- RAPPORT TD 7&8 -

- Optimization approaches -

Groupe 7 :

- MARÇAL Thomas
- KOSKAS Axel

---

1. Manufacturing process

Decision variables:

Part 1 : x_1
Part 2 : x_2

Objective function:

$$\text{Max}(f(x)) = x_1 * 50 + x_2 * 100$$

Constraints:

$$10 * x_1 + 5 * x_2 \leq 2500$$

$$4 * x_1 + 10 * x_2 \leq 2000$$

$$x_1 + 1.5 * x_2 \leq 450$$

$$x_1 > 0, x_2 > 0$$

Results:

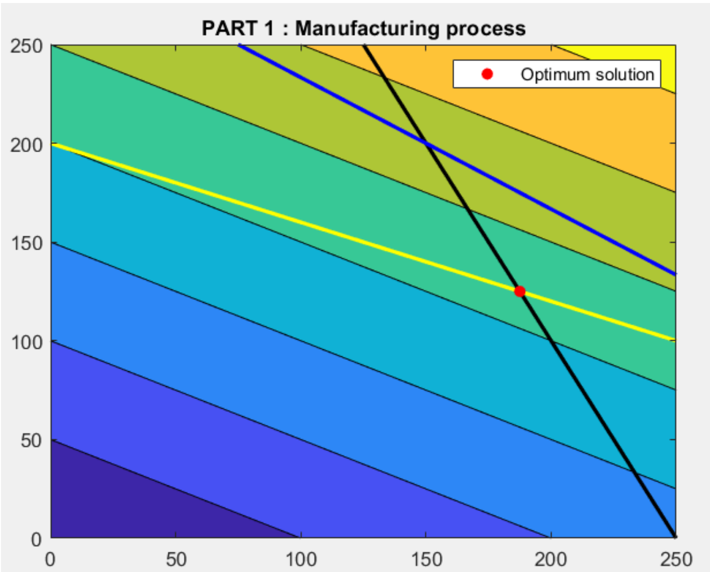
The MATLAB function returns the values :

$$f = [187{,}5 ; 125]$$

$$val = -2.1875e + 04$$

So :

$$\boldsymbol{x_1 = 187,5 \quad \& \quad x_2 = 125}$$



2. Manufacturing process

Decision variables:

Trains: t
Soldiers: s

Objective function:

$$\text{Max}\big(f(x)\big) = 3 * t + 2 * s$$

Constraints:

$$s \leq 40$$

$$s + t \leq 80$$

$$2 * t + s \leq 100$$

$$s > 0, t > 0$$

Results:

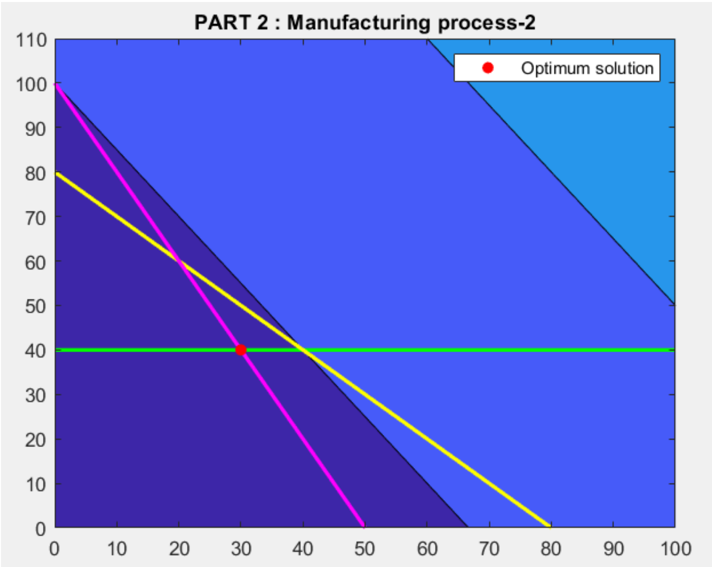
The MATLAB function returns the values :

$$f = [30\,;4]$$

$$val = -170$$

So :

$$\boldsymbol{s = 30 \quad \& \quad t = 40}$$



3. Positioning of antennas

Decision variables:

Position selon x : x_1
Position selon y : x_2

Objective function:

$$\mathrm{Min}(f(x1,x2)) = 200 \cdot ||(x_1,x_2) - \mathrm{Client1}|| + 150 \cdot ||(x_1,x_2) - \mathrm{Client2}||$$

$$+200 \cdot ||(x_1,x_2) - \mathrm{Client3}|| + 300 \cdot ||(x_1,x_2) - \mathrm{Client4}||$$

Constraints:

We have : - Antenne1 = [-5,10]

- Antenne2 = [5,0]

$$||\,[x_1,x_2] - \text{ Antenne 1 }|| \geq 10$$

$$||\,[x_1,x_2] - \text{ Antenne 2 }|| \geq 10$$

So, in MATLAB, we have :

$$g_1 = 10 - ||\,[x_1,x_2] - \text{ Antenne 1 }||$$

$$g_2 = 10 - \ ||\,[x_1,x_2] - \text{ Antenne 2 }||$$

Results:

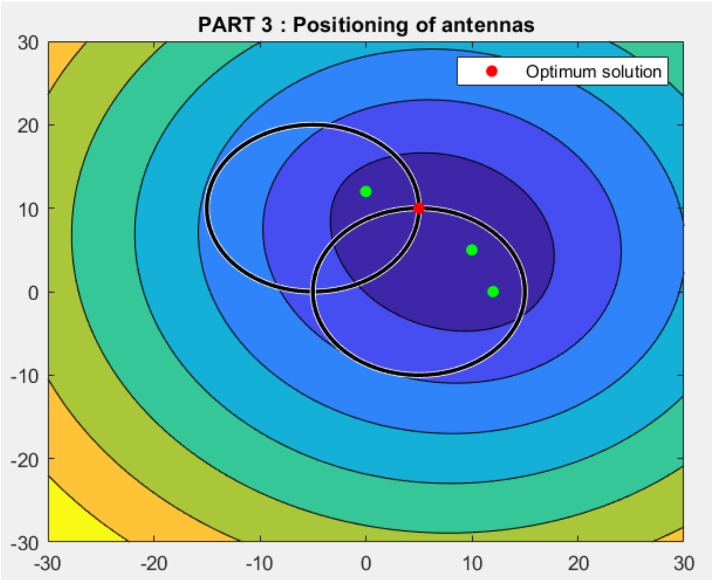
The MATLAB function returns the values :

$$f = [5\,;10]$$

$$val = 5.7997e + 03$$

So :

$$\boldsymbol{x_1 = 5} \quad \& \quad \boldsymbol{x_2 = 10}$$



4. 2R robot

Decision variables:

Angle 1: θ_1
Angle 2: θ_2

Objective function :

We have : $P = [P_x\,; P_y] = [\,2\,; 3\,]$.

So :

$$Min(f(\theta_1,\theta_2)) = ||\ [L_1 * \cos(\theta_1) + L_2 * \cos(\theta_2 + \theta_1)\,; L_1 * \sin(\theta_1) + L_2 * \sin(\theta_2 + \theta_1)] - [\,P_x\,; P_y\,]\ ||$$

Constraints:

In this problem we have no constraints.

Results:

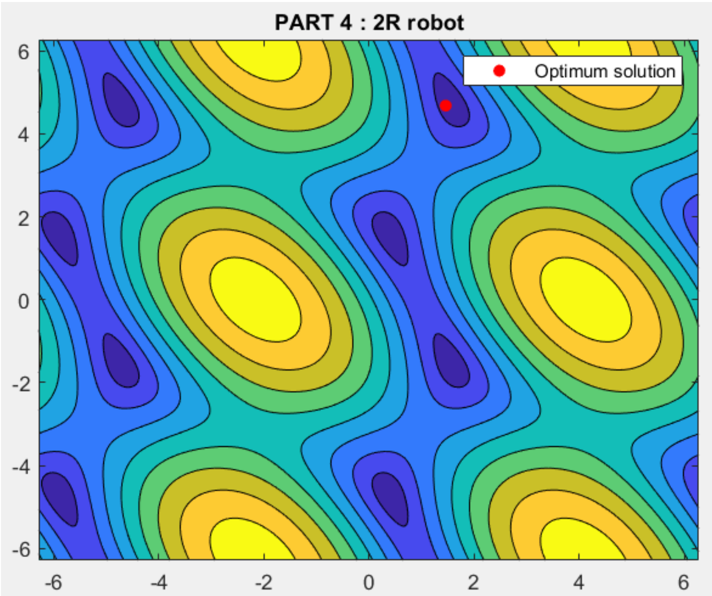
The MATLAB function returns the values :

$$f = [1{,}4566\,; 4{,}6762]$$

$$val = 1$$

So :

$$\boldsymbol{x_1 = 1,4566} \quad \& \quad \boldsymbol{x_2 = 4,6762}$$



5. Dimension synthesis of a 2R mechanism

1.

Direct kinematics of the 2R mechanism:

$$[x\,;y] = [l_1 * \cos(\theta_1) + l_2 * \cos(\theta_1 + \theta_2) \;\; ; \;\; l_1 * \sin(\theta_1) + l_2 * \sin(\theta_1 + \theta_2)]$$

Inverse kinematics of the 2R mechanism:

$$\theta_2 = \arccos\left(\frac{-{l_1}^2 - {l_2}^2 + (X^2 + Y^2)}{2 * l_1 * l_2}\right)$$

$$\theta_1 = \arctan\left(\frac{Y}{X}\right) - \arctan\,(l_2 * \frac{\sin(\theta_2)}{l_1 + l_2 * \cos(\theta_2)})$$

2. The Jacobian Matrix for the 2R robot is :

$$Jacobian = \begin{pmatrix} -l2 * sin(\theta_1 + \theta_2) - l1 * sin\,(\theta_1) & -l2 * sin\,(\theta_1 + \theta_2) \\ l2 * cos(\theta_1 + \theta_2) + l1 * cos\,(\theta_1) & l2 * cos\,(\theta_1 + \theta_2) \end{pmatrix}$$

3. The Stiffness Matrix is :

$$K_\theta = \begin{pmatrix} \frac{3 * E * \mathrm{I}}{l_1^3} & 0 \\ 0 & \frac{3 * E * \mathrm{I}}{l_2^3} \end{pmatrix} with\ I = \frac{\pi\ *\ d^4}{64}\ \ and\ \ E = 210\ GPa$$

4.

Decision variables:

Arm Length 1 : L_1 *noted* x_1
Arm Length 2 : L_2 *noted* x_2
Rayon : R *noted* x_3

Objective function:

$$Min(f(\theta_1, \theta_2)) = \rho_{steel} * \pi * R^2 * L_1 + \rho_{steel} * \pi * R^2 * L_2$$

Constraints:

In this problem, we have a total of 12 constraints, divided into 4 sets of 3 constraints for each vertex of the square that the robot arm must reach.

$$k = \sqrt{\frac{Max(Eigen(J^T J))}{Min(Eigen(J^T J))}} > 0.2$$

$$K_x^{-1} * f < 0.1 * 10^{-3}$$

$$x_1 > 0, \qquad x_2 > 0, \qquad x_3 > 0$$

So, in MATLAB, we have :

$$g_1 = 0{,}2 - \mathrm{k}$$

$$g_2 = [K_x^{-1} * f](1) - 0{,}1 * 10^{-3}$$

$$g_3 = [K_x^{-1} * f](2) - 0{,}1 * 10^{-3}$$

<u>Results:</u>

The MATLAB function returns the values :

$$f = [0{,}6594\,; 2{,}4873\,; 1{,}9905]$$

$$val = 3.0552e + 05$$

So :

$$\boldsymbol{x_1 = 0,6594} \quad \& \quad \boldsymbol{x_2 = 2,4873} \quad \& \quad \boldsymbol{x_3 = 1,9905}$$

In conclusion :

$$L_1 = 0{,}6594\, m$$

$$L_2 = 2{,}4873\, m$$

$$R = 1{,}9905\, cm$$