Notes on Immersed Boundary Method

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Section 1

Introduction

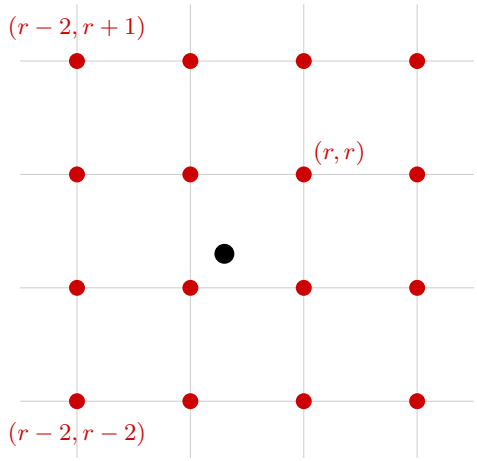
Subsection 1.1

The Interpolation Function

Given the material position X we want to obtian the velocity of fluid at that point. [1] This process is directly interpreted by

$$\mathbf{u}(\mathbf{X}(r,s,t),t) = \int \mathbf{u}(\mathbf{x},t)\delta(\mathbf{x}-\mathbf{X}(r,s,t))d\mathbf{x} \tag{1.1}$$

[1] *Note that X usually doesn't lies on the grid point.*



However in the discrete case we would need the discrete Delta Function.

In the figure on the left, black point is the material point and red points are the grid points. The δ function average all the velocity at the red points.

Something to pay attention is that in MATLAB, the convention for a matrix to represent a grid of point is to have lay points with same x coordinat in the **same row** This may be different from our intuition. However, This is reasonable since we normally define the matrix for storing points to have size

$$N_x \times N_y$$

So it has N_x rows.

Section 2

Penalty Immersed Boundary Method (PIB)

The Penalty Immersed Boundary Methods is a simple approach try to solve fluid-structure interaction with material bounadary that has mass.

Subsection 2.1

Rigid Body pIB

One kind of problem pIB can simulates is a rigid body in flow. The key of pIB is to consider two sets of Material Points, denoted as $X(q,s,t)$ and $Y(q,s,t)$. Here q, r are Lagrangian Coordinate and t is time. $\mathbf{X}$ is the true boundary points that interact masslessly with the flow, and def is called the **Ghost Mass**. It doesn't interact with the flow but only with the Material points $\mathbf{X}$ through an imaginary spring.

Explanation 1

1. At the start, the Material points' position is updated using fluid velocity interpolated at $\mathbf{X}$.

```
1  X_half_top = X_tube_top + dt / 2 * interpolation(u,
       X_tube_top, Num_b, Nx, Ny, dx, dy);
```

2. The force acting to the fluid by the Material is modeled by a spring connecting the Ghost

point and material point.

$$\mathbf{F}(q,r,t) = K(\mathbf{Y}(q,r,t) - \mathbf{X}(q,r,t)) \tag{2.1}$$

3. Then spread the force F to Eucleadian space.
4. Numerically solve the time evolution of fluid using Navier Stokes solver.
5. Update the Material point position $\mathbf{X}$.

2.1.1 Implementation : Circle in Flow