Lecture 3: The Eaton-Kortum Model

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Outline

1. Introduction
2. Detailed presentation of the Eaton-Kortum model[1]
3. Extensions
4. Some empirical applications

[1] Eaton, J. and S. Kortum, 2002, "Technology, Geography and Trade", *Econometrica* 70(5): 1741-1779

Introduction

Introduction

- The Ricardo and HO models predict:
  - Specialization along comparative advantage, including in GVCs
  - Inter-industry trade between countries that have different technologies (Ricardo) or endowments (HOS)
  - Gains from trade due to between-sector reallocation
- But their empirical performance is limited:
  - Ricardo model too stylized to deliver realistic predictions
  - HOV explains little of world trade. Empirical fit is consistently improved when introducing productivity differences.[2]
  - Neither model is directly consistent with gravity equations.

[2] Optional reading on the empirics of the HO model: Bernhofen (2010).

Introduction

- Eaton & Kortum (2002):
  - Ricardian model of international trade (differences in technology) with a role for geography (barriers to trade).
  - Trade patterns result from the interplay between comparative advantage forces and gravity forces.
  - Yields a gravity equation and can be estimated structurally.
- The model has several extensions and has been used in various quantitative exercises.
- One of the most cited International Trade articles in the last 20 years!

The Eaton-Kortum (2002) Model

Assumptions

- I countries ($i = 1...I$)
- Continuum of goods $j \in [0, 1]$
- CES Utility

$$U_i = Q_i \equiv \left[\int_0^1 Q_i(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

  with $\sigma > 1$ and where Q_i represents an aggregate consumption index in country i. Derivation of CES demand function
- Goods produced with a bundle of homogenous inputs/factors costing c_i in country i (first taken as exogenous)
- Iceberg trade costs from i to n: $d_{ni} > 1$, with $d_{ii} = 1, d_{ni} \leq d_{nk} d_{ki} \forall k$.
  Iceberg trade costs

Assumptions (ii)

- Country i's efficiency in producing good j: $z_i(j)$

$\Rightarrow$ (Minimum) CIF price of good j produced in country i, when exported in country n:

$$p_{ni}(j) = \underbrace{\frac{c_i}{z_i(j)}}_{Unit\ cost} \underbrace{d_{ni}}_{Trade\ barrier}$$

▸ optimal price

- Perfect competition across suppliers.

$\Rightarrow$ Price actually paid in country n for good j:

$$p_n(j) = min\{p_{ni}(j); i = 1...I\}$$

Assumptions (iii)

- *Probabilistic* technologies. $z_i(j)$ is the realization of a random variable Z_i drawn from a country-specific cdf:

$$F_i(z) = Pr[Z_i \leq z]$$

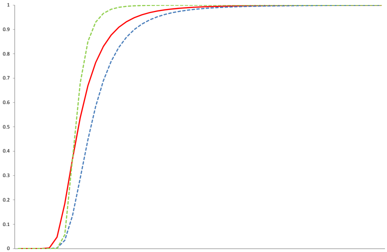
- Productivity draws assumed independent across goods and countries
- F_i assumed to be the Fréchet or Type II extreme value distribution[3]:

$$F_i(z) = e^{-T_i z^{-\theta}}$$

with $T_i > 0$ and $\theta > 0$ Fréchet distribution

[3] Eaton & Kortum (1999) micro-found the Fréchet productivity distribution as the outcome of a process of innovation and diffusion, in which T_i is a stock of ideas. EK 1999

$$F_i(z) = e^{-T_i z^{-\theta}}$$



Benchmark in red. Doubling of T_i in blue / of θ in green

- T_i "state of technology" or **absolute advantage**. A greater T_i means that country i is more likely to draw a high efficiency for any good j.
- θ heterogeneity across goods or extent of **comparative advantage** within the continuum: A greater θ implies a lower variance of productivity.

Price distribution

- Country i offers goods to country n with the price distribution:

$$G_{ni}(p) \equiv Pr[P_{ni} \leq p] = 1 - e^{-T_i(c_i d_{ni})^{-\theta} p^{\theta}}$$

- Distribution of prices actually paid in country n:

$$\begin{aligned} G_n(p) &\equiv Pr[P_n \leq p] \\ &= 1 - \prod_i [1 - G_{ni}(p)] \\ &= 1 - e^{-\Phi_n p^{\theta}} \end{aligned}$$

where $\Phi_n \equiv \sum_i T_i(c_i d_{ni})^{-\theta}$

details

Distribution of prices governed by

- States of technology around the world $\{T_i\}$,
- Input costs around the world $\{c_i\}$,
- Geographic barriers $\{d_{ni}\}$
  - If $d_{ni} = 1, \forall n, i$ then $\Phi_n = \Phi, \forall n$ (LOP)
  - If $d_{ni} \rightarrow \infty, \forall i$ then $\Phi_n = T_n c_n^{-\theta}$ (Autarky)

$\Rightarrow$ Φ_n interprets as the strength of competition that any firm will encounter in country n

Bilateral trade

- Law of large numbers: Share of goods that n buys from i = Probability that i provides the lowest price good in country n. ▸ details

$$
\begin{aligned}
\pi_{ni} &= Pr[p_{ni}(j) \leq min\{p_{ns}(j); s \neq i\}] \\
&= \int_0^\infty \prod_{s \neq i} [1 - G_{ns}(p)] dG_{ni}(p) \\
&= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n}
\end{aligned}
$$

- The price of goods actually bought has the same distribution $G_n(p)$, no matter the source. Hence the fraction of goods bought from i is also the fraction of expenditure on imports from i, or

$$
\frac{X_{ni}}{X_n} = \pi_{ni}
$$

- Taking logs, we get a gravity equation:

$$\ln X_{ni} = \underbrace{\ln\left(T_i c_i^{-\theta}\right)}_{Exporter-specific} + \underbrace{\ln\left(X_n \Phi_n^{-1}\right)}_{Importer-specific} - \theta \underbrace{\ln d_{ni}}_{Pair-specific}$$

- Bilateral trade is increasing in the export- and importer-specific terms, and decreasing with pair-specific trade barriers.
- The trade barriers elasticity is determined by the structural productivity distribution parameter θ
  - When the variance of productivity is high (low θ), the lowest-cost supplier has a strong cost advantage, and is more likely to remain the lowest-cost supplier when trade costs increase
  - Captures the tension between CA forces and trade barriers.
- Trade flows respond to geographic barriers at the **extensive margin**. More remote countries export a narrower range of goods.

Bilateral trade (iii)

- Country i's normalized import share in country n:

$$S_{ni} \equiv \frac{X_{ni}/X_n}{X_{ii}/X_i} = \frac{\Phi_i}{\Phi_n} d_{ni}^{-\theta} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\theta}$$

  Always lower than one due to the triangle inequality (the maximum value for p_n is $p_i d_{ni}$)

- As overall prices in market n fall relative to prices in market i ($\uparrow p_i/p_n$) or as n becomes more isolated from i ($\uparrow d_{ni}$), i's normalized share in n declines
- As the force of comparative advantage weakens (higher θ), normalized import shares become more elastic

$\Rightarrow$ Structural equation that provides insight into the value of comparative advantage (θ)

General Equilibrium solution

- Suppose production is linear in labor:

$$c_i = w_i$$

$\Rightarrow$ Price levels as a function of wages:

$$P_n = \gamma \left[\sum_i T_i \left(d_{ni} w_i \right)^{-\theta} \right]^{-1/\theta}$$

where $\gamma \equiv \left[\Gamma \left(\frac{\theta+1-\sigma}{\theta} \right) \right]^{1/(1-\sigma)}$ price index

$\Rightarrow$ Trade shares as a function of wages and prices:

$$\frac{X_{ni}}{X_n} = T_i \left(\frac{\gamma d_{ni} w_i}{P_n} \right)^{-\theta}$$

General Equilibrium solution

- To close the model, one needs to solve for equilibrium wages across countries.
- Use the trade balance condition for each country (as in DFS 1977).
- Requires to solve a fixed point problem $\Rightarrow$ Numerical solutions
- Several simplifying assumptions may help solve the model: exogenous labor supply, wages determined in the non-manufacturing sector...

Extensions

Extensions: Multiple sectors

- Costinot, Donaldson & Komunjer (2012) reinterpret EK's model in the context of a multi-country, multi-sector world
- There are K industries in each country ($k = 1...K$). Within each industry, a continuum of varieties is produced according to the technology described above
- Preferences are Cobb-Douglas across sectors and CES within sectors (elasticity σ^k)
- With multiple sectors, T_i now has a sector dimension (but, crucially, θ remains common across all countries and industries...):

$$F_i^k(z) = e^{-T_i^k z^{-\theta}}$$

Extensions: Multiple sectors (ii)

- With multiple industries, the model predicts what are the goods that a given country will specialize in
- Trade shares:

$$\ln \frac{\pi_{ni}^{k}\pi_{ni'}^{k'}}{\pi_{ni}^{k'}\pi_{ni'}^{k}} = \ln \frac{T_i^k T_{i'}^{k'}}{T_i^{k'} T_{i'}^{k}} - \theta \ln \frac{d_{ni}^{k} d_{ni'}^{k'}}{d_{ni}^{k'} d_{ni'}^{k}}$$

- Assuming further that $d_{ni}^k = d_{ni} d_n^k$ implies a monotonous relationship between the ranking of relative fundamental productivities and the ranking of exports:

$$\frac{T_i^1}{T_{i'}^1} \leq \ldots \leq \frac{T_i^K}{T_{i'}^K} \Leftrightarrow \frac{\pi_{ni}^1}{\pi_{ni'}^1} \leq \ldots \leq \frac{\pi_{ni}^K}{\pi_{ni'}^K}$$

- Without intra-industry heterogeneity (DFS, 1977), this ranking states that country i' has a comparative advantage over i in the high k goods $\Rightarrow$ Trade patterns with two countries

Extensions: Multiple sectors (iii)

- With Fréchet-distributed heterogeneity, we further have:

$$\frac{z_i^1(j)}{z_{i'}^1(j)} \preceq \dots \preceq \frac{z_i^K(j)}{z_{i'}^K(j)}$$

  where $\preceq$ denotes first-order stochastic dominance
- Country i' is not expected to produce high-k goods *only* (as in DFS 1977), but to produce and export *relatively more* of these goods.
- Randomness within a good implies that one cannot predict trade patterns for each variety of each good. But assumptions on the distribution imply clear predictions at the good level.

Extensions: Imperfect Competition

- Bernard, Eaton, Jensen & Kortum (2003) extend the EK model to allow for imperfect competition à la Bertrand.
- Consumer prices are above marginal costs. Model predicts a distribution of mark-ups in each market, that is bounded above by the Dixit-Stiglitz constant mark-up.
- Additional predictions on within-country heterogeneity in prices, productivities, etc[4]

[4] See BEJK's appendix on the AER website for further details.

Extensions: Trade in intermediate goods

- Caliendo & Parro (2015) build a Ricardian multi-sector model:

$$\pi_{ni}^{k} = \frac{T_i^k (c_i^k d_{ni}^k)^{-\theta^k}}{\Phi_n^k}$$

- Allow for IO linkages and trade in intermediate goods:

$$c_i^k = \Gamma_i^k w_i^{\beta_i^k} \left[\prod_l p_i^{l\,\gamma_i^{lk}} \right]^{1-\beta_i^k}$$

  where β_i^k measures the share of intermediate goods in production and γ_i^{lk} the share of intermediates from sector l used in the production of sector k

- Welfare impact of trade now comes from consumers having access to cheaper goods *and* producers being able to produce at lower costs

Extensions: Trade in frictional markets

- Lenoir, Martin, Mejean (2020) build a Ricardian model of trade in frictional product markets
- A discrete number of suppliers and buyers in each country (S_i and B_n)
- Each supplier from i has a probability λ_{ni} to meet with a buyer of n:

$$p_{b_n} = min_{s_j \in \Omega_{b_n}} \left\{ \frac{c_j d_{nj}}{z_{s_j}} \right\}$$

Extensions: Trade in frictional markets

- Price distribution:

$$G_n(p) = 1 - e^{-p^\theta \Phi_n \kappa_n}$$

with $\kappa_n \equiv \frac{\sum_j \lambda_{nj} T_j (c_j d_{nj})^{-\theta}}{\sum_j T_j (c_j d_{nj})^{-\theta}}$ the expected share of worldwide suppliers met

- Trade shares:

$$\pi_{ni} = \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} \frac{\lambda_{ni}}{\kappa_n}$$

- Individual trade patterns:

$$\rho_{ni}(z_{s_i}) = \lambda_{ni} e^{-(c_i d_{ni})^\theta z_{s_i}^{-\theta} \Phi_n \kappa_n}$$

Increasing in z_{s_i}, log super-modular in frictions and firm's productivity

Empirical Applications

Empirical analysis

- The model can be estimated to quantitatively assess the role of Ricardian advantage in driving international trade
- Crucial parameters: θ and $\{T_i\}$, which drive the distribution of productivities and comparative advantage
- Once those coefficients are estimated, it is possible to run various counterfactuals.[5]

[5] Optional reading: See Costinot and Donaldson (2017) for a non-technical introduction to the estimation of the EK model.

Estimating θ: EK Method 1

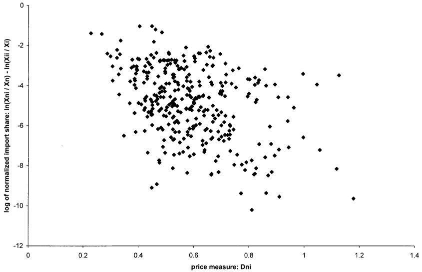
- Bilateral trade in manufactures among 19 OECD countries in 1990 (342 bilateral relationships X_{ni})
- Absorption of manufactures as a measure of X_n (STAN, OECD)
- Proxy for trade barriers:
  - Distance and other geographic barriers
  - Retail price differentials measured at the product level (WB): Interpreted as a sample of $p_i(j)$, used to calculate relative prices, which are theoretically bounded above by bilateral trade costs:

$$ln\frac{p_i d_{ni}}{p_n} \simeq= \frac{max2_j\{r_{ni}(j)\}}{mean\{r_{ni}(j)\}} = D_{ni}$$

$r_{ni}(j) = \ln p_n(j) - \ln p_i(j)$ relative price of commodity j
$\Rightarrow exp(D_{ni})$ price index in destination n that would prevail if everything was purchased from i, relative to the actual price index in n

Estimating θ, EK Method 1

Normalized import shares and relative prices



Source: Eaton & Kortum, 2002. Unconditional correlation -0.4

- Model: $\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{p_i d_{ni}}{p_n}\right)^{-\theta}$
- Estimated θ (method-of-moments)

$$\hat{\theta} = \frac{\sum_n \sum_i \ln \frac{X_{ni}/X_n}{X_{ii}/X_i}}{\sum_n \sum_i [-\ln d_{ni} - \ln P_i + \ln P_n]} \Rightarrow \hat{\theta} = 8.28$$

$\rightarrow$ Standard deviation in efficiency at given T = 15%

Estimating θ, CDK method

- Model:

$$\ln \frac{\pi_{ni}^{k}\pi_{ni'}^{k'}}{\pi_{ni}^{k'}\pi_{ni'}^{k}} = \ln \frac{T_i^k T_{i'}^{k'}}{T_i^{k'} T_{i'}^{k}} - \theta \ln \frac{d_{ni}^{k} d_{ni'}^{k'}}{d_{ni}^{k'} d_{ni'}^{k}}$$

  where $T_i^k = {z_i^k}^{\theta}$ is related to the *fundamental* productivity in country i and sector k

- *Identification problem*: $z_i^k \propto E[z_i^k(j)]$ is not observed. What is observed to the econometrician is $\tilde{z}_i^k \propto E[z_i^k(j)|\Omega_i^k]$ where Ω_i^k is the set of varieties *actually* produced in country i, sector k

- Under the Frechet assumption, one can however show:

$$\frac{z_i^k}{z_{i'}^k} = \left(\frac{\tilde{z}_i^k}{\tilde{z}_{i'}^k}\right)\left(\frac{\pi_{ii}^k}{\pi_{i'i'}^k}\right)^{1/\theta}$$

  (A more open country tend to produce a smaller, more productive set of varieties)

Estimating θ, CDK method

- Consequence:

$$\ln \frac{S^k_{ni} S^{k'}_{ni'}}{S^{k'}_{ni} S^k_{ni'}} = \theta \ln \frac{\tilde{z}^k_i \tilde{z}^{k'}_{i'}}{\tilde{z}^{k'}_i \tilde{z}^k_{i'}} - \underbrace{\theta \ln \frac{d^k_{ni} d^{k'}_{ni'}}{d^{k'}_{ni} d^k_{ni'}}}_{\text{residual if } d^k_{ni} = d_{ni} d^k_i} \quad \text{where} \quad S^k_{ni} = \frac{\pi^k_{ni}}{\pi^k_{ii}}$$

- Data:
  - Trade flows and import penetration indices $(1 - \pi^k_{ii})$ from standard sources (here: OECD STAN, Manuf)
  - Variations in relative productivity levels proxied by differences in producer price indices (source: GGDC Productivity Level Database):

$$\frac{\tilde{z}^k_i \tilde{z}^{k'}_{i'}}{\tilde{z}^{k'}_i \tilde{z}^k_{i'}} = \frac{E[p^k_{i'}(j)|\Omega^k_{i'}] E[p^{k'}_i(j)|\Omega^{k'}_i]}{E[p^k_i(j)|\Omega^k_i] E[p^{k'}_{i'}(j)|\Omega^{k'}_{i'}]}$$

Cross-sectional results

Dependent variable	log (corrected exports)	log (exports)	log (corrected exports)	log (exports)
	(1)	(2)	(3)	(4)
log (productivity based on producer prices)	1·123***	1·361***	6·534***	11·10***
	(0·0994)	(0·103)	(0·708)	(0·981)
Estimation method	OLS	OLS	IV	IV
Exporter × importer fixed effects	YES	YES	YES	YES
Industry × importer fixed effects	YES	YES	YES	YES
Observations	5652	5652	5576	5576
R^2	0·856	0·844	0·747	0·460

Notes: Regressions estimating equation (18) using data from 21 countries and 13 manufacturing sectors (listed in Table 1) in 1997. "Exports" is the value of bilateral exports from the exporting country to the importing country in a given industry. "Corrected exports" is "exports" divided by the share of the exporting country's total expenditure in the given industry that is sourced domestically (equal to one minus the country and industry's IPR). "Productivity based on producer prices" is the inverse of the average producer price in an exporter–industry. Columns (3) and (4) use the log of 1997 R&D expenditure as an instrument for productivity. Data sources and construction are described in full in Section 4.1. Heteroskedasticity-robust standard errors are reported in parentheses. ***Statistically significantly different from zero at the 1% level.

Source: Costinot, Donaldson & Komunjer, 2012. Table 3

- Endogeneity: i) Simultaneity bias (Agglomeration $\rightarrow \uparrow$ exports causes $\uparrow$ productivity), ii) attenuation bias due to measurement error in productivity
- Instruments: R&D expenditures, by country$\times$sector

Summary on $\hat{\theta}$

Table: Estimated θ parameters

	$\hat{\theta}$
EK, Method of moments	8.28
EK, 2Stages GLS+OLS	2.84
EK, 2Stages GLS+2SLS	3.60
EK, OLS Trade Eq.	2.44
EK, 2SLS Trade Eq.	12.86
SW, SMM	4.12
CDK, IV	6.53
CP, (mean)	8.22

EK=Eaton & Kortum (2002), CDK=Costinot, Donaldson & Komunjer (2012)

SW=Simonovska & Waugh (2011), CP=Caliendo & Parro (2014)

Further empirics

- Evidence of Ricardian comparative advantage using *strong* structural assumptions (basically Fréchet)
- Recent literature estimates non-parametric Ricardian models...
- Costinot and Donaldson (2012, 2017), Costinot et al. (2016): focus on agriculture where unconditional productivities (by field and crop) can be predicted.
  - → Correlation between predicted and observed output patterns is as high as .21 (CD, 2012)
  - → Gains from specialization in agriculture are substantial, as measured by average productivity gains and a reduction in the spatial dispersion of prices (CD, 2017)
  - → Large negative impact of future decreases in productivities due to climate change. Welfare impact depends strongly on how production reallocates over space as a consequence of changes in CA (CDS, 2016)

Counterfactuals

Once estimated, the model can be used to run counterfactuals:

- What are the welfare gains from trade? (Arkolakis et al, 2012)
- What is the impact of multilateral/unilateral tariff eliminations? (Caliendo & Parro, 2015)
- How much does trade spread the benefit of local improvements in technology? (Levchenko & Zhang, 2011)
- How will climate change affect the patterns of production and trade? (CDS, 2016)
- How does specialization affect the volatility of GDPs? (Caselli et al, 2015)

Application: Evaluating the gains from NAFTA

- Caliendo & Parro (2015) use a variant of the Eaton-Kortum model to evaluate the trade and welfare impact of NAFTA.
- NAFTA: A free trade area between the US, Mexico and Canada
  - Enhance trade within the area / Divert existing trade between the area and the RoW
  - Increase welfare: Access to cheaper consumption goods plus increased competitiveness through a drop in input prices
  - → Potentially important Ricardian gains since the 3 countries have very different production structures
- Main insights:
  - ▸ Important role of sectoral IO linkages to amplify the trade and welfare effect of the partnership

Theoretical framework

i. Multiple sectors:

$$U_i \;=\; \prod_{k=1}^{K} Q_i^{k\;\alpha_i^k}, \quad \sum_{k=1}^{K} \alpha_i^k = 1, \quad Q_i^k = \left[\int_0^1 Q_i^k(j)^{\frac{\sigma^k-1}{\sigma^k}} dj\right]^{\frac{\sigma^k}{\sigma^k-1}}$$

ii. Input-Output linkages:

$$c_i^k = w_i^{\gamma_i^k} \prod_{k'=1}^{K} P_i^{k'\;\gamma_i^{k,k'}}, \quad \sum_{k=1}^{K} \gamma_i^{k,k'} = 1 - \gamma_i^k$$

iii. Non tradable sectors:

$$d_{ni}^k = +\infty \textit{ for some } k$$

iv. Sector-specific productivity distributions (Fréchet):

$$F_i^k(z) = e^{-T_i^k z^{-\theta^k}}$$

Analytical predictions

- Equilibrium prices under perfect competition:

$$p^k_{ni}(j) = \frac{c^k_i}{z^k_i(j)} d^k_{ni} \quad \Rightarrow \quad P^k_n = A^k \left[\sum_{h=1}^{I} T^k_h \left[c^k_h d^k_{nh} \right]^{-\theta^k} \right]^{-1/\theta^k}$$

- Expenditure shares:

$$\begin{aligned} \pi^k_{ni} &= Pr[p^k_{ni}(j) \leq min_s \{p^k_{ns}(j); s \neq i\}] \\ &= \frac{T^k_i \left[c^k_i d^k_{ni} \right]^{-\theta^k}}{\sum_{h=1}^{I} T^k_h \left[c^k_h d^k_{nh} \right]^{-\theta^k}} \end{aligned}$$

Changes in tariffs affect π^k_{ni} directly (through d^k_{ni}) and indirectly (through the price of inputs encapsulated in c^k_i)

- GE solution under the assumption of balanced trade at the world level (but country-specific trade deficits) gives the vector of equilibrium wages **w** which is specific to a tariff vector

Impact of trade liberalization

- Equilibrium in relative changes implies:

$$\ln \frac{\hat{w}_n}{\hat{P}_n} = -\underbrace{\sum_{k=1}^{K} \frac{\alpha_n^k}{\theta^k} \ln \hat{\pi}_{nn}^k}_{\textit{Final goods}} - \underbrace{\sum_{k=1}^{K} \frac{\alpha_n^k}{\theta^k} \frac{1-\gamma_n^k}{\gamma_n^k} \ln \hat{\pi}_{nn}^k}_{\textit{Intermediate goods}} - \underbrace{\sum_{k=1}^{K} \frac{\alpha_n^k}{\gamma_n^k} \ln \prod_{l=1}^{J} \left(\hat{P}_n^l / \hat{P}_n^k\right)^{\gamma_n^{l,k}}}_{\textit{Sectoral Linkages}}$$

$$\ln \hat{\pi}_{ni}^k = -\theta^k \left[\ln \hat{c}_i^k + \ln \hat{d}_{ni}^k - \ln \hat{P}_n^k\right]$$

where $\hat{x} = dx/x$, $\{\hat{c}_i^k\}$ and $\{\hat{P}_n^k\}$ are non-linear functions of $\{\hat{w}_n\}$ and $\{\hat{d}_{ni}^k\}$

- Impact of trade liberalization on real wages can be summarized by the impact it has on domestic shares ($\{\pi_{nn}^k\}$) and sectoral price indices ($\{P_n^k\}$)

Impact of trade liberalization (ii)

- Trade liberalization increases real wages by reducing the sectoral shares of domestic consumption ($\ln \hat{\pi}_{nn}^k$), i.e.

  i. Giving consumers access to cheaper imported goods
  ii. Reducing the cost of same sector imported inputs (Only role of intermediates if $\gamma_n^k \neq 1$ and $\gamma_n^{k,k} = 1 - \gamma_n^k$
  iii. Reducing the cost of imported inputs for other sectors (when $\gamma_n^{k,k} \neq 1 - \gamma_n^k$)

- Changes in real wages do not directly map into changes in welfare in this model because of trade deficits (D_n) and tariff revenues (R_n):

$$\ln \hat{W}_n = \ln \frac{\hat{I}_n}{\hat{P}_n} = \frac{w_n L_n}{I_n} \ln \frac{\hat{w}_n}{\hat{P}_n} + \frac{R_n}{I_n} \ln \frac{\hat{R}_n}{\hat{P}_n} + \frac{D_n}{I_n} \ln \frac{\hat{D}_n}{\hat{P}_n}$$

Welfare Impact

- Using the equilibrium conditions of the model:

$$\ln \frac{\hat{I}_n}{\hat{P}_n} = \underbrace{\sum_{h=1}^{I}\sum_{k=1}^{K}\left(\frac{E_{hn}^k}{I_n}\ln \hat{c}_n^k - \frac{M_{nh}^k}{I_n}\ln \hat{c}_h^k\right)}_{Terms\ \ of\ \ trade} + \underbrace{\sum_{h=1}^{I}\sum_{k=1}^{K}\frac{d_{nh}^k M_{nh}^k}{I_n}\left(\ln \hat{M}_{nh}^k - \ln \hat{c}_h^k\right)}_{Volume\ \ of\ \ trade}$$

- Terms of trade effect due to an increase in exporter prices relative to the change in importer prices
- Volume of trade effect due to the creation of additional trade flows following trade liberalization

Empirical strategy

- Calibration of the observed parameters:
  - $\{\pi_{ni}^k\}$ calibrated using trade and production data
  - $\{\alpha_i^k\}$ fitted to data on sectoral absorption
  - $\{\gamma_i^k\}$ and $\{\gamma_i^{k,k'}\}$ fitted to IO tables
- Estimation of the unobserved parameters $\{\theta^k\}$:

$$\ln \frac{X_{ni}^k X_{im}^k X_{mn}^k}{X_{in}^k X_{mi}^k X_{nm}^k} = -\theta^k \ln \frac{d_{ni}^k d_{im}^k d_{mn}^k}{d_{in}^k d_{mi}^k d_{nm}^k}$$

Assume $\ln d_{ni}^k = \ln(1+\tau_{ni}^k) + \nu_{ni}^k + \mu_n^k + \delta_i^k + \varepsilon_{ni}^k, \quad \nu_{ni}^k = \nu_{in}^k$.
Then

$$\ln \frac{X_{ni}^k X_{im}^k X_{mn}^k}{X_{in}^k X_{mi}^k X_{nm}^k} = -\theta^k \ln \frac{(1+\tau_{ni}^k)1+\tau_{im}^k)(1+\tau_{mn}^k)}{(1+\tau_{in}^k)(1+\tau_{mi}^k)(1+\tau_{nm}^k)} + \varepsilon_{nim}^k$$

Only data on bilateral trade and tariffs by product are needed.

Sectoral trade elasticities

Table 1. Dispersion-of-productivity estimates

	Full sample			99% sample			97.5% sample		
Sector	θ^j	s.e.	N	θ^j	s.e.	N	θ^j	s.e.	N
Agriculture	8.11	(1.86)	496	9.11	(2.01)	430	16.88	(2.36)	364
Mining	15.72	(2.76)	296	13.53	(3.67)	178	17.39	(4.06)	152
Manufacturing									
Food	2.55	(0.61)	495	2.62	(0.61)	429	2.46	(0.70)	352
Textile	5.56	(1.14)	437	8.10	(1.28)	314	1.74	(1.73)	186
Wood	10.83	(2.53)	315	11.50	(2.87)	191	11.22	(3.11)	148
Paper	9.07	(1.69)	507	16.52	(2.65)	352	2.57	(2.88)	220
Petroleum	51.08	(18.05)	91	64.85	(15.61)	86	61.25	(15.90)	80
Chemicals	4.75	(1.77)	430	3.13	(1.78)	341	2.94	(2.34)	220
Plastic	1.66	(1.41)	376	1.67	(2.23)	272	0.60	(2.11)	180
Minerals	2.76	(1.44)	342	2.41	(1.60)	263	2.99	(1.88)	186
Basic metals	7.99	(2.53)	388	3.28	(2.51)	288	-0.05	(2.82)	235
Metal products	4.30	(2.15)	404	6.99	(2.12)	314	0.52	(3.02)	186
Machinery n.e.c.	1.52	(1.81)	397	1.45	(2.80)	290	-2.82	(4.33)	186
Office	12.79	(2.14)	306	12.95	(4.53)	126	11.47	(5.14)	62
Electrical	10.60	(1.38)	343	12.91	(1.64)	269	3.37	(2.63)	177
Communication	7.07	(1.72)	312	3.95	(1.77)	143	4.82	(1.83)	93
Medical	8.98	(1.25)	383	8.71	(1.56)	237	1.97	(1.36)	94
Auto	1.01	(0.80)	237	1.84	(0.92)	126	-3.06	(0.86)	59
Other Transport	0.37	(1.08)	245	0.39	(1.08)	226	0.53	(1.15)	167
Other	5.00	(0.92)	412	3.98	(1.08)	227	3.06	(0.83)	135
Test equal parameters		F(17, 7294) = 7.52					Prob > F = 0.00		
Aggregate elasticity	4.55	(0.35)	7212	4.49	(0.39)	5102	3.29	(0.47)	3482

Source: Caliendo & Parro, 2015. The “99% sample” and “97.5% sample” drop the 'smallest' countries in each sector from the estimation.

Counterfactual analysis

i. Introduce the change in tariffs from 1993 to 2005 between NAFTA members, fix tariffs for the RoW to 1993 levels

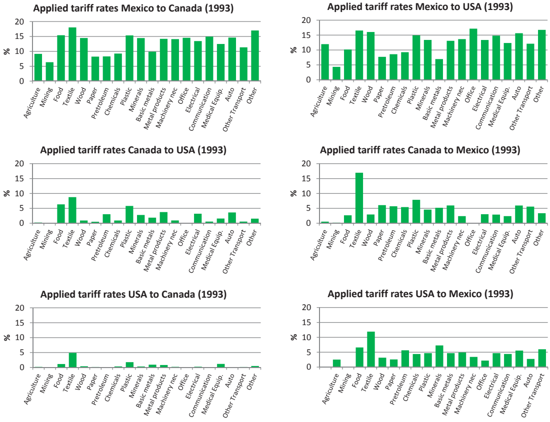
ii. Introduce the change in tariffs from 1993 to 2005 between NAFTA members as well as observed changes in world tariffs

iii. Introduce the change in world tariffs from 1993 to 2005, fixing NAFTA tariffs to 1993 levels.

The difference between (ii) and (iii) measures gains from world tariff reductions with and without NAFTA.[6]

[6] The model only takes exogenous trade imbalances into account. But in principle, trade liberalization can affect trade deficits. This is a limit of the analysis.

Pre-NAFTA tariffs



Source: UNCTAD-TRAINS)

Source: Caliendo & Parro, 2015. In 1993, sectoral tariff rates applied by Mexico, Canada and the US to NAFTA members were on average 12.5, 4.2 and 2.7%. By 2005, they dropped to almost zero between NAFTA members but tariffs that Mexico, Canada and the US applied to the RoW were on average 7.1, 2.2 and 1.7%, respectively

The role of intermediate goods and sectoral linkages

- In 1993, the role of intermediate goods is already substantial...
  - Respectively 68, 61.5 and 64.6% of Mexico's, Canada's and the US imports from non-NAFTA countries were intermediate goods
  - Respectively 82.1, 72.3 and 72.8% of Mexico's, Canada's and the US imports from NAFTA countries were intermediate goods
- ... As is the extent of cross-sectoral linkages:
  - In the IO Tables, the mean share of own-sector inputs is around 15-20%
  - More than 70% of intermediate consumption is addressed to other sectors
  - Average share of tradables in the production of non-tradables is 23% for the US and 32% for Mexico / Average shares of non-tradables in the production of tradables are 34% for the US and 26% for Mexico

Welfare effect from NAFTA's Tariff reductions

Table 2. Welfare effects from NAFTA's tariff reductions

Country	Welfare: Total	Terms of trade	Volume of Trade	Real wages
Mexico	1.31%	-0.41%	1.72%	1.72%
Canada	-0.06%	-0.11%	0.04%	0.32%
U.S.	0.08%	0.04%	0.04%	0.11%

Source: Caliendo & Parro, 2015. Analysis holds RoW tariffs unchanged

- Mexico gains the most, both in terms of welfare and in terms of real wages.
- Most important source of gains is increase in the volume of trade (mostly within NAFTA; trade vis-à-vis the RoW decreases, trade diversion).
- US terms-of-trade improved, both vis-à-vis NAFTA members and the RoW.
- Welfare effects widely vary across sectors.

Trade effect from NAFTA's Tariff reductions

Table 5. Trade effects from NAFTA's tariff reductions

	Mexico	Canada	U.S.
Mexico's imports	-	116.60%	118.31%
Canada's imports	58.57%	-	9.49%
U.S.'s imports	109.54%	6.57%	-

Source: Caliendo & Parro, 2015. Analysis holds RoW tariffs unchanged

- Large aggregate effects for all members
- Canada and the US increased a lot their imports from Mexico: role as a supplier of intermediates to NAFTA
- Strong impact on the specialization of countries: Mexico becomes more specialized

Specialization due to NAFTA

Table 6. Export shares by sector before and after NAFTA's tariff reductions

Sector	Mexico Before	Mexico After	Canada Before	Canada After	United States Before	United States After
Agriculture	4.72%	3.03%	4.99%	5.04%	6.91%	6.35%
Mining	15.53%	7.85%	8.99%	8.96%	1.72%	1.52%
Manufacturing						
Food	2.33%	1.48%	4.82%	4.68%	5.09%	4.73%
Textile	4.42%	6.92%	1.05%	1.49%	2.68%	3.49%
Wood	0.59%	0.52%	8.12%	8.05%	2.02%	1.98%
Paper	0.62%	0.51%	8.34%	8.44%	4.99%	4.89%
Petroleum	1.62%	5.28%	0.59%	0.78%	4.30%	5.71%
Chemicals	4.40%	2.53%	5.58%	5.40%	10.00%	9.25%
Plastic	0.80%	0.48%	2.06%	2.06%	2.28%	2.43%
Minerals	1.32%	0.84%	0.81%	0.78%	0.94%	0.92%
Basic metals	3.24%	2.00%	10.29%	10.19%	3.05%	3.11%
Metal products	1.22%	1.03%	1.47%	1.53%	2.23%	2.59%
Machinery n.e.c.	4.30%	2.53%	4.69%	4.49%	10.37%	9.70%
Office	3.34%	5.07%	2.44%	2.54%	7.70%	7.29%
Electrical	20.79%	34.07%	2.50%	2.35%	6.07%	7.97%
Communication	8.57%	7.08%	3.11%	3.02%	7.19%	6.81%
Medical	2.48%	3.28%	0.98%	1.03%	5.16%	4.79%
Auto	16.43%	13.05%	24.42%	24.07%	8.20%	8.09%
Other Transport	0.28%	0.26%	3.21%	3.58%	7.32%	6.65%
Other	3.02%	2.20%	1.55%	1.52%	1.77%	1.74%
Normalized Herfindahl	0.092	0.138	0.083	0.081	0.042	0.040

Source: Caliendo & Parro, 2015. Analysis holds RoW tariffs unchanged

Decomposition of trade and welfare effects

Table 11. Trade and welfare effects from NAFTA across different models

	Welfare			Imports growth from NAFTA members			
		Multi sector			Multi sector		
Country	One sector	No materials	No I-O	One sector	No materials	No I-O	Benchmark
Mexico	0.41%	0.50%	0.66%	60.99%	88.08%	98.96%	118.28%
Canada	-0.08%	-0.03%	-0.04%	5.98%	9.95%	10.14%	11.11%
U.S.	0.05%	0.03%	0.04%	17.34%	26.91%	30.70%	40.52%

Source: Caliendo & Parro, 2015. Analysis holds RoW tariffs unchanged

- Welfare gains are always reduced in comparison to benchmark

$\Rightarrow$ Trade in intermediates, Sectoral heterogeneity and Sectoral linkages all matter

Concluding remarks

- A very elegant way of introducing Ricardo into a multi-country (potentially multi-sector) model
- Multiple extensions and empirical applications.
- Analytics strongly rely on some assumptions: Fréchet distribution, same variance of productivities in all industries.
- Estimates are quite sensitive to the estimation strategy, with important consequences for gains from trade...

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Demand functions

- Consumers solves:

$$\begin{cases} \max_{\{Q_i(j)\}_{j\in[0,1]}} \left[\int_0^1 Q_i(j)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}} \\ s.t. \int_0^1 P_i(j)Q_i(j)dj \leq R_i \end{cases}$$

- Solution of the maximization program is:

$$Q_i(j) = \left(\frac{P_i(j)}{P_i}\right)^{-\sigma} \frac{R_i}{P_i}$$

with P_i the ideal price index $(R_i/P_i = U_i, \quad \forall R_i)$:

$$P_i = \left[\int_0^1 P_i(j)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}}$$

Back to assumptions

Iceberg trade costs

Definition: *A cost of transporting a good that uses up some fraction of the good itself, rather than other resources. By analogy with floating icebergs, costless except for the part of the iceberg that melts. Far from realistic, but a tractable way of modeling transport costs since it impacts no other market.* (Alan Deardorff's glossary)

- A short-cut: Trade cost is just a deadweight loss to the economy
- Ad valorem equivalent of trade costs about 170% in rich countries ($d_{ni} = 2.7$), including transport, border-related and local distribution costs (Anderson and van Wincoop 2004).

Back to assumptions

Optimal Prices

- Firms' profit:

$$\Pi_i(j) = \sum_n \left[p_{ni}(j) Q_{ni}(j) - \frac{c_i}{z_i(j)} d_{ni} Q_{ni}(j) \right] = \sum_n \Pi_{ni}(j)$$

- Under perfect competition:

$$p_{ni}(j) = \frac{c_i}{z_i(j)} d_{ni}$$

and

$$Q_{in}(j) = 0 \; if \;\; p_{in}(j) > p_n(j) / Q_n(j) \; otherwise$$

Back to assumptions

Price distribution

- $p_{ni}(j) = \frac{c_i}{z_i(j)} d_{ni}$ is the realization of random variable P_{ni}, that has cdf

$$\begin{aligned} G_{ni}(p) &= Pr[P_{ni} \leq p] = Pr\left[Z_i \geq \frac{c_i d_{ni}}{p}\right] \\ &= 1 - F_i\left(\frac{c_i d_{ni}}{p}\right) = 1 - e^{-T_i\left(\frac{c_i d_{ni}}{p}\right)^{-\theta}} \end{aligned}$$

- $p_n(j) = min\{p_{ni}(j); i = 1...I\}$ is the realization of a random variable $P_n = min\{P_{ni}; i = 1...I\}$ which cdf is:

$$G_n(p) = Pr[P_n \leq p] = 1 - \prod_{i=1}^{I} Pr[P_{ni} > p]$$

$$= 1 - \prod_{i=1}^{I}[1 - G_{ni}(p)] = 1 - e^{-p^{\theta}\sum_{i=1}^{I} T_i(c_i d_{ni})^{-\theta}}$$

Back to the model

Price index

- Using:

$$G_n(p) = 1 - e^{-\Phi_n p^\theta} \quad \textit{and} \quad g_n(p) = \Phi_n \theta p^{\theta-1} e^{-\Phi_n p^\theta}$$

- one can derive the price index:

$$\begin{aligned}
P_n &= \left[\int_0^1 p_n(j)^{1-\sigma} dj\right]^{\frac{1}{1-\sigma}} = \left[\int_0^1 p^{1-\sigma} dG_n(p)\right]^{\frac{1}{1-\sigma}} \\
&= \left[\int_0^1 p^{1-\sigma} \theta p^{\theta-1} \Phi_n e^{-\Phi_n p^\theta} dp\right]^{\frac{1}{1-\sigma}} \\
&= \Phi_n^{-1/\theta} \left[\int_0^1 u^{\frac{1-\sigma}{\theta}} e^{-u} du\right]^{\frac{1}{1-\sigma}} \quad \textit{where} \quad u = \Phi_n p^\theta \\
&= \Phi_n^{-1/\theta} \left[\Gamma\left(\frac{1-\sigma}{\theta} - 1\right)\right]^{\frac{1}{1-\sigma}}
\end{aligned}$$

(well-defined iif $\sigma - 1 < \theta$)

Back to the model

Fréchet distribution

- Generalized extreme value distribution: A family of continuous probability distributions usually used as an approximation to model the maxima of long (finite) sequences of random variables
- CDF:

$$F(x;\mu,\sigma,\xi) = exp\left\{-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-\frac{1}{\xi}}\right\}$$

μ a location parameter, $\sigma > 0$ the scale parameter, ξ the shape parameter

Fréchet distribution

- In particular:
  - Gumbel or type I extreme value: $\xi = 0$

$$F(x;\mu,\sigma) \to exp\left\{-exp\left[-\frac{x-\mu}{\sigma}\right]\right\}, \quad x \in R$$

  - Frechet or type II extreme value: $\xi > 0$

$$F(x;\mu,\sigma,\alpha) = \begin{cases} 0, & x \leq \mu \\ exp\left\{-\left[\frac{x-\mu}{\sigma}\right]^{-\alpha}\right\}, & x > \mu \end{cases}$$

  - Reversed Weibull or type III extreme value: $\xi < 0$

$$F(x;\mu,\sigma,\alpha) = \begin{cases} exp\left\{-\left[-\frac{x-\mu}{\sigma}\right]^{\alpha}\right\}, & x < \mu \\ 1, & x \geq \mu \end{cases}$$

Back to the model

A model of technology diffusion (EK, IER 1999)

- A model of endogenous growth in which technology is the result of research effort
- Flow of ideas diffusing to country i $\dot{\mu}_{it}$ depends on the stock of researchers in each country, their productivity and the rate at which ideas diffuse across countries
- Quality of an idea is a random variable drawn in a Pareto $F(z) = 1 - z^{-\theta}$
- New ideas adopted at a rate $\dot{\mu}_{it} z^{-\theta}$

$\Rightarrow$ Proba that no idea is adopted in the time interval $[t, t + dt] = e^{-\dot{\mu}_{it} z^{-\theta} dt}$

Evolution of the production frontier:
$H_i(z, t + dt) = H_i(z, t) e^{-\dot{\mu}_{it} z^{-\theta} dt} \Rightarrow \frac{\partial \ln H_i(z, t+dt)}{\partial t} = -\dot{\mu}_{it} z^{-\theta} \Rightarrow$
$H_i(z, t) = e^{-\mu_{it} z^{-\theta}}$

Back to the model

Details on trade shares

- Probability that country i is the lowest-cost supplier:

$$\begin{aligned}
\pi_{ni} &= \int_0^\infty \prod_{s \neq i} Pr[P_{ns} \geq p] dG_{ni}(p) \\
&= \int_0^\infty e^{-p^\theta \sum_{s \neq i} T_s (c_s d_{ns})^{-\theta}} dG_{ni}(p) \\
&= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} \int_0^\infty \Phi_n e^{-p^\theta \Phi_n} \theta p^{\theta - 1} dp \\
&= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} [1 - G_n(p)]_0^\infty \\
&= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n}
\end{aligned}$$

Details on trade shares

- Distribution of prices conditional on being the lowest cost supplier:

$$
\begin{aligned}
\tilde{G}_{ni}(p) &= Pr[P_{ni} \leq p | P_{ni} \leq min_{s \neq i}\{P_{ns}\}] \\
&= \int_0^p \prod_{s \neq i} Pr[P_{ns} \geq q] dG_{ni}(q) \\
&= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} \int_0^p \Phi_n e^{-p^\theta \Phi_n} \theta p^{\theta - 1} dp \\
&= \pi_{ni} G_n(p)
\end{aligned}
$$

For goods that are purchased, conditioning on the source has no bearing on the good's price $\rightarrow$ Trade shares only depends on π_{ni}

Back to the model