SECTION 1

Simulation Set Up

In this section, I briefly describe the simulation set up from the most trivial case all the way up to the true problem. We start with a brief introduction about how the inversion process proceed.

SUBSECTION 1.1

Inversion Process

From hydrostatic balance, the surface pressure is propotional to the surface sea height (SSH).

$$\eta = \frac{p\{z=0\}}{g} \tag{1.1}$$

In all the QG models, the pressure is expanded asymptotically in the form of

$$p_{total} \sim p^0 + \epsilon p^1 \tag{1.2}$$

The first order pressure is directly related to the first order potential Φ^0 by the formula

$$p^0 = f\Phi^0$$

As from geostrophic balance we have

$$p^0_y = -fu^0 = -f\partial_y\Phi^0 \qquad p^0_x = fv^0 = f\partial_x\Phi^0$$

Now we have

$$\eta^0 = \frac{f\Phi^0}{g}$$

Similarly from the first order correction we have

$$\eta^1 = \frac{p^1}{g}$$

So in general, considering the first order correction to the hydrostatic balance Eq 1.1, we have

$$\eta \sim \eta^0 + \epsilon\eta^1 = \frac{f\Phi^0}{g} + \epsilon\frac{p^1}{g} \tag{1.3}$$

This is the equation 40 in Ryan's note.

Now remember our goal is to use η to invert for Φ^0. What is the relationship between p^1 and Φ^0? The relationship is given by eq **??**. We copy it here

$$\boxed{\nabla^2 p^1 - f\zeta^1 = 2J(\Phi^0_x, \Phi^0_y)}$$

Where J is the Jacobian operator. This is a non-linear relationship, recall that ζ^1 is related to the potentials via Eq **??**. So

$$\nabla^2 p^1 = f\left(\nabla^2\Phi^1 + F^1_{zy} - G^1_{zx}\right) + 2J(\Phi^0_x, \Phi^0_y)$$

Here the relationship between Φ^1, F^1 and G^1 are given by Eq **??**, **??** and **??**. Then the whole inversion problem is clear.

SUBSECTION 1.2

Pre-Defined Velocity Fields

We start with a very simple case to play around with this model.

1.2.1 Toy Model Setup

The toy model start with a pre-defined potential Φ^0.