a 300 nm wide and 5 nm tick NiFe soft nanostrip. This dimensions are widely used for experimental use.

Therefore, a simultaneous solution of the diffusive Zhang and Li model together with the magnetization dynamics equation has uncovered a qualitatively new feature of the spin-transfer torque effect in the presence of spin diffusion.

Advances in spintronics recognized by 2007 Nobel Prize in Physics have enable over the last decade advances in computer memory, in hard drives, this is a metal based structures which utilize magnetoresisite effects to save and read data from a magnetic disk. [27]

Some application include racetrack technology by the IBM fellow scientific Parkin [19]

Base on this study numeric applications have been unfold.

### 3.3 Numerical Methods

#### 3.3.1 Laplacian

[12]

The differential evaluation in one-dimension, The Second order Taylor expansion readily yields expressions for the first and seconds central derivatives

$$\frac{df}{dx} = \frac{f_{i+1} - f_{i-1}}{2a}$$

and

$$\frac{d^2f}{dx^2} = \frac{f_{i+1} - 2f_i + f_{i-1}}{a^2}$$

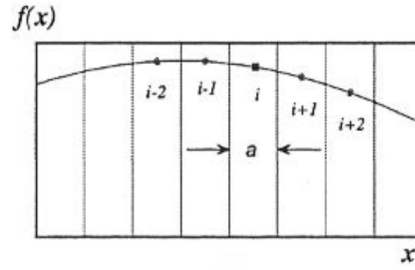


FIGURE 3.2: Sampled at regular intervals  $a$

Taylor expansion of the function  $f(x)$  around  $x = x_i$  yields where  $f^{(k)}(x_i) = f^{(k)}$  if  $k = 0$

$$f(x) = \sum_{k=0}^{\infty} \frac{(x - x_i)^k}{k!} f^{(k)}(x_i) = \sum_{k=0}^{\infty} \frac{(x - x_i)^k}{k!} f^{(k)}$$

Applying the previous equation to nearest and next nearest neighbor to grid point  $i$  and truncation to the 4th order yields a set of four equations:

$$\begin{bmatrix} -2a & \frac{(-2a)^2}{2!} & \frac{-(2a)^3}{3!} & \frac{(-2a)^4}{4!} \\ -a & \frac{(-a)^2}{2!} & \frac{(-a)^3}{3!} & \frac{(-a)^4}{4!} \\ a & \frac{(a)^2}{2!} & \frac{(a)^3}{3!} & \frac{(a)^4}{4!} \\ 2a & \frac{(2a)^2}{2!} & \frac{(2a)^3}{3!} & \frac{(2a)^4}{4!} \end{bmatrix} \begin{bmatrix} f_i^{(1)} \\ f_i^{(2)} \\ f_i^{(3)} \\ f_i^{(4)} \end{bmatrix} = \begin{bmatrix} f_{i-2} - f_i \\ f_{i-1} - f_i \\ f_{i+1} - f_i \\ f_{i+2} - f_i \end{bmatrix} \quad (3.2)$$

The set of linear equations provide numerical estimates for the first, second, third and fourth derivatives of  $f$  at any given point  $i$ .

The general form of the first and second derivate based on second nearest neighbors expansion reads:

$$f_i^{(1)} = \frac{f_{i-2} - 8f_{i-1} + 8f_{i+1} - f_{i+2}}{12a}$$

$$f_i^{(2)} = \frac{f_{i-2} + 16f_{i-1} - 30f_i + 16f_{i+1} - f_{i+2}}{12a^2}$$

$$\begin{bmatrix} -2a & \frac{(-2a)^2}{2!} & \frac{-(2a)^3}{3!} & \frac{(-2a)^4}{4!} \\ -a & \frac{(-a)^2}{2!} & \frac{(-a)^3}{3!} & \frac{(-a)^4}{4!} \\ a & \frac{(a)^2}{2!} & \frac{(a)^3}{3!} & \frac{(a)^4}{4!} \\ 1 & \frac{(3a)}{2} & \frac{(3a/2)^3}{3!} & \frac{(3a/2)^4}{4!} \end{bmatrix} \begin{bmatrix} f_i^{(1)} \\ f_i^{(2)} \\ f_i^{(3)} \\ f_i^{(4)} \end{bmatrix} = \begin{bmatrix} f_{i-2} - f_i \\ f_{i-1} - f_i \\ f_{i+1} - f_i \\ f_{i+2}(x_R) \end{bmatrix} \quad (3.3)$$

[6]

### 3.3.2 Finite Differences in the Time Domain

Modern numerical algorithms for the solution of ordinary differential equations are also base on the method of the Taylor series. Each algorithm such as Runge-Kutta method are constructed so they give an expression depending of the parameter ( $h$ ), in other works the step as an approximate solution of the first terms of the Taylor series. [25] The method can acurately tackle a wide range of problems as weel as can solve complicated problem, but it is generally computacinally expensive. Solutions requiere large amount of memory and computacional time.

The FDTD method employs finite differences as approximations to both the spatial and temporal derivates that appear in Maxwell's equations

### 3.3.3 Fourth order Runge and Kutta method

There exist several computational numeric methods to solver such equations, methods like Euler, Midpoint Method and Runge-Kutta integrator method can solve this type

of equations. The RG4 this method is used for the simulation because its numerically more accurate when compared to the others.

The RG4 method differs widely from the Euler method and the Midpoint method. The euler method is the simplest, the derivative at the starting point of each interval is extrapolated to find the next function value, see figure 3.3. Euler method only has first order accuracy while the RG4 its fourth order integrator.

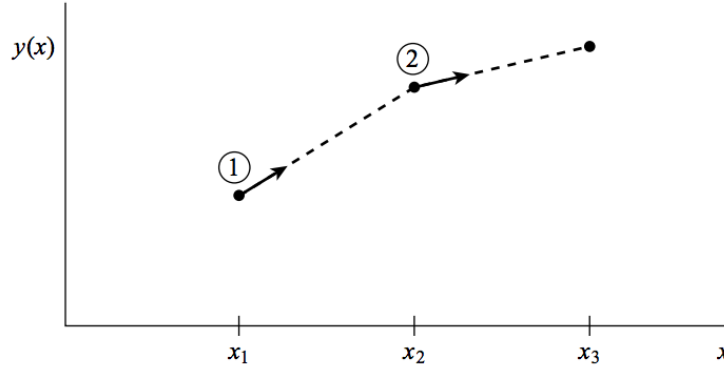


FIGURE 3.3: Euler Method, Is the simplest approximate to solver differential equation or numerically solve equations.

RK4 goes as follows:

$$y_{n+1} = y_n + 1/6K_1 + 1/3K_2 + 1/3K_3 + 1/6K_4 \quad (3.4)$$

where

$$\begin{aligned} K_1 &= h\dot{f}(x_n, y_n) \\ K_2 &= h\dot{f}(x_n + h/2, y_n + k_1/2) \\ K_3 &= h\dot{f}(x_n + h/2, y_n + k_2/2) \\ K_4 &= h\dot{f}(x_n + h, y_n + k_3) \end{aligned}$$

As the equations shows, each step, the derivative is evaluated four times, once at the initial point, twice at trial midpoints, and once at a trial endpoint. From these four values, the final value is calculated, just like the following equation 3.4

[21]

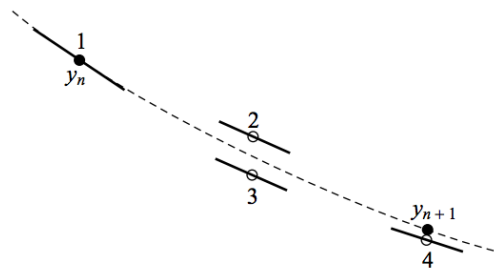


FIGURE 3.4: Fourth-order Runge and Kutta method, Each step the derivative is evaluated four times.

## Chapter 4

# CUDA implementation

### 4.1 Numerical Methods on the GPUs

#### 4.1.1 Finite Difference Time Domain

#### 4.1.2 Fourth order Runge and Kutta

The GPU implementation. the application reads

At initialize the applications first it allocates all the CUDA and c arrays.

To allocate a big chunk of memory in the Device

```
cudaMalloc
```

And C with

```
malloc
```

In the initialization function it also reads the magnetization data from a specific file in this specific case from “upVW-magn-2.5nm.data”

The initial values for the simulation are

### 4.2 structure

The number of threads that are allocated within each kernel is a 2d grid.

Depending on the hardware configuration, each GPU can allocate different threads per block. To make a homogeneous grid space for each GPU a simple calculation is made.

NX	480
NY	120
NZ	1
TX	1200.0
TY	300.0
TZ	5.0
u	1
D	$1.0e^3 \text{ nm}^2/\text{ns}$
tau sd	$1.0e^{-3} \text{ ns}$
tau sf	$25.0e^{-3} \text{ ns}$

Threads per block X	32
Threads per block Y	32

```

NXCUDA = (int)powf(2,ceilf(logf(NX)/logf(2)));

NYCUDA = (int)powf(2,ceilf(logf(NY)/logf(2)));

if((int)powf(2,ceilf(logf(NZ)/logf(2))) < 1)
    NZCUDA = 1;
else
    NZCUDA = (int)powf(2,ceilf(logf(NZ)/logf(2)));

//Setup optimum number of blocks
XBLOCKS_PERGRID = (int)ceil((float)NX/(float)XTHREADS_PERBLOCK);
printf("XBLOCKS_PERGRID = %i\n",XBLOCKS_PERGRID);

YBLOCKS_PERGRID = (int)ceil((float)NY/(float)YTHREADS_PERBLOCK);

```

The calculations are divided into two parts, the CPU code and the GPU code. All the intense computation and simulation is done on the GPU, while on the CPU only minor calculations are done, such as I/O data.

### 4.2.1 CPU

In the *initial\_calculations* functions it calculates the terms on magnetization components the read magnetization data. This function basically reads data from a .dat file and allocates the memory for each blocks, it reads

The file is divide into two blocks of data, the first block of 57600 rows are the coordinate X and the coordinate Y. Then the next 57600 rows by 3 columns are the magnetization data. Base on the information read the matrices of data is created.

here two data sets are created. The coordinates point data  $(x, y)$  and the magnetization data  $(x, y, z)$ .



After this initialization data, the next step is to send this data, that is actually read on the CPU (host) to the GPU(device).

First we print the Initial and final coordinates that read, this is to ensure that the values are read successfully.

### 4.2.2 GPU

Arrays are created on the Host and sent to the Device using

In the type it can be either `cudaMemcpyHostToDevice` or `cudaMemcpyDeviceToHost`, depending, if the memory that is being copied is sent to the host or to the device.

```
cudaMemcpy(dst, src, size_in_bytes, type);
```

After initialization the coordinates points and the magnetization data in the device are done, these values are sent to the GPU, with the function `cudaMemcpy()` and value set to `cudaMemcpyHostToDevice`.

In the Initialization of the calculations most of the arrays are filled up with values based on the data read from the .dat magnetization.

```
--global__ void gsource(double *sm_out, double *matrix_in, double u, int grid_width);

--global__ void gm_x_source(double *temp_x, double *temp_y, double *temp_z,
                           double *mx, double *my, double *mz,
                           double *sm_x, double *sm_y, double *sm_z,
                           int grid_width);
```

The function `gsource`

Makes the following calculation of the *double \* matrix\_in* or *m*

$$out[i] = (m[i - 2] - 8.0 * m[i - 1] + 8.0 * m[i + 1] - m[i + 2]) * \frac{u}{12 * \delta X} \quad (4.1)$$

where

$$\delta X = \frac{TX}{NX}$$

This calculation is done for the arrays read from the .dat file, for `dev_mx`, `dev_my` and `dev_mz` and are saved in temporary arrays `dev_sm_x`, `dev_sm_y`, and `dev_sm_z`.

The method.

*gm\_x\_source* calculates the coss producto of the array  $m_{xyx}$  and  $sm_{xyz}$ , this is done twice.

THis data is saved on the arrays  $dev_s m_{xyz}$ ,

After launching this two kernels the initial setup is done, the next step is the actual simulation using runge and kutta integrator.

### 4.2.3 KG4

As seed in Runge and Kutta section, this method is implementation to numerically solve the differential equation. Intuitively the implementation on CUDA code is done with 4 kernls, where each kernel calculates respectivelly the order of the integrator. In the last term calculation is where all the magic occur, the sum of the previous 3 calculated terms.

```
--global__ void gterm1_RK1( . . . );
--global__ void gterm2_RK2( . . . );
--global__ void gterm3_RK3( . . . );
--global__ void gterm4_RK4( . . . );
```

Between each term calculation of RG4 laplacian calculation kernels are launched.

```
--global__ void glaplacianx( . . . );
--global__ void glaplacianyboundaries( . . . );
--global__ void glaplaciany( . . . );
```

The final kernel is launched *voidgterm4<sub>RK4</sub>*() obtain the array  $deltam_{xyz}$ , which is the final result of the RK4 integrator. This array is sent to the last step.

### 4.2.4 effective values

When the rg4 integretaor is done effective values are calculatated, this values sirve the porpese of calculation the.

```
--global__ void gm_x_sm( . . . );
--global__ void gu_eff( . . . );
--global__ void gu_eff_beta_eff( . . . );
--global__ void gbeta_eff( . . . );
--global__ void gbeta_diff( . . . );
```

The last kernel *voidgbeta\_diff(...)*; is where the two final arrays are obtain, which then are sent to the CPU for the final calculation.

The final calculation is just the sum of all the elements of  $\beta_{diffnum}$  and  $\beta_{diffden}$ , there divided. This final single values tells us...

This is the final step of the simulations this is where *beta\_diff* is obtained.

The final data is saved

#### 4.2.5 time

When the simulation is done, it will repeat the process until the values converges.

### 4.3 Validation

The validate the code, that is obtained from the simulation

Once obtain the results from the simulation, the results are written into two separated data files; .eff and .spin. depending of the configuration of the application is possible to obtain the uVW or the. Because CUDA framework is highly parallel system is fairly easy to obtain erroneous data from the calculations, even setting up the threads per block incorrectly is possible to get data set that a wrong, or results that don't diverge. It is necessary that when finishing making changes to the code validating the results with a valid data set is done.

The validation is done by checking the output the simulation with a valid data set, the output of the validation application tells us the error factor of the current data with the valid set. So for each data set there is a threshold value, that can tell if the that is close enough to the results. A example of the validation performed.

```
//validation float error precision
```

#### 4.3.1 Validation

### 4.4 Data Flow

The initial data flow of the kernels goes as follow, Fi

## Chapter 5

# Optimization Results

This chapter is the results of the CUDA code implementation launched on several different GPUs nodes. The test are performed on various GPUs architectures, which, has different hardware characteristics. Each GPU node is analyzed using the NVIDIA's Visual Profiler, in addition the CUDA kernels are evaluated in performance; throughput, bandwidth, executing and parallel time. Furthermore the results, are analyzed and optimized using the schemes from chapter 3. Lastly the code is executed remotely on the supercomputer "Piritakua" at the Department of Multidisciplinary Studies Yuriria, University of Guanajuato

### 5.1 Supercomputer "Piritakua"

The experiments are carried out using the supercomputer Piritakua. The massive GPU cluster was design and built by Dr. Claudio from the University of Guanajuato Multidisciplinary Studies Yuriria. The GPU cluster is located at a small town of Mexico, Yuriria. The supercomputer at the Front-end has a eight core Intel Xeon at 2.4 Ghz, at the back-end several GPU are connected, one NVIDIA Tesla K20, two Tesla M2070 and a GTX 580.

A GNU LINUX distribution installed on the system, the CentOS 64 bits version 6.4. CentOS stands for Community Enterprise Operating System which is free operating system and one of the most popular GNU Linux distribution for web servers and as well is supported by RHEL (Red Hat Enterprise Linux). [2]

The specifications of the front-end cluster.

Processor	Number	Cores	RAM
Server Dell Intel Xeon E5620 2.4 GHz	1	8	12 GB
Server HP Proliant SL 350s Gen3 Intel Xeon X5650 2.67 GHz	2	24	32 GB
Server HP Proliant SL 250s Gen8 Intel Xeon E5-2670 2.60 GHz	3	48	104 GB
CPU Xeon Phi 5110p	1	8	8 GB
CPU Xeon Phi 7120p	1	8	16 GB

The CUDA Code was launched on only two CPUs, a laptop with a eight core intel i7-3630QM and a high-end CPU Xeon Phi 7120p from the cluster. In addition the Xeon Phi was used for all the experiments for the Cluster's GPUs. The Xeon Phi 720p is capable of achieving f 1.2 teraflops of double precision floating point instructions with 352 GB/sec memory bandwidth at 300 W.

When accessing "Piritakua" remotely is possible to use all the GPUs available on the cluster. The specifications of the GPU connected to the back-end are as follow, CC stands for compute capability.

Model	Cores	RAM	DP	SP	Bandwidth	CC
Tesla K20m	2496	5GB	1.17 Tflops	3.52 Tflops	208 GB/s	3.5
Tesla M2070	448	6GB	515 Gflops	1030 Gflops	150 GB/s	2.0
Tesla C2050	448	2.5GB	512 Gflops	1030 Gflops	144 GB/s	2.0
GeForce 580	512	1.5GB	520 Gflops	1,154 Gflops	192.2 GB/s	2.0
GeForce 670MX	960	3GB	520 Gflops	1,154 Gflops	67.2 GB/s	3.0

The code was launched on all Piritakua's GPUs and on external GeForce GTX 670m, located on a laptop. The "m" stands for the mobil graphic card version. In addition the 670m card is design for less power usage, but with high graphics power, it even has more cores than some Tesla models, however this types of cards has way more less Bandwidth than standard versions.

### 5.1.1 Architecture Differences

Architecture dependent technical differences of NVIDIA GPUs. During CUDA development a lot of internal features have been improved, but most paradigms for the programmer stayed the same. For example a streaming processor can now handle 2048 threads at a time, but the maximum block size stayed at 1024. This results in a 100% theoretical occupancy for block sizes of 1024 compared to 66% of Fermi. Another example is the use of Shared Memory. Maxwell has 64KB dedicated Shared Memory. The maximum amount of Shared Memory per Block is 48KB for all three architectures. [9]

There are two GPU architectures that code was launched, the Fermi and the Kepler. The Tesla K20m is base on “Kepler” GPU architecture and the Tesla M2070, M2050 and GeForce GTX 580 on the Fermi architecture. The Kepler is a newer architecture than the Fermi. The big difference between the is the number of CUDA cores per SM. Roughly summarized as:

Name	Fermi		Kepler		Maxwell
Compute Capability	2.0	2.1	3.0	3.5	5.0
Single Precision Operation per Clock/SM	32	48	192		128
Double Precision Operation per Clock/SM	4/16 <sup>1</sup>	4	8	8/64 <sup>2</sup>	1 <sup>3</sup>
Max Number of Threads per SM / SM	16		32		
Max Number of Registers per Thread/SM	1536			2048	
Max Number of Threads per Block	1024				
Active Thread Blocks per SM / SM	8		16		32
Max Warps per Multiprocessor/ SM	48		64		
Registers / SM	32K		64K		
Level 1 Cache	16/48 KB		16/32/48 KB		64 KB
Shared Memory / SM	16/48 KB		16/32/48 KB		64 KB
Warp Size	32				

### 5.1.2 Experiment metrics

The initial implementation was launched on several GPU nodes as well as several times on the same node. The implementation was launched 10 times on each node, For example the following table are the results of the initial implementation launched on the NVIDIA Tesla K20m.

number	time
1	64846.0
2	59010.4
3	67332.4
4	68171.8
5	61818.0
6	65111.3
7	67346.4
8	65666.6
9	67343.3
10	65681.8

As we can see each time the code is executed, different time results. The order in which operations are evaluated can significantly affect the final value of a floating point computation; t It’s possible that the first issued block might be issued to different multiprocessor each time, and if there are an uneven number of blocks across multiprocessors, then the execution order could vary slightly between runs [5].

As we know there is no guaranty that threads will execute at the same order. To accomplish such task thread synchronization is needed.

### 5.1.3 Validation

## 5.2 Results

The CUDA code was launched on each GPU of the "Piritakua" supercomputer. As we know the supercomputer has different GPU, as well as several architectures and different number of CUDA cores.

mention CUDA versions. as well as the compiler used for the test.

Node	initial Avg time	1st optimization	2nd optimization
Tesla K20m	64846.0	55	33
Tesla M2070	64846.0	55	33
GeForce GTX 580	64846.0	55	33
GeForce 670m	64846.0	55	33

### 5.2.1 Initial Test

The initial implementation was launched on several GPU nodes, as previous mentioned. As we know. Se figure.5.1.

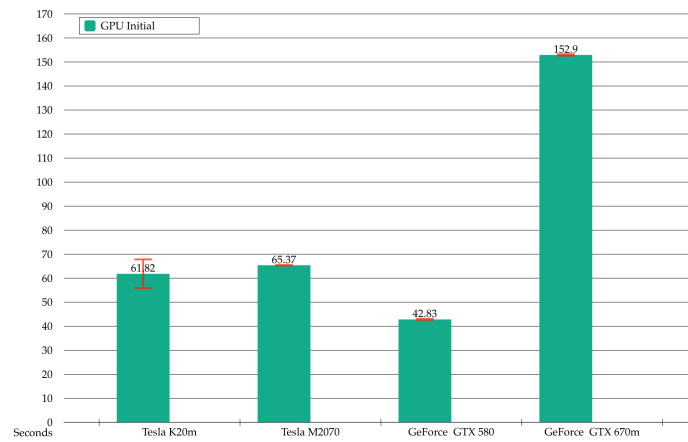


FIGURE 5.1: Initial results of the implementation running on several different GPU nodes

### 5.2.2 Finite Precision

We all know summing one-third three times yields to one.

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

However this is not always true on a computer. It actually depends of the floating-point precision.

### 5.2.2.1 Visual profiler

The visual profiler.

The visual profiler was used on Laptop with GeForce GTX 670m with the intel eight core i7-3630QM.

the

command

```
nvprof o nvprof.log ./command
```

## 5.3 Branching

CUDA follows the Single Instruction Multiple Thread architecture. This means that there are running several threads executing the same code. Each thread can operate on its own data and has its own address counter. They are free to use each data dependent path. But also each thread is executing the same operation at the same time. When a thread within a warp branches differently the other threads get deactivated. This can be described by the following listing and illustrated by [9].

### 5.2

---

```
__global__ void kernel(int* out){

    idx = threadIdx.x;
    int result;

    if(idx == 0){
        result = foo();
    } else {
        result = bar();
        out[idx] = result;
    }
}
```

---

LISTING 5.1: CPU Vector Addition

The optimization



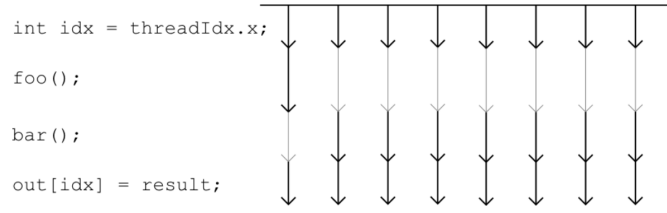


FIGURE 5.2: The execution flow of a branching code, with warp size 8. Black arrows are active threads, and the grey ones are disabled.

### 5.3.1 Occupancy

### 5.3.2 Concurrent Kernels

Initially each kernel was launched in the default stream 0, the figure 5.3 illustrates such result in the Visual Profiler. Each kernel that is being launched cannot run simultaneously because the next kernel launch needs to compute data, in other words the kernels are not independent from each other.



FIGURE 5.3: Kernels running in the default 0 Stream.

The gsolution kernel computes the Zhang an Li model for x, y, z coordinates using extensively the global memory from the device. The kernel which is launched in the default stream cannot run in parallel with others kernels. To manage concurrency the kernel is divided into three kernels which computes individual each coordinate.

```
__global__ void gsolution(double *sfrelax_x, double *sfrelax_y, double *sfrelax_z,
                        double *sm_x, double *sm_y, double *sm_z,
                        double *sdex_x, double *sdex_y, double *sdex_z,
                        double *lapl_x, double *lapl_y, double *lapl_z,
                        double *deltam_x, double *deltam_y, double *deltam_z,
                        int grid_width)
{
    int i = blockIdx.x * blockDim.x + threadIdx.x + 2;
    int j = blockIdx.y * blockDim.y + threadIdx.y;

    int index = j * grid_width + i;

    if (i > 1 && i < NX + 2 && j >= 0 && j < NY)
    {
        deltam_x[index] = sfrelax_x[index] + sdex_x[index] + lapl_x[index]
                        - sm_x[index];
        deltam_y[index] = sfrelax_y[index] + sdex_y[index] + lapl_y[index]
                        - sm_y[index];
        deltam_z[index] = sfrelax_z[index] + sdex_z[index] + lapl_z[index]
                        - sm_z[index];
    }
}
```

}  
}

LISTING 5.2: Evaluation of x, y, z coordinates of the Zhang and Li model in a single kernel

To achieve concurrent kernels the streams need to access memory blocks that are pinned to a stream. So each memory block corresponding x, y, z are mapped to 3 streams, furthermore all the matrices corresponding for example x are mapped to stream 1.

```
--global__ void gsolution(double *deltam,
                        double *sfrelax, double *sm, double *sdex, double *lapl)
{
    int i = blockIdx.x * blockDim.x + threadIdx.x + 2;
    int j = blockIdx.y * blockDim.y + threadIdx.y;

    int index = j * NXCUDA_CONST + i;

    if (i > 1 && i < NX + 2 && j >= 0 && j < NY)
    {
        deltam[index] = sfrelax[index] + sdex[index] + lapl[index] - sm[index];
    }
}
```

LISTING 5.3: Evaluation of individual coordinates of the Zhang and Li model

This same method is applied to every kernel that can be separated into three kernel calls. Some kernels cannot be separated such as the cross product, because the product uses memory block from the other streams. Instead of running one big kernel, three individual kernels are launched simultaneously. The figure 5.4 shows the results of concurrent kernels in the Tesla K20.

```
for (int i = 0; i < 3; i++)  
{  
    gsolution<<<(blocks, threads, 0, stream[i]>>>(  
        spinAccXYZ[i]->getDev_deltam(),  
        spinAccXYZ[i]->getDev_sfrelax(),  
        spinAccXYZ[i]->getDev_sm(),  
        spinAccXYZ[i]->getDev_sdex(),  
        spinAccXYZ[i]->getDev_lapl());  
}
```

LISTING 5.4: Evaluation of individual coordinates of the Zhang and Li model



FIGURE 5.4: Concurrent kernels in the Tesla K20

### 5.3.2.1 Results

graphic.

k20 6.0 speed up.

However

### 5.3.3 Shared Memory

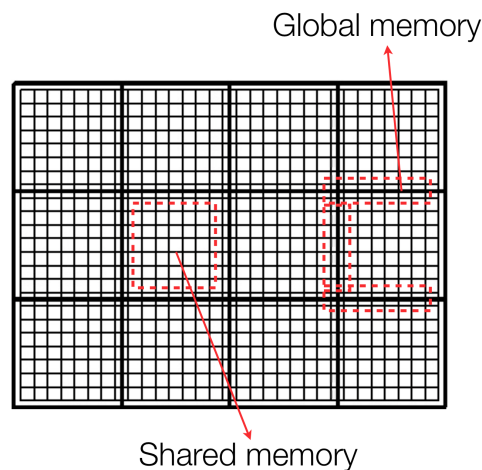
Shared memory was

To increase ocupancy

---

LISTING 5.5: Evaluation of individual coordinates of the Zhang and Li model

---




---

FIGURE 5.5: Shared Memory Strategy

<http://www.bu.edu/pasi/files/2011/07/Lecture31.pdf>

### 5.3.3.1 Results

### 5.3.4 Structure of Arrays (SAO)

AoS and SoA refer to "Array of Structures" and "Structure of Arrays" respectively. These two terms refer to two different ways of laying out your data in memory. This is illustrated in figure 5.6 and 5.7 respectively. AOS, grouping properties of an object together and making an array of those objects in memory, whereas a structure of arrays would be a single structure in which you make an array for each property. The structure

of arrays can allow for better cache utilization, easier to access continues data, making better use of each read you make from memory, providing a more effective route to memory.

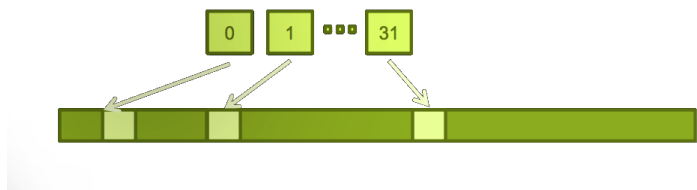


FIGURE 5.6: AOS memory layout

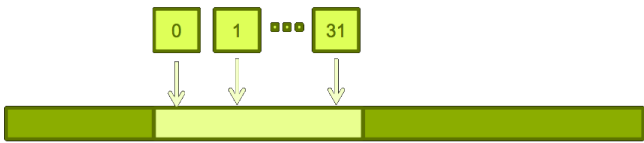


FIGURE 5.7: SAO memory layout

The code changed from.

5.3.4.1 Results

5.4 Optimized

Summarize the data

GPU	Original	Constant	Streams	Shared	SAO	Shared 2
Tesla K20m	0	0	0	0	0	0
Tesla M2070	107468.0	0	0	0	0	0
Tesla C2050	0	0	0	0	0	0
GeForce GTX 580	0	0	76407.4	0	0	0
GeForce 670m	0	0	0	0	0	0

seed-up

GPU	Original	Constant	Streams	Shared	SAO	Shared 2
Tesla K20m	0	0	0	0	0	0
Tesla M2070	3.512x	0	0	0	0	0
Tesla M2070	0	0	0	0	0	0
GeForce GTX 580	0	0	76407.4	0	0	0
GeForce 670m	0	0	0	0	0	0

## Chapter 6

# Conclusions and future work

### 6.1 Main Section 1

## Appendix A

# Appendix Title Here

Write your Appendix content here.

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