

LECTURE NOTES ON SURFACE QUASI-GEOSTROPHIC

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THOMAS LI

Undergraduate Student at New York University Shanghai

jl15535@nyu.edu

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Table 1: Glossary of Variables and Operators

Variables and Operators			
Symbol	Variable and Operators	Symbol	Description
$\mathbf{v}(x, y, z, t) = (u, v, w)$	Full 3-dimensional Velocity	∇	2D Gradient Operator
$\mathbf{u}(x, y, z, t) = (u, v)$	2-dimensional velocity, in x and y direction	∇_3	3D Gradient
$\hat{\cdot}$	Dimensionless Variable	$\frac{D}{Dt}$	Material Derivative

SECTION 1

Literature Review

SUBSECTION 1.1

Ryan et.al.

Year: 2025

The QG^{+1} model incorporates the first-order corrections that were neglected in the basic QG approximation. It essentially refines the QG equations by accounting for non-geostrophic (ageostrophic) flow components that are dependent on the Rossby number (ϵ).

In **Chapter 2**, the QG^{+1} model is introduced. In this paper,

$$N = f \equiv \text{Constant}^1$$

¹page 8

To facilitate the asymptotic approximation, a potential field is introduced.

$$\mathbf{A} = (-G, -F, \Phi)$$

By Incompressible condition we have

$$\mathbf{v} = \nabla_3 \times \mathbf{A}^2$$

²In this paper ∇_3 is 3D gradient. 2D is just ∇

Some Physical implications of the model

1. Breaking Symmetry of QG model.
2. It Captures Cyclogeostrophic balance.

Cyclogeostrophic balance is a fundamental force balance approximation used in meteorology and physical oceanography to describe the motion of fluids (like air and water) in curved paths, where the Coriolis force is balanced by the pressure gradient force and the centrifugal force. It is an essential extension of the simpler geostrophic balance, which only considers straight flow. This balance is particularly important in systems with high curvature and strong winds, such as tropical cyclones (hurricanes/typhoons), mid-latitude low-pressure systems, and strong ocean eddies. The Governing Equation is

$$\underbrace{f\mathbf{v}}_{\text{Coriolis Force}} + \underbrace{\frac{|\mathbf{v}|^2}{R}}_{\text{Centrifugal Force}} = \underbrace{-\frac{1}{\rho} \frac{\partial p}{\partial n}}_{\text{Pressure Gradient Force}} \quad (1.1)$$

Here n is the normal direction pointing toward the center of curvature.

3. Inclusion of **Frontogenesis** ³.

³Generation of Ocean Fronts

In **Chapter 3**. A simulation for QG^{+1} is conducted, showing several features:

1. More Vigorous due to capturing ageostrophic frontogenesis.
2. Since the Ageostrophic effects creates stronger surface velocity. Finer structure can be seen on surface using QG^{+1} . ⁴.

⁴See Figure 4 in page 24

In summary, this paper provides a very detailed derivation to the QG^{+1} equation which is introduced more detailed in 4. This paper also demonstrate two simulation to show how QG^{+1} model captures balanced submesoscale dynamics and frontogenesis.

SUBSECTION 1.2

J.Wang et.al. Reconstructing the Ocean's Interior from Surface Data

Year : 2013

In the **Introduction**, the author discussed the current challenge of using SSH and SST ⁵ measurement to reconstruct subsurface dynamics.

⁵*Surface Sea Height and
Surface Sea Temperature*

- Traditional studies assume the signal is dominated by barotropic and first baroclinic modes. However, these modes are typically calculated by **assuming buoyancy anomalies vanish at the surface**.
- SQG theory works as well. But it normally assume 0 interior PV.

The author introduced the **Interior plus surface QG** method. It is quasigeostrophic

SECTION 2

Quasi-Geostrophic Equations

In this section, I introduce some basic equations in QG theories. Building a foundation for SQG, eSQG and other variations introduced later.

SUBSECTION 2.1

Governing Equations

Momentum Equation

$$\frac{Dt}{Dt} \tag{2.1}$$

SECTION 3

Surface Quasi-Geostrophic Balance

SECTION 4

QG⁺¹ Model
