A FRAMEWORK FOR GEOECONOMICS

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Governments use their countries' economic strength from financial and trade relationships to achieve geopolitical and economic goals. We provide a model of the sources of geoeconomic power and how it is wielded. The source of this power is the ability of a hegemonic country to coordinate threats across disparate economic relationships as a mean of enforcement on foreign entities. The hegemon wields this power to demand costly actions out of the targeted entities, including mark-ups, import restrictions, tariffs, and political concessions. The hegemon uses its power to change targeted entities' activities to manipulate the global equilibrium in its favor and increase its power. A sector is strategic either in helping the hegemon form threats or in manipulating the world equilibrium via input-output amplification. The hegemon acts a global enforcer, thus adding value to the world economy, but destroys value by distorting the equilibrium in its favor.

KEYWORDS: Geopolitics, Economic Coercion, Economic Statecraft.

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1. INTRODUCTION

Hegemonic countries use their financial and economic strength to extract economic and political surplus from other countries around the world. This practice, referred to as geoeconomics, is not as blunt as the direct threat to go to war, as it operates through commercial channels like the threat to interrupt the supply or purchase of goods, the sharing of technology, or financial relationships and services. Despite its importance and practical relevance, the deeper foundations of geoeconomic power have remained elusive.

We provide a formal model of the sources of geoeconomic power and how it is wielded. We identify the source of the power to be the ability of countries like the US (or China), which we refer to as hegemons, to coordinate threats across disparate economic relationships as a means of enforcement for their demands on foreign entities over which they have no direct legal control. Such coordinated "joint threats" – for example, suspending access to the dollar-based financial system and blocking technological inputs such as semiconductors – are particularly effective because they threaten punishment across many relationships for deviations on any one of them. Indeed, geoeconomic power operates in areas in which complete contracts are not feasible either because of limited enforceability or because for political and legal reasons formal contracts are unpalatable (e.g. government to government relationships). The hegemon's ability to act as a global enforcer using joint threats can add value by reducing commitment issues and expanding the set of feasible economic activity.

The hegemon wields its power to demand costly actions from the targeted entities. This notion of power is broader than market power and also includes the ability to demand changes in economic activities and political concessions. We show how the hegemon uses its demands not only to extract direct monetary benefits but also to shape the global equilibrium in its favor by asking targeted entities to alter their activities vis à vis other entities. For example, the hegemon may demand that foreign banks stop lending to a geopolitical rival, such as when the US demanded that European commercial banks stop financing trade between Iran and third-party countries. A hegemon may also ask a foreign firm to stop using sensitive technology sourced from a rival, such as when the US pressured European firms to stop purchasing telecommunication technology and infrastructure supplied by Huawei.

Formally, we model a collection of countries and productive sectors with an input-output network structure. Sectors are collections of firms operating in a specific country and industry (e.g. Russian oil extraction and American oil extraction are two distinct sectors). The model features limited enforceability of contracts, as well as externalities both in production functions and in the objective functions of country-level representative consumers. Production externalities, whereby an individual sector's productivity can depend on what other sectors are producing both within and across countries, can capture external economies of scale and strategic complementarities. The externalities entering directly in the representative consumer's objective help us capture political affinity between countries' governments as well as externalities that are traditionally outside of the domain of economics, such as national security. We model threats as trigger strategies that firms and governments employ to punish other entities for deviating from contracts through exclusion from an economic relationship in the future. Joint threats are trigger strategies in which the punishment of exclusion from multiple economic relationships is triggered by an entity's deviation on any one of them. In our model, a hegemon is a country that is able to coordinate many such threats both via its national entities and via their economic network abroad.

We allow targeted entities to be firms or governments. In practice both are relevant: hegemons pressure foreign governments to obtain political concessions or pressure foreign firms for specific actions often against the wishes of those firms' governments. A key feature of our model is that the targeted foreign entities voluntarily comply with the hegemon's demands. They do so if the value of commitment derived from the hegemon's joint threats outweighs the costs of acceding to the hegemon's demands. In practice, these threats are crucial in the conduct of secondary sanctions to induce foreign entities to stop activities that are legal in their own jurisdictions. For example, foreign banks comply with US secondary sanctions given the value generated by their business with the US. Formally, voluntary compliance is described by the participation constraint of the targeted entity that tracks the limits to the hegemons' power: i.e. the maximal private cost to the entity of the actions the hegemon can demand. We refer to this as the hegemon's Micro-Power.

We show that the hegemon always maximizes global enforcement by coordinating punishment along as many relationships as feasible. In doing so, the hegemon maximizes its Micro-Power. From a micro perspective, a sector is strategic to the extent that the hegemon can use it to build its Micro-Power by forming threats on other entities. In this sense, strategic sectors are those that supply inputs that are widely used, with high value added for targets, and with poor available substitutes. Some goods may have these properties due to physical constraints: rare earths, oil and gas. Others have them in equilibrium due to increasing returns to scale and natural monopolies. For example, the dollar-based financial infrastructure of payment and clearing systems (like SWIFT) is a strategic asset that the US often uses in geoeconomic threats.

We allow for a rich set of costly actions that the hegemon can demand. Formally, they include both monetary transfers and a complete set of revenue neutral taxes (wedges) on targeted entities' input purchases. These instruments can be specialized to take the form of mark-ups, bilateral import-export quantity restrictions, tariffs, and political concessions. Many of these instruments are used in practice in economic coercion and sanctions policy. Given its limited power, the hegemon optimally trades-off the use of each of the instruments to maximize its country's welfare. All else equal, it favors monetary extraction from sectors that have little influence on the global equilibrium. It favors wedges to alter a target's economic activities whenever those activities impact other sectors that the hegemon cares about. We show that this input-output propagation of the production externalities is summarized by a generalized Leontief inverse matrix and that the hegemon manipulates the transmission in its favor. We define Macro-Power to be the social value to the hegemon of the costly actions it demands of the targeted entities. From a macro perspective, a sector is strategic if demanding costly actions from it is particularly effective at shaping the world equilibrium in the hegemon's favor. In this sense, strategic sectors tend to be those that have a high influence on world output due to endogenous amplification (in the generalized Leontief-inverse). Sectors like finance, research and development, and information technology are good candidates for being strategic in this sense.

Crucially, Micro- and Macro-Power interact since the hegemon can use demands on one part of the network to shape the equilibrium in ways that increase its power over other parts. The hegemon values having Micro-Power over sectors that generate its Macro-Power because it can exploit the difference between the private costs to targeted entities and the social benefit to itself. In accepting the hegemon's demands, the targeted entities consider only their private costs, but the hegemon enjoys the social benefits of the outcomes of these actions. As a result, we show that allocations with a hegemon are constrained inefficient from a global perspective. The hegemon acts as a global enforcer, echoing the public good provision highlighted in “hegemonic stability theory" in political science, and some of its policies correct negative externalities. The global planner also provides the same enforcement (maximal joint threats) and, in some dimensions, corrects externalities similarly to the hegemon. However, the hegemon destroys value at the global level compared to the global planner by demanding transfers and manipulating the equilibrium in its favor.

Finally, we specialize the model to two simple applications that illustrate recent examples of geoeconomics in practice. In the first example, we focus on the US demand to European governments and firms that they stop using information technology (IT) infrastructure produced by China’s Huawei. Since this technology has strategic complementarities in its adoption, the example illustrates the Macro-Power notion of a strategic sector. Indeed, we show that the pressure that the US applied to European sectors that it could influence was higher because, by causing these sectors not to adopt the technology, the US can also induce lower adoption by sectors and countries that it could not directly pressure.

Our second example focuses on the Chinese Belt and Road Initiative (BRI), an official lending program that aims to join borrowing and trade decisions. The example illustrates the value of joint threats in an economic relationship, government to government lending, in which enforcement is typically limited. In this example, profitable trade relationships act as an endogenous cost of default. Our model explains how China’s BRI can enhance borrowing capacity in developing countries while allowing China to demand political concessions from these governments in return.

Literature Review. In two landmark contributions Hirschman (1945, 1958) relates the structure of international trade to international power dynamics and sets up forward and backward linkages in input-output structures as a foundation for structural economic development. Much of our model is inspired by this work and aims to provide a formal framework for the power structures. We connect to three broad strands of literature.

First, the paper connects to the literature in political science on economic statecraft. The notion of economic statecraft was explored in depth by Baldwin (1985) and Blackwill and Harris (2016). Our modeling of power and the distinction between Micro and Macro Power are related to the levels or faces of power as in Bachrach and Baratz (1962), Cohen (1977), and Strange (1988). The literature on hegemonic stability theory debated whether hegemons, by providing public goods globally, can generate better world outcomes than multipolar configurations (Kindleberger (1973), Krasner (1976), Gilpin (1981), Keohane (1984)). Keohane and Nye (1977) analyze the relationship between power and economic interdependence. Waltz (1979) analyzes how economic interdependence relates to anarchic and hierarchical power systems of the international order. Farrell and Newman (2019) and Drezner et al. (2021) investigate how interdependence can be "weaponized." We relate to the rationalist approach of Fearon (1995) in focusing on hegemonic power in a rational expectations full information model in which the targets voluntarily engage with an hegemon.

Second, the paper relates to the literature on networks, industrial policy, and trade. The literature on networks includes Gabaix (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Blanchard et al. (2016), Bigio and La'O (2020), Baqaee and Farhi (2019, 2022), Liu (2019), Elliott, Golub, and Leduc (2022), Bachmann et al. (2022), and Hausmann et al. (2024). In trade, we relate to the study of global value chains (Grossman et al. (2021), Antràs and Chor (2022)), optimal tariffs and trade agreements (Bagwell and Staiger (1999), Grossman and Helpman (1994)), issue linkage (Limão (2005), Maggi (2016)), and sanctions (Eaton and Engers (1992)). Antràs and Miquel (2023) explore how foreign influence affects tariff and capital taxation policy, and Kleinman, Liu, and Redding (2020) explore whether countries become more politically aligned as they trade more with each other. We also relate to the literate on whether closer trade relationships promote peace (Martin, Mayer, and Thoenig (2008, 2012), Thoenig (2023)).

Third, the paper uses tools developed in economic theory and macroeconomics. We employ grim trigger strategies to build a subgame perfect equilibrium building on Abreu et al. (1986, 1990). Our notion of joint triggers relates to the literature on multi-market contact (Bernheim and Whinston (1990)) and multitasking (Holmstrom and Milgrom (1991)) in which the presence of multiple activities or tasks can help to provide higher powered incentives. We introduce externalities a la Greenwald and Stiglitz (1986) and our study of the hegemon's optimal usage of wedges and transfers is related to the analysis of inefficiency in the presence of externalities (Geanakoplos and Polemarchakis (1985)) and the macro-prudential tools that can be used to improve welfare (Farhi and Werning (2016)).

2. MODEL SETUP

Time is discrete and infinite, $t = 0, 1, \ldots$ Each period is a stage game, described below. All agents have subjective discount factor β.

2.1. *Stage Game*

There are N countries in the world. Each country n is populated by a representative consumer and a set of productive sectors $\mathcal{I}_n$, and is endowed with a set of local factors $\mathcal{F}_n$. We define $\mathcal{I}$ to be the union of all productive sectors across all countries, $\mathcal{I} = \bigcup_{n=1}^{N} \mathcal{I}_n$, and define $\mathcal{F}$ analogously. Each sector, populated by a continuum of identical firms, produces a differentiated good indexed by $i \in \mathcal{I}$ out of local factors and intermediate inputs. The good produced by sector i is sold on world markets at price p_i, with good 1 as the numeraire. Factor f has price p^{ℓ}_f. Factors are internationally immobile. Denote the vector of intermediate goods prices (excluding the numeraire) as p, the vector of factor prices as p^{ℓ}, and the vector of all prices (excluding the numeraire) as $P = (p, p^{\ell})$. Online Appendix Table B.1 references the paper's frequently used notation.

Representative Consumer. The representative consumer in country n has preferences $U_n(C_n) + u_n(z)$, where $C_n = \{C_{ni}\}_{i\in\mathcal{I}}$ and where z is a vector of aggregate variables used to capture externalities a la Greenwald and Stiglitz (1986). Consumers take z as given. We assume U_n is increasing, concave, and continuously differentiable. The term $u_n(z)$ can capture non-economic objectives, such as national security or diplomatic concessions, or direct utility weight on activities in foreign countries. Representative consumer n owns domestic firms and the endowments of local factors, yielding a budget constraint:

$$\sum_{i\in\mathcal{I}} p_i\, C_{ni} \leq \sum_{i\in\mathcal{I}_n} \Pi_i + \sum_{f\in\mathcal{F}_n} p^{\ell}_f \bar{\ell}_f,$$

where Π_i are the profits of sector i and $p^{\ell}_f \bar{\ell}_f$ is factor income. We define the consumer's Marshallian demand function as $C_n(p, w_n)$, where $w_n = \sum_{i\in\mathcal{I}_n} \Pi_i + \sum_{f\in\mathcal{F}_n} p^{\ell}_f \bar{\ell}_f$, indirect utility function from consumption in the stage game as $W_n(p, w_n) = U_n(C_n(p, w_n))$, and total indirect utility in the stage game is $W_n(p, w_n) + u_n(z)$.

Firms. A firm in sector i located in country n produces output y_i using a subset $\mathcal{J}_i \subset \mathcal{I}$ of intermediate inputs and the country n local factors. Firm i's production is $y_i = f_i(x_i, \ell_i, z)$, where $x_i = \{x_{ij}\}_{j\in\mathcal{J}_i}$ is the vector of intermediate inputs used, x_{ij} is use of intermediate input j, $\ell_i = \{\ell_{if}\}_{f\in\mathcal{F}_n}$ is the vector of factors used, and ℓ_{if} is use of local factor f. Firms take the aggregate vector z as given. For simplicity, we assume that for production functions that can use both factors and intermediate inputs we have $f_i(0, \ell_i, z) = 0$, so that a firm that has no ability to source intermediate inputs cannot produce.[1] We assume that f_i is increasing, strictly concave, and satisfies the Inada conditions in (x_i, ℓ_i), and is continuously differentiable in all its arguments. We use the language of firms and sectors, but this is not meant to restrict the focus to private actors exclusively. Many of these entities might be part of, owned, or operated by the government (e.g., a state-owned enterprise).

The stage game has three subperiods: Beginning, Middle, and End. Since each sector has a continuum of identical firms and we restrict to symmetric equilibria, we consider a representative firm per sector. We refer to firm i when clarity necessitates distinguishing an individual firm from the rest of the firms in the same sector, and sector i when describing representative firm outcomes (see Online Appendix B.1). The game described below unfolds between an individual firm in sector i and the continuum of firms (suppliers) in sector j.

In the Beginning, firm i places an order x_{ij} to suppliers in sector $j \in \mathcal{J}_i$ and an order ℓ_i for local factors. The order x_{ij} is placed in equal proportion to each firm in sector j. Factor orders are always accepted and factors cannot be stolen.

In the Middle, each firm in sector j decides to Accept, $a_{ij} = 1$, or Reject, $a_{ij} = 0$, the order of firm i. We assume all firms within a given sector j play the same pure strategy. If the order x_{ij} is Rejected by suppliers in sector j, firm i receives none of that input and owes no payment to suppliers in sector j. If the order is Accepted by suppliers in sector j, the suppliers immediately deliver the entire order x_{ij} to firm i.

In the End, if the order was Accepted, firm i owes the payment $p_j x_{ij}$ to suppliers in sector j. Firm i can choose to Pay suppliers, or Steal from them. If firm i chooses to Steal, suppliers in sector j are only able to recover an exogenous fraction $1 - \theta_{ij} \in [0, 1]$ of the sale order value $p_j x_{ij}$. We denote $S_i \subset \mathcal{J}_i$ the subset of sectors from which firm i steals. For example, $S_i = \{1, 2\}$ denotes the action of stealing inputs provided by suppliers in sectors 1 and 2 and not any others, and $S_i = \emptyset$ denotes no stealing.

[1] We allow for the presence of sectors that simply repackage the factors and use no intermediate inputs. As we describe below, since factors cannot be stolen, these sectors are treated separately from the main analysis and only used in some examples to sharpen the characterization.

For an order (x_i, ℓ_i) in the Beginning, a vector $a_i \in \{0,1\}^{J_i}$ of acceptance choices in the Middle ($J_i = |\mathcal{J}_i|$), and a stealing action $S_i \subset \mathcal{J}_i$ in the End, firm i's stage game payoff is:

$$p_i f_i(x_i \cdot a_i, \ell_i, z) - \sum_{j \in \mathcal{J}_i} p_j a_{ij} x_{ij} - \sum_{f \in \mathcal{F}_n} p_f^\ell \ell_{if} + \sum_{j \in S} \theta_{ij} p_j a_{ij} x_{ij}.$$

Correspondingly, suppliers in sector j lose $\theta_{ij} p_j a_{ij} x_{ij}$ if firm i steals from them. The stage game captures many economic relationships that are based on repeated transactions and limited enforceability: a lender-borrower relationship in finance or a supplier-customer relationship in goods or services. The enforceability parameters θ_{ij} are flexible, and for example might be lower for international than domestic relationships.

2.2. *Repeated Game*

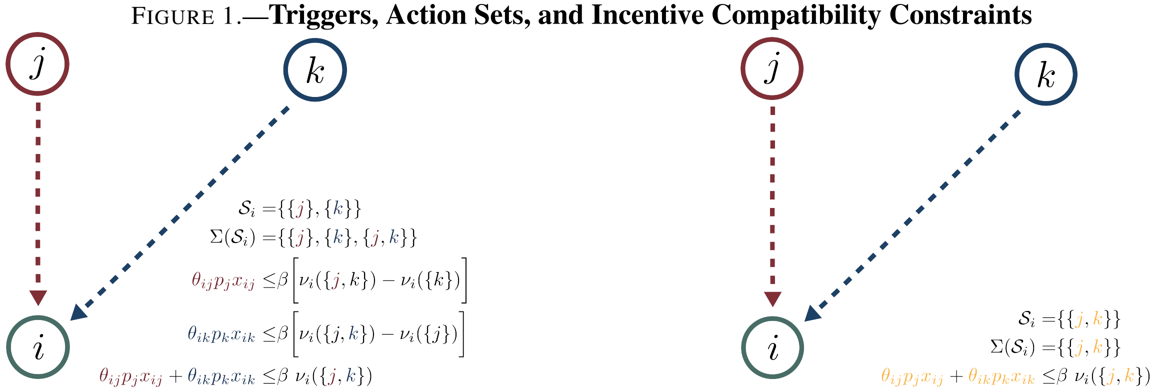
We assume suppliers play trigger strategies that involve switching to Rejecting any future order by an individual firm following some Stealing actions by that firm. We track permanent exclusion by $B_{ij} \in \{0,1\}$. If $B_{ij} = 0$, then suppliers in sector j will Reject any order placed by firm i. If $B_{ij} = 1$, then suppliers in sector j will Accept an incentive compatible order (defined below) and Reject an order that is not incentive compatible. For expositional convenience, we say that suppliers in j "Trust" firm i if $B_{ij} = 1$ and "Distrust" firm i if $B_{ij} = 0$. We define $\mathcal{B}_i = \{j \mid B_{ij} = 1\}$ to be the set of supplying sectors that Trust firm i. Exclusion off-path is tracked at the level of the specific firm within a sector that deviates, taking as given that on path the other firms in the same sector did not deviate and thus retained access. This means that equilibrium prices and quantities do not change based on the deviation of an individual atomistic firm.

The proof of Lemma 1 and Online Appendix B.1 formally characterize trigger strategies and we focus here on an intuitive presentation. We study subgame perfect equilibria that are Markov in $\mathcal{B}_i$, and restrict attention to pure strategies that are symmetric within a sector. In principle, one could allow for non-stationary (front-loaded) punishments in an attempt to worsen the off-path equilibrium and sustain a better equilibrium than Markov and potentially implement the Ramsey plan (Ray (2002); Acemoglu et al. (2008)). Our purpose is not to explore the best sustainable equilibrium but to focus on a simple Markov one that provides much economics while minimizing the theoretical complexity.

A strategy of firm i in the Beginning is $\sigma_i^-(\mathcal{B}_i)$, mapping $\mathcal{B}_i$ into an order (x_i, ℓ_i). A strategy of suppliers in sector j in the Middle with regard to firm i is $\sigma_{ij}(x_i, \ell_i, \mathcal{B}_i)$, mapping an order size and $\mathcal{B}_i$ into an acceptance decision a_{ij}. A strategy of firm i in the End is $\sigma_i^+(a_i, x_i, \ell_i, \mathcal{B}_i)$, mapping acceptance decisions, order size, and $\mathcal{B}_i$ into stealing action S_i. We build a value function starting from an exogenous continuation value $\nu_i(\mathcal{B}_i)$ assumed to be non-decreasing and with $\nu_i(\emptyset) = 0$. We focus the exposition on the on-path strategies and values, $\mathcal{B}_i = \mathcal{J}_i$, with Online Appendix B.1 detailing the rest of the off-path strategies and equilibrium value function following the iterative process of Abreu et al. (1990).

Trigger Strategies and Incentive Compatibility. We study triggers that take two forms: individual and joint. In the case of an individual trigger, if firm i Steals from suppliers in sector j, then suppliers in sector j Distrust individual firm i in all future periods. In the case of a joint trigger between suppliers in sectors j and k with respect to firm i, if firm i Steals from suppliers in either sector j or k, then suppliers in both sectors j and k Distrust individual firm i in all future periods. We assume that joint triggers are symmetric and note that they can be chained. For example, firm i stealing from suppliers h triggers suppliers j if h has a joint trigger with k and k has a joint trigger with j.

FIGURE 1.—**Triggers, Action Sets, and Incentive Compatibility Constraints**



(a) Individual Triggers

(b) Joint Triggers

Notes: Panels focus on a firm in sector i with suppliers in sectors j and k. Action sets and related incentive constraints are from the perspective of firm i under different configurations. Panel (a) illustrates the case in which suppliers in sectors j and k have individual triggers only. Panel (b) illustrates the case in which suppliers in sectors j and k have a joint trigger.

Figure 1 illustrates a simple case of two sectors j and k supplying to firm i. In building the incentive compatibility constraint for firm i, we know by backward induction that suppliers never Accept an order that will be stolen since their payoff is strictly negative from doing so. Hence, we focus on a constraint for orders that are Accepted and not stolen. In Panel (a), the suppliers in sectors j only have individual triggers, resulting in an IC constraint $\theta_{ij}p_jx_{ij} \leq \beta[\nu_i(\{j,k\}) - \nu_i(\{k\})]$. Firm i compares the one-off Stealing gain $\theta_{ij}p_jx_{ij}$ with the continuation value loss of not being able to use input j again. Suppliers in sector k have an identical set-up and constraint. Finally, the firm could Steal from both suppliers generating the constraint $\theta_{ij}p_jx_{ij} + \theta_{ik}p_kx_{ik} \leq \beta\nu_i(\{j,k\})$. Panel (b) illustrates joint triggers between sectors j and k. Intuitively, firm i would never Steal from only one of sectors j or k, since both would retaliate anyway. $\mathcal{S}_i$ is the set of the smallest undominated stealing actions. In Panel (a) this included stealing from j and k separately, but in Panel (b) only stealing from both at the same time is undominated. The set $\Sigma(\mathcal{S}_i)$ then considers all possible combinations of these undominated actions. In Panel (a) this includes Stealing from j and k separately and Stealing from both at the same time. In Panel (b) this only includes Stealing from both. Therefore, under joint triggers in Panel (b) there is only one IC left, the joint stealing constraint: $\theta_{ij}p_jx_{ij} + \theta_{ik}p_kx_{ik} \leq \beta\nu_i(\{j,k\})$.

Lemma 1 provides a full characterization of the logic above in the general case. Let $P(\mathcal{J}_i)$ denote the power set of $\mathcal{J}_i$, that is all subsets of $\mathcal{J}_i$, and let $\Sigma(\mathcal{S}) = \{\bigcup_{X\in\mathcal{X}} X \mid \emptyset \neq \mathcal{X} \subset \mathcal{S}\}$ be all possible unions of elements of $\mathcal{S}$. Given the firm's incentive problems, suppliers' strategy in the Middle is to Accept an order if and only if equation (1) is satisfied for all $S \in \Sigma(\mathcal{S}_i)$.

LEMMA 1: *There is a partition $\mathcal{S}_i$ of $\mathcal{J}_i$ such that the order (x_i, ℓ_i) is incentive compatible with respect to all stealing actions, $P(\mathcal{J}_i)$, if and only if it is incentive compatible with respect to $\Sigma(\mathcal{S}_i)$. The incentive compatibility constraint for $S_i \in \Sigma(\mathcal{S}_i)$ is*

$$\sum_{j\in S_i} \theta_{ij}p_jx_{ij} \leq \beta\Big[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S_i)\Big]. \tag{1}$$

Since conditional on the IC holding the continuation value does not depend on order size, firm i's strategy in the Beginning is an order size (x_i, ℓ_i) to maximize its stage game payoff $\Pi_i(x_i, \ell_i, \mathcal{J}_i) = p_if_i(x_i, \ell_i, z) - \sum_{j\in\mathcal{J}_i} p_jx_{ij} - \sum_{f\in\mathcal{F}_n} p^\ell_f \ell_{if}$, subject to incentive compatibility (equation (1)). Since Π_i is a concave function and equation (1) describes a convex set, this optimization problem is convex, the solution of which is the optimal order size.

2.3. *Market Clearing, Externalities, and Equilibrium*

Denote $D_j = \{i \in \mathcal{I} \mid j \in \mathcal{J}_i\}$ the set of sectors that source from sector j. Market clearing for good j is $\sum_{n=1}^{N} C_{nj} + \sum_{i \in D_j} x_{ij} = y_j$ and for local factor f is $\sum_{i \in \mathcal{I}_n} \ell_{if} = \bar{\ell}_f$. We assume that the vector of aggregates is $z = \{z_{ij}\}$ where $z_{ij}^* = x_{ij}^*$, where the $*$ notation denotes an equilibrium value. That is, z-externalities are based on the quantities of inputs in bilateral sectors i and j relationships. This general formulation can be specialized to cover strategic complementarities, either at the sector level (e.g., external economies of scale) or across sectors (e.g., thick market externalities).

An equilibrium of the model is prices for goods and factors P and allocations $\{x_i, C_n, y_i, \ell_i, z_{ij}\}$ such that: (i) firms maximize profits, given prices; (ii) households maximize utility, given prices; (iii) markets clear.

3. HEGEMONIC POWER

Our main analysis focuses on when and how a hegemon can build power and wield it to demand costly actions. We consider a single country that is a hegemon and Online Appendix B.2 provides an extension to competition between multiple hegemons. We define the hegemon to be country m and for it to be uniquely able to: (i) coordinate firms in its network to create joint threats; (ii) propose take-it-or-leave-it offers to its own firms and *all* downstream sectors of its firms, where contract terms specify joint threats, transfers, and restrictions on inputs purchased. Unlike individual firms and consumers, the hegemon internalizes how the terms of its contract affect the aggregates z and prices P. Since we focus on Markov equilibria, the hegemon offers a contract only for the current stage game and takes the future decisions of itself and of firms as given (i.e., the hegemon cannot commit to future contracts).

A joint threat is a coordination of trigger strategies among multiple supplying sectors of the same firm. Formally, a **joint threat** $\mathcal{S}_i'$ is a partition of $\mathcal{J}_i$ such that $\mathcal{S}_i'$ is coarser than $\mathcal{S}_i$. As an example, returning to Figure 1, a joint threat on a firm in sector i is the suppliers in j and k adopting a joint trigger (essentially moving from the configuration in Panel (a) to that in Panel (b)). Joint threats generically generate value for the firm being threatened because they relax incentive constraints. They embed in our model the view of the hegemon as a global enforcer or policeman of economic activity (Waltz (1979), Gilpin (1981)).

We assume that hegemon m can propose a take-it-or-leave-it contract to each of its domestic sectors and their foreign downstream sectors. Formally, this set is $\mathcal{C}_m = \mathcal{I}_m \cup \mathcal{D}_m$ where $\mathcal{D}_m = \bigcup_{i \in \mathcal{I}_m} D_i \backslash \mathcal{I}_m$ is the set of foreign downstream sectors.[2] The hegemon's contract to firm $i \in \mathcal{C}_m$ specifies: (i) a feasible joint threat $\mathcal{S}_i'$; (ii) nonnegative transfers $\mathcal{T}_i = \{T_{ij}\}_{j \in \mathcal{J}_{im}}$ from firm i to the hegemon's representative consumer, where $\mathcal{J}_{im} = \mathcal{I}_m \cap \mathcal{J}_i$ is the set of inputs that sector i sources from the hegemon; (iii) revenue-neutral taxes $\tau_i = \{\{\tau_{ij}\}_{j \in \mathcal{J}_i}, \{\tau_{if}^{\ell}\}_{f \in \mathcal{F}_n}\}$ on purchases of inputs and factors, with equilibrium revenues $\tau_{ij} x_{ij}^*$ and $\tau_{if}^{\ell} \ell_{if}^*$ raised from sector i rebated lump sum to firms in sector i that accept the contract. We denote $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}$ the contract offered to firm i and $\Gamma = \{\Gamma_i\}_{i \in \mathcal{C}_m}$ the set of all contracts.

We restrict the joint threats that the hegemon can make to involve sectors that are at most one step removed from the hegemon. Formally, we assume it is feasible for the hegemon to use $S \in \mathcal{S}_i$ in forming a joint threat $\mathcal{S}_i'$ if $\exists j \in S$ with $j \in \mathcal{C}_m$. We impose this restriction to prevent unrealistic situations in which the hegemon threatens a firm that it has no (immediate) relationship with.[3]

Taxes adjust the effective price firm i faces to $p_j + \tau_{ij}$ for inputs and $p_f^{\ell} + \tau_{if}^{\ell}$ for factors. Because taxes are revenue-neutral, without loss of generality we assume that tax payments and rebates do not enter the Pay/Steal decision. Instead, we assume that transfer T_{ij} is not paid if j is Stolen.[4] Transfers T_{ij} can cover different interpretations: direct monetary payments, a firm-specific mark-up charged by the hegemon on sales of its goods, or the extraction of value in some other action the firm takes on behalf of the hegemon (e.g., lobbying for political concessions). The revenue-neutral taxes τ_{ij} are typical in the macro-prudential literature that focuses on pecuniary and demand externalities (Farhi and Werning

[2]Appendix B.3.5 extends the analysis to allow the hegemon to directly control domestic firms.

[3]Online Appendix Figure B.2 provides an illustration along the line of Figure 1 of which threats by the hegemon are feasible. Online Appendix B.3.6 illustrates how to incorporate farther indirect trade into our setup.

[4]Under the contract, if firm i Pays suppliers in sector j, then it pays $p_j x_{ij}$ to suppliers in sector j and pays $\tau_{ij}(x_{ij} - x_{ij}^*) + T_{ij}$ to the hegemon's consumer. If firm i Steals from suppliers in sector j, its only payment is $\tau_{ij}(x_{ij} - x_{ij}^*)$ to the hegemon's consumer (which is zero in equilibrium). In this case, suppliers in sector j only recover an amount $(1 - \theta_{ij}) p_j x_{ij}$, while hegemon m's representative consumer recovers none of the transfer.

(2016)). Given our rebate rule, they are best thought of as quantity restrictions (see for example Clayton and Schaab (2022)). Importantly, these instruments target relationships between two sectors, covering for example restrictions on energy imports from Russia but not from other countries; or restrictions on imports of Chinese goods.[5]

Firm Participation Constraint. In deciding whether or not to accept the hegemon's contract, firm i, being small, does not internalize the effect of its decision on the prevailing aggregate vector z and prices. If firm i accepts the contract, it chooses allocations to maximize profits given the contract terms, achieving value

$$V_i(\Gamma_i) = \max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{J}_i) - \sum_{j\in\mathcal{J}_i}[\tau_{ij}(x_{ij}-x^*_{ij}) + T_{ij}] - \sum_{f\in\mathcal{F}_m} \tau^{\ell}_{ij}(\ell_{if}-\ell^*_{if}) + \beta\nu_i(\mathcal{J}_i)$$

$$s.t. \sum_{j\in S}\left[\theta_{ij}p_j x_{ij} + T_{ij}\right] \le \beta\left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i\backslash S)\right] \quad \forall S\in\Sigma(\mathcal{S}'_i) \tag{2}$$

Because transfers are associated with the firm decision to Pay, they tighten the incentive constraints, all else equal. At the level of the individual firm, taxes have two effects: (i) they affect the firm's optimal allocation because they alter the perceived price of the input good; (ii) they affect firm profits directly. In equilibrium, this latter effect washes out since taxes are rebated lump sum (i.e., $x_{ij} = x^*_{ij}$). The optimal allocation x^*_{ij}, and hence remitted revenues, are defined implicitly as a function of contract terms, prices, and z-externalities by the above optimization problem.

If firm i rejects the hegemon's contract, it retains its original action set and achieves the value $V_i(\mathcal{S}_i)$.[6] For firm i to accept the contract, it must be better off under the contract than by rejecting it. This gives rise to the **participation constraint** of firm i,

$$V_i(\Gamma_i) \ge V_i(\mathcal{S}_i), \tag{3}$$

[5]We focus on restrictions (costly actions) imposed on firms on buying inputs from other suppliers. In principle, we could also allow for bilateral taxes on sales by firm i. In equilibrium, any sales taxes would be fully passed through to the buyer and, in this sense, would be captured by the input taxes that we already consider. However, a difference is that the input taxes on firm i that arise from sales taxes on firm j would not in principle require firm i to agree to the contract. Similarly, we could also allow bilateral taxes on sales by firm i to consumers.

[6]We abuse notation and write $V_i(\mathcal{S}_i)$ as short hand for $V_i(\Gamma_i)$ when $\Gamma_i = \{\mathcal{S}_i, 0, 0\}$.

where recall that $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}$ so that the participation constraint is comparing the hegemon’s contract with joint threats, transfers, and wedges to the outside option.

The hegemon creates slack in the participation constraint by proposing a joint threat, and then can use that slack to demand costly actions. We define a **pressure point** on firm i as a joint threat $\mathcal{S}_i'$ that strictly increases firm i’s profits, that is $V_i(\mathcal{S}_i') > V_i(\mathcal{S}_i)$. This is the source of hegemon’s power over firm i. Online Appendix B.3.1 shows how to extend the model to allow the hegemon to also generate slack by making the outside option worse by threatening to cut off firms that reject the contract from its inputs. Our focus on voluntary participation in an environment with rational expectations and full information relates to the rationalist school in international relations (Fearon (1995)), although our framework can accommodate biases in the governments’ objective functions (for example, via $u_n(z)$).

Hegemon Maximization Problem. The hegemon’s objective function is the utility of its representative consumer, to whom all domestic firm profits and all transfers accrue:

$$\mathcal{U}_m = W_m(p, w_m) + u_m(z), \quad w_m = \sum_{i\in\mathcal{I}_m} \Pi_i(\Gamma_i) + \sum_{f\in\mathcal{F}_m} p_f^\ell \bar{\ell}_f + \sum_{i\in\mathcal{D}_m}\sum_{j\in\mathcal{J}_{im}} T_{ij}. \tag{4}$$

Since transfers from domestic sectors to the hegemon’s consumer net out from the consumer’s wealth, we need only keep track of operating profits $\Pi_i(\Gamma_i) = V_i(\Gamma_i) + \sum_{j\in\mathcal{J}_{im}} T_{ij}$ of the hegemon’s domestic sectors. Similarly, taxes on all sectors are revenue neutral for the hegemon, and therefore net out. However, transfers from foreign sectors do not net out, precisely because the hegemon’s consumer has no claim to foreign sectors’ profits.

The hegemon’s maximization problem is choosing a contract Γ to maximize its consumer utility (equation (4)), subject to the participation constraints of firms (equation (3)), the feasibility of joint threats, the determination of aggregates $z_{ij}^* = x_{ij}^*$, and determination of prices via market clearing.

3.1. *Optimality of Maximal Joint Threats*

We solve the hegemon’s problem in two steps. First, we prove that the hegemon offers a “maximal” joint threat that joins together all feasible threats. Second, we characterize transfers and wedges under the optimal contract.

Starting from the existing set $\mathcal{S}_i$, we show that the hegemon optimally consolidates all feasible threats at its disposal, $\mathcal{S}_i^D = \{S \in \mathcal{S}_i \,|\, \exists j \in S \, s.t. \, j \in \mathcal{C}_m\}$, into a single stealing action $S_i^D = \bigcup_{S \in \mathcal{S}_i^D} S$. The maximal joint threat is then the single action S_i^D and the remaining threats that the hegemon could not feasibly consolidate: $\overline{\mathcal{S}}_i' = \{S_i^D\} \cup (\mathcal{S}_i \backslash \mathcal{S}_i^D)$.

PROPOSITION 1: *It is weakly optimal for the hegemon to offer a contract with maximal joint threats to every firm it contracts with, that is $\mathcal{S}_i' = \overline{\mathcal{S}}_i'$ for all $i \in \mathcal{C}_m$.*

Intuitively, Proposition 1 follows from the observation that joint threats expand the set of feasible allocations, and so weakly increase targeted entities' profits. A hegemon that chose a contract that did not involve maximal joint threats could always implement the same transfers and allocations while offering a contract with maximal joint threats. The hegemon, therefore, wants to maximize its global enforcer capabilities.

Since the hegemon's contract involves all of its domestic sectors that supply to sector i entering a single joint threat, transfers can be tracked in total at the sector level, that is $\overline{T}_i = \sum_{j \in \mathcal{J}_{im}} T_{ij}$. We therefore abuse notation and track only $\overline{T}_i$ in the contract.

3.2. *Leontief Inverse and Network Propagation with Externalities*

In demanding costly actions and transfers out of targeted entities, the hegemon takes into consideration their impact on aggregate prices P and quantity-based externalities z. Therefore, to analyze the hegemon optimal contract we need to first characterize how changes in firms' allocations x_{ij} propagate through the global network. The proposition below shows that the entire propagation can be characterized in terms of a generalized Leontief inverse.[7]

PROPOSITION 2: *The aggregate response of z^* and P to a perturbation in an exogenous variable e is*

$$\frac{dz^*}{de} = \Psi^z \left(\frac{\partial x^*}{\partial e} + \frac{\partial x^*}{\partial P} \frac{dP}{de} \right)$$

$$\frac{dP}{de} = -\left(\frac{\partial ED}{\partial P} + \frac{\partial ED}{\partial z^*} \Psi^z \frac{\partial x^*}{\partial P} \right)^{-1} \left(\frac{\partial ED}{\partial e} + \frac{\partial ED}{\partial z^*} \Psi^z \frac{\partial x^*}{\partial e} \right),$$

[7]We assume that excess demand ED and firm demand x are continuously differentiable in the relevant range of allocations.

where $\Psi^z = \left(\mathbb{I} - \frac{\partial x^*}{\partial z^*}\right)^{-1}$ *and* ED *is the vector of excess demand in every good (excluding the numeraire) and factor.*

The matrix Ψ^z keeps track of all the successive amplification via the z-externalities of the original perturbation. The term $\frac{dP}{de}$ keeps track of the input-output amplification occurring via changes in equilibrium prices. Network amplification is a standard tool of macroeconomic theory that we embed in our framework since it is crucial to geoeconomics. Most of the existing literature focuses on input-output amplification via equilibrium prices (e.g. Baqaee and Farhi (2019, 2022)), while we additionally stress the importance of production externalities. To provide intuition here and in the rest of the paper, it is useful to consider special cases which we define formally below. First, consider an environment in which all prices are constant in equilibrium as defined below:

DEFINITION 1: *The* ***constant prices*** *environment assumes that consumers have identical linear preferences over goods,* $U_n = \sum_{i \in \mathcal{I}} \tilde{p}_i C_{ni}$*, and that each country has a local-factor-only firm with linear production* $f_i(\ell_i) = \sum_{f \in \mathcal{F}_n} \frac{1}{\tilde{p}_i} \tilde{p}^\ell_f \ell_{if}$*. It assumes that consumers are marginal in every good and factor-only firms are marginal in every local factor so that* $p_i = \tilde{p}_i$ *and* $p^\ell_f = \tilde{p}^\ell_f$*.*

In this simplified environment the term $\frac{\partial x^*}{\partial P} \frac{dP}{de}$ would be zero and amplification would only occur via the z-externalities: $\frac{dz^*}{de} = \Psi^z \frac{\partial x^*}{\partial e}$. Here the matrix Ψ^z captures all endogenous amplification since prices are constant, and is akin to a Leontief inverse. Intuitively, the perturbation to e changes production in a sector, leading to re-optimization in other sectors given the production externalities, which in turn filters to other sectors, and so on.

Second, consider switching off the z-externalities as defined below:

DEFINITION 2: *The* ***no z-externalities*** *environment assumes that* $u_n(z)$ *and* $f_i(x_i, \ell_i, z)$ *are constant in* z*.*

In this simplified environment the term $\frac{\partial x^*}{\partial z^*}$ would be zero and the matrix Ψ^z would reduce to the identity matrix. Amplification would only occur via prices: $\frac{dz^*}{de} = \frac{\partial x^*}{\partial e} + \frac{\partial x^*}{\partial P} \frac{dP}{de}$, where $\frac{dP}{de} = -\left(\frac{\partial ED}{\partial P}\right)^{-1} \frac{\partial ED}{\partial e}$. Intuitively, the perturbation to e changes excess demand in each market as a result of reoptimization by firms and consumers. These changes in excess demand must then be counteracted through price changes to equilibrate markets, with $\frac{\partial ED}{\partial P}$ giving the response of excess demand to prices. This is the standard input-output amplification mechanism via prices.

3.3. *Hegemon's Optimal Contract and Efficiency*

In characterizing the hegemon's optimal contract, we set up the following notation (see the proof of Proposition 3 for details). We denote $\eta_i \geq 0$ the hegemon's Lagrange multiplier on the participation constraint of firm i and $\Lambda_{iS} \geq 0$ the hegemon's multiplier on the incentive constraint of firm i for stealing action S. We also define $\overline{\Lambda}_i = \sum_{S \in \Sigma(\overline{\mathcal{S}}_i') | S_i^D \subset S} \Lambda_{iS}$, which sums all multipliers involving a stealing action included in the hegemon's maximal joint threat. We track the hegemon's perceived externalities from an increase in z_{ij}^* as $\mathcal{E}_{ij}$, and similarly we track the hegemon's perceived externalities from a transfer of wealth from consumers in country n to those in country m as Ξ_{mn}. The proposition below and its proof characterize an optimal contract and provide formal definitions of these perceived externalities.[8]

PROPOSITION 3: *An optimal contract of the hegemon has the following terms:*

1. *For foreign firms $i \in \mathcal{D}_m$ located in country n, if $\overline{\mathcal{S}}_i'$ is a pressure point on i:*
   (a) *Input wedges satisfy: $\eta_i \tau_{ij}^* = -\mathcal{E}_{ij}$.*
   (b) *Transfers satisfy: $\overline{\Lambda}_i + \eta_i \geq \frac{\partial W_m}{\partial w_m} + \Xi_{mn}$, with equality if $\overline{T}_i^* > 0$.*
2. *For domestic firms $i \in \mathcal{I}_m$, if $\overline{\mathcal{S}}_i'$ is a pressure point on i:*
   (a) *Input wedges satisfy: $(\frac{\partial W_m}{\partial w_m} + \eta_i)\tau_{ij}^* = -\mathcal{E}_{ij}$.*
   (b) *Transfers are zero: $\overline{T}_i^* = 0$.*
3. *If $\overline{\mathcal{S}}_i'$ is not a pressure point on firm i, then $\overline{T}_i^* = 0$ and $\tau_i^* = 0$.*

[8] Proposition 3 provides necessary conditions for optimality, and we assume that an equilibrium exists. Formally, if for a foreign firm i we have $\eta_i = 0$, it instead characterizes the limit of a sequence of wedges, each of which is part of a (different) optimal contract (see the proof for details). For technical reasons, we assume that if $\overline{\mathcal{S}}_i'$ is not a pressure point on firm i at the optimal (z^*, P), then it is also not a pressure point on i in a neighborhood of (z^*, P). Finally to streamline analysis we assume that every foreign country contains at least one firm that the hegemon cannot contract with, meaning that the hegemon cannot directly mandate factor prices in foreign countries.

To provide intuition for the hegemon's optimal contract, consider a foreign firm i with a binding participation constraint. We can expand the optimal tax formula in 1(a) above to:

$$\tau_{ij}^* = -\frac{1}{\eta_i}\mathcal{E}_{ij} = -\frac{1}{\eta_i}\Bigg[\underbrace{\varepsilon_{ij}^z}_{\text{Direct Impact}} + \underbrace{\overbrace{\varepsilon^{zNC}\frac{dz^{*NC}}{dz_{ij}}}^{\text{Aggregate Quantities}} + \overbrace{\varepsilon^{P^m}\frac{dP^m}{dz_{ij}}}^{\text{Prices}}}_{\text{Indirect Impact: Input-Output Amplification}} \Bigg]. \tag{5}$$

The hegemon uses the wedges to manipulate externalities in its favor. Activities that generate positive (negative) externalities $\mathcal{E}_{ij} > 0$ are subsidized (taxed). The first term in equation (5), ε_{ij}^z, measures the direct value to the hegemon of increasing sector i's use of input j:

$$\varepsilon_{ij}^z = \underbrace{\frac{\partial W_m}{\partial w_m}\sum_{k\in\mathcal{I}_m}\frac{\partial \Pi_k}{\partial z_{ij}} + \frac{\partial u_m(z)}{\partial z_{ij}}}_{\text{Externalities on Hegemon's Economy}} + \underbrace{\sum_{k\in\mathcal{C}_m}\eta_k\left[\frac{\partial \Pi_k}{\partial z_{ij}} - \frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}}\right]}_{\text{Building Power}} \tag{6}$$

The hegemon wants to increase foreign activity x_{ij} if it directly benefits one of the sectors in the hegemon's economy or if the consumer directly cares about that activity. An example of the first is a foreign firm's R&D activity that has a positive knowledge spillover on the productivity of domestic sectors. An example of the second is a foreign firm's R&D activity that is used by the military of a country hostile to the hegemon. The hegemon also cares about how its demands on activity x_{ij} affect the amount of power it has over the all sectors. All else equal, the hegemon asks for actions that make it less attractive on the margin for a firm to reject its contract (decrease the outside option $V_k(\mathcal{S}_k)$ or increase on-path profits Π_k) thus binding it tighter to the hegemon and increasing its power. As an example, in the presence of strategic complementarities, the US demands that more foreign firms rely on US financial institutions, making it harder for any one firm to deviate from US demands.

The second term in equation (5), $\varepsilon^{zNC}\frac{dz^{*NC}}{dz_{ij}}$, measures the indirect value of altering production via input-output amplification in sectors that the hegemon does not control. The term ε^{zNC} is analogous to equation (6) but for firms not in the hegemon's network. The term $\frac{dz^{*NC}}{dz_{ij}}$ summarizes the Leontief amplification impact and is given by Proposition 2 taking z_{ij} to be the exogenous variable e. The hegemon demands more action in the x_{ij} relationship the more, via the network, these actions propagate and affect activities that the hegemon does not control but values. An example is the US demanding European banks to curb financing of legitimate (from a European regulatory perspective) commercial activities of Iranian entities in order to affect the overall Iranian economy and in particular Iran's government budget and military sector.

The third term in equation (5), $\varepsilon^{P^m} \frac{dP^m}{dz_{ij}}$, is the indirect value of the induced changes in equilibrium prices. The term $\frac{dP^m}{dz_{ij}}$ summarizes the Leontief amplification impact and is given by Proposition 2 taking z_{ij} to be the exogenous variable e. Isolating the component of the vector ε^{P^m} corresponding to the value from changes in the price of input j, we have

$$\varepsilon_j^{P^m} = \underbrace{\frac{\partial W_m}{\partial w_m} X_{m,j}}_{\text{Terms of Trade}} - \underbrace{\sum_{k \in \mathcal{C}_m} \overline{\Lambda}_{kj} \theta_{kj} x_{kj}}_{\text{Pecuniary Externalities (IC)}} + \underbrace{\sum_{k \in \mathcal{C}_m} \eta_k \left[\frac{\partial \Pi_k}{\partial p_j} - \frac{\partial V_k(\mathcal{S}_k)}{\partial p_j} \right]}_{\text{Building Power}} \tag{7}$$

where $X_{m,j}$ is exports of good j by country m (negative, i.e. imports, if $j \notin \mathcal{I}_m$). Much of the trade and international macroeconomics literature has focused on terms of trade manipulation as the motive for imposing tariffs, capital controls, and entering multilateral trade agreements. Similarly, the macro-finance literature has focused on pecuniary externalities, which are also present in our framework since prices enter the incentive constraints. The last term, "Building Power", is analogous to the last term in equation (6) and key to our analysis of international power. The hegemon takes into consideration how its demands change prices and how those affect the marginal willingness of firms to accept its demands.

Proposition 3 part 1(b) shows that the hegemon has an incentive to extract transfers from foreign firms, but is limited because higher transfers tighten both the participation constraint and the incentive constraint. The hegemon also internalizes how shifting wealth between consumers alters equilibrium prices and aggregates z.

Consider next a domestic firm. The hegemon's optimal wedge formula (Proposition 3 part 2(a)) is almost identical to that for foreign firms, except that the magnitude of wedges (whether tax or subsidy) is lower because the hegemon values the profits of domestic firms. Domestic firms are never charged transfers since the firms are owned by the hegemon's consumers and transfers tighten the incentive constraints.[9]

[9] The wedges applied to domestic firms are akin to industrial policy, and in our framework this policy can be driven by domestic (e.g. education and R&D) or foreign considerations. In particular the hegemon uses the wedges

Our theory gives a way to think about who are the “friends or enemies” of the hegemon based on the sign of the spillover term $\mathcal{E}_{ij}$ (see Online Appendix B.3.9 for a formal treatment). Friendship occurs when the spillovers are positive, which can be driven by direct or indirect linkages, and by economic or non-economic motives (the term $\frac{\partial u_m(z)}{\partial z_{ij}}$ in equation (6)). For example, we think of defense alliances such as NATO as the hegemon placing positive utility on defense sectors of allied countries. In our framework the hegemon would use its global enforcement power to push those allies to increase those activities, making them internalize more of the national security externalities, and might do so at the expense of its own firms’ profits or consumers’ consumption. Indeed, the hegemon might leave surplus to allied countries’ sectors and (optimally) not fully exercise its coercive power on them.

3.4. *Strategic Sectors and The Nature of Geoeconomic Power*

Controlling, defending from foreign influence, and growing strategic sectors is a core government policy in democracies and autocracies alike. While governments frequently protect or control industries claiming they are strategic for the "national interest," there is a concern that the "strategic" label is in reality a cover for protectionism or for subsidies to politically connected entities. This ambiguity is possible because of a lack of clarity on what makes an activity strategic and a clear framework for policy evaluation.[10]

In our framework, a sector is strategic in two dimensions. First, a sector can be strategic because the hegemon can use it to form (off-path) threats on other entities. Second, because the hegemon can demand (on-path) costly actions from this sector that shape the world equilibrium in the hegemon’s favor. We distinguish two notions of power that are what makes sectors strategic: Micro-Power and Macro-Power.

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to build up domestic industries that increase the country’s power. For example, the U.S. recently imposed export restrictions on U.S. semiconductor firms (such as Nvidia and Intel) selling their output to certain Chinese sectors. While the U.S. government overall subsidizes the American semiconductor industry to build hegemonic power, it also restricts its exports to Chinese firms given the technology (even indirect) usage in the military sector.

[10]See Baldwin (1985)["Strategic Goods" section, pages 223-233] for a review of many informal definitions of strategic goods, including a quote from Soviet leader Nikita Khrushchev: "Anything one pleases can be regarded as strategic material, even a button, because it can be sewn onto a soldier’s pants. A solider will not wear pants without buttons, since otherwise he would have to hold them up with his hands. And then what can he do with his weapon?".

Micro-Power: Strategic Sectors in Threatening Target Output. Micro-Power is the maximum private cost to the target of the hegemon's demanded costly actions. It is the most the hegemon could demand before its contract gets rejected. This notion of power is related in political science to the Dahl (1957) conception of power as: "A has power over B to the extent that he can get B to do something that B would not otherwise do" (p. 202-203). The source of this power in our framework is the value to the targeted entity of the hegemon's threats, that is whether the hegemon has a pressure point on that entity. The amount of Micro-Power is given by $V_i(\overline{\mathcal{S}}_i') - V_i(\mathcal{S}_i)$, holding fixed equilibrium aggregate quantities and prices. The hegemon maximizes its Micro-Power by making maximal joint threats (Proposition 1), and then uses it to demand costly actions (Proposition 3).

To isolate Micro-Power, consider a special case in which equilibrium prices are constant and z-externalities are switched off both in the firms' production functions and consumers' utility functions (Definitions 1 and 2). Then, by Online Appendix Proposition 8, all foreign sectors are neutral and no wedges are applied. Instead, the hegemon uses all its Micro-Power to extract transfers from foreign firms until their participation constraint binds.[11]

A crucial source of Micro-Power arises from the loss for the target from the hegemon cutting off access to some of it inputs. Despite the hegemon threats being off-path, this loss in continuation value can be computed, using the model structure, as a counterfactual based on the observed on-path data. While a full empirical analysis is beyond the scope of this paper, in Online Appendix B.3.3, we offer some initial empirical guidance by specializing the production function to be Cobb-Douglass across industries and CES within industries. With this standard production function, the counterfactual loss can be measured using available estimates of the elasticity of substitution within sectors and trade data on a country's expenditure share on a sector and the expenditure share on goods each country buys from the hegemon as a share of spending on the sector. These losses are in the spirit of Hirschman (1945) notion of asymmetric power in trade relationships with the hegemon.

Goods that are strategic in this micro-sense are those widely used, with high value added for targets, and with poor substitutes. Some goods have these properties due to physical constraints like rare earths, oil, and gas. However, in identifying Micro-Power it is neces-

[11] Proposition 8 assumes identical homothetic preferences. In the case of constant prices, we do not need this restriction since wealth transfers across consumers do not cause terms of trade movements.

sary to know the parameters of the production function, but also which inputs the hegemon controls. As emphasized by Schelling (1958), the notion of strategic has to be defined in the context of an equilibrium and cannot be determined solely from ex-ante characteristics of a sector. For example, controlling one variety of natural gas is ineffective since there is a high degree of substitutability in production with other types of natural gas. However, if the hegemon controls a joint threat among all varieties of natural gas, that threat is very valuable since the input is essential for many sectors. This logic also applies to joint threats for inputs that might seem rather unrelated without guidance from a theoretical framework. For example, a joint threat involving loans and manufacturing inputs.

Macro-Power: Strategic Sectors in General Equilibrium. Macro-Power is the social value to the hegemon’s country of the costly actions it demands of targeted entities. It arises from the hegemon’s ability to extract value from the world economy indirectly, via shaping the externalities and prices. By collectively asking entities that it can pressure to take costly actions, such as curbing the usage of some inputs, the hegemon indirectly influences a larger part of the input-output network than what it directly controls. The propagation and amplification through the network structure, our externality based Leontief-inverse, is key to this effect. In this macro sense, strategic sectors tend to be those that have a high influence on world output due to endogenous amplification. Sectors like research and development, finance, and information technology are good candidates for being strategic in this sense.

Proposition 3 shows that the marginal value to the hegemon of having more power over sector i is given by the Lagrange multiplier η_i on that sector’s participation constraint. This multiplier reflects the benefit to exerting both Micro- and Macro-Power over sector i. A hegemon particularly values having Micro-Power over sectors that increase its Macro-Power because it can exploit the difference between the private costs to targeted entities and the social benefit to itself. In accepting the hegemon’s demands, the targeted entities consider only their private costs, but the hegemon internalizes the social benefits of the outcomes of these actions. Our notion of Macro-Power is related in political science to the notion of structural power (Bachrach and Baratz (1962), Cohen (1977), Strange (1988)), in which an actor is powerful because it influences an entire environment; as opposed to a lower (relational) aspect of power in which an actor induces a target to take a desired action taking the environment as given.

Re-arranging equation (5) into $\eta_i = -\frac{\mathcal{E}_{ij}}{\tau^*_{ij}}$ highlights that the marginal value of power over a sector, η_i, is related to the ratio of how much the hegemon wants to control activities in that sector, $\mathcal{E}_{ij}$, versus how much the hegemon actually controls activities in that sector, τ^*_{ij}. When desired control $\mathcal{E}_{ij}$ is high relative to actual control τ^*_{ij}, the hegemon has little correction in place over an activity that it perceives to have high general equilibrium influence. Macro-Power is thus highly valuable in such circumstances.[12]

Finally, the theory helps interpret a type of reduced-form empirical analysis that has become common in both economics and political science: regressing measures of political affinity among countries on bilateral trade or investment. The loose prediction being that as geopolitical tensions rise between two countries, one must observe a fall in bilateral economic activity. In terms of equation (5), the loose prediction appears to rely on the direct term ε^z_{ij} and in particular the direct representative consumer disliking activity in a geopolitical rival (the $u_m(z)$ term). Our analysis makes clear that indirect effects might well dominate the direct ones and increases in geopolitical rivalry might still generate more bilateral trade in some sectors.

3.5. *Efficient Allocations*

We provide an efficiency benchmark by taking the perspective of a global planner that has exactly the same powers and constraints as the hegemon, but cares about global welfare. Formally, the planner chooses a contract Γ to maximize global welfare:

$$\sum_{n=1}^{N} \Omega_n \Big[W_n(p, w_n) + u_n(z) \Big], \quad w_n = \sum_{i \in \mathcal{I}_n} V_i(\Gamma_i) + \sum_{f \in \mathcal{F}_i} p^\ell_f \bar{\ell}_f + \mathbf{1}_{n=m} \sum_{i \in \mathcal{C}_m} \sum_{j \in \mathcal{J}_{im}} T_{ij}, \quad (8)$$

subject to the participation constraints of firms (equation (3)), the feasibility of joint threats, the determination of aggregates, and the determination of prices via market clearing. The Pareto weight placed on the welfare of country n's consumer is Ω_n. As is common in the literature, we mute the planner's motive to redistribute wealth between countries by setting

[12]Our framework can be extended to allow the hegemons to buy controlling stakes (FDI) in foreign sectors. We think of purchasing a controlling stake as a way to bypass the participation constraint since then the hegemon can simply dictate the actions. Interestingly, the private market value of such stake should be lower than the social value to the hegemon that internalizes its geoeconomic use thus providing a rationale for the investment screening policies such as CFIUS in the US.

the welfare weights to equalize the social marginal value of wealth across consumers. The following proposition characterizes the global planner's solution.

PROPOSITION 4: *An optimal contract of the hegemon from the global planner's perspective features maximal joint threats $\mathcal{S}_i' = \overline{\mathcal{S}}_i'$, zero transfers $\overline{T}_i = 0$, and wedges given by $(\Omega_n \frac{\partial W_n}{\partial w_n} + \eta_i)\tau_{ij}^* = -\mathcal{E}_{ij}^p$ for all sectors $i \in \mathcal{C}_m$ on which the hegemon has a pressure point. Wedges and transfers are zero if $\overline{\mathcal{S}}_i'$ is not a pressure point on i.*

The planner and the hegemon agree that supplying maximal joint threats is optimal since it relaxes the targeted entities' incentive problems and in principle allows more economic activity to take place. The planner and the hegemon, however, disagree on the value of transfers and on the optimal wedges to be applied.

Both the planner and the hegemon understand that the transfers are negative-sum globally since they tighten incentive problems. The planner, therefore, chooses never to demand transfers. The hegemon, instead, values receiving positive transfers from foreign firms.[13] Both the global planner and the hegemon want to use the wedges in equilibrium to affect externalities. However, the global planner implements wedges that are different from those implemented by the hegemon. Intuitively, the planner and hegemon might disagree on who their friends and enemies are. Formally, $\mathcal{E}_{ij}^p$ tracks the impact of activity x_{ij} on the planner's Lagrangian rather than the hegemon's one.

In political science, Kindleberger (1973), Krasner (1976), Gilpin (1981), and Keohane (1984) debated whether hegemons can generate better world outcomes by providing public goods globally than configurations with no hegemons (or multiple hegemons). In our framework, the hegemon acts as a global enforcer, echoing the public good provision, and some of its policies correct negative externalities. Indeed, the global planner also provides the same enforcement (maximal joint threats) and, in some dimensions, might correct externalities similarly to the hegemon. However, the hegemon destroys value at the global level compared to the global planner by demanding transfers and manipulating the externalities in its favor. Because of the externalities, the equilibrium with the hegemon can even be worse for some entities than the equilibrium without the hegemon depending on

[13] If we allowed hegemon consumers to own foreign sectors this would contribute to aligning the hegemons' incentives with those of the planner by making the hegemon care about the profits of foreign sectors that it owns. Exogenous ownership of foreign sectors would be easy to introduce in this framework.

whether the enforcement and positive correction of externalities are more then offset by the externality manipulation.

The view of the hegemon as a global enforcer also brings up the scope and focus of this paper. There are alternative means of enforcement via military actions that also have a long history (Findlay and O'Rourke (2009)). Military build-up as an enforcement deterrent could be accommodated in the framework, for example by the hegemon being able to offer lower θs, thus expanding the target inside option, but at a fiscal (resource) cost to the hegemon (see also Section 4.2 and Appendix B.3.7). Other aspects of military power and war are instead further removed from the focus of this paper (Fearon (1995), Powell (2006)).

4. APPLICATIONS

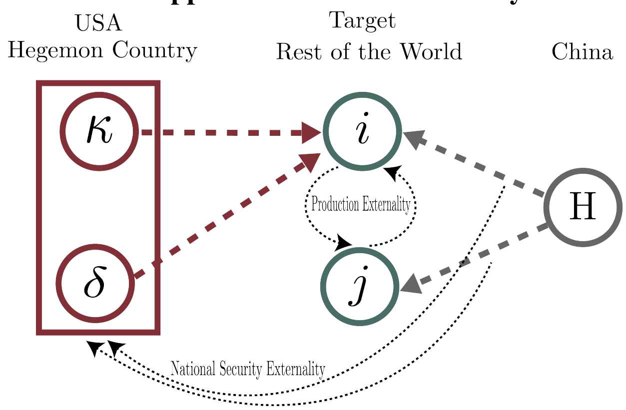
We specialize the model to capture two leading applications of geoeconomics in practice.

4.1. *National Security Externalities*

In this application we take as inspiration the US government demand to European governments and firms that they stop using information technology infrastructure produced by China's Huawei (Farrell and Newman (2023)). We assume the hostile technology is a national security threat from the perspective of the hegemon, but a positive production externality for firms in third party countries. This captures the notion that this infrastructure could be used for spying and or military uses, but that for a private firm the technology is attractive (privately profitable) and the more so the more other firms are using it. That is, the technology has a strategic complementary in its adoption capturing interoperability. The application is both of practical interest and helps us illustrate the importance of production externalities and network amplification in how a hegemon pressures strategic sectors.

There are three regions: the hegemon country m, a hostile foreign country h, and "rest of world" RoW which may comprise multiple countries. Figure 2 illustrates the set-up of this application. We assume constant prices (Definition 1). The hostile foreign country h has a single sector, which we denote by H. We take the output of this sector to be the numeraire, $p_H = 1$. Sector H and sectors in the hegemon country are not subject to externalities from z, that is $f_H(x_H, \ell_H, z)$ and $f_k(x_k, \ell_k, z)$ for $k \in \mathcal{I}_m$ are constant in z. We assume that sectors in the hegemon country do not source from the hostile country's sector H and vice-versa, ensuring that H cannot be used by the hegemon as part of a joint threat.

FIGURE 2.—**Application: National Security Externality**



Notes: Figure depicts the model set-up for the application on national security as described in Section 4.1.

The main action in this application comes from RoW sectors. We assume that all RoW sectors source from H, and define $z^H \equiv \{z_{iH}\}_{i \in \mathcal{I}_{RoW}}$ to be the vector of purchases by RoW sectors of input H. For simplicity, we assume sectors in RoW have production that is separable in H: $f_i(x_i, \ell_i, z) = f_{i,-H}(x_{i,-H}, \ell_i) + f_{iH}(x_{iH}, z^H)$, where $x_{i,-H}$ denotes the vector of all inputs except input H. We introduce external economies of scale by setting:

$$f_{iH}(x_{iH}, z^H) = A_{iH}(z^H) g_{iH}(x_{iH}). \tag{9}$$

We assume that $\frac{\partial A_{iH}}{\partial z_{jH}} > 0$ for all $i, j \in \mathcal{I}_{RoW}$, so that there are positive spillovers from greater usage of H. This helps us capture technologies, such as 5G infrastructure, that have strategic complementarities in adoption and usage. We further assume that $A_{iH}(z^H) g_{iH}(z_{iH})$ is concave in z^H. Observe that $f_{i,-H}$ is constant in z. For simplicity we assume $\theta_{iH} = 0$, so that firms are unconstrained in their use of input H. We assume that in absence of a hegemon, there are no joint triggers.

Hegemon Negative Externality from H. We assume that the hegemon's representative consumer's utility function has a negative externality from rest-of-world production using H, that is $u_m(z) = u_m(z^H)$ and $\frac{\partial u_m}{\partial z_{iH}} < 0$ for all $i \in \mathcal{I}_{RoW}$. This simple reduced-form utility term in the objective function of the hegemon helps us capture a direct disutility from the RoW usage of the technology of a hostile country. In practice, the US government concerns regarding Huawei technology stemmed from the possibility that it could be used for spying or in military applications; we capture the direct US government goal of shrinking the usage of the technology.

From Proposition 1, maximal joint threats are optimal for the hegemon. Since there are no z-externalities in production by domestic firms and prices are constant, Proposition 3 tells us $\overline{T}_i = 0$ and $\tau_i = 0$ is an optimal contract for all domestic sectors. To characterize the optimal contracts for sectors in the RoW, the relevant part of the objective function (equation (4)) reduces to $\mathcal{U}_m = u_m(z^H) + \sum_{i \in \mathcal{D}_m} \overline{T}_i$.

Network Amplification. Network amplification occurs due to the strategic complementarity in the use of H. We can capture the interesting economics even considering only two sectors in RoW: one sector, which we denote i, that the hegemon can contract with; and one sector, which we denote j, that the hegemon cannot contract with. In this environment, employing Proposition 2 we have $\Psi^{z,NC} = \left(1 - \frac{\partial x^*_{jH}}{\partial z_{jH}}\right)^{-1} = \frac{\gamma_j}{\gamma_j - \xi_{jj}}$, where $\xi_{ij} = \frac{z_{jH}}{A_{iH}(z^H)} \frac{\partial A_{iH}(z^H)}{\partial z_{jH}}$ is the elasticity of productivity A_{iH} with respect to the externality z_{jH}, so that ξ_{jj} are sector j external economies of scale, and where $\gamma_i = \frac{-x^*_{iH} g''_{iH}(x^*_{iH})}{g'_{iH}(x^*_{iH})}$. Applying Proposition 2 we have that the total transmission of a change in the targeted sector i usage of input H to the usage by sector j of the same input is given by:

$$\frac{dz^{NC}}{dz_{iH}} = \frac{dz_{jH}}{dz_{iH}} = \Psi^{z,NC} \frac{\partial x^*_{jH}}{\partial z_{iH}} = \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \frac{z_{jH}}{z_{iH}}.$$

Optimal Contract. The hegemon's optimal tax formula of Proposition 3 and equation (5) reduces to $\tau_{iH} = -\frac{1}{\eta_i} \varepsilon^z_{iH} - \varepsilon^z_{jH} \frac{dz_{jH}}{dz_{iH}}$ since the term $\varepsilon^{P^m} \frac{dP^m}{dz_{ij}}$ is zero given constant prices. Using equation (6) we can further unpack this formula to write:

$$\begin{aligned}
\tau_{iH} = - & \overbrace{\frac{1}{\eta_i} \frac{\partial u_m}{\partial z_{iH}}}^{\text{Externalities on Hegemon's Economy}} \overbrace{+ p_i A_{iH}(z^H) \Big[g_{iH}(x^o_{iH}(z^H)) - g_{iH}(x^*_{iH}) \Big] \xi_{ii} \frac{1}{z_{iH}}}^{\text{Building Power}} \\
& \underbrace{+ \left[- \frac{1}{\eta_i} \frac{\partial u_m}{\partial z_{jH}} + p_i A_{iH}(z^H) \Big[g_{iH}(x^o_{iH}(z^H)) - g_{iH}(x^*_{iH}) \Big] \xi_{ij} \frac{1}{z_{jH}} \right] \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \frac{z_{jH}}{z_{iH}}}_{\text{Network Amplification}}
\end{aligned}$$

where $x^o_{iH}(z^H)$ is what firm i's optimal usage of input H would be if it rejected the hegemon contract. In the presence of national security externalities, the optimal tax is positive, $\tau_{iH} > 0$, reflecting the hegemon's desire to mitigate the negative externality. Three key forces underlie the tax formula.

The first term in the tax formula is the direct externality from an increase in z_{iH} on representative consumer m. The negative externality contributes to a positive tax. This tax is higher when η_i is lower, that is when the marginal cost of using the hegemon's power over firm i (the slack in that firm's participation constraint) is lower.

The second term captures the hegemon's desire to build Micro-Power over firms in sector i by leveraging the external economies of scale. Each firm that accepts the hegemon's contract and reduces its usage of input of H increases on the margin the hegemon's power over other firms in the same sector by lowering productivity A_{iH} and making it less attractive to reject the contract to use more of the H input. The hegemon is manipulating the external economies of scale to get firms to downscale the undesirable technology. Once it is successfully downscaled, no individual firm has a high desire to use it on the margin.

Finally, the third term is the indirect effect of the hegemon's demands on the sector it can pressure (sector i) on the sector it cannot pressure (sector j). As sector i usage of input H falls, that is z_{iH} falls, the productivity A_{jH} of firms in sector j in using input H also falls, prompting firms in sector j to reduce the use of H. This leads to a fall in z_{jH}, which has a positive externality effect on the hegemon consumer and also increases Micro-Power over firms in sector i. Both effects mirror those described in the previous two paragraphs but are now arising from the equilibrium choices of a sector the hegemon does not directly control. The Leontief amplification $\Psi^{z,NC} = \frac{\xi_{ji}}{\gamma_j - \xi_{jj}}$ captures the magnitude of this response by sector j. This effect contributes towards a higher tax rate, since reducing usage by sector i of input H has a positive externality by also reducing demand by sector j for input H.

In this application, sector i is strategic from a Macro-Power perspective because by influencing its actions the hegemon impacts the actions of sectors it could not pressure directly. As a consequence the hegemon makes higher demands (more positive τ_{iH}) and manipulates the difference between the private cost to the target of the actions (Micro-Power) and the social value to the hegemon (Macro-Power) to build more power over other targets within and across sectors. In practice, this explains that the strong pressure applied by the

US on European firms to prevent usage of Huawei technology aimed at making the technology less valuable to adopt for other entities, that the US could not directly pressure, once European entities were also not using it.

4.2. *Official Lending, Infrastructure Projects, and Political Concessions*

China's flagship Belt and Road Initiative (BRI) has sought to jointly provide official loans and manufacturing inputs often in exchange for political concessions (Dreher et al. (2022)). Our model explains how China acts as a global enforcer in emerging economies by providing pressure jointly across lending and manufacturing relationships while extracting surplus in terms of political concessions.

We specialize the model to the configuration in Figure 3. The hegemon country, in this application China, has two sectors: sector k is a lender and sector j is a manufacturer. For simplicity, both sectors produce only using local factors. The target country, in this application a developing economy, has a single sector i that uses both inputs from China to produce. To focus the application on the essentials, we further assume constant prices (Definition 1), no z-externalities (Definition 2), and that sector i has a separable production function $f_i(x_i) = f_{ij}(x_{ij}) + f_{ik}(x_{ik})$. We think of the lending sector, k, as providing a loan to or buying a bond issued by sector i. The loan is for amount $x_{ik} = b$ and the gross interest rate is $p_k = R$. Like in the sovereign default literature, we assume limited or nonexistent loan legal enforceability, so that $\theta_{ik} > 0$.

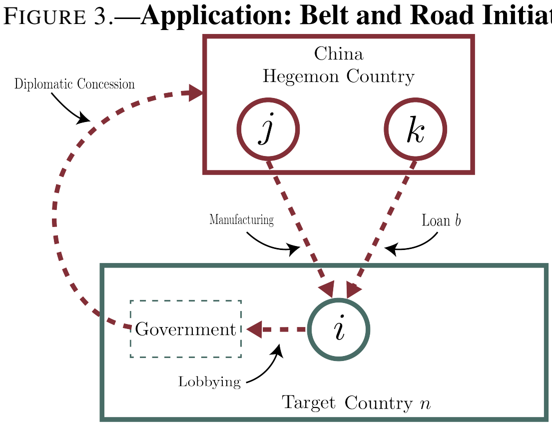
To build intuition, consider a configuration with no hegemon, only individual triggers on j and k, and no loan legal enforceability $\theta_{ik} = 1$. Lending can be sustained by the future surplus of the lending relationship, along the lines of the sovereign default model of Eaton and Gersovitz (1981). In particular, $Rb \leq \beta\left[\nu_i(\{j,k\}) - \nu_i(\{j\})\right] = \beta\nu_i(\{k\})$, where the latter equality follows from the separable production function and individual triggers. The Markov equilibrium value of $\frac{p_i f_{ik}(b^*) - Rb^*}{1-\beta}$ is the present discounted value of all future borrowing by sector i. Solving for the borrowing limit, we obtain $b \leq \left(\beta\frac{p_i}{R}\right)^{\frac{1}{1-\xi}}$ under the assumption that $f_{ik}(b) = b^{\xi}$ for $\xi \in (0,1)$. The IC (borrowing limit) binds whenever $\xi > \beta$.

To sharpen the application, we assume that $\theta_{ij} = 0$ so that firms in sector i can never steal input j. Thus the incentive constraint for stealing j does not bind. Without a hegemonic China, the equilibrium features limited lending and an unconstrained manufacturing relationship. As a hegemon, China can impose a joint threat that links together the provision of lending and manufacturing goods. If the target country defaults on either input, both are withdrawn in the future. Under the joint threat the incentive constraint of the target country sector i is:

$$\theta_{ik}Rb + \overline{T}_i \leq \beta\nu_i(\{j,k\}) = \beta\frac{p_i f_{ik}(b') - Rb'}{1-\beta} - \beta\frac{\overline{T}_i'}{1-\beta} + \beta\frac{p_i f_{ij}(x_{ij}^*) - p_j x_{ij}^*}{1-\beta},$$

where $'$ variables are exogenous continuation values from the perspective of this period, with equilibrium consistency condition $b = b' = b^*$, and $\overline{T}_i = \overline{T}_i' = \overline{T}_i^*$. The present value of the manufacturing relationship provides additional incentives to repay the debt in the joint threat, an endogenous cost of default on the loan. Under the joint threat, and assuming a binding IC under individual threats, the equilibrium features the same level of manufacturing activity but an increase in borrowing. The surplus is extracted by China via a transfer $\overline{T}_i^* > 0$. Appendix B.3.8 fully characterizes this equilibrium.

FIGURE 3.—**Application: Belt and Road Initiative**



Notes: Figure depicts the model set-up for the application on the Belt and Road Initiative as described in Section 4.2.

Our mechanism is related to that proposed in Bulow and Rogoff (1989), whereby lenders seize the exports of a country conditional on a default, thereby generating a cost of default.[14] It is also related to Cole and Kehoe (1998), where government reputation is com-

[14]Under isolated threats, our model features positive borrowing. The impossibility result of Bulow and Rogoff (1989) does not kick in because we are not allowing inter-temporal saving and up-front payment contracts as in Eaton and Gersovitz (1981).

mon across multiple relationships. In Mendoza and Yue (2012) a country faces an endogenous productivity loss in case of default due to being shut off from trade finance, hence losing the ability to import intermediate goods and being forced to switch to imperfect domestic substitutes. In our framework, joint threats offer a means for a country to voluntarily raise its cost of default, thereby allowing it to borrow more. In particular, the more input varieties and the more profitable those input varieties that are sourced from China, the more the borrowing constraint is relaxed. This application also helps us to visualize economic enforcement versus a military one. Historically, enforcement of sovereign debt could include sending the navy to threaten the blockade or shelling of a foreign port. Such military threats share the enforcement aspect of our commercial joint threats, but might differ substantially in their resource and human cost.

One interpretation of the transfers is mark-ups on the manufacturing goods being sold by China to the target country, or equivalently an interest rate on the loan above the market rate R. This application cautions against empirical work that assesses China's lending programs in isolation, i.e. focusing only on the loans and their returns. Both the sustainability of the loans and the economic returns from the lending have to be assessed jointly with other activities, such as manufacturing exports, that are occurring jointly with the lending. The benefits to China might not even accrue in monetary form as we explore below.

Transfers as Costly Actions and Political Concessions. Our framework could be extended to allow for a rich model of political lobbying and influence (Grossman and Helpman (1994), Bombardini and Trebbi (2020)). The costly actions that the hegemon demands can take the form of political lobbying or diplomatic concessions. In this case, the transfer $\overline{T}_i$ represents the private cost to the firm of an action. Here we focus on a leading example for geoeconomics in which China asks the firms to lobby their governments for a political concession. We necessarily keep the modeling reduced form, but it provides a starting point for future research interested in introducing a deeper model of lobbying.

We assume that a bilateral geopolitical concession can be made from country n to China. We let the concession be the element $z_n^c \in \{0, 1\}$ of aggregate vector z and assume that it enters positively in China's utility, $u_m(z_n^c)$ with $u_m(1) > u_m(0)$, and negatively in the target's country utility, $u_n(z_n^c)$ with $u_n(0) > u_n(1)$. We assume that no utility is derived by either country from all other elements of z. Governments care about consumer welfare and therefore internalize these utility costs and benefits. Governments also care about the profits of the firms in their country net of transfers. We assume that a hegemon asking a firm to make a positive transfer can alternatively ask that firm to transfer part or all of that transfer to the government in exchange for the government undertaking the geopolitical action, with any money not transferred being paid as usual to the hegemon. The geopolitical action is feasible to implement as long as country level transfer exceed the government utility cost of the concession. These concessions can account, for example, for China asking countries that are part of the Belt and Road Initiative not to recognize Taiwan (Dreher et al. (2022)).

5. CONCLUSION

Geoeconomics is a topic of practical importance but for which a formal treatment has proven elusive. This paper provides a general and formal framework that derives precise economic concepts to analyze this important topic. We show how concepts such as pressure, economic coercion, power, interdependence, strategic sectors, and third party sanctions emerge based on three core ingredients: limited enforceability and trigger punishments, input-output amplification, and externalities. Voluntary compliance with the hegemon's demands gives rise to a participation constraint that reflects its limits to power. We show how the framework can be used to make sense of many geoeconomic activities in practice like the US demands that European firms not use Huawei's technology, or China's flagship Belt and Road Initiative. The framework is flexible and can be extended for future analyses of a rich set of issues in geoeconomics as well as guide the necessary empirical measurement.

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APPENDIX A: PROOFS

Proof of Lemma 1. Let $M_{ij} \subset \mathcal{J}_i$ be the (possibly empty) set of joint triggers of suppliers in j in its relationship with firm i. The assumption that joint triggers are symmetric means $k \in M_{ij} \iff j \in M_{ik}$. Let $B'_{ij}(S) \in \{0,1\}$ indicate whether suppliers in j Trust firm i in the next period following stealing action S. Trigger strategies are formally defined by

$$B'_{ij}(S) = \begin{cases} B_{ij}, S \cap K_{ij} = \emptyset \\ 0, \qquad o.w. \end{cases}, \quad K_{ij} = \{j\} \cup \bigcup_{k \in M_{ij}} K_{ik} \tag{10}$$

where $K_{ij} \subset \mathcal{J}_i$ is the full set of individual and joint trigger relationships (including chaining) and is constructed below. Chaining of joint triggers is reflected by $K_{ik} \subset K_{ij}$ for $k \in M_{ij}$. Following Stealing action S, suppliers $\mathcal{B}'_i(S) = \mathcal{B}_i \backslash (\bigcup_{j \in S} K_{ij})$ Trust firm i.

To construct the partition $\mathcal{S}_i$, we first construct the smallest sets K_{ij} consistent with equation (10), that is involving minimal retaliation. Let $\{X^n_{ij}\}_{n=0}^{\infty}$ be a sequence of sets constructed iteratively as follows. Let $X^0_{ij} = \{j\}$ and, for $n \geq 1$, let $X^n_{ij} = X^{n-1}_{ij} \cup \bigcup_{x \in X^{n-1}_{ij}} M_{ix}$.[15] Since $\mathcal{J}_i$ is a finite set, since $X^{n-1}_{ij} \subset X^n_{ij} \subset \mathcal{J}_i$, and since $X^n_{ij} = X^{n-1}_{ij} \Rightarrow$

[15]The first element $X^0_{ij} = \{j\}$ is the individual trigger. The second element, $X^1_{ij} = \{j\} \cup M_{ij}$, adds in the joint triggers of suppliers in j, and so on.

$X_{ij}^{n+1} = X_{ij}^n$, then $\exists \overline{N}_{ij} > 0$ such that $X_{ij}^{\overline{N}_{ij}} = X_{ij}^n$ for all $n \geq \overline{N}_{ij}$. Define the *minimum retaliation set* as $X_{ij}^* = X_{ij}^{\overline{N}_{ij}}$.

We next show that $k \in X_{ij}^*$ if and only if $X_{ik}^* = X_{ij}^*$. The if statement is immediate since $k \in X_{ik}^*$ by construction. Consider then the only if statement and let $k \in X_{ij}^*$. Since $k \in X_{ij}^*$, then $\exists N > 0$ s.t. $k \in X_{ij}^N$ and therefore $X_{ik}^* \subset X_{ij}^*$. Moreover since $k \in X_{ij}^*$, by construction there is a sequence $x_0, .., x_N$, with $x_0 = j$ and $x_N = k$, such that $x_n \in M_{ix_{n-1}}$ for $n = 1, ..., N$. Reversing that sequence and using symmetry of joint triggers, we have a sequence $x_N, ..., x_0$ such that $x_{n-1} \in M_{ix_n}$. Hence, $j \in X_{ik}^N$, and hence $j \in X_{ik}^*$. But then $X_{ij}^* \subset X_{ik}^*$, and hence $X_{ij}^* = X_{ik}^*$.

Next, consistent with equation (10) we define $K_{ij} = X_{ij}^*$. Define $\mathcal{S}_i = \bigcup_{j \in \mathcal{J}_i} \{K_{ij}\}$. Observe that $\mathcal{S}_i$ is a partition of $\mathcal{J}_i$ since: (i) $\bigcup_{j \in \mathcal{J}_i} X_{ij}^* = \mathcal{J}_i$; (ii) $\forall j, k \in \mathcal{J}_i$, either $X_{ij}^* = X_{ik}^*$ or $X_{ij}^* \cap X_{ik}^* = \emptyset$. For any $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$, by definition of $\Sigma(\cdot)$ there exists a $\mathcal{S}_i(\mathcal{B}_i) \subset \mathcal{S}_i$ such that $\mathcal{B}_i = \bigcup_{X \in \mathcal{S}_i(\mathcal{B}_i)} X$.[16] Since $\mathcal{S}_i$ is a partition of $\mathcal{J}_i$, then $\mathcal{S}_i(\mathcal{B}_i)$ is a partition of $\mathcal{B}_i$.

Consider a firm at the (on or off path) node $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$ at which it is Trusted by suppliers $\mathcal{B}_i$ and Distrusted by suppliers $\mathcal{J}_i \backslash \mathcal{B}_i$. The incentive compatibility constraint associated with firm i preferring no stealing over stealing action $S \in P(\mathcal{B}_i)$ is

$$\Pi_i(x_i, \ell_i, \mathcal{B}_i) + \sum_{j \in S} \theta_{ij} p_j x_{ij} + \beta \nu_i(\mathcal{B}_i'(S)) \leq \Pi_i(x_i, \ell_i, \mathcal{B}_i) + \beta \nu_i(\mathcal{B}_i),$$

which reduces to $\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta[\nu_i(\mathcal{B}_i) - \nu_i(\mathcal{B}_i'(S))]$. Parallel to main text, the notation $\Pi_i(x_i, \ell_i, \mathcal{B}_i)$ indicates that $x_{ij} = 0$ for $j \notin \mathcal{B}_i$.

We now complete the proof of the Lemma (at all $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$). The only if statement holds trivially since $\Sigma(\mathcal{S}_i(\mathcal{B}_i)) \subset \Sigma(\mathcal{B}_i) = P(\mathcal{B}_i) \backslash \{\emptyset\}$ since $\mathcal{S}(\mathcal{B}_i)$ is a partition of $\mathcal{B}_i$. Thus consider the if statement. Suppose that (x_i, ℓ_i) is incentive compatible with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Let $S \in P(\mathcal{B}_i)$. If $S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ then incentive compatibility holds by assumption, so let $S \notin \Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Given a stealing action S, all suppliers $k \in \bigcup_{j \in S} X_{ij}^*$ Distrust firm i. Since elements of $\mathcal{S}_i(\mathcal{B}_i)$ are disjoint and since $X_{ij}^* = X_{ik}^* \iff j \in X_{ik}^*$, there is a unique subset $\mathcal{X}_i(S) \subset \mathcal{S}_i(\mathcal{B}_i)$ such that $\bigcup_{X \in \mathcal{X}_i(S)} X = \bigcup_{j \in S} X_{ij}^*$. Define $\Xi_i(S) = \bigcup_{X \in \mathcal{X}_i(S)} X$. For any $S \in P(\mathcal{B}_i)$, the stealing choice S is weakly dominated by the steal-

[16] We slightly abuse notation by denoting $\mathcal{S}_i = \mathcal{S}_i(\mathcal{J}_i)$.

ing choice $\Xi_i(S)$, since S and $\Xi_i(S)$ yield the same continuation value $\nu_i(\mathcal{B}_i \backslash \Xi_i(S))$ but $\Xi_i(S)$ yields higher flow payoff. Since $\Xi_i(S) \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ and since $\Xi_i(S)$ weakly dominates S, then if (x_i, ℓ_i) is incentive compatible with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$ it is also incentive compatible with respect to S. But since S was generic, then incentive compatibility with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$ implies incentive compatibility with respect to $P(\mathcal{B}_i)$.

Proof of Proposition 1. Consider a hypothetical optimal contract $\Gamma^o = \{\mathcal{S}_i'^o, \mathcal{T}_i^o, \tau_i^o\}_{i \in \mathcal{C}_m}$ that is feasible and satisfies firms' participation constraints, and suppose that $\mathcal{S}_i' \neq \overline{\mathcal{S}_i}'$. We use (x^o, ℓ^o) to denote firm allocations under this contract (and so on). The proof is one of implementability: we show that the hegemon can achieve the same allocations, prices, and transfers using a feasible contract with maximal joint threats, $\Gamma^* = \{\overline{\mathcal{S}}_i', \mathcal{T}_i^o, \tau_i^*\}$.

We first construct τ^* by $\tau_{ij}^* = \frac{\partial \Pi_i(x_i^o, \ell_i^o, \mathcal{J}_i)}{\partial x_{ij}}$ and $\tau_{if}^{\ell *} = \frac{\partial \Pi_i(x_i^o, \ell_i^o, \mathcal{J}_i)}{\partial \ell_{if}}$. The relaxed problem (not subject to incentive compatibility) of firm i is

$$\max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{J}_i) - \sum_{j \in \mathcal{J}_i} [\tau_{ij}^*(x_{ij} - x_{ij}^*) + T_{ij}] - \sum_{f \in \mathcal{F}_m} \tau_{if}^{\ell *}(\ell_{if} - \ell_{if}^*),$$

which yields solution $\frac{\partial \Pi_i}{\partial x_{ij}} = \tau_{ij}^*$ and $\frac{\partial \Pi_i}{\partial \ell_{if}} = \tau_{if}^*$, that is $x_i = x_i^o$ and $\ell_i = \ell_i^o$. It remains to verify this allocation is incentive compatible. Since $\overline{\mathcal{S}}_i'$ is a joint threat of $\mathcal{S}_i'^o$, then $\Sigma(\overline{\mathcal{S}}_i') \subset \Sigma(\mathcal{S}_i'^o)$, and hence (x_i^o, ℓ_i^o) is incentive compatibility under contract Γ_i^*. Since (x_i^o, ℓ_i^o) solves firm i's relaxed problem and is incentive compatible, it is optimal for firm i.

Next, conjecturing $(z^*, P^*) = (z^o, P^o)$, then every firm $i \notin \mathcal{C}_m$ and every consumer n faces the same decision problem as under the original contract. Hence, every firm and every consumer has the same optimal policy. Hence $x^* = z^o$ and markets clear at prices $P^* = P^o$, consistent with the conjecture.

Finally, since allocations, transfers, and prices are the same, then since firm i's participation constraint is satisfied under contract Γ^o it is also satisfied under contract Γ^*. Since prices, allocations, and transfers are unchanged, the hegemon's objective attains the same value as under the original contract. Thus the hegemon is indifferent between feasible contracts Γ^o and Γ^*, completing the proof.

Proof of Proposition 2. To clarify the ordering for matrix algebra, $z_i^* = (z_{i,\min \mathcal{J}_i}^*, \ldots, z_{i,\max \mathcal{J}_i}^*)^T$ is a $|\mathcal{J}_i| \times 1$ vector and $z^* = (z_1^{*T}, \ldots, z_{|\mathcal{I}|}^{*T})^T$ is a $\sum_{i \in \mathcal{I}} |\mathcal{J}_i| \times 1$ vector. Let $|z^*| = \sum_{i \in \mathcal{I}} |\mathcal{J}_i|$. We stack x^* from x_{ij}^* in the same manner.

Since $x^*(\Gamma, z^*, P) = z^*$, then totally differenting yields $\frac{\partial x^*}{\partial e} + \frac{\partial x^*}{\partial P}\frac{dP}{de} + \frac{\partial x^*}{\partial z^*}\frac{dz^*}{de} = \frac{dz^*}{de}$, where $\frac{\partial x^*}{\partial e}$ is a $|z^*| \times 1$, and $\frac{\partial x^*}{\partial z^*}$ is a $|z^*| \times |z^*|$ matrix with each row corresponding to the vector $\frac{\partial x^*_{ij}}{\partial z^*}$. Rearranging yields $\frac{dz^*}{de} = \Psi^z \frac{\partial x^*}{\partial e} + \Psi^z \frac{\partial x^*}{\partial P}\frac{dP}{de}$, where $\Psi^z = \left(\mathbb{I} - \frac{\partial x^*}{\partial z^*}\right)^{-1}$.

Next, define the excess demand for good i as $ED_i = \sum_{n=1}^N C_{ni} + \sum_{j \in \mathcal{D}_i} x_{ji} - y_i$ and the excess demand for market f as $ED^\ell_f = \sum_{i \in \mathcal{I}_n} \ell_{if} - \bar{\ell}_f$. Define $ED = (ED_2, \ldots, ED_{|\mathcal{I}|}, ED^\ell_1, \ldots, ED_{|\mathcal{F}|})^T$ which is a $(|\mathcal{I}| + |\mathcal{F}| - 1) \times 1$ vector (excluding the numeraire). Market clearing requires $ED(\Gamma, z^*, P) = 0$, so that totally differentiating in e yields $\frac{\partial ED}{\partial e} + \frac{\partial ED}{\partial z^*}\frac{dz^*}{de} + \frac{\partial ED}{\partial P}\frac{dP}{de} = 0$. Substituting in for $\frac{dz^*}{de}$, rearranging, and inverting yields $\frac{dP}{de} = -(\frac{\partial ED}{\partial P} + \frac{\partial ED}{\partial z^*}\Psi^z\frac{\partial x^*}{\partial P})^{-1}(\frac{\partial ED}{\partial e} + \frac{\partial ED}{\partial z^*}\Psi^z\frac{\partial x^*}{\partial e})$, which completes the proof.

Proof of Proposition 3. For any prices and aggregates $Q = (P, z^*)$, define the subset $\mathcal{P}(Q) \subset \mathcal{C}_m$ of sectors that the hegemon has pressure points on. We divide the proof into the three regions in which the hegemon's optimal contract could lie: (i) the hegemon has no pressure points, $\mathcal{P} = \emptyset$; (ii) the hegemon has pressure points on all sectors, $\mathcal{P} = \mathcal{C}_m$; (iii) the hegemon has pressure points on some (but not all) sectors, $\mathcal{P} \neq \emptyset$ and $\mathcal{P} \neq \mathcal{C}_m$. Note that some of these regions may be empty and some points Q cannot be part of an equilibrium.

Case (i): Pressure points on no sectors. Suppose that $\mathcal{P}(Q) = \emptyset$. Then $V_i(\overline{S}'_i) = V_i(\mathcal{S}_i)$ for all $i \in \mathcal{C}_m$, and hence the hegemon must set $\overline{T}_i = 0$ and $\tau_i = 0$ for all i.

Case (ii): Pressure points on all sectors $i \in \mathcal{C}_m$. Suppose that $\mathcal{P}(Q) = \mathcal{C}_m$. As is common in the literature (e.g., Farhi and Werning (2016)), we assume the hegemon is able to select its preferred equilibrium (P, z^*) when there are multiple equilibria consistent with its contract. Since the hegemon has complete instruments for $i \in \mathcal{C}_m$, we adopt the primal approach whereby the hegemon directly selects allocations of firms $i \in \mathcal{C}_m$, and derive the wedges that implement these allocations.

The Lagrangian of firm i, with choice variables (x_i, ℓ_i), is

$$\mathcal{L}_i = \Pi_i(x_i, \ell_i, \mathcal{J}_i) - \sum_{j \in \mathcal{J}_i} [\tau_{ij}(x_{ij} - x^*_{ij})] - \sum_{f \in \mathcal{F}_m} \tau^\ell_{if}(\ell_{if} - \ell^*_{if}) - \overline{T}_i + \beta\nu_i(\mathcal{J}_i)$$
$$+ \sum_{S \in \Sigma(\overline{S}'_i)} \lambda_{iS}\left[\beta\Big(\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S)\Big) - \sum_{j \in S}\left[\theta_{ij}p_j x_{ij} - \mathbf{1}_{S^D_i \subset S}\overline{T}_i\right]\right]$$

Denoting $\overline{\lambda}_{ij} \equiv \sum_{S \in \Sigma(\overline{\mathcal{S}}'_i) | j \in S} \lambda_{iS}$, the FOCs are

$$\tau_{ij} = \frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\lambda}_{ij} \theta_{ij} p_j; \qquad \tau^{\ell}_{if} = \frac{\partial \Pi_i}{\partial \ell_{if}}$$

Given that the firm's optimization problem is convex, given an incentive compatible allocation (x_i, ℓ_i), and given nonnegative Lagrange multipliers $\lambda_{iS} \geq 0$ such that complementary slackness holds, these equations define wedges that implement (x_i, ℓ_i).

Next consider the hegemon's Lagrangian. Under the primal approach of choosing $\{x_i, \ell_i, \overline{T}_i\}_{i \in \mathcal{C}_m}, P, z$, the hegemon's Lagrangian is

$$\begin{aligned}
\mathcal{L}_m = & W_m\Bigg(p, \sum_{i \in \mathcal{I}_m} \Pi_i(x_i, \ell_i, \mathcal{J}_i) + \sum_{f \in \mathcal{F}_m} p^{\ell}_f \overline{\ell}_f + \sum_{i \in \mathcal{D}_m} \overline{T}_i\Bigg) + u_m(z) \\
& + \sum_{i \in \mathcal{C}_m} \eta_i \Big[\Pi_i(x_i, \ell_i, \mathcal{J}_i) - \overline{T}_i + \beta \nu_i(\mathcal{J}_i) - V_i(\mathcal{S}_i)\Big] + \sum_{i \in \mathcal{C}_m} \gamma_i \overline{T}_i \\
& + \sum_{i \in \mathcal{C}_m} \sum_{S \in \Sigma(\overline{\mathcal{S}}'_i)} \Lambda_{iS} \Big[\beta \Big[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S)\Big] - \sum_{j \in S} \theta_{ij} p_j x_{ij} - \mathbf{1}_{S^D_i \subset S} \overline{T}_i\Big] \\
& + \sum_{f \in \mathcal{F}_m} \kappa_f ED_f + ED^m \phi + \sum_{i \in \mathcal{C}_m} \sum_{j \in \mathcal{J}_i} \psi_{ij} \Big[z_{ij} - x_{ij}\Big] + \Big[z^{NC} - x^{NC}\Big] \psi^{NC}.
\end{aligned}$$

where for factor f located in country n, $ED_f = \overline{\ell}_f - \sum_{i \in \mathcal{I}_n \cap \mathcal{C}_m} \ell_{if} - \sum_{i \in \mathcal{I}_n \backslash \mathcal{C}_m} \ell_{if}(P, z)$, and for good i

$$ED_i = \begin{cases} \sum_{n=1}^N C_{ni}(P, w_n) + \sum_{j \in D_i \cap \mathcal{C}_m} x_{ji} + \sum_{j \in D_i \backslash \mathcal{C}_m} x_{ji}(P, z) - f_i(x_i, \ell_i, z), i \in \mathcal{C}_m \\ \sum_{n=1}^N C_{ni}(P, w_n) + \sum_{j \in D_i \cap \mathcal{C}_m} x_{ji} + \sum_{j \in D_i \backslash \mathcal{C}_m} x_{ji}(P, z) - y_i(P, z), \quad i \notin \mathcal{C}_m \end{cases}$$

where $y_i(P, z) = f_i(x_i(P, z), \ell_i(P, z), z)$. We defined $\phi = (\phi_2, \ldots, \phi_N, \{\phi^{\ell}_f\}_{f \notin \mathcal{F}_m})$ (Lagrange multipliers on market clearing) and ED^m analogously.[17] We defined $\psi^{NC} = \{\psi_{ij}\}_{i \notin \mathcal{C}_m}$ (Lagrange multipliers on determination of aggregates) and z^{NC}, x^{NC} analogously. $\gamma_i \geq 0$ is the Lagrange multiplier on transfer non-negativity. In principle we should

[17] It is technically convenient to separate market clearing for factors in the hegemon's country from those in other countries since the hegemon contracts with all its domestic firms, that is $\mathcal{I}_m \backslash \mathcal{C}_m = \emptyset$.

also include non-negativity constraints on allocations and prices ($x_{ij}, z_{ij}, \ell_{ij}, P \geq 0$). When taking first order conditions for allocations to determine equilibrium tax rates, we focus on cases in which these non-negativity constraints do not bind. Incorporating binding constraints adds terms related to these Lagrange multipliers to the planner's FOCs.[18]

We structure the proof by first deriving expressions for the Lagrange multipliers $\psi_{ij}, \psi^{NC}, \phi$ and then taking FOCs in contract terms. The FOC in z_{ij} for $i \in \mathcal{C}_m$ is

$$-\psi_{ij} = \varepsilon^z_{ij} + \frac{\partial ED^m}{\partial z_{ij}}\phi - \frac{\partial x^{NC}}{\partial z_{ij}}\psi^{NC} \tag{11}$$

where $\varepsilon^z_{ij} = \frac{\partial W_m}{\partial w_m}\sum_{k\in\mathcal{I}_m}\frac{\partial \Pi_k}{\partial z_{ij}} + \frac{\partial u_m(z)}{\partial z_{ij}} + \sum_{k\in\mathcal{C}_m}\eta_k\left[\frac{\partial \Pi_k}{\partial z_{ij}} - \frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}}\right]$. Next, defining $\Psi^{z,NC} = (\mathbb{I} - \frac{\partial x^{NC}}{\partial z^{NC}})^{-1}$, the (block) FOC in z^{NC} is

$$0 = \Psi^{z,NC}\varepsilon^{z,NC} + \Psi^{z,NC}\frac{\partial ED^m}{\partial z^{NC}}\phi + \psi^{NC} \tag{12}$$

Defining $P^m = (p_2, \ldots, p_N, p^\ell_{-m})$ to be the price vector (excluding country m factors and the numeraire), the block FOC in P^m is

$$0 = \varepsilon^{P^m} + \frac{\partial ED^m}{\partial P^m}\phi - \frac{\partial x^{NC}}{\partial P^m}\psi^{NC} \tag{13}$$

where

$$\varepsilon^{P^m} = \frac{\partial W_m}{\partial P^m} + \frac{\partial W_m}{\partial w_m}\frac{\partial w_m}{\partial P^m} + \sum_{i\in\mathcal{C}_m}\eta_i\left[\frac{\partial \Pi_i}{\partial P^m} - \frac{\partial V_i}{\partial P^m}\right] - \sum_{i\in\mathcal{C}_m}\sum_{S\in\Sigma(\overline{\mathcal{S}}'_i)}\Lambda_{iS}\sum_{j\in S}\theta_{ij}\frac{\partial p_j}{\partial P^m}x_{ij}.$$

Equation (7) obtains since by Envelope Theorem $\frac{\partial W_m}{\partial P^m} = -\frac{\partial W_m}{\partial w_m}\sum_{i\in\mathcal{I}}\frac{\partial p_i}{\partial P^m}C_{mi}$ and $\frac{\partial w_m}{\partial P^m} = \sum_{i\in\mathcal{I}_m}[\frac{\partial p_i}{\partial P^m}y_i - \sum_{j\in\mathcal{J}_i}\frac{\partial p_j}{\partial P^m}x_{ij}]$, with $X_{m,j} = \mathbf{1}_{j\in\mathcal{I}_m}y_j - \sum_{i\in\mathcal{I}_m}x_{ij} - C_{mj}$.

[18]Given Inada conditions, it will also push the planner towards using arbitrarily large wedges to prevent use of goods held to corner solutions.

Substituting equation 12 into equation 13 and rearranging,

$$\phi = -\left(\frac{\partial ED^m}{\partial P^m} + \frac{\partial x^{NC}}{\partial P^m}\Psi^{z,NC}\frac{\partial ED^m}{\partial z^{NC}}\right)^{-1}\left(\varepsilon^{P^m} + \frac{\partial x^{NC}}{\partial P^m}\Psi^{z,NC}\varepsilon^{z,NC}\right)$$

Substituting into equation 12,

$$\begin{aligned}\psi^{NC} =& \left[-\mathbb{I} + \Psi^{z,NC}\frac{\partial ED^m}{\partial z^{NC}}\left(\frac{\partial ED^m}{\partial P^m} + \frac{\partial x^{NC}}{\partial P^m}\Psi^{z,NC}\frac{\partial ED^m}{\partial z^{NC}}\right)^{-1}\frac{\partial x^{NC}}{\partial P^m}\right]\Psi^{z,NC}\varepsilon^{z,NC} \\ &+ \Psi^{z,NC}\frac{\partial ED^m}{\partial z^{NC}}\left(\frac{\partial ED^m}{\partial P^m} + \frac{\partial x^{NC}}{\partial P^m}\Psi^{z,NC}\frac{\partial ED^m}{\partial z^{NC}}\right)^{-1}\varepsilon^{P^m}.\end{aligned}$$

Using Proposition 2, $\frac{\partial P^m}{\partial z_{ij}} = -\left[\frac{\partial ED^m}{\partial z_{ij}} + \frac{\partial x^{NC}}{\partial z_{ij}}\Psi^{z,NC}\frac{\partial ED^m}{\partial z^{NC}}\right]\left(\frac{\partial ED^m}{\partial P^m} + \frac{\partial x^{NC}}{\partial P^m}\Psi^{z,NC}\frac{\partial ED^m}{\partial z^{NC}}\right)^{-1}$ (after transposition). Substituting ϕ and ψ^{NC} into equation 11,

$$\psi_{ij} = -\varepsilon^z_{ij} - \frac{dP^m}{dz_{ij}}\varepsilon^{P^m} - \left(\frac{\partial x^{NC}}{\partial z_{ij}} + \frac{dP^m}{dz_{ij}}\frac{\partial x^{NC}}{\partial P^m}\right)\Psi^{z,NC}\varepsilon^{z,NC}.$$

and again using Proposition 2, we have (after transposition)

$$\psi_{ij} = -\varepsilon^z_{ij} - \frac{dP^m}{dz_{ij}}\varepsilon^{P^m} - \frac{dz^{NC}}{dz_{ij}}\varepsilon^{z,NC}. \tag{14}$$

Next, since the hegemon contracts with every domestic firm (i.e., $\partial ED_f/\partial p^\ell_f = 0$) and since factors are internationally immobile (i.e., $\partial x^{NC}/\partial p^\ell_f = 0$), the FOC in factor price p^ℓ_f ($f \in \mathcal{F}_m$) is $0 = \sum_{i\in\mathcal{I}_m}\eta_i\left[\frac{\partial\Pi_i(x_i,\ell_i,\mathcal{J}_i)}{\partial p^\ell_f} - \frac{\partial V_i(\mathcal{S}_i)}{\partial p^\ell_f}\right] + \frac{\partial ED^m}{\partial p^\ell_f}\phi$. Consumer n wealth is unaffected by the factor price owing to market clearing (i.e., $\frac{\partial w_m}{\partial p^f_\ell} = \sum_{i\in\mathcal{I}_m}\ell_{if} - \overline{\ell}_f = 0$), so $\partial ED^m/\partial p^\ell_f = 0$ and therefore

$$0 = \sum_{i\in\mathcal{I}_m}\eta_i\left[\frac{\partial\Pi_i(x_i,\ell_i,\mathcal{J}_i)}{\partial p^\ell_f} - \frac{\partial V_i(\mathcal{S}_i)}{\partial p^\ell_f}\right] = \sum_{i\in\mathcal{I}_m}\eta_i\left[\ell_{if} - \ell^{\text{Outside}}_{if}\right]$$

where the second equality follows by Envelope Theorem, and $\ell^{Outside}_{if}$ is factor usage of a firm that deviates to the outside option.

We are now ready to take FOC with respect to allocations $\{x_i, \ell_i, \overline{T}_i\}_{i\in\mathcal{C}_m}$. To streamline analysis, taking e to be one of the allocations $\{x_i, \ell_i, \overline{T}_i\}_{i\in\mathcal{C}_m}$ so that e is a scalar, we have

$$
\begin{aligned}
&\frac{\partial}{\partial e}\left[ED^m\phi + \sum_{i\in\mathcal{C}_m}\sum_{j\in\mathcal{J}_i}\psi_{ij}\left[z_{ij}-x_{ij}\right] + \left[z^{NC}-x^{NC}\right]\psi^{NC}\right] \\
=&\frac{\partial ED^m}{\partial e}\phi - \sum_{i\in\mathcal{C}_m}\sum_{j\in\mathcal{J}_i}\psi_{ij}\frac{\partial x_{ij}}{\partial e} \\
=&\sum_{i\in\mathcal{C}_m}\sum_{j\in\mathcal{J}_i}\varepsilon^z_{ij}\frac{\partial x_{ij}}{\partial e} + \sum_{i\in\mathcal{C}_m}\sum_{j\in\mathcal{J}_i}\frac{dz^{NC}}{dz_{ij}}\frac{\partial x_{ij}}{\partial e}\varepsilon^{z,NC} + \sum_{i\in\mathcal{C}_m}\sum_{j\in\mathcal{J}_i}\frac{dP^m}{dz_{ij}}\frac{\partial x_{ij}}{\partial e}\varepsilon^{P^m} \\
&-\frac{\partial ED^m}{\partial e}\left(\frac{\partial ED^m}{\partial P^m} + \frac{\partial x^{NC}}{\partial P^m}\Psi^{z,NC}\frac{\partial ED^m}{\partial z^{NC}}\right)^{-1}\frac{\partial x^{NC}}{\partial P^m}\Psi^{z,NC}\varepsilon^{z,NC} \\
&-\frac{\partial ED^m}{\partial e}\left(\frac{\partial ED^m}{\partial P^m} + \frac{\partial x^{NC}}{\partial P^m}\Psi^{z,NC}\frac{\partial ED^m}{\partial z^{NC}}\right)^{-1}\varepsilon^{P^m} \\
=&\sum_{i\in\mathcal{C}_m}\sum_{j\in\mathcal{J}_i}\varepsilon^z_{ij}\frac{\partial x_{ij}}{\partial e} + \frac{dz^{NC}}{de}\varepsilon^{z,NC} + \frac{dP^m}{de}\varepsilon^{P^m}
\end{aligned}
\tag{15}
$$

FOC for ℓ_{if} for a Domestic Firm. The hegemon's FOC for (domestic) ℓ_{if} is (using 15)

$$0 = \frac{\partial W_m}{\partial w_m}\frac{\partial \Pi_i}{\partial \ell_{if}} + \eta_i\frac{\partial \Pi_i}{\partial \ell_{if}} - \kappa_f + \frac{dz^{NC}}{d\ell_{if}}\varepsilon^{z,NC} + \frac{dP^m}{d\ell_{if}}\varepsilon^{P^m}$$

Defining $\mathcal{E}^\ell_{if} = \varepsilon^{zNC}\frac{dz^{*NC}}{d\ell_{if}} + \varepsilon^{P^m}\frac{dP^m}{d\ell_{if}}$, since the firm's problem yields a tax rate $\tau^\ell_{if} = \frac{\partial \Pi_i}{\partial \ell_{if}}$, then we have $(\frac{\partial W_m}{\partial w_m} + \eta_i)\tau^\ell_{if} = -\mathcal{E}^\ell_{if} + \kappa_f$.

FOC for ℓ_{if} for a Foreign Firm. The hegemon's FOC for (foreign) ℓ_{if} is $0 = \eta_i\frac{\partial \Pi_i}{\partial \ell_{if}} + \mathcal{E}^\ell_{if}$, so $\eta_i\tau^\ell_{if} = -\mathcal{E}^\ell_{if}$.

FOC for x_{ij} for a domestic firm. Let $\mathcal{E}_{ij} = \varepsilon^z_{ij} + \varepsilon^{zNC}\frac{dz^{*NC}}{dz_{ij}} + \varepsilon^{P^m}\frac{dP^m}{dz_{ij}}$ and let $\overline{\Lambda}_{ij} = \sum_{S\in\Sigma(\overline{\mathcal{S}}'_i)|j\in S}\Lambda_{iS}$. For a domestic sector, the hegemon's FOC for x_{ij} is (using 15)

$$0 = \frac{\partial W_m}{\partial w_m}\frac{\partial \Pi_i}{\partial x_{ij}} + \eta_i\frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij}\theta_{ij}p_j + \mathcal{E}_{ij}$$

To obtain the implementing taxes, construct the firm nonnegative Lagrange multiplier as $\lambda_{iS} = \frac{\Lambda_{iS}}{\frac{\partial W_m}{\partial w_m}+\eta_i}$. The firm's FOC is therefore $\tau_{ij}(\frac{\partial W_m}{\partial w_m} + \eta_i) = (\frac{\partial W_m}{\partial w_m} + \eta_i)\frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij}\theta_{ij}p_j$, which combined with the planner's FOC yields $\tau_{ij}(\frac{\partial W_m}{\partial w_m} + \eta_i) = -\mathcal{E}_{ij}$.

FOC for x_{ij} for a foreign sector. The hegemon's FOC for (foreign) x_{ij} is $0 = \eta_i \frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij}\theta_{ij}p_j + \mathcal{E}_{ij}$. For a positive constant $\alpha > 0$, we add and subtract $\alpha\frac{\partial \Pi_i}{\partial x_{ij}}$ to obtain $(\eta_i + \alpha)\frac{\partial \Pi_i}{\partial x_{ij}} - \overline{\Lambda}_{ij}\theta_{ij}p_j = -\mathcal{E}_{ij} + \alpha\frac{\partial \Pi_i}{\partial x_{ij}}$. Constructing the nonnegative firm Lagrange multiplier $\lambda_{iS} = \frac{\Lambda_{iS}}{\eta_i+\alpha}$ and combining the firm's FOC with the planner's FOC obtains $\tau_{ij}(\eta_i + \alpha) = -(\mathcal{E}_{ij} - \alpha\frac{\partial \Pi_i}{\partial x_{ij}})$. If $\eta_i > 0$, we set $\alpha = 0$ and obtain $\eta_i\tau_{ij} = -\mathcal{E}_{ij}$. If $\eta_i = 0$, at an interior value of x_{ij} either $\overline{\Lambda}_{ij} = \mathcal{E}_{ij} = 0$ or $\overline{\Lambda}_{ij}, \mathcal{E}_{ij} > 0$. In the former case we can write $\eta_i\tau_{ij} = -\mathcal{E}_{ij}$ trivially. In the latter case, as $\alpha \to 0$ we have $-(\mathcal{E}_{ij} - \alpha\frac{\partial \Pi_i}{\partial x_{ij}}) \to -\mathcal{E}_{ij}$, and so we heuristically represent optimal wedges, taking very small α, by $\eta_i\tau_{ij} = -\mathcal{E}_{ij}$.

FOC for $\overline{T}_i$ for a domestic sector. Holding fixed allocations, a transfer $\overline{T}_i$ for a domestic sector has no impact on excess demand in any market, since it redistributes from country m's firms to country m's consumer. The FOC is $0 = -\eta_i - \overline{\Lambda}_i + \gamma_i$, so that $\overline{T}_i = 0$.

FOC for $\overline{T}_i$ for a foreign sector. Holding fixed allocations, a transfer $\overline{T}_i$ reallocates wealth from consumers in country n to consumers in country m. The FOC is $0 = \frac{\partial W_m}{\partial w_m} - \eta_i - \overline{\Lambda}_i + \Xi_{mn} + \gamma_i$ (for $\Xi_{mn} = \varepsilon^{z,NC}(\frac{dz^{*NC}}{dw_m} - \frac{dz^{*NC}}{dw_n}) + \varepsilon^{P^m}(\frac{dP^m}{dw_m} - \frac{dP^m}{dw_n})$), and the inequality follows from $\gamma_i \geq 0$.

Case (iii): Pressure points on a subset of firms. Suppose $\mathcal{I}_m^p \subset \mathcal{I}_m$ and $\mathcal{D}_m^p \subset \mathcal{D}_m$ (one of which may be empty) with $(\mathcal{I}_m^p \cup \mathcal{D}_m^p) \cap \mathcal{P}(Q) = \emptyset$. As in case (i), $\overline{T}_i = 0$ and $\tau_i = 0$ for $i \in \mathcal{I}_m^p \cup \mathcal{D}_m^p$. Redefine the contractible set as $\mathcal{C}_m^{new} = \mathcal{C}_m \backslash (\mathcal{I}_m^p \cup \mathcal{D}_m^p)$ and $\mathcal{D}_m^{new} = \mathcal{D}_m \backslash \mathcal{D}_m^p$ (one of $\mathcal{D}_m^{new}$ or $\mathcal{C}_m^{new} \cap \mathcal{I}_m$ may be empty). The hegemon's Lagrangian over $\{x_i, \ell_i, \overline{T}_i\}_{i\in\mathcal{C}_m^{new}}, P, z$ is

$$\begin{aligned}\mathcal{L}_m =& W_m\left(p, \sum_{i\in\mathcal{I}_m\backslash\mathcal{I}_m^p} \Pi_i(x_i, \ell_i, \mathcal{J}_i) + \sum_{i\in\mathcal{I}_m^p} V_i(\mathcal{S}_i) + \sum_{f\in\mathcal{F}_m} p_f^\ell \overline{\ell}_f + \sum_{i\in\mathcal{D}_m^{new}} \overline{T}_i\right) + u_m(z)\\ &+ \sum_{i\in\mathcal{C}_m^{new}} \eta_i \left[\Pi_i(x_i, \ell_i, \mathcal{J}_i) - \overline{T}_i + \beta(\nu_i(\mathcal{J}_i) - V_i(\mathcal{S}_i))\right]\\ &+ \sum_{i\in\mathcal{C}_m^{new}} \sum_{S\in\Sigma(\overline{\mathcal{S}}_i')} \Lambda_{iS}\left[\beta\left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i\backslash S)\right] - \sum_{j\in S}\theta_{ij}p_j x_{ij} - \mathbf{1}_{S_i^D\subset S}\overline{T}_i\right]\end{aligned}$$

$$+ ED\phi + \sum_{i\in\mathcal{C}_m^{new}} \sum_{j\in\mathcal{J}_i} \psi_{ij} \left[z_{ij} - x_{ij} \right] + \left[z^{NC} - x^{NC} \right] \psi^{NC} + \sum_{i\in\mathcal{C}_m^{new}} \gamma_i \overline{T}_i$$

Analysis parallels case (ii) and we highlight the differences. We have

$$\varepsilon_{ij}^{z} = \frac{\partial W_m}{\partial w_m} \left[\sum_{k\in\mathcal{I}_m\setminus\mathcal{I}_m^p} \frac{\partial \Pi_k}{\partial z_{ij}} + \sum_{k\in\mathcal{I}_m^p} \frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}} \right] + \frac{\partial u_m(z)}{\partial z_{ij}} + \sum_{k\in\mathcal{C}_m^{new}} \eta_k \left[\frac{\partial \Pi_k}{\partial z_{ij}} - \frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}} \right].$$

Let P^m be defined as before if $\mathcal{I}_m^p = \emptyset$ and otherwise let $P^m = P$. ε^{P^m} is formally defined as before (with $\mathcal{C}_m^{new}$ replacing $\mathcal{C}_m$) under the definition of P^m. The FOCs for x_{ij} and $\overline{T}_i$ are identical to case (ii) up to the new definitions. The FOCs for ℓ_{if} are identical to case (ii) up to the new definitions with ϕ_f in place of κ_f in the case that $\mathcal{I}_m^p = \emptyset$, and is identical with $\kappa_f = 0$ up to the new definitions otherwise.

Proof of Proposition 4. Proposition 1 holds for the global planner by the same argument. The firm Lagrangian and FOCs are same as in the proof of Proposition 3. The global planner's Lagrangian is the same as the hegemon's up to the new objective function, $\sum_{n=1}^{N} \Omega_n [W_n(p, w_n) + u_n(z)]$. Formal analysis proceeds as in the proof of Proposition 3 up to the new objective function. Absent a pressure point on sector i, $\overline{T}_i = 0$ and $\tau_i = 0$. For any sector i located in country n, the same derivations yield input wedges satisfying $(\Omega_n \frac{\partial W_n}{\partial w_n} + \eta_i)\tau_{ij} = -\mathcal{E}_{ij}^p$ (note this sector is valued by n's consumer). The externality vector $\mathcal{E}_{ij}^p$ is formally defined by the same equation, but replacing ε_{ij}^z and ε^{P^m} with:

$$\varepsilon_{ij}^{zp} = \sum_{n=1}^{N} \Omega_n \left[\frac{\partial W_n}{\partial w_n} \frac{\partial w_n}{\partial z_{ij}} + \frac{\partial u_n}{\partial z_{ij}} \right] + \sum_{k\in\mathcal{C}_m} \eta_k \left[\frac{\partial \Pi_k}{\partial z_{ij}} - \frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}} \right]$$

$$\varepsilon^{P^m p} = \sum_{n=1}^{N} \Omega_n \frac{dW_n}{dP^m} + \sum_{i\in\mathcal{C}_m} \left[\eta_i \left[\frac{\partial \Pi_i}{\partial P^m} - \frac{\partial V_i(\mathcal{S}_i)}{\partial P^m} \right] - \sum_{S\in\Sigma(\overline{\mathcal{S}}_i')} \Lambda_{iS} \sum_{j\in S} \theta_{ij} \frac{\partial p_j}{\partial P^m} x_{ij} \right]$$

where $\frac{dW_n}{dP^m} = \frac{\partial W_n}{\partial P^m} + \frac{\partial W_n}{\partial w_n} \frac{\partial w_n}{\partial P^m}$. The spillover Ξ_{mn}^p is defined as before, replacing $\varepsilon_{ij}^z, \varepsilon^{P^m}$ with $\varepsilon_{ij}^{zp}, \varepsilon^{P^m p}$. The condition for no redistributive motive is therefore $\Omega_m \frac{\partial W_m}{\partial w_n} - \Omega_n \frac{\partial W_n}{\partial w_n} + \Xi_{mn}^p = 0$. Finally, the FOC for a transfer $\overline{T}_i$ for a firm in country n is $0 = -\eta_i - \overline{\Lambda}_i + \Omega_m \frac{\partial W_m}{\partial w_n} - \Omega_n \frac{\partial W_n}{\partial w_n} + \Xi_{mn}^p + \gamma_i = -\eta_i - \overline{\Lambda}_i + \gamma_i$, so that $\overline{T}_i = 0$. This completes the proof.

SUPPLEMENT TO "A FRAMEWORK FOR GEOECONOMICS"

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B.1. FURTHER DETAILS ON STRATEGIES AND SPE IN REPEATED GAME

In the main text we presented the equilibrium starting from exogenous continuation values of the stage game $\nu_i(\mathcal{B}_i)$. To build an SPE we conjecture and verify a value function $\mathcal{V}_i(\mathcal{B}_i)$ of firm i in the repeated game that is non-decreasing in $\mathcal{B}_i$, that is $\mathcal{V}_i(\mathcal{B}_i) \leq \mathcal{V}_i(\mathcal{B}_i')$ if $\mathcal{B}_i \subset \mathcal{B}_i'$.[1] We first construct the value function in an equilibrium without a hegemon, and then extend the construction to an equilibrium with a hegemon. Finally, Appendix B.1.0.0.3 helps to clarify notationally the difference between individual firms and sectors.

We complete construction of a subgame perfect equilibrium (SPE) for firm i by constructing the associated value function $\mathcal{V}_i(\mathcal{B}_i)$ at each set $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$. This construction follows an iterative process (Abreu et al. (1990)). Note that in this paper we build an SPE but do not focus on sustaining the best possible SPE. In each step, the value function $\mathcal{V}_i(\mathcal{B}_i)$ is given as a fixed point of the equation

$$\begin{aligned} \mathcal{V}_i(\mathcal{B}_i) \;&=\; \max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{B}_i) + \beta\mathcal{V}_i(\mathcal{B}_i) \\ &s.t. \quad \sum_{j\in S}\theta_{ij}p_j x_{ij} \leq \beta\Big[\mathcal{V}_i(\mathcal{B}_i) - \mathcal{V}_i(\mathcal{B}_i\backslash S)\Big] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i)). \end{aligned}$$

[1] In the SPE that we construct, suppliers that Distrust individual firm i, i.e. $j \notin \mathcal{B}_i$, Reject any positive order. If hypothetically suppliers in $j \notin \mathcal{B}_i$ Accepted a positive order, firm i would still believe that suppliers in j will reject every future order, given $B_{ij} = 0$. Firm i would then Steal from suppliers in j. Hence, suppliers in j Reject the order. For $\theta_{ij} = 0$ this is an assumption given indifference for the suppliers, and otherwise a strict preference.

In this iterative process, the value function constructed in the SPE with no stealing in steps $n = 0, \ldots, N$ is subsequently used as the off-path continuation values of the SPE at step $N+1$, until the final step with $\mathcal{B}_i = \mathcal{J}_i$ is reached.

We do not restrict to renegotation-proof equilibria in the sense of (for example) Farrell and Maskin (1989), Bernheim and Ray (1989), and Abreu et al. (1993). As highlighted by Fudenberg and Tirole (1991) and Mailath and Samuelson (2006), the practical relevance of this restriction is not obvious since it involves players coordinating onto a more efficient equilibrium, but many economic problems focus attention on inefficient equilibria. We do not impose such requirements both because the coordination required appears unlikely in practice and because we do not want to embed a notion of efficiency in an equilibrium concept, given our paper is fundamentally about an inefficient world economy.

B.1.0.0.1. *Iterative Construction.* Recall that individual firms are atomistic. Therefore, a single firm deviating and being excluded by (a subset of) future suppliers does not require recomputing equilibrium prices and aggregates at that off-path point. That firm simply produces facing the same prices and aggregates as all other firms, but with only a subset of the available inputs.

Since f_i is increasing, concave, and satisfies Inada conditions, then defining

$$\overline{v}_i \equiv \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{J}_i)$$

we have $\overline{v}_i < +\infty$. Thus we must have $\mathcal{V}_i(\mathcal{B}_i) \leq \frac{1}{1-\beta}\overline{v}_i$ for all $\mathcal{B}_i$.

That $\mathcal{V}_i(\emptyset) = 0$ follows trivially from $f_i(0, \ell_i, z) = 0$. Consider first an element $\mathcal{B}_i \in \mathcal{S}_i$, so that the continuation value from the stealing action is zero. To construct an SPE, we define for $u \geq 0$ a function $\mathcal{V}_i(\mathcal{B}_i|u)$ as solving

$$\mathcal{V}_i(\mathcal{B}_i|u) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i) + \beta \mathcal{V}_i(\mathcal{B}_i|u) \quad s.t. \quad \sum_{j \in \mathcal{B}_i} \theta_{ij} p_j x_{ij} \leq \beta u. \tag{B.1}$$

Since $\overline{v}_i < +\infty$, we can for any $u \geq 0$ define the unique finite value $v_i(u)$ by

$$v_i(u) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{B}_i) \quad s.t. \quad \sum_{j \in \mathcal{B}_i} \theta_{ij} p_j x_{ij} \leq \beta u$$

Then, $\mathcal{V}_i(\mathcal{B}_i|u) = \frac{1}{1-\beta}v_i(u)$ is the unique solution to equation (B.1). Therefore, there is an SPE without stealing with value $\mathcal{V}_i(\mathcal{B}_i) = u$ if $\frac{1}{1-\beta}v_i(u) = u$. Consider the function $\Delta(u) = \frac{1}{1-\beta}v_i(u) - u$. Zeros of this function provide values in SPEs with no stealing. First, $\Delta(0) \geq 0$ (which is thus an SPE if it holds with equality).[2] There is also a positive SPE: from the Inada condition, $\Delta'(0+) = +\infty$, and hence $\Delta(\epsilon) > 0$ for sufficiently small ϵ (note if $\Delta(0) > 0$ this is trivial). Likewise since $v_i(u) \leq \overline{v}_i$, then $\Delta(u) < 0$ for $u > \frac{1}{1-\beta}\overline{v}_i$. Hence by continuity, there is at least one positive SPE $u > 0$. Finally, since f_i is concave and $\sum_{j\in\mathcal{B}_i}\theta_{ij}p_jx_{ij} \leq u$ describes a convex set, then $v_i(u)$ is increasing and concave in u, and hence $\Delta(u)$ is concave. Therefore, there is exactly one positive value of u.

Next consider the iterative construction. Suppose we have constructed, either as SPEs or with reversion, values for all $\hat{\mathcal{B}}_i \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))\backslash\{\mathcal{B}_i\}$. That is we know the continuation values $\mathcal{V}_i(\mathcal{B}_i\backslash S)$ $\forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ from previously constructed SPEs. We can then use these values to construct $\mathcal{V}_i(\mathcal{B}_i)$ as follows. First, for $u \geq 0$ we define

$$\begin{aligned}\mathcal{V}_i(\mathcal{B}_i|u) = &\max_{x_i,\ell_i}\Pi_i(x_i,\ell_i,\mathcal{B}_i) + \beta\mathcal{V}_i(\mathcal{B}_i|u)\\ &\qquad s.t. \quad \sum_{j\in S}\theta_{ij}p_jx_{ij} \leq \beta\Big[u - \mathcal{V}_i(\mathcal{B}_i\backslash S)\Big] \quad \forall S\in\Sigma(\mathcal{S}_i(\mathcal{B}_i))\end{aligned}$$

Thus defining stage game payoff as

$$v_i(u) = \max_{x_i,\ell_i}\Pi_i(x_i,\ell_i,\mathcal{B}_i) \quad s.t. \quad \sum_{j\in S}\theta_{ij}p_jx_{ij} \leq \beta\Big[u - \mathcal{V}_i(\mathcal{B}_i\backslash S)\Big] \quad \forall S\in\Sigma(\mathcal{S}_i(\mathcal{B}_i))$$

then we have $\mathcal{V}_i(\mathcal{B}_i|u) = \frac{1}{1-\beta}v_i(u)$. We construct the fixed points, if any, of $\frac{1}{1-\beta}v_i(u) = u$. As before, $v_i(u)$ is increasing and concave, and therefore there are at most two positive fixed points (for given continuation values).

If an element $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)$ has no SPE associated with no stealing, then we assume that at the beginning of a period in which firm i faces $\mathcal{B}_i$, the suppliers that Trust firm i automatically update to an element $\hat{\mathcal{B}}_i \in \Sigma(\mathcal{S}_i(\mathcal{B}_i))$ such that $\hat{\mathcal{B}}_i$ results in an SPE with no stealing. As a result, $\mathcal{V}_i(\mathcal{B}_i) = \mathcal{V}_i(\hat{\mathcal{B}}_i)$. That is to say, suppliers understand that if the suppliers that

[2]For example, it holds with equality if $\theta_{ij} > 0$ for all j.

Trust firm i were $\mathcal{B}_i$, the firm would in fact Steal from a subset with probability 1, and therefore suppliers update accordingly. We assume throughout the paper that $\mathcal{B}_i = \mathcal{J}_i$ has an SPE with no stealing.

B.1.0.0.2. *Continuation Value Functions in Hegemon Problem.* The hegemon's optimal contract was also characterized for a given set of continuation value functions ν_i. We now provide the equilibrium consistency conditions for a Markov equilibrium. Consider a set of continuation value functions $\nu = \{\nu_i\}$ for firms. Given these continuation value functions, let (Γ, P, z) be the hegemon's optimal contract, prices, and aggregates when the continuation value functions are ν. Then, (Γ, P, z, ν) is an equilibrium if: (i) $\nu_i(\mathcal{B}_i) = \mathcal{V}_i(\mathcal{B}_i)$ for $\mathcal{B}_i \in \Sigma(\mathcal{S}_i)\backslash\{\mathcal{J}_i\}$; and, (ii) $\nu_i(\mathcal{J}_i) = V_i(\Gamma_i)$. To clarify, ν_i are exogenous continuation value functions, $\mathcal{V}_i$ is the off-path value function in the construction of the SPE, and V_i is the equilibrium on-path value function.

B.1.0.0.3. *Individual Firms versus Sectors.* Our paper somewhat abuses notation by identifying i as both a sector of suppliers and an individual firm within that sector. More completely, we could write that sector i has a unit continuum of firms, indexed by $h \in [0, 1]$, and denote $\mathcal{B}_{ih}$ the set of sectors that "Trust" firm ih (i.e., firm h in sector i). The equilibrium would then be determined by the full collection $\mathcal{B} = \{\mathcal{B}_{ih}\}_{i,h}$ and so we could write value functions accordingly. We abused notation as follows. First, we are studying a Markov equilibrium in which all firms will be Trusted on the equilibrium path. In our Nash structure, firm ih chooses its behavior taking as given that all other firms will remain Trusted. If it goes off the equilibrium path by Stealing, it becomes Distrusted by certain sector(s). Since firm ih is infinitesimal, the equilibrium prices P and aggregates z^* do not change once it goes off the equilibrium path (since every other firm, including others within its sector, have chosen not to Steal). Since (P, z^*) are not affected by firm ih's choice, we can write firm ih's value function as $\mathcal{V}_{ih}(\mathcal{B}_{ih})$, leaving implicit the dependence on (the path of) (P, z^*). And since all firms in sector i are identical and we are studying a symmetric equilibrium, we further abuse notation by dropping the h subscript.

B.2. EXTENDING THE FRAMEWORK: HEGEMONIC COMPETITION FOR DOMINANCE

We now consider the possibility that multiple countries are hegemons. For simplicity, we focus on the case in which two countries, m_1 and m_2, are hegemons. To streamline analysis, we focus on competition over transfers, and assume constant prices and no z-externalities (Definitions 1 and 2).

Each hegemon offers a contract as described in Section 3, taking as given the contract offered by the other hegemon. As usual, we begin by taking as given continuation value functions ν_i of firms.

Competition Setup. Let $\mathcal{C} = \mathcal{C}_{m_1} \cup \mathcal{C}_{m_2}$ be the set of firms that contract with at least one hegemon. Hegemon $m \in \{m_1, m_2\}$ offers a contract $\{\Gamma_i^m\}_{i \in \mathcal{C}_m}$, where $\Gamma_i^m \equiv \{\mathcal{S}_i'^m, \mathcal{T}_i^m, \tau_i^m\}_{i \in \mathcal{C}_m}$ denotes the contract offered to firm $i \in \mathcal{C}_m$. As in Section 3, the joint threat $\mathcal{S}_i'$ must be feasible. In the analysis that follows, it will be notationally convenient to designate a hypothetical trivial contract $\Gamma_i^m = \{\mathcal{S}_i, 0, 0\}$ offered by hegemon m to firms $i \in \mathcal{C} \backslash \mathcal{C}_m$. This reduces cumbersome notation of tracking which firms are offered one or two contracts by ensuring all firms in $\mathcal{C}$ are offered two contracts (one of which may be trivial, purely hypothetical, and equivalent to their outside option). We let $\Gamma^m = \{\mathcal{S}_i'^m, \mathcal{T}_i, \tau_i^m\}_{i \in \mathcal{C}}$ be hegemon m's contract, including trivial contracts offered to firms $i \in \mathcal{C} \backslash \mathcal{C}_m$.

Firm i faces revenue-neutral wedges and transfers from both hegemons that are added together when both contracts are accepted.[3] Anticipating that a best response to hegemon $-m$ setting $\tau_i^{-m} = 0$ is for hegemon m to set $\tau_i^m = 0$, we will solve the model assuming all wedges to be zero, and then verify that neither hegemon has an incentive to deviate to nonzero wedges. Therefore, we write the contract $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i^{m_1} + \mathcal{T}_i^{m_2}, 0\}$ as the combined contract when firms accept both contracts.

The joint threat $\mathcal{S}_i'$ arising when firm i accepts both contracts is constructed by taking the union of joint trigger sets and applying Lemma 1 (see the proof of Lemma 1 for details on triggers). Here we detail the special case where both hegemons offer maximal joint threats, as indeed they will in equilibrium. Recall that $S_i^{Dm} = \bigcup_{S \in \mathcal{S}_i^{Dm}} S$ and $\overline{\mathcal{S}}_i'^m = \{S_i^{Dm}\} \cup (\mathcal{S}_i \backslash \mathcal{S}_i^D)$, where we define $S_i^{Dm} = \emptyset$ if $i \notin \mathcal{C}_m$. Then, maximal (combined) joint threats,

[3] Each hegemon takes as given the other hegemon's equilibrium rebates when both contracts are accepted. If firm i chooses to only accept one contract, equilibrium rebates by the hegemon whose contract is accepted are those that maintain revenue neutrality under the single contract, while there are no rebates by the hegemon whose contract was rejected. If neither contract is accepted, there are no rebates.

$\overline{S}'_i$, is given by

$$\overline{S}'_i = (\mathcal{S}_i \backslash (\mathcal{S}_i^{Dm_1} \cup \mathcal{S}_i^{Dm_2})) \cup \mathcal{X}_i, \quad \mathcal{X}_i = \begin{cases} \{S_i^{Dm_1}, S_i^{Dm_2}\} & S_i^{Dm_1} \cap S_i^{Dm_2} = \emptyset \\ \{S_i^{Dm_1} \cup S_i^{Dm_2}\} & \text{otherwise} \end{cases} \tag{B.2}$$

Intuitively, $\overline{\mathcal{S}}'_i$ combines both hegemon's maximal joint threats into a single maximal joint threat if the two have any common threats. If there are no common threats, the two hegemon's maximal joint threats are separate actions within $\overline{\mathcal{S}}'_i$.

Finally, we define the participation constraints of all firms. In particular, hegemon m's contract is accepted by firm i if

$$\max\{V_i(\Gamma_i), V_i(\Gamma_i^m)\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\} \tag{B.3}$$

Both contracts are accepted by firm i if

$$V_i(\Gamma_i) \geq \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}. \tag{B.4}$$

Existence of an Equilibrium. We show existence of an equilibrium in which both hegemons offer maximal joint threats, and both hegemons' contracts are accepted. We then discuss how competition shapes the transfers extracted.

The model with two hegemons has to account for the fact that if hegemon m's contract is rejected by firm i, then hegemon m can no longer use firm i in joint threats.[4] This is important because a best response of hegemon m to a contract Γ^{-m} might involve offering a contract to firm i that leads firm i to reject the contract of hegemon $-m$. To make progress, we restrict the form of the network structure as follows. Let $\mathcal{P} = \{i \in \mathcal{C} \mid V_i(\overline{\mathcal{S}}'_i) > V_i(\mathcal{S}_i)\}$ denote the set of firms for which the two hegemons can, possibly only jointly, generate a pressure point. Formally, define hegemon pressure points as **isolated** if: $i \in \mathcal{P} \Rightarrow \mathcal{J}_i \cap \mathcal{P} = \emptyset$. This definition implies that if the two hegemons can generate a pressure point on i, then the two hegemons cannot generate a pressure point on any firm $j \in \mathcal{J}_i$ that is immediately upstream from i. It ensures that two firms with pressure points from the set of hegemons

[4] This was not an issue in the model with a single hegemon because that hegemon always ensured its contract satisfied the participation constraint.

they contract with are not directly linked to one another. Using this condition, we prove that an equilibrium exists in which both hegemons offer maximal joint threats with no wedges.

PROPOSITION 5: *Suppose that hegemon pressure points are isolated. An equilibrium of the model with competition exists in which each hegemon m offers a contract featuring maximal joint threats and no wedges, $\Gamma_i^m = \{\overline{\mathcal{S}}_i'^m, \overline{T}_i^{m*}, 0\}$, to each $i \in \mathcal{C}_m$. Transfers from all firms $i \notin \mathcal{P}$ are zero. Each firm $i \in \mathcal{C}$ accepts the contract(s) it is offered.*

The proof of Proposition 5 proceeds by constructing transfers $\overline{T}_i^{m*}$ such that each contract Γ_i^m is a best response to contract Γ_i^{-m}, and such that both contracts are accepted, that is $V_i(\Gamma_i) \geq \max\{V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2}), V_i(\mathcal{S}_i)\}$. The transfers extracted by each hegemon from a foreign firm $i \notin \mathcal{I}_{m_1} \cup \mathcal{I}_{m_2}$ depend on the degree to which they can provide different threats. In the limit where hegemon threats have no overlap, $\mathcal{S}_i^{Dm_1} \cap \mathcal{S}_i^{Dm_2} = \emptyset$, competition is limited because each hegemon offers a different set of threats. By contrast when threats have full overlap, $\mathcal{S}_i^{Dm_1} = \mathcal{S}_i^{Dm_2}$, the two hegemons offer the same set of threats, and so bid each other down to zero transfers, $\overline{T}_i^m = 0$. In this case, firms receive full surplus from the relationships. This result is reminiscent of the Bertrand paradox, in which two firms competing on prices bid each other down to the perfect competition price. This outcome is also efficient ex post, since all joint threats are supplied and no transfers are extracted.

For a firm that is domestic to hegemon m, that is $i \in \mathcal{I}_m$, it remains optimal for hegemon m to demand no transfers, $\overline{T}_i^{m*} = 0$. Hegemon $-m$ then extracts the largest transfer that leaves firm i indifferent between accepting both contracts and accepting only that of hegemon m: $V_i(\overline{S}_i', \overline{T}_i^{-m*}) = V_i(\overline{\mathcal{S}}_i'^m)$. Thus the joint threats that the firm's own hegemon can provide become that firm's outside option, to which that firm is held by the other hegemon.

B.2.0.1. *Proof of Proposition 5*

Given constant prices and no z externalities (Definitions 1 and 2), the objective function of hegemon m is to maximize its country's wealth level, $w_m = \sum_{i \in \mathcal{I}_m} \Pi_i(\Gamma_i) + \sum_{i \in \mathcal{D}_m} \sum_j T_{ij}$. We assume that $\tau_i = 0$ for both hegemons, and then verify that neither hegemon has an incentive to deviate.

Given hegemons do not have a pressure point on firm $i \notin \mathcal{P}$, both hegemons must offer a trivial contract $\Gamma_i^m = \{\mathcal{S}_i, 0, 0\}$ to such firms to avoid having their contract rejected. Since all firms $i \notin \mathcal{P}$ therefore trivially accept the contracts they are offered, given isolated pressure points then the decision problem of each hegemon becomes separable across sectors $i \in \mathcal{P}$. This is due not only to separability of the objective function, but also because the joint threat remains feasible even if some firms in $\mathcal{P}$ reject hegemon m's contract, given that every firm $i \in \mathcal{P}$ has $\mathcal{J}_i \cap \mathcal{P} = \emptyset$ (i.e., $S_i^D \subset \mathcal{J}_i \backslash \mathcal{P}$).

We begin by providing the analog of Proposition 1: both hegemons offer contracts featuring maximal joint threats to all firms $i \in \mathcal{P}$.

LEMMA 2: *Fix a contract Γ^{-m} of hegemon $-m$. Then for all $i \in \mathcal{P}$, it is weakly optimal for hegemon m to offer maximal joint threats, $\mathcal{S}_i'^m = \overline{\mathcal{S}}_i'^m$.*

Proof of Lemma 2. Fix a contract $\Gamma_i^{-m} = \{\mathcal{S}_i'^{-m}, \mathcal{T}_i^{-m}, 0\}$ of hegemon $-m$. The proof strategy is to show that if a contract $\Gamma_i^m \equiv \{\mathcal{S}_i'^m, \mathcal{T}_i^m, 0\}$ is accepted by firm i, then the contract $\Gamma_i^{m\prime} = \{\overline{\mathcal{S}}_i'^m, \mathcal{T}_i^m, 0\}$ is also accepted by firm i. Let $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i^m + \mathcal{T}_i^{-m}, 0\}$ be the joint contract if hegemon m offers Γ_i^m, and $\Gamma_i' = \{\mathcal{S}_i'', \mathcal{T}_i^m + \mathcal{T}_i^{-m}, 0\}$ be the joint contract if hegemon m offers $\Gamma_i^{m\prime}$. Since the contract Γ_i^m is accepted by firm i, then

$$\max\{V_i(\Gamma_i), V_i(\Gamma_i^m)\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}.$$

Since $\overline{\mathcal{S}}_i'^m$ is a joint threat of $\mathcal{S}_i'^m$, then $\mathcal{S}_i''$ is a joint threat of $\mathcal{S}_i'$. Therefore, $V_i(\Gamma_i^{m\prime}) \geq V_i(\Gamma_i^m)$ and $V_i(\Gamma_i') \geq V_i(\Gamma_i)$. Therefore,

$$\max\{V_i(\Gamma_i'), V_i(\Gamma_i^{m\prime})\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\},$$

and hence contract $\Gamma_i^{m\prime}$ is also accepted by firm i. Finally, firm i is weakly better off (which is valued by hegemon m if firm i is domestic). Thus, maximal joint threats is a weak best response, concluding the proof of Lemma 2.

From Lemma 2, $\mathcal{S}_i'^m = \overline{\mathcal{S}}_i'^m$ is a best response to any contract Γ_i^{-m}, and therefore all transfers of m appear under the joint threat. Thus we will focus on the total transfer $\overline{T}_i^m$ for firms $i \in \mathcal{P}$. The optimal contract for firm i is characterized by Propositions 3 and 8 if only one hegemon contracts with i, so assume $i \in \mathcal{C}_{m_1} \cap \mathcal{C}_{m_2}$.[5]

[5]In the language of Online Appendix B.3.9, firm i is a neutral firm.

Let $\Gamma_i^m = \{\overline{\mathcal{S}}_i'^m, \overline{T}_i^m, 0\}$ be a candidate optimal contract of hegemon m, and let $\Gamma_i = \{\overline{\mathcal{S}}_i', \overline{T}_i^{m_1}, \overline{T}_i^{m_2}, 0\}$ be the joint contract. We split the remainder of the proof between domestic and foreign firms.

Foreign Firms. Let $i \in \mathcal{P} \backslash (\mathcal{I}_{m_1} \cup \mathcal{I}_{m_2})$ be a firm foreign to both hegemons. We begin with the following intermediate result.

LEMMA 3: $(\Gamma_i^m, \Gamma_i^{-m})$ *is part of an equilibrium in which firm i accepts both contracts if and only if one of the following holds:*

1. *Firm i is held to its outside option, with*

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\} \tag{B.5}$$

2. *Firm i exceeds its outside option, with*

$$V_i(\Gamma_i) = V_i(\Gamma_i^{m_1}) = V_i(\Gamma_i^{m_2}) > V_i(\mathcal{S}_i) \tag{B.6}$$

Proof of Lemma 3. Supposing that both contracts are accepted, then

$$V_i(\Gamma_i) \geq \max\{V_i(\mathcal{S}_i), V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2})\}.$$

First, suppose that firm i is held to its outside option, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$. Then,

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\}.$$

Finally, suppose that we have two contracts that satisfy equation B.5. Then, if either hegemon increased its transfer, the firm would reject both contracts and revert to the outside option. Likewise, a hegemon that lowered its transfer would have its contract accepted, but be strictly worse off. Therefore we have an equilibrium.

Suppose, second, that firm i exceeds its outside option, $V_i(\Gamma_i) > V_i(\mathcal{S}_i)$. By way of contradiction, suppose that $V_i(\Gamma_i) > \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m})\}$. Then, hegemon m could increase its transfer without its contract being rejected, and so be strictly better off. Therefore, $V_i(\Gamma_i) = \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m})\}$. Again by way of contradiction suppose that (without loss) $V_i(\Gamma_i) = V_i(\Gamma_i^m) > V_i(\Gamma_i^{-m})$. Then again, hegemon m could increase its transfer without its contract being rejected, and so be strictly better off. Therefore,

$$V_i(\Gamma_i) = V_i(\Gamma_i^{m_1}) = V_i(\Gamma_i^{m_2}) > V_i(\mathcal{S}_i).$$

Finally, supposing equation B.6 holds, then if either hegemon increased its transfer, the firm would reject its contract and accept only that of the other hegemon. A hegemon that lowered its transfer would have its contract accepted, but be strictly worse off. Therefore, neither hegemon deviates, and we have an equilibrium, concluding the proof of Lemma 3.

We use Lemma 3 to construct an equilibrium. Since $i \in \mathcal{P}$, $V_i(\overline{\mathcal{S}}_i') > V_i(\mathcal{S}_i)$. Without loss of generality, let $V_i(\overline{\mathcal{S}}_i'^m) \geq V_i(\overline{\mathcal{S}}_i'^{-m})$. We begin by constructing the minimal transfer $t_0^m \geq 0$ such that $V_i(\overline{\mathcal{S}}_i'^m, t_0^m) = V_i(\overline{\mathcal{S}}_i'^{-m}, 0)$. Since $\overline{S}_i'$ is a joint threat of $\overline{\mathcal{S}}_i'^m$, then $V_i(\overline{\mathcal{S}}_i', t_0^m) \geq V_i(\overline{\mathcal{S}}_i'^m, t_0^m)$. If $V_i(\overline{\mathcal{S}}_i', t_0^m) = V_i(\overline{\mathcal{S}}_i'^m, t_0^m)$, then $V_i(\Gamma_i) = V_i(\Gamma_i^m) = V_i(\Gamma_i^{-m})$, and hence either equation (B.5) or (B.6) is satisfied and we have an equilibrium.

Suppose instead $V_i(\overline{\mathcal{S}}_i', t_0^m) > V_i(\overline{\mathcal{S}}_i'^m, t_0^m)$. Then, we construct a function $t^{-m}(t)$ by

$$V_i(\overline{\mathcal{S}}_i'^m, t_0^m + t) = V_i(\overline{\mathcal{S}}_i'^{-m}, t^{-m}(t)).$$

We can construct this function from $t = 0$ to $t = \bar{t}$, where $\bar{t}$ solves $V_i(\overline{\mathcal{S}}_i'^m, t_0^m + t) = V_i(\mathcal{S}_i)$ (note it is possible for $\bar{t} = 0$).

Suppose first $\exists t^* \in [0, \bar{t}]$ such that

$$V_i(\overline{\mathcal{S}}_i', t_0^m + t^*, t^{-m}(t^*)) = V_i(\overline{\mathcal{S}}_i'^m, t_0^m + t^*).$$

Then, equation (B.6) is satisfied if $t^* < \bar{t}$, and equation (B.5) is satisfied if $t^* = \bar{t}$. Therefore, by Lemma 3 we have found an equilibrium.

Suppose instead that no such t^* exists, and therefore $V_i(\overline{\mathcal{S}}_i', t_0^m + \bar{t}, t^{-m}(\bar{t})) > V_i(\mathcal{S}_i)$. Then, define $\overline{T}_i^m$ and $\overline{T}_i^{-m}$ such that $\overline{T}_i^m \geq t_0^m + \bar{t}$, $\overline{T}_i^{-m} \geq t^{-m}(\bar{t})$, and $V_i(\overline{\mathcal{S}}_i', \overline{T}_i^m, \overline{T}_i^{-m}) = V_i(\mathcal{S}_i)$. Then, equation (B.5) is satisfied, and hence we have found an equilibrium.

Therefore, an equilibrium exists as described, assuming both hegemons impose zero wedges. Observe that imposing nonzero wedges cannot increase the value of its objective, and leads to its contract being (weakly) rejected. Thus, zero wedges is a best response of each hegemon, concluding this portion of the proof.

Domestic Firms. Let $i \in \mathcal{P} \cap \mathcal{I}_m$ be a domestic firm of hegemon m.

LEMMA 4: $(\Gamma_i^m, \Gamma_i^{-m})$ *is part of an equilibrium in which firm* $i \in \mathcal{P} \cap \mathcal{I}_m$ *accepts both contracts if and only if one of the following holds:*

1. *Firm* i *is held to its outside option, with* $\overline{T}_i^m = 0$ *and*

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\} \tag{B.7}$$

2. *Firm* i *exceeds its outside option, with* $\overline{T}_i^m = 0$ *and*

$$V_i(\Gamma_i) = V_i(\Gamma_i^m) \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\} \tag{B.8}$$

Proof of Lemma 4. Supposing that both contracts are accepted, then

$$V_i(\Gamma_i) \geq \max\{V_i(\mathcal{S}_i), V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2})\}.$$

Suppose first that firm i is held to its outside option, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$. Then, since both contracts are accepted, $V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\}$. Moreover, if $\overline{T}_i^m > 0$, hegemon m could reduce its transfer while having its contract accepted, strictly improving welfare. Thus, $\overline{T}_i^m = 0$. Finally, suppose that we have two contracts that satisfy equation B.7 and that $\overline{T}_i^m = 0$. If hegemon $-m$ increased its transfer, then its contract would be rejected. Hegemon m has no incentive to increase its transfer, and so we have an equilibrium.

Suppose, second, that firm i exceeds its outside option, $V_i(\Gamma_i) > V_i(\mathcal{S}_i)$. Suppose, hypothetically, that $V_i(\Gamma_i) > V_i(\Gamma_i^m)$. Then, hegemon $-m$ could increase its transfer without its contract being rejected, and so be strictly better off. Therefore, $V_i(\Gamma_i) = V_i(\Gamma_i^m)$, and therefore $V_i(\Gamma_i) = V_i(\Gamma_i^m) \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}$. If this condition holds, and $\overline{T}_i^m > 0$, then hegemon m could decrease its transfer for its domestic firm without its contract being rejected, and so be strictly better off. Therefore, $\overline{T}_i^m = 0$. Finally, suppose this condition holds and $\overline{T}_i^m = 0$. Then, if hegemon $-m$ increased its transfer, its contract would be rejected. Hegemon m cannot further decrease its transfer. Therefore, neither hegemon deviates, and we have an equilibrium. □

Lemma 4 shows that $\overline{T}_i^m = 0$ in any equilibrium, that is a domestic firm is not charged a transfer by its hegemon. Since $\overline{T}_i^m = 0$, then $V_i(\Gamma_i^{-m}) \leq V_i(\Gamma_i)$. We can construct the transfer of hegemon $-m$ as the solution to $V_i(\overline{\mathcal{S}_i}', \overline{T}_i^{-m}) = V_i(\overline{\mathcal{S}}_i'^m)$. If $V_i(\overline{\mathcal{S}}_i'^m) = V_i(\mathcal{S}_i)$, then equation (B.7) is satisfied and we have an equilibrium. If $V_i(\overline{\mathcal{S}}_i'^m) > V_i(\mathcal{S}_i)$, then equation (B.8) is satisfied and we have an equilibrium. In both cases, zero wedges is part of an optimal policy for both hegemons. Therefore, we have an equilibrium.

This concludes the proof of Proposition 5.

B.3. ADDITIONAL RESULTS AND DERIVATIONS

B.3.1. *Manipulating the Outside Option*

We show how to extend our setup to allow the hegemon to make threats conditional on a firm rejecting the contract. We think of these threats as being a blunt "do what I say or else..." approach to hegemonic power. These threats amount to manipulating the outside option of targeted entities by threatening to cut off access to the inputs controlled by the hegemon if the contract is rejected. We show that in a repeated game these threats are not as powerful as those that instead increase the inside option, such as the joint threats we study in the main text. The intuition is that, for given continuation values, these threats incentivize the target to accept the hegemon's demands by increasing the cost of not doing so. However, in equilibrium, the continuation values are lowered since the same outside option threats will be made in the future. Low continuation values in turn feed back into the current period problem lowering the target incentives to comply with the hegemon's demands. In this sense, such blunt outside option threats are in part self-defeating and threats that increase the inside option are more powerful.

In addition to specifying a joint threat $\mathcal{S}_i'$, transfers $\mathcal{T}_i$, and wedges τ_i, the hegemon can also impose a *punishment* $\mathcal{P}_i$, which is a restriction that the firm permanently loses access to inputs contained in $\mathcal{P}_i \subset \mathcal{S}_i$ if it rejects the hegemon's contract.[6] As with feasibility of joint threats, it is feasible for the hegemon to use $S \in \mathcal{S}_i$ to form a punishment if $\exists j \in S$ such that $j \in \mathcal{C}_m$.[7] Let $\mathcal{B}_i(\mathcal{P}_i) = \mathcal{J}_i \backslash (\bigcup_{S \in \mathcal{P}_i} S)$ be the set of retained inputs given punishment

[6] It is also straight-forward to allow for the punishment to entail only loss of access for the current period.

[7] For brevity we state punishments as losing access to elements of $\mathcal{S}_i$, but it is easy to extend to allow for punishments of losing access to specific inputs (while potentially retaining access to other inputs of $S \in \mathcal{S}_i$).

$\mathcal{P}_i$. The outside option of firm i is therefore

$$
\begin{aligned}
V_i^o(\mathcal{P}_i) \;&=\; \max_{x_i,\ell_i} \Pi_i(x_i,\ell_i,\mathcal{B}_i(\mathcal{P}_i)) + \beta \nu_i(B_i(\mathcal{P}_i)) \\
&\qquad s.t. \quad \sum_{j\in S} \theta_{ij} p_j x_{ij} \leq \beta \Big[\nu_i(B_i(\mathcal{P}_i)) - \nu_i(\mathcal{B}_i(\mathcal{P}_i)\backslash S) \Big] \quad \forall S \in \Sigma(\mathcal{S}_i(\mathcal{B}_i(\mathcal{P}_i))).
\end{aligned}
$$

which leaves implicit that $x_{ij} = 0$ for $j \notin \mathcal{B}_i(\mathcal{P}_i)$. The participation constraint is therefore

$$V_i(\Gamma_i) \geq V_i^o(\mathcal{P}_i).$$

Similar in spirit to Proposition 1, the optimal punishment is the one that minimizes the outside option, and we denote this minimal value $\underline{V}_i^o$.[8] The optimal contract can be analyzed as in the baseline model but with with $\underline{V}_i^o$ replacing $V_i(\mathcal{S}_i)$ in the participation constraint.

A one-off threatened punishment at date t is (weakly) effective for the hegemon taking continuation values as given. However, in a Markov equilibrium the threat would be made in every period. For exposition, suppose the participation constraint binds. Lowering the future value of retaining access to the hegemon's inputs therefore lowers the continuation value $\nu_i(\mathcal{J}_i) = V_i^o(\mathcal{P}_i)$, which tightens the participation constraint this period. Therefore, threats of punishment for contract rejection are partially self-defeating. In contrast, with joint threats the hegemon first increases the inside option to $V_i(\overline{\mathcal{S}}_i')$, and then uses demands for costly actions to lower it to $V_i(\mathcal{S}_i)$ up to the participation constraint. Moving the inside option up has a positive feedback into the problem since it makes it less attractive in equilibrium not to deal with the hegemon in the future. This is so especially for friendly firms that might be left some surplus in equilibrium. Even for neutral and unfriendly targets, threats that increase the inside option are particularly powerful for the hegemon by providing a positive feedback loop with future continuation values.

[8]That is, $\underline{V}_i^o = \min_{\mathcal{P}_i \subset \mathcal{S}_i | \mathcal{P}_i \text{ is feasible}} V_i^o(\mathcal{P}_i)$. Although the economically intuitive case is the one in which the outside option is minimized with the threat to cut off as many inputs as possible, translating optimality of a maximal utility punishment into cutting off the most varieties requires that V_i^o be a nonincreasing function, which cannot be guaranteed in general due to incentive problems.

B.3.2. *Specializing the Model to Nested CES Production Functions*

Assume there are only two periods and that in the second period there are no incentive problems (i.e. all θ's are set to zero in the second period). Each sector uses a two-tier nested constant elasticity of substitution (CES) production function. Firm i produces using input vector x_i with length $|\mathcal{J}_i|$ and, for simplicity, no local factors. The inputs are partitioned into bundles, where $\tilde{x} \in \tilde{X}_i$ denotes the varieties of inputs used in a given bundle, and $\tilde{X}_i$ is the set of all bundles. We assume each input only enters one bundle. The production function is then given by:

$$f_i(x_i) = \left(\sum_{\tilde{x} \in \tilde{X}_i} \tilde{\alpha}_{i\tilde{x}} \left(\sum_{j \in \tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}} \right)^{\frac{\rho_i}{\chi_{i\tilde{x}}}} \right)^{\frac{\xi_i}{\rho_i}} . \tag{B.9}$$

We allow CES parameters $\chi_{i\tilde{x}}$ to vary across bundles. At time zero, the loss in continuation value arising from stealing variety k is given by:

$$\log \nu_i(\mathcal{J}_i) - \log \nu_i(\mathcal{J}_i \backslash \{k\}) = -\frac{\xi_i}{1-\xi_i} \frac{1-\rho_i}{\rho_i} \log \left[1 - \Omega_{i\tilde{x}_k} \left(1 - \left(1 - \omega_{ik} \right)^{\frac{1-\chi_{i\tilde{x}_k}}{\chi_{i\tilde{x}_k}} \frac{\rho_i}{1-\rho_i}} \right) \right], \tag{B.10}$$

where $\Omega_{i\tilde{x}_k}$ is the expenditure share of firm i on the bundle that contains input k denoted by $\tilde{x}_k$, and ω_{ik} is the expenditure share on input k within that bundle. We provide a step-by-step derivation of this equation below and definitions of the expenditure shares, but we first provide some intuition.

All else equal, losing varieties with bigger expenditure shares leads to a greater loss. Intuitively, losing inputs that are cheap (low p_k) or are technologically a large fraction of production (i.e. high related α's) increases the loss. Losing a variety k is more costly the closer the production function is to constant returns to scale $\xi \uparrow 1$ because a more scalable production suffers more from one of its inputs being constrained at zero.

To understand the role of substitutability within and across buckets, consider the specific bucket that contains variety k. Fix a within-bundle expenditure share ω_{ik}. If that bucket has a parameter $\chi_{i\tilde{x}_k} \leq 0$ (i.e. more complementarity than Cobb-Douglas), then losing variety k amounts to the same as losing the entire bucket. Intuitively, this occurs because the absence of input k makes strictly positive production from that bucket impossible. For parameters

$\chi_{i\tilde{x}_k} > 0$, the loss decreases the more the varieties are substitutable. A similar logic applies across baskets and is governed by the parameter ρ_i.

This example illustrates the role of "alternatives" in diminishing the value of threats to shut off a firm from a particular input. Intuitively, the existence of closely substitutable inputs or the fact that a particular input accounts for a small expenditure share, decreases this input’s strategic value in threats.

Derivation of Equation (B.10). Starting from the nested CES production function in equation (B.9), we first solve the expenditure minimization problem associated with bundle $\tilde{x}$, given by

$$\min \sum_{j\in\tilde{x}} p_j x_{ij} \quad s.t. \quad \left(\sum_{j\in\tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}}\right)^{\frac{1}{\chi_{i\tilde{x}}}} \geq \overline{x}$$

Letting λ denote the Lagrange multiplier on the production constraint, the FOCs are

$$0 = p_j - \lambda\left(\sum_{j\in\tilde{x}} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}}\right)^{\frac{1}{\chi_{i\tilde{x}}}-1} \alpha_{ij} x_{ij}^{\chi_{i\tilde{x}}-1} \Rightarrow \left(\frac{p_j}{\alpha_{ij}}\frac{\alpha_{ik}}{p_k}\right)^{\frac{1}{1-\chi_{i\tilde{x}}}} x_{ij} = x_{ik}$$

Substituting into the production constraint yields

$$\overline{x} = \left(\sum_{j\in\tilde{x}} \alpha_{ij}^{\frac{1}{1-\chi_{i\tilde{x}}}} p_j^{-\frac{\chi_{i\tilde{x}}}{1-\chi_{i\tilde{x}}}}\right)^{\frac{1}{\chi_{i\tilde{x}}}} \left(\frac{p_k}{\alpha_{ik}}\right)^{\frac{1}{1-\chi_{i\tilde{x}}}} x_{ik}.$$

Therefore, the expenditure function is $e_i(p,\overline{x}) = P_{i\tilde{x}}\overline{x}$ where the price index of the consumption basket $\tilde{x}$ is $P_{i\tilde{x}} = \left(\sum_{j\in\tilde{x}} \alpha_{ij}^{\frac{1}{1-\chi_{i\tilde{x}}}} p_j^{-\frac{\chi_{i\tilde{x}}}{1-\chi_{i\tilde{x}}}}\right)^{-\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}}$.

The optimization problem over bundles, abusing notation by using $\tilde{x}$ as aggregate consumption of bundle $\tilde{x}$, is

$$\max p_i \left(\sum_{\tilde{x}\in\tilde{X}_i} \alpha_{i\tilde{x}} \tilde{x}^{\rho_i}\right)^{\frac{\xi_i}{\rho_i}} - \sum_{\tilde{x}\in\tilde{X}_i} P_{i\tilde{x}}\tilde{x}$$

B.16

This yields FOCs

$$p_i \left(\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}} \tilde{x}^{\rho_i} \right)^{\frac{\xi_i}{\rho_i} - 1} \alpha_{i\tilde{x}} \xi_i \tilde{x}^{\rho_i - 1} = P_{i\tilde{x}} \Rightarrow \tilde{x} = \left(\frac{P_{i\tilde{x}_k}}{P_{i\tilde{x}}} \frac{\alpha_{i\tilde{x}}}{\alpha_{i\tilde{x}_k}} \right)^{\frac{1}{1-\rho_i}} \tilde{x}_k$$

$$\tilde{x}_k = \left(\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}} \right)^{\frac{\xi_i - \rho_i}{\rho_i (1-\xi_i)}} \left(\frac{\alpha_{i\tilde{x}_k}}{P_{i\tilde{x}_k}} \right)^{\frac{1}{1-\rho_i}} (p_i \xi_i)^{\frac{1}{1-\xi_i}}.$$

Therefore, expenditures are

$$\sum_{\tilde{x}} P_{i\tilde{x}} \tilde{x} = (p_i \xi_i)^{\frac{1}{1-\xi_i}} \left(\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}} \right)^{\frac{\xi_i}{\rho_i} \frac{1-\rho_i}{1-\xi_i}}$$

while revenues from production are

$$p_i \left(\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}} \tilde{x}^{\rho_i} \right)^{\frac{\xi_i}{\rho_i}} = p_i (p_i \xi_i)^{\frac{\xi_i}{1-\xi_i}} \left(\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}} \right)^{\frac{\xi_i}{\rho_i} \frac{1-\rho_i}{1-\xi_i}}.$$

If firm i has all inputs left, we therefore have

$$\nu_i(\mathcal{J}_i) = p_i^{\frac{1}{1-\xi_i}} \left[(\xi_i)^{\frac{\xi_i}{1-\xi_i}} - (\xi_i)^{\frac{1}{1-\xi_i}} \right] \left(\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}^{-\frac{\rho_i}{1-\rho_i}} \right)^{\frac{\xi_i}{\rho_i} \frac{1-\rho_i}{1-\xi_i}}.$$

Off-path, the price index is $P_{i\tilde{x}}(\mathcal{B}_i) = \left(\sum_{j \in \tilde{x} \cap \mathcal{B}_i} \alpha_{ij}^{\frac{1}{1-\overline{\chi}_{i\tilde{x}}}} p_j^{-\frac{\overline{\chi}_{i\tilde{x}}}{1-\overline{\chi}_{i\tilde{x}}}} \right)^{-\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}}$ if firm i has inputs $\mathcal{B}_i$ remaining. Therefore,

$$\nu_i(\mathcal{B}_i) = p_i^{\frac{1}{1-\xi_i}} \left[(\xi_i)^{\frac{\xi_i}{1-\xi_i}} - (\xi_i)^{\frac{1}{1-\xi_i}} \right] \left(\sum_{\tilde{x} \in \tilde{X}_i} \alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}} P_{i\tilde{x}}(\mathcal{B}_i)^{-\frac{\rho_i}{1-\rho_i}} \right)^{\frac{\xi_i}{\rho_i} \frac{1-\rho_i}{1-\xi_i}}.$$

Therefore, we have

$$\log \nu_i(\mathcal{J}_i) - \log \nu_i(\mathcal{J}_i \backslash \{k\})$$

$$= \frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}\log\left(\frac{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}(\mathcal{J}_i\backslash\{k\})^{-\frac{\rho_i}{1-\rho_i}}}\right)$$

$$= -\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}\log\left(1-\frac{\alpha_{i\tilde{x}_k}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}_k}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}\left[1-\left(\frac{P_{i\tilde{x}_k}(\mathcal{J}_i\backslash\{k\})}{P_{i\tilde{x}_k}(\mathcal{J}_i)}\right)^{-\frac{\rho_i}{1-\rho_i}}\right]\right)$$

$$= -\frac{\xi_i}{\rho_i}\frac{1-\rho_i}{1-\xi_i}\log\left(1-\Omega_{i\tilde{x}_k}\left[1-\left(1-\omega_{ik}\right)^{\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}\frac{\rho_i}{1-\rho_i}}\right]\right)$$

given the definitions of expenditure shares, $\Omega_{i\tilde{x}_k} = \frac{\alpha_{i\tilde{x}_k}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}_k}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}^{\frac{1}{1-\rho_i}}P_{i\tilde{x}}(\mathcal{J}_i)^{-\frac{\rho_i}{1-\rho_i}}}$ and $\omega_{ik} = \frac{\alpha_{ik}^{\frac{1}{1-\chi_{i\tilde{x}_k}}}p_k^{-\frac{\chi_{i\tilde{x}_k}}{1-\chi_{i\tilde{x}_k}}}}{\sum_{j\in\tilde{x}_k}\alpha_{ij}^{\frac{1}{1-\chi_{i\tilde{x}_k}}}p_j^{-\frac{\chi_{i\tilde{x}_k}}{1-\chi_{i\tilde{x}_k}}}}$.

In the case of Cobb-Douglas ($\rho=0$),

$$\log\nu_i(\mathcal{B}_i) = \log\left(p_i^{\frac{1}{1-\xi_i}}\left[(\xi_i)^{\frac{\xi_i}{1-\xi_i}}-(\xi_i)^{\frac{1}{1-\xi_i}}\right]\right)-\frac{\xi_i}{1-\xi_i}\sum_{\tilde{x}\in\tilde{X}_i}\frac{\alpha_{i\tilde{x}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}}\log P_{i\tilde{x}}(\mathcal{B}_i)$$

For the illustrative empirical work below in Appendix B.3.3, we assume that each nested basket has exactly one hegemon input $|\mathcal{I}_m\cap\tilde{x}|=1$, and then

$$\sum_{k\in\mathcal{I}_m}[\log\nu_i(\mathcal{J}_i)-\log\nu_i(\mathcal{J}_i\backslash\{k\})] = \log\nu_i(\mathcal{J}_i)-\log\nu_i(\mathcal{J}_i\backslash\mathcal{I}_m).$$

Therefore, we can recover $\log\nu_i(\mathcal{J}_i)-\log\nu_i(\mathcal{J}_i\backslash\mathcal{I}_m)$ by adding up

$$\log\nu_i(\mathcal{J}_i)-\log\nu_i(\mathcal{J}_i\backslash\{k\}) = \frac{\xi_i}{1-\xi_i}\frac{\alpha_{i\tilde{x}}}{\sum_{\tilde{x}\in\tilde{X}_i}\alpha_{i\tilde{x}}}\left[\log P_{i\tilde{x}}(\mathcal{J}_i\backslash\{k\})-\log P_{i\tilde{x}}(\mathcal{J}_i)\right]$$

B.18

$$
\begin{aligned}
&= \frac{\xi_i}{1-\xi_i}\Omega_{i\tilde{x}_k} \log\left(1-\omega_{ik}\right)^{-\frac{1-\chi_{i\tilde{x}}}{\chi_{i\tilde{x}}}} \\
&= -\frac{\xi_i}{1-\xi_i}\frac{1}{\sigma_{i\tilde{x}}-1}\Omega_{i\tilde{x}_k} \log\left(1-\omega_{ik}\right)
\end{aligned}
$$

which uses that $\sigma_{i\tilde{x}} = \frac{1}{1-\chi_{i\tilde{x}}}$.

B.3.3. *Measuring The Loss in Continuation Value*

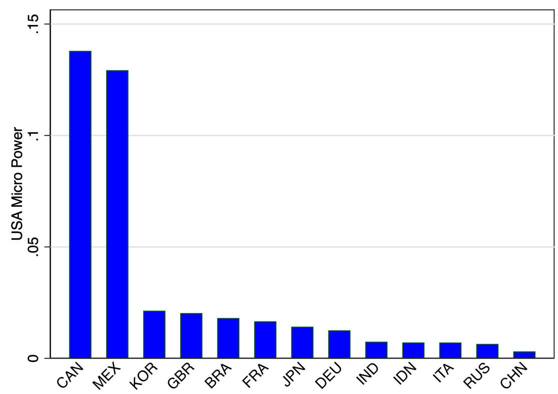
Suppose country n has a representative final goods producer that produces out of domestic and foreign inputs using a nested CES production function as in Appendix B.3.2. From the previous subsection, with a Cobb-Douglas outer nest, we can write the loss to country n's final goods producer from losing access to all the hegemon's goods as

$$
\tilde{\nu}_n \equiv \frac{\xi}{1-\xi}\sum_{k\in\mathcal{I}_m}\left(\log \nu_n(\mathcal{J}_n) - \log \nu_n(\mathcal{J}_n\backslash\{k\})\right) \tag{B.11}
$$

To measure this loss, we start by considering the loss from losing all $k \in \mathcal{I}_m$, that is all firms based in country m, and abstract from threats using firms based outside of m. We set the returns to scale parameter ξ = 0.8, which is within the range of estimates in the literature. We use trade and production data from the OECD Inter-Country Input-Output (ICIO) tables (OECD (2023, http://oe.cd/icio)). We use sectoral elasticities from Fontagné et al. (2022) and restrict our loss calculations to the ICIO goods sectors for which elasticities estimates are available.

Our exercise is in the spirit of Hirschman (1945), evaluating which countries a hegemon has power over based on the nature of the bilateral trade relationship. We focus here on the case where only the hegemon can cut off goods. For every country n, we estimate the loss in continuation value that the United States can cause as in equation B.11. We the estimate losses for the year 2019. While the ICIO data is available until 2020, we use 2019 to avoid effects of Covid on the data. In Figure B.1, we see that the United States has the potential to cause much higher losses to neighbors like Canada and Mexico than to China or Russia. Our measure of the loss of continuation value is related to the Hausmann et al. (2024) estimation of the economic costs that the United States and Europe could impose on Russia via export controls in the Baqaee and Farhi (2022) framework. More generally, our measure parallels the sufficient statistics for welfare gains from international trade in Arkolakis et al. (2012) while focusing on the loss of exports from a single country.

FIGURE B.1.—**USA Ability to Induce Continuation Value Losses in Various Countries, 2019**



Notes: This plots the loss in continuation value calculation following Equation B.11. Trade and production data OECD ICIO tables and trade elasticities from Fontagné et al. (2022).

B.3.4. *Identifying Pressure Points: A Special Case*

In this appendix, we consider an environment in which firms have separable production and provide a necessary and sufficient condition for identifying pressure points. We start by defining the environment:

DEFINITION 3: *The* ***separable production*** *environment assumes that firms that use intermediate inputs have* $f_i(x_i, \ell_i, z) = \sum_{j\in\mathcal{J}_i} f_{ij}(x_{ij}, z)$.

We assume separable production. We write $\Pi_i(x_i, \mathcal{B}_i) = \sum_{j\in\mathcal{B}_i} \pi_{ij}(x_{ij})$, where $\pi_{ij}(x_{ij}) = p_i f_{ij}(x_{ij}, z) - p_j x_{ij}$. Now, suppose that continuation value ν_i is separable across elements of $\mathcal{S}_i(\mathcal{B}_i)$, that is we can write $\nu_i(\mathcal{B}_i) = \sum_{S\in\mathcal{S}_i(\mathcal{B}_i)} v_i(S)$. Then, the incentive constraint associated with $S \in S_i(\mathcal{B}_i)$ is $\sum_{j\in S} \theta_{ij} p_j x_{ij} \leq \beta v_i(S)$. Therefore, if the incentive constraint holds for $S_1, S_2 \in \mathcal{S}_i(\mathcal{B}_i)$, it also holds for $S_1 \cup S_2$. Thus incentive compatibility with respect to $\mathcal{S}_i(\mathcal{B}_i)$ implies incentive compatibility with respect to $\Sigma(\mathcal{S}_i(\mathcal{B}_i))$. Thus the decision problem of firm i becomes separable over elements of the action set $\mathcal{S}_i(\mathcal{B}_i)$, leading to a value function that is separable over elements of the basis, consistent with the assumption.

Now, we move to characterizing pressure points. As a preliminary, the optimization problem of firm i has a corresponding Lagrangian

$$\mathcal{L}(x_i, \lambda|\mathcal{S}_i) \equiv \sum_{j\in\mathcal{J}_i} \pi_{ij}(x_{ij}) + \sum_{S\in\mathcal{S}_i} \lambda_{iS}\left[\beta v_i(S) - \sum_{j\in S}\theta_{ij}p_j x_{ij}\right],$$

where $\lambda_{iS} \geq 0$ is the Lagrange multiplier on the incentive compatibility constraint associated with $S \in \mathcal{S}_i$. We obtain the following result.

PROPOSITION 6: *$S_1, \ldots, S_n \in \mathcal{S}_i$ is a pressure point on firm i if and only if $\lambda_{iS} \neq \lambda_{iS'}$ for some $S, S' \in \{S_1, \ldots, S_n\}$.*

Proposition 6 proves that a necessary and sufficient condition for a pressure point is that the Lagrange multipliers of the existing equilibrium differ among those input relationships that enter the joint threat. To build intuition, return to the example in Figure 1. Consider the equilibrium under individual triggers $\mathcal{S}_i = \{\{j\}, \{k\}\}$, then firms in sector i have a pressure point resulting from the joint threat actions $\{j\}, \{k\}$ if and only if $\lambda_{ij} \neq \lambda_{ik}$. Intuitively, if $\lambda_{ij} > \lambda_{ik}$, then the marginal value of slack in the incentive compatibility constraint for (stealing) good j is higher than for slack in the incentive compatibility constraint for good k. The joint threat creates value by consolidating the two constraints and altering relative production of the two goods, a means of redistributing slack. Heuristically, the joint threat facilitates a *decrease* in production using k in order to create slack that allows for an *increase* in production using j under the joint threat. By contrast if $\lambda_j = \lambda_k$, then slack is equally valuable across goods j and k, even when both multipliers are strictly positive and both constraints bind. In this case, no value is created by forming a joint threat: production under the joint threat is precisely the same as under isolated threats. The proof of Proposition 6 formalizes these intuitions. This result is useful because, in separable production environments, it is possible to identify pressure points based on the existing equilibrium without having to re-compute the firm's optimization problem.

Proof of Proposition 6. We break the proof into the if and only if statements.

If. Suppose that there exist $S', S'' \in \{S_1, \ldots, S_n\}$ such that $\lambda_{iS'} > \lambda_{iS''}$ (without loss of generality). Suppose that we augment the incentive compatibility constraint for S to be

$$\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq \beta v_i(S) + \tau_S,$$

where τ_S is a constant (that is set equal to zero in the baseline). Observe that since $S' \cap S'' = \emptyset$, then joint threat constructed from S' and S'' yields the incentive constraint

$$\sum_{j \in S' \cup S''} \theta_{ij} p_j x_{ij} \leq \beta[v_i(S') + v_i(S'')] + \tau_{S'} + \tau_{S'\prime}.$$

Therefore, a weaker expansion of incentive compatible allocations than achieved by a joint threat is to instead increase $\tau_{S'}$ and decrease $\tau_{S''}$ in such a manner that $\tau_{S'} + \tau_{S''} = 0$. If such a perturbation strictly increases value, then creating a joint threat also strictly increases value.

Since $V_i(\mathcal{S}_i, \tau) = \mathcal{L}$, then the welfare effect of a perturbation to τ_S, by Envelope Theorem, is $\frac{\partial V_i}{\partial \tau_S} = \lambda_{iS}$. Therefore, the total profit impact on firm i of the perturbation $d\tau_{S'} = 1$ and $d\tau_{S''} = -1$ is

$$\frac{\partial V_i}{\partial \tau_{S'}} - \frac{\partial V_i}{\partial \tau_{S'\prime}} = \lambda_{iS'} - \lambda_{iS''} > 0.$$

Therefore, there is an $\epsilon > 0$ such that when defining τ by $\tau_{S'} = \epsilon$, $\tau_{S''} = -\epsilon$, and $\tau_S = 0$ otherwise, we have $V_i(\mathcal{S}_i, \tau) > V_i(\mathcal{S}_i, 0)$. But since $V_i(\mathcal{S}_i') \geq V_i(\mathcal{S}_i, \tau)$, then $V_i(\mathcal{S}_i') > V_i(\mathcal{S}_i)$, and hence $(S_1, \ldots, S_n)$ is a pressure point on i.

Only If. Because the decision problem of firm i is separable across elements of the action set, and because elements $S \notin \{S_1, \ldots, S_n\}$ are unchanged, the same allocations x_{ij}^* for $j \in \bigcup_{S \in \mathcal{S}_i \setminus \{S_1, \ldots, S_n\}} S$ remain optimal. It remains to show that optimal allocations are unchanged for $j \in \bigcup_{S \in \{S_1, \ldots, S_n\}} S$. Suppose first that $\lambda_{iS_1} = \ldots = \lambda_{iS_n} = 0$. Then, x_{ij} is produced at first-best scale, $x_{ij} = x_{ij}^u$. But then since $x_{ij}^* = x_{ij}^u$ is also implementable under joint threats, then the optimal allocation under joint threats is again $x_{ij}^* = x_{ij}^u$, and hence $(S_1, \ldots, S_n)$ is not a pressure point on i.

Suppose next that $\lambda_{iS_1} = \ldots = \lambda_{iS_n} > 0$ and let x_i^* be optimal production under $\mathcal{S}_i$. Because the decision problem of firm i is separable across elements of the action set, let us focus on the subset $\mathcal{X} = \{S_1, \ldots, S_n\}$ of elements in the joint threat. Denoting $\mathcal{L}(x_i, \hat{\lambda}|\mathcal{X})$ the Lagrangian associated with elements $\mathcal{X}$,

$$\mathcal{L}(x_i, \hat{\lambda}_i|\mathcal{X}) = \sum_{j \in \bigcup_{S \in \mathcal{X}} S} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \hat{\lambda}_{iS} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij} \right].$$

Recalling that the firm's objective function is concave while each constraint is convex, the Lagrangian has a saddle point at (x_i^*, λ_i).

Next, consider the decision problem of firm i when faced with a joint threat, so that $\mathcal{S}_i'$ has an element $S' = \bigcup_{S \in \mathcal{X}} S$. As again the decision problem of the firm is separable across elements of $\mathcal{S}_i'$, then we can define the Lagrangian of firm i with respect to element S' by

$$\mathcal{L}(x_i, \mu_i|S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \mu_{iS'} \left[\beta \sum_{S \in \mathcal{X}} v_i(S) - \sum_{j \in S'} \theta_{ij} p_j x_{ij} \right].$$

Observe that once again, the objective function is concave while the constraint is convex. Since $S \cap S' = \emptyset$ for all $S, S' \in \mathcal{X}$, then we can write

$$\mathcal{L}(x_i, \mu_i|S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \mu_{iS'} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij} \right].$$

Finally, let us define $\mu_{iS'} = \lambda_{iS_1}$. Since $\lambda_{iS_1} = \ldots = \lambda_{i,S_n}$, then we have

$$\mathcal{L}(x_i, \mu_i|S') = \sum_{j \in S'} \pi_{ij}(x_{ij}) + \sum_{S \in \mathcal{X}} \lambda_{iS} \left[\beta v_i(S) - \sum_{j \in S} \theta_{ij} p_j x_{ij} \right].$$

As a result, we have $\mathcal{L}(x_i, \mu_i|S') = \mathcal{L}(x_i, \lambda_i|\mathcal{X})$ for all x_i. More generally since for any μ_i' there is a corresponding vector $\lambda_{iS}' = \mu_i'$, then since $\mathcal{L}(x_i, \hat{\lambda}_i|\mathcal{X})$ has a saddle point at (λ_i, x_i^*), then $\mathcal{L}(x_i, \hat{\mu}_i|S')$ has a saddle point at (μ_i, x_i^*). Therefore, x_i^* is also an optimal policy under joint threat $\mathcal{S}_i'$. Therefore, $V_i(\mathcal{S}_i') = V_i(\mathcal{S}_i)$ and hence $(S_1, \ldots, S_n)$ is not a pressure point. This concludes the proof.

B.3.5. *Compliance by Domestic Firms*

Our baseline model assumes that the hegemon contracts with both its own domestic firms and also foreign firms. This means voluntary compliance is also required of domestic firms. Governments typically have much more control over their own firms, through domestic law, than over foreign firms, over which they have no direct legal jurisdiction. Within a domestic economy, power over domestic firms may vary. A hegemon seeking to project geoeconomic power abroad has to find ways to co-opt or coerce its domestic firms into action. There are a number of political economy constraints at home, including legal restrictions, domestic political objectives, interest groups, and other forces that make this domestic power projection less than immediate. This is especially true since private interests of the firms being spurred to action and those of the government might differ. We show how to extend the model so that the hegemon can mandate the choices of its own firms, subject only to a profit positivity constraint.

For the hegemon's domestic firms $i \in \mathcal{I}_m$, we replace the participation constraint (3) with a profit non-negativity constraint, given by

$$V_i(\Gamma_i) \geq 0. \tag{B.12}$$

Relative to the baseline model, there is first a difference in Micro-Power. In particular, the hegemon's Micro-Power over a domestic firm i is now the full value of its operations, $V_i(\Gamma_i)$, rather than just the difference between its inside and outside options, $V_i(\Gamma_i) - V_i(\mathcal{S}_i)$. Intuitively, the hegemon is making the outside option of the firm being shut down completely and having no value. This expands the set of feasible asks of the hegemon of its own firms (all else held equal). Because the hegemon directly values the private profits of its domestic firms, it has no incentive to extract transfers from them, and domestic firms might be left with positive profits (i.e., their non-negativity constraint would not be binding).[9] Second, there is also an effect on the hegemon's optimal asks. In particular, in deriving the

[9] In our repeated game, a domestic firm held to its non-negativity constraint would face added difficulty in sourcing goods because its incentives to deviate would be high in a Markov equilibrium in which it was also held to profit non-negativity in the future. It might therefore be restricted towards purchasing goods that were well enforced by law ($\theta_{ij} = 0$), a calibration likely to apply in particular for many domestic inputs.

optimal contract, equation 6 would be replaced by

$$\varepsilon_{ij}^{z} = \underbrace{\frac{\partial W_m}{\partial w_m}\sum_{k\in\mathcal{I}_m}\frac{\partial \Pi_k}{\partial z_{ij}} + \frac{\partial u_m(z)}{\partial z_{ij}}}_{\text{Externalities on Hegemon's Economy}} + \underbrace{\overbrace{\sum_{k\in\mathcal{D}_m}\eta_k\left[\frac{\partial \Pi_k}{\partial z_{ij}} - \frac{\partial V_k(\mathcal{S}_k)}{\partial z_{ij}}\right]}^{\text{Foreign Firms}} + \overbrace{\sum_{k\in\mathcal{I}_m}\eta_k\frac{\partial \Pi_k}{\partial z_{ij}}}^{\text{Domestic Firms}}}_{\text{Building Power}},$$

and equation 7 (price-based) would be similarly replaced. For any domestic firms held to the profit non-negativity constraint, the hegemon would be incentivized to build power over them by increasing their inside option, $\partial\Pi_k/\partial z_{ij}$, but not by lowering their outside option, which is now fixed at 0.

B.3.6. *Indirect Trade*

In practice, export restrictions have limits as firms find ways around the restriction, a phenomenon referred to as leakage. Restrictions can often be eluded through re-routing trade, for example routing through a third country that simply re-packages the goods. In this appendix, we sketch two ways in which our model can capture or be extended to capture re-routed trade.

B.3.6.0.1. *Re-routed Trade by Introducing Repackaging Sectors.* One way to capture re-routing of trade is through a reinterpretation of some of the productive sectors in our model. To illustrate this possibility, consider the following simple sketch. Consider an individual firm i that can purchase (among others) intermediate goods k and h. Suppose that good h itself is produced out of good k, that is the production technology of sector h is $f_h(x_{hk})$. If sector h is not subject to incentive problems (i.e., $\theta_{hk}=0$), then optimization yields $p_h f_h'(x_{hk}) = p_k$, and the price of good h is linked to that of k based on the efficiency with which sector h can repackage good k. We think of sector h as being outside the hegemon's control (i.e. cannot be used to make hegemonic threats), while sector k is under the hegemon's control. If goods h and k are substitutable in i's production, then off the equilibrium path when i is cut off by k, firm i is likely to rebalance towards purchasing h, even if h is a costly alternative due to some repackaging and re-routing costs (embedded in h's production function). In this sense, one might expect off-path that indirect trade would be a leading source of rebalancing for a firm that was cut off from direct suppliers of a good.

The above sketch is deliberately simple and illustrative. One could generalize it by having h be a repackager of a set of goods that are similar to k using (for example) a CES technology. One could also assume that firm i uses a CES aggregator to combine good k with the repackaged bundle h.

This form of re-routed trade helps to motivate our assumption on threat feasibility, which restricts joint threats to involve sectors that are at most one step removed from the hegemon. If the hegemon could form threats along longer chains, it could disrupt not only direct but also more indirect trade. We limited the feasibility of threats precise to allow for limited control of economic sectors.

B.3.6.0.2. *Leakage of Threats.* A second possibility is that punishments are imperfectly enforced. In particular, some firms in sector k may continue selling to firm i even when a punishment is supposed to be carried out. We capture this by assuming there is a probability π that firm i continues to be Trusted by suppliers in sectors in S even after it Steals from them and a probability $1-\pi$ that firm i becomes Distrusted. The case $\pi = 0$ is the baseline model. This assumption modifies the incentive constraint (equation 1) of individual firm i to be

$$\sum_{j \in S} \theta_{ij} p_j x_{ij} \leq (1-\pi)\beta \left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S) \right].$$

This tightens any given incentive constraint in proportion to the probability the firm is able to evade being cut off.

B.3.7. *Incorporating Military Enforcement*

Political scientists have highlighted different forms of “hard” power (e.g., military threats) and “soft” power (e.g., cultural attraction) (Nye (2004)). The economic threats that we study are certainly softer than military ones and it is interesting to consider how the two types of threats are related. Historically military threats have also been a means of enforcement of private and government relations (see Findlay and O’Rourke (2009) for an historical overview). For example, a hegemon can use a threat of a naval blockade to enforce repayment of sovereign debt or prevent expropriation of a local investment. Both economists (e.g. Bulow and Rogoff (1989)) and political scientists (e.g. Tomz (2012)) have pointed out the limits of military threats in the modern context, but they have surely not disappeared.

We sketch a reduced-form extension of our model to incorporate the military as an enforcement mechanism. We think of the hegemon as a country exogenously endowed with a technology (its military strength) that allows it to serve a role as a global enforcer.[10] In particular, the hegemon is able to directly increase the contract enforceability, that is lower the parameters θ_{ij}, for firms in its economic network. Formally, for each $i \in \mathcal{C}_m$, the hegemon can lower θ_{ij} to $\underline{\theta}_{ij} \leq \theta_{ij}$ for $j \in \mathcal{J}_i$.

The hegemon offers a contract $\Gamma_i = \{\mathcal{S}'_i, \mathcal{T}_i, \tau_i, \theta'_i\}$, and a firm that rejects the contract reverts to the outside option with $(\mathcal{S}_i, \theta_i)$. The value function, parallel to equation (2), is

$$V_i(\Gamma_i) = \max_{x_i, \ell_i} \Pi_i(x_i, \ell_i, \mathcal{J}_i) - \sum_{j \in \mathcal{J}_i} [\tau_{ij}(x_{ij} - x^*_{ij}) + T_{ij}] - \sum_{f \in \mathcal{F}_m} \tau^{\ell}_{ij}(\ell_{if} - \ell^*_{if}) + \beta \nu_i(\mathcal{J}_i)$$

$$s.t. \sum_{j \in S} \left[\theta'_{ij} p_j x_{ij} + T_{ij}\right] \leq \beta \left[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S)\right] \quad \forall S \in \Sigma(\mathcal{S}'_i)$$

The participation constraint (equation (3)) is now

$$V_i(\Gamma_i) \geq V_i(\mathcal{S}_i, \theta_i).$$

The hegemon can build Micro-Power through both economic threats and military enforcement. Interestingly, military and economic power can be either complements or substitutes. For example, if military power is able to lower to $\underline{\theta}_{ij} = 0$, then military power is able to obviate the need for economic threats. Similarly, and as highlighted by the BRI example, economic threats can substitute for military enforcement. For example, a joint threat linking a poorly enforced activity (high θ_{ij}) with a well enforced activity (low θ_{ik}) can serve much the same role as directly raising the enforceability of j through a military threat. Finally, it is also clear that these two threats can be complements. Consider a case in which both j and k are similarly poorly enforced, and so a joint threat alone creates

[10] To maintain parallel treatment with economic threats, we think of threats of military enforcement as being cheap in the sense that the hegemon has already paid the sunk cost of having a large military and threats are only carried out off path against specific entities. An important step for future work is to consider the relative costs of the two types of threats, especially incorporating the human and other costs of carrying out military action.

little value. If military enforcement can raise the enforceability of one relationship, say j, then (drawing again on the BRI example) an economic threat that then links the two relationships will help transfer that greater direct enforcement of j to a greater economic enforcement of k through the joint threat.

B.3.8. *Further Characterization of the Equilibrium in the BRI Application*

We provide a solution to the equilibrium and optimal contract in the application to China's Belt and Road Initiative from Section 4.2.[11]

PROPOSITION 7: *The hegemon has a pressure point on entity i if and only if*

$$\theta_{ik} > \frac{\beta}{1-\beta}\frac{1-\xi}{\xi}. \tag{B.13}$$

If the hegemon has a pressure point, then it holds the entity to the participation constraint and sets the transfer to:

$$\overline{T}_i^* = p_i(b^{*\xi} - b^{o\xi}) - R(b^* - b^o),$$

where b^* is the borrowing level under acceptance of the hegemon's contract, and $b^o = \left(\frac{p_i}{R}\frac{\beta}{\theta_{ik}(1-\beta)+\beta}\right)^{\frac{1}{1-\xi}}$ is the borrowing level under a rejection. The borrowing level b^* is the largest value $b^* \leq b^u$ that satisfies

$$p_i b^{*\xi} - (1-\theta_{ik})Rb^* \leq \frac{1}{1-\beta}\left[p_i b^{o\xi} - Rb^o\right] + \frac{\beta}{1-\beta}\pi_{ij}^*, \tag{B.14}$$

where $b^u = \left(\frac{p_i\xi}{R}\right)^{\frac{1}{1-\xi}}$ is the optimal unconstrained borrowing level in the stage game and $\pi_{ij}^* = p_i f_{ij}(x_{ij}^*) - p_j x_{ij}^*$ are the equilibrium profits of the manufacturing relationship.

Proposition 7 characterizes a necessary and sufficient condition, the inequality in equation B.13, for the hegemon to have a pressure point. The inequality is intuitive: (i) a pressure point does not exist if discount rates are too low (high β) because, as in a standard Folk

[11] We throughout this application discard the more trivial possible equilibrium in which $b^o = 0$, that is debt is not sustainable under isolated threats because borrowing is not possible in the future.

Theorem argument, enforcement is easy to sustain even on the outside option given high values placed on continuation; (ii) a pressure point does not exist if direct enforcement is too strong (low θ_{ik}) since joint threats have no value if direct enforcement is possible; (iii) if the desired borrowing is too low given the decreasing returns to scale (low ξ).

If there is a pressure point, then Proposition 7 characterizes how powerful this pressure is, that is the transfers extracted, and the borrowing levels on both the inside and outside option. Intuitively, if the manufacturing relationship is very valuable, then under acceptance of the contract the threats are so powerful that the unconstrained level of borrowing b^u can be sustained. Formally, there exists a value $\overline{\pi}_{ij}$ such that $b^* = b^u$ for $\pi^*_{ij} > \overline{\pi}_{ij}$. In this regime, further lowering direct enforcement (increasing θ_{ik}) or increasing discount rates (lowering β), increase the hegemon's power and the equilibrium transfers it extracts. The hegemon's power and transfer also increase in the profitability of the lending relationship (higher p_i or lower R). These comparative statics are intuitive, since both effects lower the outside option level of borrowing b^o but do not affect the inside option borrowing.

If the manufacturing relationship is not sufficiently valuable, $\pi^*_{ij} \leq \overline{\pi}_{ij}$, then b^* is the lower solution to equation B.14 holding with equality. In this regime, the hegemon has a pressure point but the threats are not as powerful, and borrowing occurs at the constrained level (constrained by the incentive compatibility) on both the inside and outside option. In this regime, the comparative statics affect both the inside and outside option level of borrowing, and the net effect on transfers depends on the relative impact on the borrowing.

The above characterization illustrates both the economics of the BRI application and provides a complete solution in terms of the underlying parameters of an application of our general framework. The economics again stresses the role of the hegemon as a global enforcer of economic activity. This role is more powerful the more the activities that could not otherwise be sustained are valuable, the less other means of direct enforcement are present, and the more the hegemon can mix activities with differential enforcement. The conditions that sustain a pressure point and pin down the amount of power are intuitive in terms of the underlying parameters and the economic forces at play.

B.3.8.1. *Proof of Proposition 7*

Denote $\pi_{ik}(b) = p_i b^{\xi} - Rb$. The IC under joint threats is:

$$\theta_{ik} Rb + \overline{T}_i \leq \beta \nu_i(\{j,k\}) = \frac{\beta}{1-\beta}\left[\pi_{ik}(b') - \overline{T}'_i + \pi^*_{ij}\right], \tag{B.15}$$

where $'$ variables denote continuation values taken as exogenous in the current period (recall that we are studying a Markov equilibrium, so allocations are identical in all future periods). Since the manufacturing relationship is fully enforced, $\theta_{ij} = 0$, the manufacturing relationship is operated at its first best scale in all periods. The PC in equilibrium is binding by Proposition 8 (see Appendix B.3.9) and given in the current period by:

$$\pi_{ik}(b^*) - \overline{T}^*_i + \pi^*_{ij} + \beta\nu_i(\{j,k\}) = \pi_{ik}(b^o_i) + \pi^*_{ij} + \beta\nu_i(\{j,k\}). \tag{B.16}$$

Simplifying the PC, the current transfer is:

$$\overline{T}^*_i = \pi_{ik}(b^*) - \pi_{ik}(b^o). \tag{B.17}$$

Recall that $\pi_{ik}(b)$, i.e. the firm stage game profit coming from borrowing, is strictly concave and achieves its maximum at

$$b^u = \left(\frac{p_i \xi}{R}\right)^{\frac{1}{1-\xi}},$$

which is the stage game unconstrained optimal level of borrowing. If the entity rejects the hegemon’s contract, it operates under isolated threats for the current period, and the contract can be offered again next period. The IC for the current period is:

$$\theta_{ik} Rb^o \leq \beta[\nu_i(\{j,k\}) - \frac{1}{1-\beta}\pi^*_{ij}] = \frac{\beta}{1-\beta}\pi_{ik}(b') - \frac{\beta}{1-\beta}\overline{T}'_i. \tag{B.18}$$

Since by Proposition 8 in Online Appendix B.3.9 the participation constraint binds in future periods, we have $\pi_{ik}(b') - \overline{T}'_i = \pi_{ik}(b^{o\prime})$, and therefore the IC constraint after contract rejection simplifies to:

$$\theta_{ik} Rb^o \leq \beta \frac{p_i b^{o\prime\xi} - Rb^{o\prime}}{1-\beta}. \tag{B.19}$$

B.30

The equilibrium consistency conditions feature $b = b' = b^*$, $b^o = b^{o\prime}$, and $\overline{T}_i = \overline{T}'_i = \overline{T}^*_i$. Substituting in equilibrium consistency and rearranging yields[12]

$$b^o \leq \left(\frac{p_i}{R} \frac{\beta}{\beta + \theta_{ik}(1-\beta)} \right)^{\frac{1}{1-\xi}}$$

An equilibrium therefore features $b^o = b^u$ iff

$$\theta_{ik} \leq \frac{\beta}{1-\beta} \frac{1-\xi}{\xi}.$$

Therefore, there is no pressure point iff the above inequality holds.

Now suppose that we have $\theta_{ik} > \frac{\beta}{1-\beta}\frac{1-\xi}{\xi}$ and there is a pressure point. Under contract rejection, then the binding IC constraint under isolated threats yields as above

$$b^o = \left(\frac{p_i}{R} \frac{\beta}{\theta_{ik}(1-\beta) + \beta} \right)^{\frac{1}{1-\xi}}.$$

Next, under contract acceptance, using the IC and the binding PC we have

$$\theta_{ik} R b^* + \left(p_i (b^{*\xi} - b^{o\xi}) - R(b^* - b^o) \right) \leq \frac{\beta}{1-\beta} \left[p_i b^{o\xi} - R b^o + \pi^*_{ij} \right],$$

which simplifies to:

$$p_i b^{*\xi} - (1-\theta_{ik}) R b^* \leq \frac{1}{1-\beta} \left[p_i b^{o\xi} - R b^o \right] + \frac{\beta}{1-\beta} \pi^*_{ij}.$$

The RHS is a constant given the known solution for b^o. Given that $\xi < 1$, the LHS is increasing for $b^* < \left(\frac{\xi p_i}{(1-\theta_{ik})R} \right)^{\frac{1}{1-\xi}}$ and thereafter decreasing. The RHS is maximized at a point above b^u, therefore either the constraint binds or else $b^* = b^u$.[13] Thus we have a

[12] We discard the more trivial possible equilibrium in which $b^o = 0$.

[13] Suppose that in equilibrium we had $b^* > b^u$. Then, $b^* = b^u$ would be implementable for entity i (even if not as part of an equilibrium), a contradiction.

solution at b^u iff

$$p_i b^{u\xi} - (1-\theta_{ik})Rb^u \leq \frac{1}{1-\beta}\left[p_i b^{o\xi} - Rb^o\right] + \frac{\beta}{1-\beta}\pi^*_{ij},$$

which is a critical threshold on the value of the manufacturing relationship. Therefore, the solution is the largest value $b^* \leq b^u$ that satisfies

$$p_i b^{*\xi} - (1-\theta_{ik})Rb^* \leq \frac{1}{1-\beta}\left[p_i b^{o\xi} - Rb^o\right] + \frac{\beta}{1-\beta}\pi^*_{ij},$$

completing the proof.

B.3.9. *Classifying Friends and Enemies*

Our framework provides a classification of "friends and enemies" of the hegemon based on externalities. We borrow this terminology from Kleinman et al. (2020) who base it on a country's real income response to a foreign country's increase in productivity. We base it on the externalities that sector has from the hegemon's perspective.[14]

PROPOSITION 8: *If consumer preferences are identical and homothetic (i.e., $U_n(C_n) = U(C_n)$ where U is homothetic), under the hegemon's optimal contract, foreign sector i is:*

1. ***Unfriendly** to the hegemon if $\mathcal{E}_{ij} \leq 0$ for all $j \in \mathcal{J}_i$, with strict inequality for at least one j. These sectors are held to their participation constraint and have strictly negative activities $\mathcal{E}_{ij} < 0$ taxed $\tau_{ij} > 0$.*
2. ***Neutral** to the hegemon if $\mathcal{E}_{ij} = 0$ for all $j \in \mathcal{J}_i$. These sectors are held to their participation constraint and are not taxed.*
3. ***Friendly** to the hegemon if $\mathcal{E}_{ij} \geq 0$ for all $j \in \mathcal{J}_i$, with strict inequality for at least one j. These sectors have strictly positive activities $\mathcal{E}_{ij} > 0$ subsidized $\tau_{ij} < 0$.*

Proposition 8 delineates three types of relationships: friendly sectors that have only (weakly) positive spillovers from the hegemon's perspective, neutral sectors with no spillovers, and unfriendly sectors with only (weakly) negative spillovers. Sectors can in

[14]The proposition below applies to sectors on which the hegemon has a pressure point.

general have mixed activities, and we leave those sectors unclassified as mixed sectors. A friendly sector i has its strictly positive-externality activities subsidized, while an unfriendly sector has its strictly negative-externality activities taxed. A neutral sector, in contrast, is neither taxed nor subsidized.

The notion of friendship that we develop is both theoretically grounded and relevant for understanding how the hegemon interacts with these sectors in its optimal contract. Friendship can be driven by direct or indirect linkages, and by economic or non-economic motives (the term $\frac{\partial u_m(z)}{\partial z_{ij}}$ in equation (6)). Interestingly, friendship is an important driver of which sectors are held to their participation constraints and achieve no surplus under the optimal contract. Despite the hegemon having all the bargaining power, the hegemon might leave surplus to the foreign sectors (slack participation constraint), but only if it is friendly. Unfriendly and neutral retain no surplus.

These notions can be used to understand the effect of military or political alliances, such as NATO, in the presence of externalities (Olson and Zeckhauser (1966)). We think of these alliances as the hegemon placing positive utility on certain defense sectors of allied countries. In our framework the hegemon would use its global enforcement power to push those allied countries to increase those activities, making them internalize more of the national security externalities, and might also do so at the expense of its own firms' profits or consumers' consumption. Indeed, the hegemon might leave surplus to allied countries' sectors and (optimally) not fully exercise its coercive power on them.

B.3.9.1. *Proof of Proposition 8*

We first show that $\Xi_{mn} = 0$ under identical and homothetic preferences. It suffices to show that excess demand is invariant to wealth transfers between consumers. Excess demand in factor markets does not depend on consumption and so is invariant. Given identical homothetic preferences, we have $C_{ni}(p, w_n) = C_i(p)w_n$ and therefore

$$ED_i = C_i(p)\sum_{n=1}^{N} w_n + \sum_{j\in\mathcal{D}_i} x_{ji} - y_i$$

which is invariant to wealth transfers. Therefore, $\Xi_{mn} = 0$.

Next, we show that the participation constraints of unfriendly and neutral sectors bind. From Proposition 3, the first order condition for transfers yields

$$\overline{\Lambda}_i + \eta_i \geq \frac{\partial W_m}{\partial w_m} > 0$$

and therefore either $\overline{\Lambda}_i > 0$ or $\eta_i > 0$. If $\eta_i > 0$ the proof is completed, so suppose by way of contradiction that $\overline{\Lambda}_i > 0$ but $\eta_i = 0$. Recall that by definition $\overline{\Lambda}_i = \sum_{S\in\Sigma(\overline{\mathcal{S}}_i')|S_i^D\subset S}\Lambda_{iS}$. Since $\overline{\Lambda}_i > 0$, $\exists S \in \Sigma(\overline{\mathcal{S}}_i')$ such that $S_i^D \subset S$ and $\Lambda_{iS} > 0$. Define $\mathbf{S} = \{S \in \Sigma(\overline{\mathcal{S}}_i')|S_i^D \subset S, \, \Lambda_{iS} > 0\}$ to be the nonempty set of all such sets. The proof proceeds by considering two mutually exclusive cases.

Case (i). Suppose first that $\forall S \in \mathbf{S}$, $\exists k \in S$ such that $x_{ik}^* > 0$. Let $\mathbf{K} = \{k \in \bigcup_{S\in\mathbf{S}} S \,|\, x_{ik}^* > 0\}$ be the nonempty set of all such k. Returning to the hegemon's Lagrangian in the proof of Proposition 3, consider the perturbation whereby the hegemon increases $\overline{T}_i$ by ϵ arbitrarily small and decreases x_{ik} by $\frac{\epsilon}{p_k\theta_{ik}}$ for all $k \in \mathbf{K}$. Using the derivations of Proposition 3 (i.e., the FOCs for $\overline{T}_i$ and x_{ij} for a foreign firm), the impact of this perturbation on the hegemon's Lagrangian is

$$\frac{\partial \mathcal{L}_m}{\partial \overline{T}_i} - \sum_{k\in\mathbf{K}} \frac{1}{p_k\theta_{ik}}\frac{\partial \mathcal{L}_m}{\partial x_{ik}} = \frac{\partial W_m}{\partial w_m} + \gamma_i - \eta_i - \overline{\Lambda}_i + \Xi_{mn} - \sum_{k\in\mathbf{K}} \frac{1}{p_k\theta_{ik}}\left[\eta_i \frac{\partial \Pi_i}{\partial x_{ik}} - \overline{\Lambda}_{ik}\theta_{ik}p_k + \mathcal{E}_{ik}\right]$$

where recall that $\overline{\Lambda}_{ij} = \sum_{S\in\Sigma(\overline{\mathcal{S}}_i')|j\in S}\Lambda_{iS}$. Under the supposition that the participation constraint does not bind ($\eta_i = 0$) and given that $\Xi_{mn} = 0$, we have

$$\frac{\partial \mathcal{L}_m}{\partial \overline{T}_i} - \sum_{k\in\mathbf{K}} \frac{1}{p_k\theta_{ik}}\frac{\partial \mathcal{L}_m}{\partial x_{ik}} = \frac{\partial W_m}{\partial w_m} + \gamma_i - \overline{\Lambda}_i + \sum_{k\in\mathbf{K}} \overline{\Lambda}_{ik} - \sum_{k\in\mathbf{K}} \frac{1}{p_k\theta_{ik}}\mathcal{E}_{ik}$$

Given the definition of $\mathbf{S}$, we have $\overline{\Lambda}_i = \sum_{S\in\Sigma(\overline{\mathcal{S}}_i')|S_i^D\subset S}\Lambda_{iS} = \sum_{S\in\mathbf{S}}\Lambda_{iS}$. Since for each $S \in \mathbf{S}$ $\exists k \in \mathbf{K}$ such that $k \in S$, then $\sum_{k\in\mathbf{K}}\overline{\Lambda}_{ik} \geq \sum_{S\in\mathbf{S}}\Lambda_{iS} = \overline{\Lambda}_i > 0$. Therefore,

$$\frac{\partial \mathcal{L}_m}{\partial \overline{T}_i} - \sum_{k\in\mathbf{K}} \frac{1}{p_k\theta_{ik}}\frac{\partial \mathcal{L}_m}{\partial x_{ik}} \geq \frac{\partial W_m}{\partial w_m} + \gamma_i - \sum_{k\in\mathbf{K}} \frac{1}{p_k\theta_{ik}}\mathcal{E}_{ik} \geq \frac{\partial W_m}{\partial w_m} > 0$$

where the second to last inequality follows since firm i is either unfriendly or neutral, $\mathcal{E}_{ik} \leq 0$. But this contradicts that the hegemon's contract was optimal, contradicting the supposition that the participation constraint did not bind in this case.

Case (ii). Suppose instead that $\exists S_0 \in \mathbf{S}$ such that $x^*_{ik} = 0$ for all $k \in S_0$. Since $S_0 \in \mathbf{S}$, then $S_i^D \subset S_0$ and therefore $x^*_{ij} = 0$ for all $j \in S_i^D$. The strategy is to show that the allocation x^*_i is implementable for sector i at its outside option (in which it rejects the contract and faces $\mathcal{S}_i$), without having to make the transfer payment. As such, firm i's value must be higher at the outside option (owing to the Inada condition), contradicting the supposition that the participation constraint did not bind ($\eta_i = 0$).

To show that x^*_i is implementable at the outside option, we must show that

$$\sum_{j \in S} \theta_{ij} p_j x^*_{ij} \leq \beta \Big[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash S) \Big] \quad \forall S \in \Sigma(\mathcal{S}_i)$$

Taking $S \in \Sigma(\mathcal{S}_i)$, then since $\mathcal{S}_i^D \subset \mathcal{S}_i$ there exists a $\mathcal{X}_i \subset \mathcal{S}_i^D$ and $\mathcal{Y}_i \subset \mathcal{S}_i \backslash \mathcal{S}_i^D$ such that $S = X_i \cup Y_i$ for $X_i = \bigcup_{Z \in \mathcal{X}_i} Z$ and $Y_i = \bigcup_{Z \in \mathcal{Y}_i} Z$. Since by definition $\overline{\mathcal{S}}'_i \backslash \{S_i^D\} = \mathcal{S}_i \backslash \mathcal{S}_i^D$, then $Y_i \in \Sigma(\overline{\mathcal{S}}'_i)$. Because $X_i \subset S_i^D$, $x^*_{ij} = 0$ for $j \in X_i$. Because $Y_i \in \Sigma(\overline{\mathcal{S}}'_i)$, because $x^*_{ij} = 0$ for $j \in X_i$, and because x^*_i is IC in the hegemon's problem, we have

$$\sum_{j \in S} \theta_{ij} p_j x^*_{ij} = \sum_{j \in Y_i} \theta_{ij} p_j x^*_{ij} \leq \beta \Big[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash Y_i) \Big] \leq \beta \Big[\nu_i(\mathcal{J}_i) - \nu_i(\mathcal{J}_i \backslash (X_i \cup Y_i)) \Big]$$

where the final inequality follows from monotoncity of ν_i. Therefore, x^*_i is incentive compatible at the outside option, contradicting the supposition that $\eta_i = 0$.

Therefore, the participation constraint binds for unfriendly and neutral sectors ($\eta_i > 0$).

Finally, consider wedges. For an unfriendly or neutral firm, from Proposition 3 we have $\tau^*_{ij} = -\frac{\mathcal{E}_{ij}}{\eta_i}$ and the result follows. For a friendly firm, if the participation constraint binds the result follows analogously. If the participation constraint does not bind, then returning to the argument of the proof of Proposition 3 we have $\tau_{ij} < 0$ for $\mathcal{E}_{ij} > 0$ and small α.

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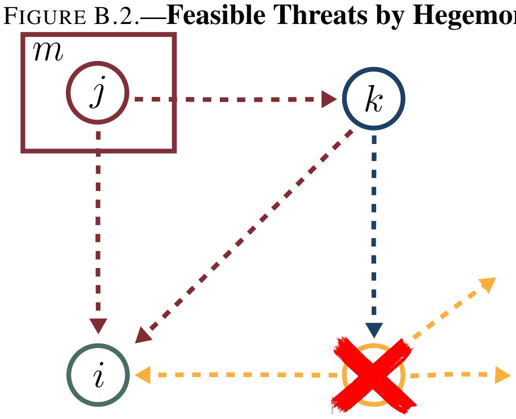
TABLE B.1
SUMMARY OF NOTATION

Symbol	Meaning	Symbol	Meaning
General Set-up		B_{ij}	Dummy for whether suppliers j Trusts firm i
$\mathcal{I}_n$	Set of sectors in country n. $\mathcal{I}$ set of all sectors	$\mathcal{B}_i$	Set of suppliers that Trust firm i
$\mathcal{F}_n$	Set of factors in country n. $\mathcal{F}$ set of all factors	**Continuation and Value Functions**	
Equilibrium Objects		$\nu_i(\mathcal{B}_i)$	Exogenous continuation value
p_i	Price of good produced by sector i	$\mathcal{V}_i(\mathcal{B}_i)$	Eqm value function of firm i in repeated game
p_f^ℓ	Price of local factor ℓ	$V_i(S_i)$	Firm's current value as a function of its action set $\mathcal{S}_i$
p, p^ℓ, P	Vector of intermediate goods, factor, all prices	$V_i(\Gamma_i)$	Value of firm i when accepting contract Γ_i
z	z vector of all externalities z_{ij}	**Hegemon**	
Consumer		D_i	Set of sectors downstream from Sector i
$U_n(C_n)$	Utility of rep.agent in country n from consumption	$\mathcal{D}_m$	Set of foreign sectors that source at least one input from hegemon's country
$u_n(z)$	Utility of rep. agent in country n from z	$\mathcal{C}_m$	Set of firms hegemon can contract with
Π_i	Profits of sector i	$\mathcal{J}_{im}$	Set of inputs that sector i sources from sectors in country m
w_n	Income of consumer in country n	T_{ij}	Transfers from i to hegemon in relationship with j. Vector $\mathcal{T}_i$. Sum $\overline{T}_i$
$W_n(p, w_n)$	Indirect utility function from consumption	τ_{ij}	Revenue-neutral tax i faces on purchases of goods from sector j
Firms		τ_{if}^ℓ	Revenue-neutral tax i faces on purchases of factor ℓ
x_{ij}	Intermediate input j used by firm i. Vector x_i	τ_i	Vector of revenue-neutal input and factor taxes faced by i
ℓ_{if}	Local factor f used by firm i. Vector ℓ_i	Γ_i	Hegemon's contract $\Gamma_i = \left\{\mathcal{S}_i', \mathcal{T}_i, \tau_i\right\}$. Vector Γ
y_i	Output of firm i, $y_i = f_i(x_i, \ell_i, z)$	Ψ^z	Matrix capturing endogenous externality amplification
$\mathcal{J}_i$	Set of suppliers to firm i	$\mathcal{L}^m$	Hegemon's Lagrangian
Stealing		η_i	Lagrange multiplier on the participation constraint of firm i
θ_{ij}	Share of order that can be stolen	Λ_{iS}	Lagrange multiplier on the IC constraint of firm i for action S
a_{ij}	Dummy for whether suppliers j Accepts order of firm i	$\bar{\Lambda}_i$	Sum of multipliers for stealing actions in hegemon's threat
$S_i \subset \mathcal{J}_i$	The subset of sectors from which firm i steals	$\mathcal{E}_{ij}$	Hegemon's perceived externalities from increase in z_{ij}^*
$\mathcal{S}_i$	Action Set: Set of firms' possible stealing decisions	Ξ_{mn}	Hegemon's perceived externalities from a transfer from n to m
$\mathcal{S}_i'$	Joint threat, Coarser partition of $\mathcal{S}_i$	ε_{ij}^z	Direct value to hegemon of increasing sector $i's$ use of input j
$\overline{\mathcal{S}}_i'$	Maximal joint threat	ε_{ij}^{zNC}	Indirect value to hegemon of increasing sector $i's$ use of input j
S_i^D	Inputs in hegemon's maximal joint threat	$\varepsilon_j^{P^m}$	Value to hegemon from changes in the price of input j
$P(\mathcal{J}_i)$	Power set of $\mathcal{J}_i$	Ω_n	Global planner's welfare weight on country n
$\Sigma(\mathcal{S})$	Set of all possible unions of elements of $\mathcal{S}$	β	Discount Factor

FIGURE B.2.—**Feasible Threats by Hegemon**



Notes: The figure illustrates the following configuration: sector j is located in the hegemon country and supplies to sector k and i. Sector k supplies to sector i and to another sector (orange and crossed-out), which itself supplies to sector i. The hegemon has a feasible joint threat on sector i via controlling the threats of j and k. The hegemon does NOT have a feasible joint threat on the orange crossed-out sector.