UNIVERSITY NAME (IN BLOCK CAPITALS)

Thesis Title

by

Author Name

A thesis submitted in partial fulfillment for the
degree of Doctor of Philosophy

in the
Faculty Name
Department or School Name

March 2018

Declaration of Authorship

I, AUTHOR NAME, declare that this thesis titled, 'THESIS TITLE' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
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- Where I have consulted the published work of others, this is always clearly attributed.
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- I have acknowledged all main sources of help.
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"Write a funny quote here."

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UNIVERSITY NAME (IN BLOCK CAPITALS)

Abstract

Faculty Name
Department or School Name

Doctor of Philosophy

by Author Name

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too…

Acknowledgements

The acknowledgements and the people to thank go here, don't forget to include your project advisor...

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Abbreviations

LAH	**L**ist **A**bbreviations **H**ere

Physical Constants

Speed of Light $c = 2.997\ 924\ 58 \times 10^8\ \mathrm{ms}^{-\mathrm{S}}$ (exact)

Symbols

a	distance	m
P	power	W (Js^{-1})
ω	angular frequency	$rads^{-1}$

For/Dedicated to/To my…

Chapter 1

Method

1.1 Data Generation

The evolution of the condensate phase was simulated using code written in C++. This defined the codensate as a square lattice consisting of N^2 points, where N is the linear size of the lattice, and each point restricted to be between $[0, 2\pi]$. Initially, the angle of each point was random, the system therefore being in the disordered phase. The simulation updated each lattice point according to the compactified and discretised equation:

$$\begin{aligned}\theta_{i,j}(t+\mathrm{d}t) = \theta_{i,j}(t) + \mathrm{d}t\,[&- D_x(\cos(\theta_{i,j} - \theta_{i+1,j}) + \cos(\theta_{i,j} - \theta_{i-1,j}) - 2) \\ &- D_y(\cos(\theta_{i,j} - \theta_{i,j-1}) + \cos(\theta_{i,j} - \theta_{i,j+1}) - 2) \\ &- \frac{\lambda_x}{2}(\sin(\theta_{i,j} - \theta_{i+1,j}) + \sin(\theta_{i,j} - \theta_{i-1,j})) \\ &- \frac{\lambda_y}{2}(\sin(\theta_{i,j} - \theta_{i,j-1}) + \sin(\theta_{i,j} - \theta_{i,j+1}))] \\ &+ 2\pi c_L \times \sqrt{\mathrm{d}t} \times \xi\end{aligned}$$

where $\theta_{i,j}(t)$ is the value of the condensate at points i, j of the lattice and $\mathrm{d}t$ is the timestep used. Periodic boundary conditions were used. The final term is the stochastic term where ξ is a uniformaly distributed random number (restricted to $[-0.5, 0.5]$) that was also added at each timestep.

The timestep was initially chosen to be $\mathrm{d}t = 0.01$ and this was the value used to generate the significant data. However, other values were considered with: namely, $\mathrm{d}t = 0.02$ and $\mathrm{d}t = 0.001$ with system size 32. Figure. 1.1 comes the Binder cumulant for this system size for the three values of $\mathrm{d}t$. The evolution for $\mathrm{d}t = 0.02$ did not follow that of $\mathrm{d}t = 0.01$, implying that $\mathrm{d}t = 0.02$ was potentially too large of a timestep, whereas the behaviour of dt = 0.01 reasonably matched that of dt = 0.001, demonstrating that dt = 0.01 was an appropriate timestep for the simulation, and a smaller one was computationally unnecessary.

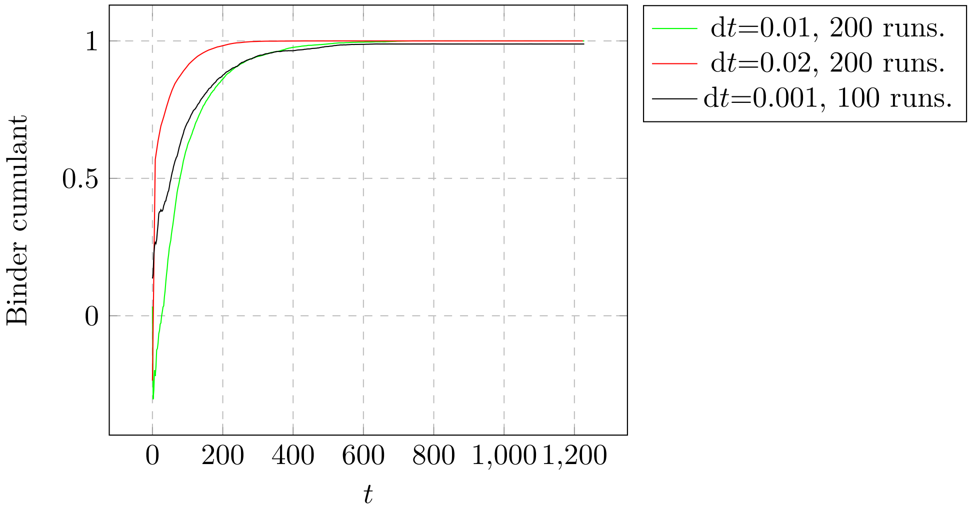

FIGURE 1.1: The Binder cumulant convergence for $L = 32$ at different values of dt. The convergence of dt = 0.001 compared to dt = 0.01 suggests that a timestep of dt = 0.001 was unnecessary, while the behaviour of dt = 0.02 although qualitatively correct, deviated from that dt = 0.02 significantly to be considered trustworthy.

The value of c_L at which the phase transition occurs was determined. Figure. 1.2 shows the number of vortices $t = 400$ for a system size of 64 at various values of c_L. The value $c_L = 0.2$ was then chosen for the remainder of project. Naively, after determining that $c_L = 0.2$ resulted in the Binder cumulant evolving from zero to near one in the linear evolution, $c_L = 0.1$ was also tested with the expectation that the convergence would occur faster. In fact, for $L = 128$, the Binder cumulant did not approach one at all, as shown in Figure. 1.3. This can be explained by the existence of vortices, which are higher in Figure. 1.2 for $c_L = 0.1$ than higher values below the transition.

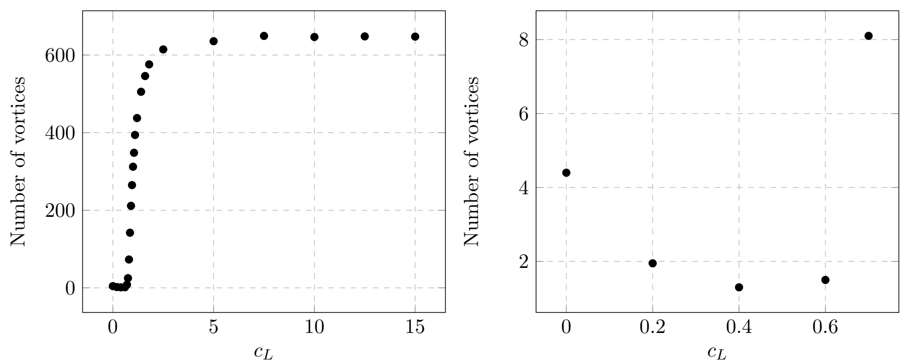

FIGURE 1.2: The number of vortices at the end of a simulation ($t = 400$) as a function of c_L for $L = 64$ with 20 runs.

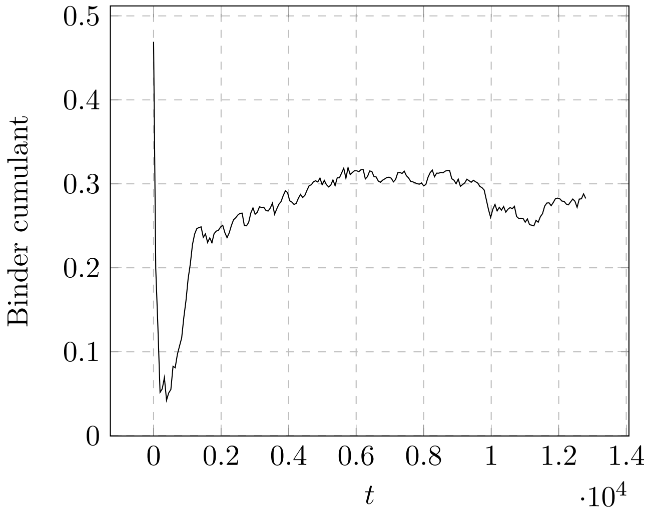

FIGURE 1.3: The Binder cumulant for $L = 128$ and $c_L = 0.1$. Despite being in the ordered phase, the Binder cumulant does not converge to near one due to the vortices.

1.2 Data Extraction

Using the generated data, the Binder cumulant was calculated as follows: for each timestep of a simulation, the magnetisation $\boldsymbol{M}$, defined by

$$\boldsymbol{M} = \frac{1}{N^2}\sum_{i,j}(\cos(\theta_{i,j}), \sin(\theta_{i,j})),$$

where the sum is over all lattice points, was calculated. The averages (over all the realisations) $\left\langle \boldsymbol{M}^2 \right\rangle$ and $\left\langle (\boldsymbol{M}^2)^2 \right\rangle$ were then used in the Binder cumulant given by

$$g = 2 - \frac{\left\langle (\boldsymbol{M}^2)^2 \right\rangle}{\left\langle \boldsymbol{M}^2 \right\rangle^2}$$

To estimate the error, the Binder cumulant was considered to be a function of the variables $\boldsymbol{M}^2 >$, i.e.

$$g = 2 - N\frac{\sum_i (\boldsymbol{M}_i^2)^2}{\left(\sum_i \boldsymbol{M}_i^2\right)^2}$$

where the sum is over every realisation and N is the number of realisations. These variables are indentical and independent, so the error of each is the same and is approximated by

$$\sigma^2_{\boldsymbol{M}^2} = \frac{1}{N-1}\sum_i(\boldsymbol{M}_i^2 - \left\langle \boldsymbol{M}^2 \right\rangle)^2 = \frac{N}{N-1}(\left\langle (\boldsymbol{M}^2)^2 \right\rangle - \left\langle \boldsymbol{M}^2 \right\rangle^2).$$

Using error propagation, the error on the Binder cumulant is

$$\begin{aligned}
\sigma_g^2 &= 4N^2 \sum_k \left(\frac{\boldsymbol{M}_k^2}{\left(-\sum_i \boldsymbol{M}_i^2\right)^2} + \frac{\sum_i (\boldsymbol{M}_i^2)^2}{\left(\sum_i \boldsymbol{M}_i^2\right)^3} \right)^2 \sigma_{\boldsymbol{M}^2} \\
&= \frac{4N^3}{N-1} \left(\left\langle (\boldsymbol{M}^2)^2 \right\rangle - \left\langle \boldsymbol{M}^2 \right\rangle^2 \right) \sum_k
\end{aligned}$$

Appendix A

An Appendix

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