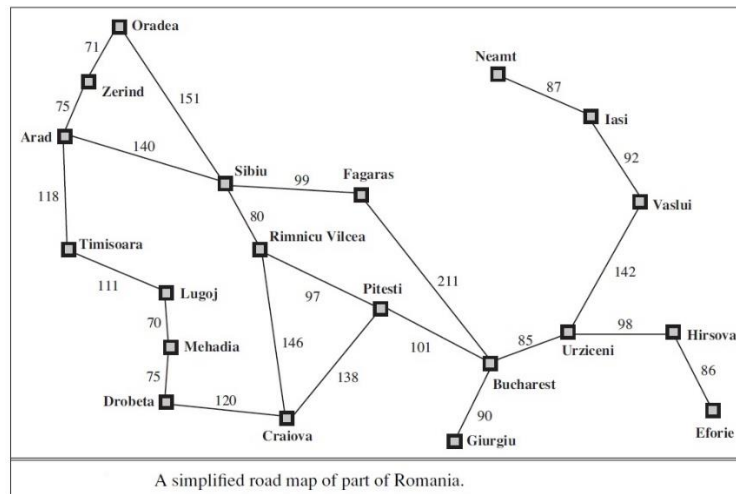


Path planning involves finding an optimal path between two locations in a known, partially known, or unknown environment. Search performance might be described by completeness, optimality, time complexity, or space complexity.

Consider the following simplified map from *Artificial Intelligence: A Modern Approach* (Third Edition) by Russell and Norvig.



Problem 0. Suppose we are in Arad and wish to find the shortest path to Bucharest. What do we do?

Dijkstra's algorithm [Dijkstra (1956/1959)]

Dijkstra's algorithm is a single-source shortest-path algorithm.

- STEP 1 Mark all nodes as *unvisited*.
- STEP 2 Set the distance for the start node to 0 and the temporary distance for all other nodes to ∞ . Set the start node as *current*.
- STEP 3 For the current node, consider each unvisited neighbour v and calculate the temporary distance through *current*. Compare the temporary distance to the current distance for v and assign the smaller of the two as the new temporary distance for v .
- STEP 4 When all the neighbours of *current* have been updated, mark *current* as *visited*.
- STEP 5 If the destination node has been marked *visited* or if the smallest temporary distance amongst unvisited nodes is ∞ then STOP.
- STEP 6 Otherwise, select the unvisited node with smallest temporary distance, set it as *current*, and go to STEP 3.

Problem 1. Implement Dijkstra's algorithm in a language of your choice that is available on *repl.it*.

Problem 2. Apply Dijkstra's algorithm to **Problem 0**. Include a visual component which shows the progress of the algorithm.

Problem 3. Modify the output of this version of Dijkstra's algorithm to find the desired path.

A* search [Hart, Nilsson, and Raphael (1968)]

A* is a generalization of Dijkstra's algorithm. It is similar to Dijkstra's algorithm in that accumulated distances are used but adds an important component to the decision-making process.

To decide which path to extend, A* chooses the path that minimises

$$f(n) = g(n) + h(n)$$

where n is the next node on the path, $g(n)$ is the cost of the path from the start node to n , and $h(n)$ is a *heuristic function* that estimates the cost of the cheapest path from n to the goal. A heuristic function provides additional knowledge of the problem to the search algorithm. A well-chosen heuristic function for the task at hand is vital to success.

A heuristic function h is *admissible* if h never overestimates the actual cost to get to the goal. If $w(x, y)$ represents the weight of edge xy , then a heuristic function h is *consistent* if $h(a) \leq w(a, b) + h(b)$ for every edge ab .

Can you give a nontrivial heuristic function that is always admissible for **Problem 0**?

If the heuristic function is admissible, then A* is guaranteed to return a least-cost path from the start node to the goal. If the heuristic function is also consistent, then A* will return an optimal path without processing any node more than once.

Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

Values of h_{SLD} —straight-line distances to Bucharest.

A* search is identical to Dijkstra's algorithm except that A* minimises the function f in STEP 3.

Problem 4. Implement the A* algorithm in a language of your choice that is available on *repl.it*.

Problem 5. Apply A* to **Problem 0**. Again, include a visual component which shows the progress of the algorithm.

Problem 6. Modify the output of this version of A* to find the desired path.