TD1 - Understanding basics of Kinematics & Reference frames

Groupe :

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PART I :

1.4.

1. Discretization is the act of cutting a segment of continuous values in a defined number of values. It enables us to do calculations with a computer as it only deals with discrete values.

2. The mechanism has 1 degree of freedom since it can only do a translation according to one axis.

3. /

4. 'Hold on', in the algorithm retains the precedent plot in the current axis therefore the new plot is added following the previous axis while not deleting the previous plot.

5. See Code.

6. See Code.

Bonus Part :

Rotation matrix: $R = \begin{pmatrix} \cos\theta & -\sin\theta \\ +\sin\theta & \cos\theta \end{pmatrix}$

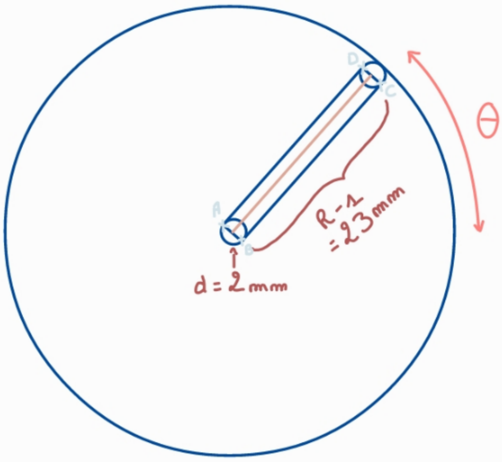


schéma de la simulation

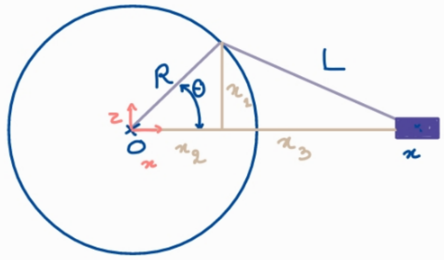


schéma complet du méchanisme

SOH CAH TOA

- $\sin\theta = \dfrac{x_1}{R}$
- $\cos\theta = \dfrac{x_2}{R}$
- $x_3 = x - x_2 = \sqrt{L^2 - R^2\sin^2\theta}$

Kinematic equation

$$x = R\cos\theta + \sqrt{L^2 - R^2\sin^2\theta}$$

Reverse kinematic equation

$$x^2 - 2xR\cos\theta + R^2\cos^2\theta = L^2 - R^2\sin^2\theta$$

$$\Leftrightarrow \quad x^2 - 2xR\cos\theta + R^2\underbrace{(\cos^2\theta + \sin^2\theta)}_{=1} = L^2$$

$$\Leftrightarrow \quad x^2 - 2xR\cos\theta = L^2 - R^2$$

$$\Leftrightarrow \quad x(2R\cos\theta - x) = -L^2 + R^2$$

$$\Leftrightarrow \quad -x + 2R\cos\theta = \frac{R^2 - L^2}{x}$$

$$\Rightarrow \quad \theta = \arccos\left(\frac{R^2 - L^2}{2Rx} + \frac{x}{2R}\right)$$

PART II :

⇒ TASK 1

In order to realize task 1, we look for the coordinates of the different points : A, B, C, ... As we are on a 2D model, we are only looking for the position of these points according to x and z.

We will therefore carry out the Pythagorean theorem several times in order to obtain the successive heights.

→ For the coordinates of A:

We have the coordinate of x evolving as a function of rho, and the component along z does not evolve.

So: - $A_x = \rho(x)$

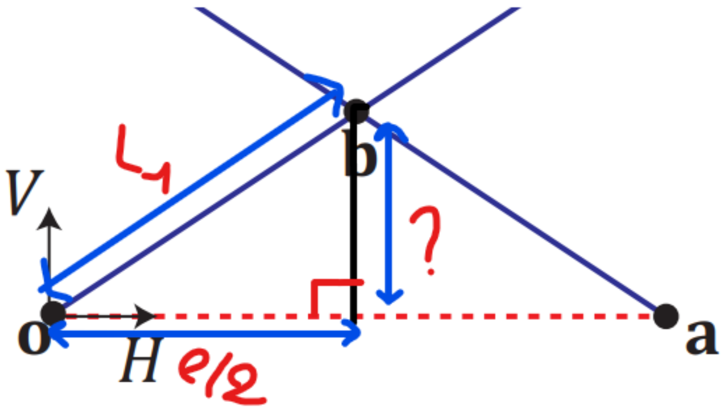
- $A_z = 0$

→ For the coordinates of B:

It is known that the coordinates as a function of x corresponds to the evolution of rho divided by two. Indeed, in order for the system to keep its equilibrium, the center of the system must follow the evolution of the distance traveled.

We thus have : $B_x = \frac{\rho(x)}{2}$

We then apply the Pythagorean theorem to our system in order to obtain the height as a function of rho.



Let "a" be the unknown, we have by application of the Pythagorean theorem :

$$L_1^{\ 2} = \frac{\rho(x)}{2}^2 + a^2$$

$$\Leftrightarrow a = \sqrt{L_1^{\ 2} - \frac{\rho(x)^2}{4}}$$

So: - $B_x = \frac{\rho(x)}{2}$

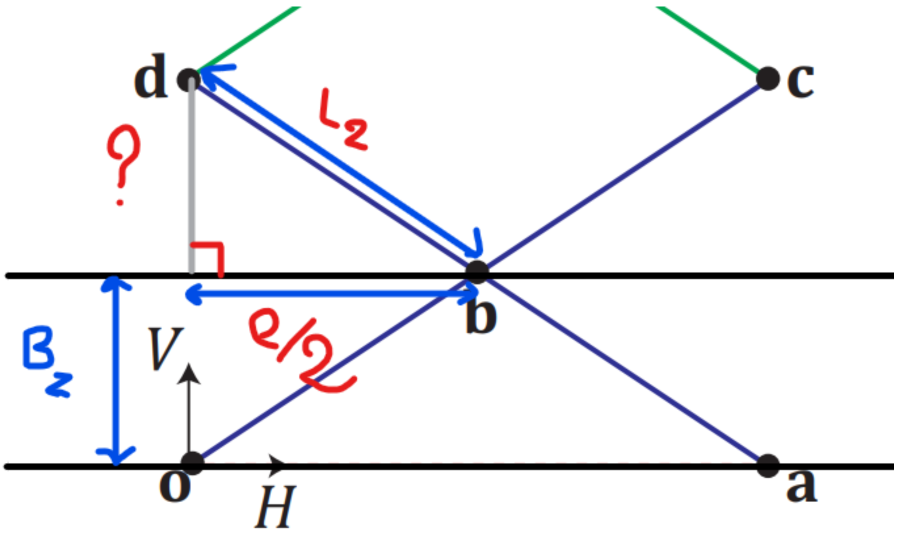
- $B_z = \sqrt{L_1^{\ 2} - \frac{\rho(x)^2}{4}}$

→ For the coordinates of D and C:

We know that: $D_x = 0$ et $C_x = \rho(x)$.

Since we have calculated the height of point B as a function of rho, it is sufficient to add the height obtained with the height between the level of B and the level of the next point. We will thus obtain C_x and D_x because they are located on the same level.

In order to calculate this unknown height, which we will note "a", we will again apply the Pythagorean theorem.



$L_2^{\ 2} = \frac{\rho(x)}{2}^2 + a^2$

$\Leftrightarrow a = \sqrt{L_2^{\ 2} - \frac{\rho(x)^2}{4}}$

So : - $D_x = 0$ et $C_x = \rho(x)$

- $C_z = D_z = \sqrt{L_1^{\ 2} - \frac{\rho(x)^2}{4}}$

→ <u>For the coordinates of E :</u>

We know that: $E_x = \frac{\rho(x)}{2}$

We apply the same method as for obtaining Bz later, except that here we will add the height of C_z or D_z, to the result obtained by applying the Pythagorean theorem.

Let "a" be the unknown, we have by application of the Pythagorean theorem :

$L_3^{\ 2} = \frac{\rho(x)}{2}^2 + a^2$

$\Leftrightarrow a = \sqrt{L_3^{\ 2} - \frac{\rho(x)^2}{4}}$

D'où : - $E_x = \frac{\rho(x)}{2}$

- $E_z = C_z + \sqrt{L_3^{\ 2} - \frac{\rho(x)^2}{4}}$

→ <u>For the coordinates of F and G:</u>

We know that: $F_x = 0$ et $G_x = \rho(x)$.

As we have calculated the height of point E as a function of rho, it is sufficient to add the height obtained with the height situated between the level of E and the level of the next point. We will thus obtain F_z and G_z because they are located on the same level.

In order to calculate this unknown height, which we will note "a", we will again apply the Pythagorean theorem.

$$L_4^{\ 2} = \frac{\rho(x)}{2}^2 + a^2$$

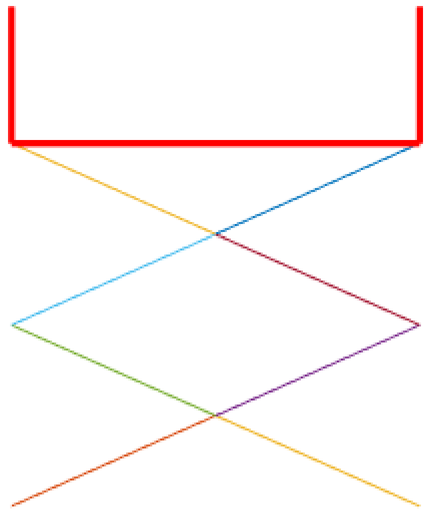
$$\Leftrightarrow a = \sqrt{L_4^{\ 2} - \frac{\rho(x)^2}{4}}$$

D'où : - $F_x = 0$ et $G_x = \rho(x)$

- $F_z = G_z = \sqrt{L_4^{\ 2} - \frac{\rho(x)^2}{4}}$

With the coordinates obtained, all that remains is to draw the lines between the points.

We thus obtain :



⇒ TASK 2

First, to obtain the joints that serve as connections between the rectangles, we must draw circles of radius 1 between the future rectangles.

Knowing that we know the coordinates of the different points : A, B, C, ... we just need to make circles at these points. By noting R the radius, and Z(i,1) the coordinates which evolve according to x and Z(i,2) the coordinates which evolve according to z, we have the following code:

$$x_0 = Z(i, 1) + R * cos(theta)$$

$$z_0 = Z(i, 2) + R * sin(theta)$$

$$p_0 = plot(x_0, z_0, 'k -- ', 'LineWidth', 1);$$

Then, in order to realize the TASK 2, we realized the following function:

```
%% Function

function [A,B,C,D] = rects(angle, coord, size)

% Code to obtain the arms of the system through the rotation matrix

    Rotz = [cos(angle), -sin(angle); sin(angle), cos(angle)]; % Rotation matrix

    A(1, 1:2) = Rotz*[-size(1);-size(2)];
    A(1, 1) = A(1,1) + coord(1);
    A(1, 2) = A(1,2) + coord(2);

    B(1, 1:2) = Rotz*[-size(1);size(2)];
    B(1, 1) = B(1,1) + coord(1);
    B(1, 2) = B(1,2) + coord(2);

    C(1, 1:2) = Rotz*[size(1);-size(2)];
    C(1, 1) = C(1,1) + coord(1);
    C(1, 2) = C(1,2) + coord(2);

    D(1, 1:2) = Rotz*[size(1);size(2)];
    D(1, 1) = D(1,1) + coord(1);
    D(1, 2) = D(1,2) + coord(2);

end
```

This function returns 4 coordinates, in order to form the requested rectangles. To make these rectangles, we will use the rotation matrix.

⇒ The parameter "size" allows us to make a rectangle with the desired dimensions. We want rectangles with length L_i and width 2, so we need to put the following sizes in a matrix $[\frac{L_i}{2}; 1]$.

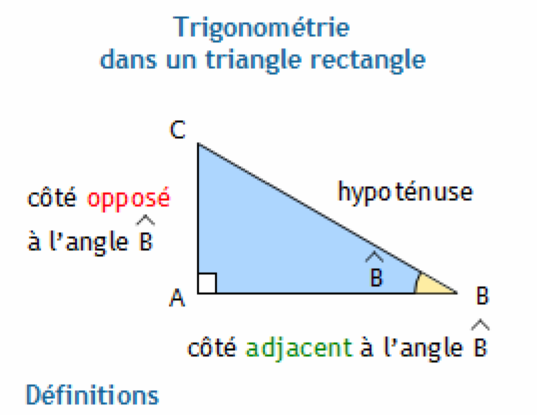
⇒ The parameter "coord" is the coordinates of the center of the rectangle.

⇒ The parameter "angle" corresponds to the angle that the rectangle must have.

This function will thus make it possible to obtain the rectangles for each link between the points.

⇒ Thus, we already know the values to put in "size" to form each rectangle. We have: $size = [\frac{L_i}{2}; 1]\ with\ i = [1, 2, 3, 4]$.

⇒ For the angle, we use the properties of right-angled triangles.



$$\sin \hat{B} = \frac{AC}{BC} = \frac{\text{côté opposé}}{\text{hypoténuse}}$$

$$\cos \hat{B} = \frac{AB}{BC} = \frac{\text{côté adjacent}}{\text{hypoténuse}}$$

$$\tan \hat{B} = \frac{AC}{AB} = \frac{\text{côté opposé}}{\text{côté adjacent}}$$

We use in the code of the TASK 2 the equation allowing to obtain the angle cos according to the adjacent side and the hypotenuse.

We have: angle = $angle \ = \ arccos(\frac{adj}{hyp})$
As we know the length of the hypotenuse and the adjacent side from the statement, we can obtain the angle for all the data.

⇒ To get "coord", we reuse the previous data and calculation methods from TASK 1, except that we must divide the arm length by two to get the center of the rectangle.

<u>→ For the rectangle OB:</u>

We easily have with the rules of kinematics :

- $OB_x = L_1 * cos(angle_{OB})/2$
- $OB_z = L_1 * sin(angle_{OB})/2$

<u>→ For the rectangle AB:</u>

It is enough to modify the length obtained for OB_x by adding the evolution of ρ.

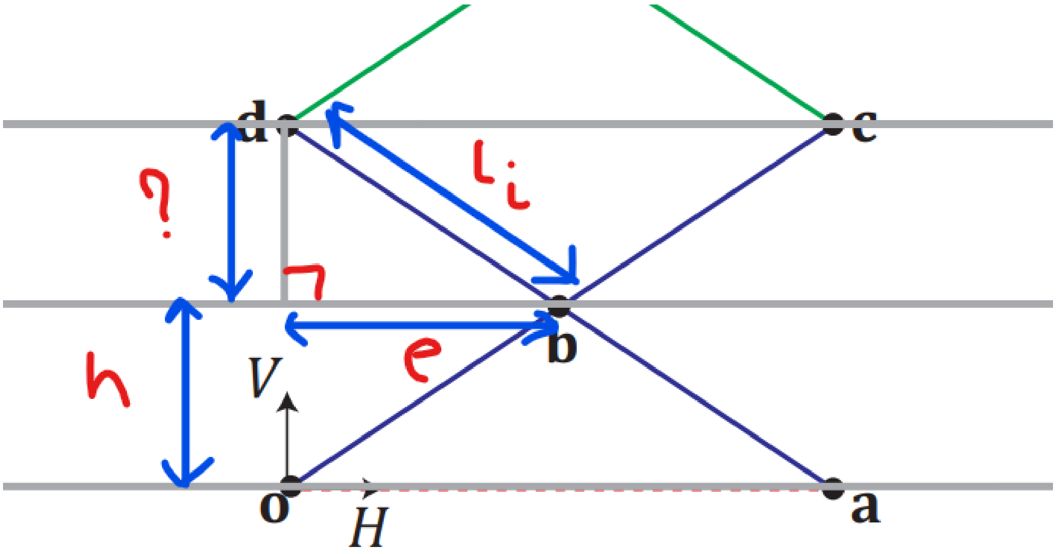
- $AB_x = L_1 * cos(angle_{OB})/2 \ + \ \rho(x)$
- $AB_z = L_1 * sin(angle_{OB})/2$

<u>→ Subsequently, for coordinates along x, we easily have :</u>

- $DB_x \ = \ DE_x \ = \ FE_x \ = \ \frac{\rho(x)}{4}$
- $CB_x \ = \ CE_x \ = \ FG \ = (1 - \frac{1}{4}) \ * \ \rho(x)$

<u>→ For coordinates along y</u>, we apply the calculations used for TASK 1 with the Pythagorean theorem.

Here's the theory:



We look for the "?" which we note "a".
By applying the Pythagorean theorem, we have :

$$L_i^2 = \rho^2 + a^2 <=> a = \sqrt{L_i^2 - \rho^2}$$

So we have the height which is a. Thus, the center of the future rectangle to be drawn, will be at a height of $\frac{a}{2}$.

Moreover, in order to have the height of the center of the final rectangle, we must add to the previously calculated length the height h, so we have :
$Height = h + \frac{a}{2}$.

So we have:

- $DB_z = CB_z = \sqrt{L_2^2 - \frac{\rho(x)^2}{4}} + B_z$
- $DE_z = CE_z = \sqrt{L_3^2 - \frac{\rho(x)^2}{4}} + D_z$
- $FE_z = GE_z = \sqrt{L_4^2 - \frac{\rho(x)^2}{4}} + E_z$

To finish, it remains to draw the lines between each point obtained by the function ***rects***.

We thus obtain :