Bayes Theorem: $P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$

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HW-1

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a)

(b) Undergrads who smoke = 0.15
Grad students who smoke = 0.23

$(.8)(\text{total students}) = \text{Ugrads}$
$(.2)(\text{totatal students}) = \text{grads}$

What is probability that student who smoke is graduate student?

$\Rightarrow$ ~~P(S|G) = P(G|X) P(S) / P(G)~~ $P(G|S) = \frac{P(S|G)\,P(G)}{P(S)}$ $\rightarrow [(.15\cdot.8)+(.23\cdot.2)]$

$$P(G|S) = \frac{(0.23)(0.2)}{(0.15\cdot(.8))+(0.23\cdot(.2))} = 0.277$$

b) A randomly chosen student has 80% chance being undergraduate so more likely undergrad.

~~c) A randomly chosen student smoker is more likely to be a grad student b/c (0.23) > (0.15)~~

~~d) P(D|G) = 0.3 P(U|D) = 0.1~~

c) $P(U|S) = \frac{P(S|U)\,P(U)}{P(S)} = \frac{(.15)(.8)}{(0.15\cdot(.8))+(0.23\cdot(.2))} = \frac{.12}{.166} = .72$

$P(G|S) = \frac{P(S|G)\,P(G)}{P(S)} = \frac{(.23)(.2)}{(0.15\cdot(.8))+(.23\cdot(.2))} = \frac{.046}{.166} = .27$

Randomly chosen smoker college student more likely to be Undergrad $(.72) > (.27)$

d) $P(D|U) = .1$ $\quad P(D|G) = .3$

$P(D) = (.8\cdot.1)+(.2\cdot.3) = .14$ $\Rightarrow$ Prob grad or Under lives in Dorm

$P(S) = (.15\cdot.8)+(.23\cdot.2) = .166$ $\Rightarrow$ Prob G or U is smoker

$P(DS|G) = (.3)\cdot(.23) = .069$ $\Rightarrow$ Prob Dorm Smoker given grad student

$P(DS|U) = (.1)\cdot(.15) = .015$ $\Rightarrow$ Prob Dorm Smoker given Undergrad student

(DS|G) $\quad P(G|DS) = \frac{P(DS)\cdot P(G)}{P(DS)} = \frac{.069\cdot(.2)}{P(DS)} = \frac{.0138}{P(DS)}$

(DS|U) $\quad P(U|DS) = \frac{P(DS)\cdot P(U)}{P(DS)} = \frac{.015\cdot.8}{DS} = \frac{.012}{P(DS)}$

$\boxed{P(DS) = (.069)(.2)+(.015)(.8)}$

$.0138 > .012$

So student who smoke & lives in dorm is most likely a Graduate student.

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(8) a) $P(A=1|+) = 3/5$ $P(A=1|-) = 2/5$
$P(B=1|+) = 2/5$ $P(B=1|-) = 2/5$
$P(C=1|+) = 4/5$ $P(c=1|-) = 1/5$

b) $\boxed{A=1, B=1, C=1}$ Class = R

$$P(+|R) = \frac{P(R|+)\,P(+)}{P(R)} = \frac{P(A=1|+) \times P(B=1|+) \times P(C=1|+)\,(.5)}{P(R)}$$

$$= \frac{(3/5)(2/5)(4/5)(1/2)}{P(R)} = \frac{.096}{P(R)}$$

$$P(-|R) = \frac{P(R|-)\,P(-)}{P(R)} = \frac{(2/5)(2/5)(1/5)(1/2)}{P(R)} = \frac{.016}{P(R)}$$

$.096 > .016$ so $\boxed{A=1\ B=1\ C=1\ \text{Class} = +}$

c) $P(A=1) = 5/10 = 1/2$ $P(A=1, B=1) = \frac{1}{2} \cdot \frac{2}{5} = \frac{2}{10} = .2$
$P(B=1) = 4/10 = 2/5$ A & B are independent probabilities. They are mutually exclusive

d) $P(A=1) = 1/2$ $P(A=1, B=0) = \frac{1}{2} \cdot \frac{3}{5} = 3/10 = .3$
$P(B=0) = 3/5$ A & B are independent & mutually exclusive

e) $P(A=1, B=1|+) = \frac{1}{5}$ $\frac{1}{5} \neq \frac{3}{5}\left(\frac{2}{5}\right)$
$P(A=1|+) = 3/5$
$P(B=1|+) = 2/5$ A & B are dependent on Class being +

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10a

$P(\text{Crocodile}) = \boxed{2 \times \boxed{P(\text{Alligator})}}$ $\cancel{A}$

$P(\text{crocodile}/X) = P(\text{Alligator}/X)$

or

Use Bayes to Expand Both sides

$$\frac{P(\cancel{X}/\text{croc})\,P(\text{croc})}{P(X)} = \frac{P(X/\text{alligator})\,P(\text{alligator})}{P(X)}$$

Substitute $\cancel{A}$ $P(X/\text{croc})\,2\,\cancel{P(\text{Alligator})} = P(X/\text{alligator})\,\cancel{P(\text{alligator})}$

$$P(X/\text{croc})\,2 = P(X/\text{alligator})$$

$$(2)\,\frac{1}{\cancel{\sqrt{2\pi}\,\cdot 2\sigma}}\,e^{\left[-\frac{1}{2}\left(\frac{X-15}{2\sigma}\right)^2\right]} = \frac{1}{\cancel{\sqrt{2\pi}\cdot 2\sigma}}\,e^{\left[-\frac{1}{2}\left(\frac{X-12}{2\sigma}\right)^2\right]}$$

$$2e^{\left[-\frac{1}{2}\left(\frac{X-15}{2}\right)^2\right]} = e^{\left[-\frac{1}{2}\left(\frac{X-12}{2}\right)^2\right]}$$

$$\log(2) + \left[-\frac{1}{2}\left(\frac{X-15}{2}\right)^2\right] = \left[-\frac{1}{2}\left(\frac{X-12}{2}\right)^2\right]$$

$$2\log 2 = \left(\frac{X-15}{2}\right)^2 - \left(\frac{X-12}{2}\right)^2$$

$$8\log 2 = (X-15)^2 - (X-12)^2$$

$$8\log 2 = \cancel{X^2} - 30x + 15^2 - \cancel{X^2} + 24x - 12^2$$

$$8\log 2 = -6x + 81$$

$$\begin{array}{r} 225 \\ -144 \\ \hline 81 \end{array}$$

$$\boxed{\frac{8\log(2) - 81}{-6} = X}$$

10b

$$2\,p(\text{crocodile}) = p(\text{alligator})$$

Apply Bayes to expand

$$2P(\text{croc}|x) = P(\text{all}|x)$$

$$\frac{2P(x|\text{croc})P(\text{croc})}{P(x)} = \frac{P(x|\text{all})P(\text{all})}{P(x)}$$

Now substitute
ignore $P(x)$ denominator

$$\frac{P(x|\text{croc})\cancel{P(\text{croc})}}{\cancel{P(x)}} = P(x|\text{all})\,2\cancel{P(\text{croc})}$$

$$P(x|\text{croc}) = 2P(x|\text{all})$$

$$\cancel{\frac{1}{\sqrt{2\pi}\cdot 2}}\,e^{\left(-\frac{1}{2}\left(\frac{x-15}{2}\right)^2\right)} = 2\cdot\cancel{\frac{1}{\sqrt{2\pi}\cdot 2}}\,e^{\left(-\frac{1}{2}\left(\frac{x-12}{2}\right)^2\right)}$$

log of both sides:

$$e^{\left(-\frac{1}{2}\left(\frac{x-15}{2}\right)^2\right)} = 2e^{\left(-\frac{1}{2}\left(\frac{x-12}{2}\right)^2\right)}$$

$$-\frac{1}{2}\left(\frac{x-15}{2}\right)^2 = \log 2 + \left[-\frac{1}{2}\left(\frac{x-12}{2}\right)^2\right]$$

$$\left(\frac{x-12}{2}\right)^2 - \left(\frac{x-15}{2}\right)^2 = 2\log 2$$

$$(x-12)^2 - (x-15)^2 = 8\log 2$$

$$-6x + 81 = 8\log 2$$

$$\boxed{x = \frac{8\log(2) - 81}{-6}}$$

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(10c) $\sigma_{crocodile}=4$ | $\sigma_{alligator}=2$

$P(\text{crocodile}) = P(\text{alligator})$

$P(\text{croc}|x) = P(\text{all}|x)$

$$\frac{1}{\cancel{\sqrt{2\pi}}\cdot 4} e^{\left[-\frac{1}{2}\left(\frac{x-15}{4}\right)^2\right]} = \frac{1}{\cancel{\sqrt{2\pi}}\cdot 2} e^{\left[-\frac{1}{2}\left(\frac{x-12}{2}\right)^2\right]}$$

$$\frac{1}{4} e^{\left[-\frac{1}{2}\left(\frac{x-15}{4}\right)^2\right]} = \frac{1}{2} e^{\left[-1/2\left(\frac{x-12}{2}\right)^2\right]}$$

Solve for X to get.

$$X = \frac{1}{3}\left(33 - 2\sqrt{3(3+8\log(2))}\right)$$

and

$$X = \frac{1}{3}\left(33 + 2\sqrt{3(3+8\log(2))}\right)$$

I USED WOLFRAM Alpha to solve