Section 1

Introduction

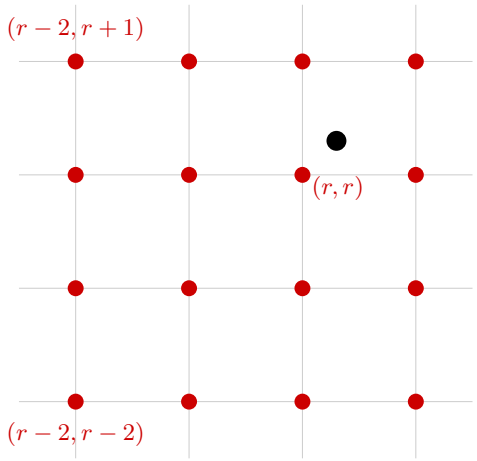
Subsection 1.1

The Interpolation Function

Given the material position X we want to obtian the velocity of fluid at that point. [1] This process is directly interpreted by

$$\mathbf{u}(\mathbf{X}(r,s,t),t) = \int \mathbf{u}(\mathbf{x},t)\delta(\mathbf{x}-\mathbf{X}(r,s,t))d\mathbf{x} \tag{1.1}$$

[1] *Note that X usually doesn't lies on the grid point.*



However in the discrete case we would need the discrete Delta Function.

In the figure on the left, black point is the material point and red points are the grid points. The δ function average all the velocity at the red points.

Something to pay attention is that in MATLAB, the convention for a matrix to represent a grid of point is to have lay points with same x coordinat in the **same row** This may be different from our intuition. However, This is reasonable since we normally define the matrix for storing points to have size

$$N_x \times N_y$$

So it has N_x rows.

Subsection 1.2

Discrete Delta Function

A commonly used discreted delta function uses a 4×4 grid. Thus the 1D delta function $\phi(x)$ should support on four points. We have the following postulates:

1. $\phi(r)$ is continuous for all real r
2. $\phi(r) = 0$ for $|r| \geq 2$
3. The following gurantee there is no difference between odd index grid point and even index grid point. Since they never exchange information between each others

$$\sum_{j \text{ even}} \phi(r-j) = \sum_{j \text{ odd}} \phi(r-j) = \frac{1}{2} \text{ for all real } r$$

4. The delta function should centered at the origin

$$\sum_{j}(r-j)\phi(r-j) = 0 \text{ for all real } r$$

5. The L_2 norm shoudl be given.

$$\sum_j (\phi(r-j))^2 = C \text{ for all real } r$$

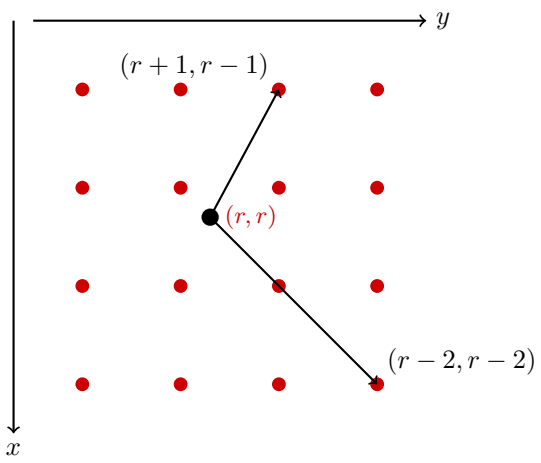
Here $r \in [0,1)$. This is a standard **4 point delta function**. We can also have 6 points delta function for higher accuracy. C can be determined by setting $r = 0$. We get

$$\phi(r) = \frac{3 - 2r + \sqrt{\Delta}}{8} \qquad \phi(r+1) = \frac{3 - 2r - \sqrt{\Delta}}{8}$$

and

$$\phi(r-1) = \frac{1 + 2r + \sqrt{\Delta}}{8} \qquad \phi(r-2) = \frac{1 + 2r - \sqrt{\Delta}}{8}$$

Here $\Delta = 1 + 4r - 4r^2$.



However, in MATLAB, grid points are stored in a particular way. Entries with same x coordinate is in same row and $(0,0)$ is at the upper-right corner.

Since the argument for ϕ is the position vector of grid point. The value for point $(r-2, r-2)$ is actually at the lower right as shown in the picture on the left. [2]

[2] *I made a mistake here when writing the function* `twoD_delta_function`. *The order is incorrect. Time: 251212*