Formal Method Mod. 2 (Model Checking)

Laboratory 9

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Outline

UNIVERSITÀ DEGLI STUDI DI TRENTO

Planning Problem

Planning Problem

Given $\langle I, G, T \rangle$, where

- **I**: (representation of) initial state
- **G**: (representation of) goal state
- **T**: transition relation

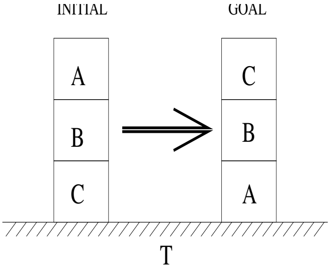
find a sequence of transitions $t_1, ..., t_n$ leading from the initial state to the goal state.

Idea

Encode planning problem as a model checking problem, such that plan is provided as counter-example for the property.

1. impose **I** as initial state
2. encode **T** as transition relation system
3. verify the LTL property **! (F goal_state)**

Example: blocks [1/9]



$$
\begin{aligned}
&Init: && On(A,B), On(B,C), On(C,T), Clear(A)\\
&Goal: && On(C,B), On(B,A), On(A,T)\\
&Move(a,b,c) && \\
&\quad Precond: && Block(a) \wedge Clear(a) \wedge On(a,b) \wedge\\
& && (Clear(c) \vee Table(c)) \wedge\\
& && a \neq b \wedge a \neq c \wedge b \neq c\\
&\quad Effect: && Clear(b) \wedge \neg On(a,b) \wedge\\
& && On(a,c) \wedge \neg Clear(c)
\end{aligned}
$$

Example: blocks [2/9]

```
MODULE block(id, ab, bl)
VAR
  above : {none, a, b, c}; -- the block above this one
  below : {none, a, b, c}; -- the block below this one
DEFINE
  clear := (above = none);
INIT
  above = ab &
  below = bl
-- a block can't be above or below itself
INVAR below != id & above != id


MODULE main
VAR
  -- at each step only one block moves
  move : {move_a, move_b, move_c};
  block_a : block(a, none, b);
  block_b : block(b, a, c);
  block_c : block(c, b, none);
...
```

Example: blocks [3/9]

- a `moving` `block` changes location and remains clear

```
TRANS
  (move = move_a -> next(block_a.clear) &
                    next(block_a.below) != block_a.below) &
  (move = move_b -> next(block_b.clear) &
                    next(block_b.below) != block_b.below) &
  (move = move_c -> next(block_c.clear) &
                    next(block_c.below) != block_c.below)
```

- a `non-moving` `block` does not change its location

```
TRANS
  (move != move_a -> next(block_a.below) = block_a.below) &
  (move != move_b -> next(block_b.below) = block_b.below) &
  (move != move_c -> next(block_c.below) = block_c.below)
```

Example: blocks [4/9]

- a block remains connected to any non-moving block

```
TRANS
  (move != move_a & block_b.above = a
                         -> next(block_b.above) = a) &
  (move != move_a & block_c.above = a
                         -> next(block_c.above) = a) &
  (move != move_b & block_a.above = b
                         -> next(block_a.above) = b) &
  (move != move_b & block_c.above = b
                         -> next(block_c.above) = b) &
  (move != move_c & block_a.above = c
                         -> next(block_a.above) = c) &
  (move != move_c & block_b.above = c
                         -> next(block_b.above) = c)
```

Example: blocks [4/9]

- a block remains connected to any `non-moving` block

```
TRANS
  (move != move_a & block_b.above = a
                    -> next(block_b.above) = a) &
  (move != move_a & block_c.above = a
                    -> next(block_c.above) = a) &
  (move != move_b & block_a.above = b
                    -> next(block_a.above) = b) &
  (move != move_b & block_c.above = b
                    -> next(block_c.above) = b) &
  (move != move_c & block_a.above = c
                    -> next(block_a.above) = c) &
  (move != move_c & block_b.above = c
                    -> next(block_b.above) = c)
```

- **Q:** what about "below block"?

Example: blocks [4/9]

- a `block` remains connected to any `non-moving` block

```
TRANS
  (move != move_a & block_b.above = a
                  -> next(block_b.above) = a) &
  (move != move_a & block_c.above = a
                  -> next(block_c.above) = a) &
  (move != move_b & block_a.above = b
                  -> next(block_a.above) = b) &
  (move != move_b & block_c.above = b
                  -> next(block_c.above) = b) &
  (move != move_c & block_a.above = c
                  -> next(block_a.above) = c) &
  (move != move_c & block_b.above = c
                  -> next(block_b.above) = c)
```

- **Q:** what about "below block"?
  A: covered in previous slide!

Example: blocks [5/9]

- positioning of blocks is symmetric: above and below relations must be symmetric.

```
INVAR
  (block_a.above = b <-> block_b.below = a)
& (block_a.above = c <-> block_c.below = a)
& (block_b.above = a <-> block_a.below = b)
& (block_b.above = c <-> block_c.below = b)
& (block_c.above = a <-> block_a.below = c)
& (block_c.above = b <-> block_b.below = c)

& (block_a.above = none ->
      (block_b.below != a & block_c.below != a))
& (block_b.above = none ->
      (block_a.below != b & block_c.below != b))
& (block_c.above = none ->
      (block_a.below != c & block_b.below != c))

& (block_a.below = none ->
      (block_b.above != a & block_c.above != a))
& (block_b.below = none ->
      (block_a.above != b & block_c.above != b))
& (block_c.below = none ->
      (block_a.above != c & block_b.above != c))
```

Example: blocks [6/9]

- a block cannot move if it has some other block above itself

```
...
TRANS
  (!next(block_a.clear) -> next(move) != move_a) &
  (!next(block_b.clear) -> next(move) != move_b) &
  (!next(block_c.clear) -> next(move) != move_c)
...
```

Example: blocks [6/9]

- a block cannot move if it has some other block above itself

```
...
TRANS
  (!next(block_a.clear) -> next(move) != move_a) &
  (!next(block_b.clear) -> next(move) != move_b) &
  (!next(block_c.clear) -> next(move) != move_c)
...
```

- **Q:** what's wrong with following formulation?

```
...
TRANS
  (next(block_a.clear) -> next(move) = move_a) &
  (next(block_b.clear) -> next(move) = move_b) &
  (next(block_c.clear) -> next(move) = move_c)
...
```

Example: blocks [6/9]

- a block cannot move if it has some other block above itself

```
...
TRANS
  (!next(block_a.clear) -> next(move) != move_a) &
  (!next(block_b.clear) -> next(move) != move_b) &
  (!next(block_c.clear) -> next(move) != move_c)
...
```

- **Q:** what's wrong with following formulation?

```
...
TRANS
  (next(block_a.clear) -> next(move) = move_a) &
  (next(block_b.clear) -> next(move) = move_b) &
  (next(block_c.clear) -> next(move) = move_c)
...
```

A:

- `move` can only have **one** valid value $\Longrightarrow$ inconsistency whenever there are two clear blocks at the same time
- any `non-clear block` would still be able to `move`
- same for "iff" formulation

Example: blocks [7/9]

Remark

A **plan** is a sequence of transitions/actions leading from the initial state to an accepting/goal state.

Idea

- assert property p: "goal state is not reachable"
- if a plan **exists**, nuXmv produces a counterexample for p
- the counterexample for p is a plan to reach the goal

Example: blocks [8/9]

Examples

- get a plan for reaching “goal state”

```
LTLSPEC
  ! F(block_a.below = none & block_a.above = b &
      block_b.below = a & block_b.above = c &
      block_c.below = b & block_c.above = none)
```

Example: blocks [8/9]

Examples

- get a plan for reaching "goal state"

```
LTLSPEC
  ! F(block_a.below = none & block_a.above = b &
      block_b.below = a & block_b.above = c &
      block_c.below = b & block_c.above = none)
```

- get a plan for reaching a configuration in which all blocks are placed on the table

```
LTLSPEC
  ! F(block_a.below = none & block_b.below = none &
      block_c.below = none)
```

Example: blocks [9/9]

- at any given time, at least one block is placed on the table

```
INVARSPEC
  block_a.below = none | block_b.below = none |
  block_c.below = none
```

Example: blocks [9/9]

- at any given time, at least one block is placed on the table

```
INVARSPEC
  block_a.below = none | block_b.below = none |
  block_c.below = none
```

- at any given time, at least one block has nothing above

```
INVARSPEC
  block_a.above = none | block_b.above = none |
  block_c.above = none
```

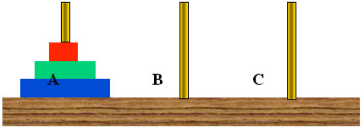
Outline

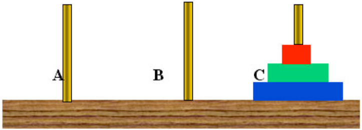
1. Planning problem
2. Examples
   - The Tower of Hanoi
   - Ferryman
   - Tic-Tac-Toe
3. Exercises

Example: tower of hanoi [1/5]

Game with 3 `poles` and N `disks` of different sizes:

- **initial state:** stack of disks with decreasing size on pole A
- **goal state:** move stack on pole C
- **rules:**
  - only one disk may be moved at each transition
  - only the upper disk can be moved
  - a disk can not be placed on top of a smaller disk





Example: tower of hanoi [2/5]

- base system model

```
MODULE main
VAR
  d1 : {left,middle,right}; -- smallest
  d2 : {left,middle,right};
  d3 : {left,middle,right};
  d4 : {left,middle,right}; -- largest
  move : 1..4; -- possible moves
```

Example: tower of hanoi [2/5]

- base system model

```
MODULE main
VAR
  d1 : {left,middle,right}; -- smallest
  d2 : {left,middle,right};
  d3 : {left,middle,right};
  d4 : {left,middle,right}; -- largest
  move : 1..4; -- possible moves
```

- disk *i* is moving

```
DEFINE
  move_d1 := (move = 1);
  move_d2 := (move = 2);
  move_d3 := (move = 3);
  move_d4 := (move = 4);
...
```

Example: tower of hanoi [2/5]

- base system model

```
MODULE main
VAR
  d1 : {left,middle,right}; -- smallest
  d2 : {left,middle,right};
  d3 : {left,middle,right};
  d4 : {left,middle,right}; -- largest
  move : 1..4; -- possible moves
```

- disk i is moving

```
DEFINE
  move_d1 := (move = 1);
  move_d2 := (move = 2);
  move_d3 := (move = 3);
  move_d4 := (move = 4);
...
```

- disk d_i can move if a smaller disk is above him (i.e. they share the same column)

```
  clear_d1 := TRUE;
  clear_d2 := d2!=d1;
  clear_d3 := d3!=d1 & d3!=d2;
  clear_d4 := d4!=d1 & d4!=d2 & d4!=d3;
```

Example: tower of hanoi [3/5]

- initial state

```
INIT
  d1 = left &
  d2 = left &
  d3 = left &
  d4 = left & move = 1;
```

Example: tower of hanoi [3/5]

- initial state

```
INIT
  d1 = left &
  d2 = left &
  d3 = left &
  d4 = left & move = 1;
```

- move description for `disk` 4

```
TRANS
    move_d4 ->
    -- disks location changes
    next(d1) = d1 &
    next(d2) = d2 &
    next(d3) = d3 &
    next(d4) != d4 &
    -- d4 can not move on top of smaller disks
    next(d4) != d1 &
    next(d4) != d2 &
    next(d4) != d3
```

Example: tower of hanoi [4/5]

- If in the next iteration a disk is not clear, you cannot move it.

```
TRANS
(next(clear_d3) = FALSE) -> (next(move) != 3)
TRANS
(next(clear_d2) = FALSE) -> (next(move) != 2)
TRANS
(next(clear_d1) = FALSE) -> (next(move) != 1)
TRANS
(next(clear_d4) = FALSE) -> (next(move) != 4)
```

Example: tower of hanoi [4/5]

- If in the next iteration a disk is not clear, you cannot move it.

```
TRANS
(next(clear_d3) = FALSE) -> (next(move) != 3)
TRANS
(next(clear_d2) = FALSE) -> (next(move) != 2)
TRANS
(next(clear_d1) = FALSE) -> (next(move) != 1)
TRANS
(next(clear_d4) = FALSE) -> (next(move) != 4)
```

- If all columns are being used, do not choose as next move the largest disk (or we would reach a deadlock).

```
TRANS
(next(clear_d1) & next(clear_d2) & next(clear_d3)) -> next(move) != 3
TRANS
(next(clear_d1) & next(clear_d2) & next(clear_d4)) -> next(move) != 4
TRANS
(next(clear_d4) & next(clear_d2) & next(clear_d3)) -> next(move) != 4
TRANS
(next(clear_d1) & next(clear_d3) & next(clear_d4)) -> next(move) != 4
```

Example: tower of hanoi [5/5]

- get a plan for reaching “goal state”

```
LTLSPEC
 ! F(d1=right & d2=right & d3=right & d4=right)
INVARSPEC
  !(d1=right & d2=right & d3=right & d4=right)
```

Example: ferryman [1/4]

A ferryman has to bring a sheep, a cabbage, and a wolf safely across a river.

- **initial state:** all animals are on the right side
- **goal state:** all animals are on the left side
- **rules:**
  - the ferryman can cross the river with at most one passenger on his boat
  - the cabbage and the sheep can not be left unattended on the same side of the river
  - the sheep and the wolf can not be left unattended on the same side of the river

Q: can the ferryman transport all the goods to the other side safely?

Example: ferryman [2/4]

- base system model

```
MODULE main
VAR
  cabbage : {right,left};
  sheep   : {right,left};
  wolf    : {right,left};
  man     : {right,left};
  move    : {c, s, w, e}; -- possible moves

DEFINE
  carry_cabbage := (move = c);
  carry_sheep   := (move = s);
  carry_wolf    := (move = w);
  no_carry      := (move = e);
```

Example: ferryman [2/4]

- base system model

```
MODULE main
VAR
  cabbage : {right,left};
  sheep   : {right,left};
  wolf    : {right,left};
  man     : {right,left};
  move    : {c, s, w, e}; -- possible moves

DEFINE
  carry_cabbage := (move = c);
  carry_sheep   := (move = s);
  carry_wolf    := (move = w);
  no_carry      := (move = e);
```

- initial state

```
ASSIGN
  init(cabbage) := right;
  init(sheep)   := right;
  init(wolf)    := right;
  init(man)     := right;
```

Example: ferryman [3/4]

- ferryman carries cabbage

```
TRANS
  carry_cabbage ->
    next(cabbage) != cabbage &
    next(man) != man &
    next(sheep) = sheep &
    next(wolf) = wolf
```

Example: ferryman [3/4]

- ferryman carries cabbage

```
TRANS
  carry_cabbage ->
    next(cabbage) != cabbage &
    next(man) != man &
    next(sheep) = sheep &
    next(wolf) = wolf
```

- ferryman carries sheep

```
TRANS
  carry_sheep ->
    next(sheep) != sheep &
    next(man) != man &
    next(cabbage) = cabbage &
    next(wolf) = wolf
```

Example: ferryman [3/4]

- ferryman carries cabbage

```
TRANS
  carry_cabbage ->
    next(cabbage) != cabbage &
    next(man) != man &
    next(sheep) = sheep &
    next(wolf) = wolf
```

- ferryman carries sheep

```
TRANS
  carry_sheep ->
    next(sheep) != sheep &
    next(man) != man &
    next(cabbage) = cabbage &
    next(wolf) = wolf
```

- ferryman carries wolf

```
TRANS
  carry_wolf ->
    next(wolf) != wolf &
    next(man) != man &
    next(sheep) = sheep &
    next(cabbage) = cabbage
```

Example: ferryman [3/4]

- ferryman carries cabbage

```
TRANS
  carry_cabbage ->
    next(cabbage) != cabbage &
    next(man) != man &
    next(sheep) = sheep &
    next(wolf) = wolf
```

- ferryman carries sheep

```
TRANS
  carry_sheep ->
    next(sheep) != sheep &
    next(man) != man &
    next(cabbage) = cabbage &
    next(wolf) = wolf
```

- ferryman carries wolf

```
TRANS
  carry_wolf ->
    next(wolf) != wolf &
    next(man) != man &
    next(sheep) = sheep &
    next(cabbage) = cabbage
```

- ferryman carries nothing

```
TRANS
  no_carry ->
    next(man) != man &
    next(sheep) = sheep &
    next(cabbage) = cabbage &
    next(wolf) = wolf
```

Example: ferryman [4/4]

- If the man is not in the same side of an animal, we cannot choose it for the next movement (otherwise deadlock).

```
TRANS
    next(man) != next(cabbage) -> next(move) != c
TRANS
    next(man) != next(sheep) -> next(move) != s
TRANS
    next(man) != next(wolf) -> next(move) != w
```

- get a plan for reaching "goal state"

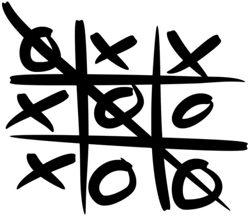
```
DEFINE
  safe_state := (sheep = wolf | sheep = cabbage) -> sheep = man;
  goal := cabbage = left & sheep = left & wolf = left;

LTLSPEC
  ! (safe_state U goal)
```

Example: tic-tac-toe [1/5]

Tic-tac-toe is a turn-based game for two adversarial players (X and O) marking the squares of a board ($\rightarrow$ a 3×3 grid). The player who succeeds in placing three respective marks in a horizontal, vertical or diagonal row wins the game.

- Example: `O` wins



- we model tic-tac-toe puzzle as an array of size nine

```
 1  | 2  | 3
____|____|____
 4  | 5  | 6
____|____|____
 7  | 8  | 9
    |    |
```

Example: tic-tac-toe [2/5]

- base system model

```
MODULE main
VAR
  B : array 1..9 of {0,1,2};
  player : 1..2;
  move : 0..9;
```

Example: tic-tac-toe [2/5]

- base system model

```
MODULE main
VAR
  B : array 1..9 of {0,1,2};
  player : 1..2;
  move : 0..9;
```

- initial state

```
INIT
  B[1] = 0 &
  B[2] = 0 &
  B[3] = 0 &
  B[4] = 0 &
  B[5] = 0 &
  B[6] = 0 &
  B[7] = 0 &
  B[8] = 0 &
  B[9] = 0;
INIT
  move = 0;
```

Example: tic-tac-toe [3/5]

- turns modeling

```
ASSIGN
  init(player) := 1;
  next(player) :=
    case
      player = 1 : 2;
      player = 2 : 1;
    esac;
```

Example: tic-tac-toe [3/5]

- turns modeling

```
ASSIGN
  init(player) := 1;
  next(player) :=
    case
      player = 1 : 2;
      player = 2 : 1;
    esac;
```

- move modeling

```
TRANS
  B[1] != 0 -> next(move) != 1
TRANS
  next(move) = 1 ->
    next(B[1]) = player &
    next(B[2])=B[2] &
    next(B[3])=B[3] &
    next(B[4])=B[4] &
    next(B[5])=B[5] &
    next(B[6])=B[6] &
    next(B[7])=B[7] &
    next(B[8])=B[8] &
    next(B[9])=B[9]
```

Example: tic-tac-toe [4/5]

- “end” state

```
DEFINE
 win1 := (B[1]=1 & B[2]=1 & B[3]=1) | (B[4]=1 & B[5]=1 & B[6]=1) |
         (B[7]=1 & B[8]=1 & B[9]=1) | (B[1]=1 & B[4]=1 & B[7]=1) |
         (B[2]=1 & B[5]=1 & B[8]=1) | (B[3]=1 & B[6]=1 & B[9]=1) |
         (B[1]=1 & B[5]=1 & B[9]=1) | (B[3]=1 & B[5]=1 & B[7]=1);

 win2 := (B[1]=2 & B[2]=2 & B[3]=2) | (B[4]=2 & B[5]=2 & B[6]=2) |
         (B[7]=2 & B[8]=2 & B[9]=2) | (B[1]=2 & B[4]=2 & B[7]=2) |
         (B[2]=2 & B[5]=2 & B[8]=2) | (B[3]=2 & B[6]=2 & B[9]=2) |
         (B[1]=2 & B[5]=2 & B[9]=2) | (B[3]=2 & B[5]=2 & B[7]=2);

 draw := !win1 & !win2 &
         B[1]!=0 & B[2]!=0 & B[3]!=0 & B[4]!=0 &
         B[5]!=0 & B[6]!=0 & B[7]!=0 & B[8]!=0 & B[9]!=0;

TRANS
  (win1 | win2 | draw) <-> next(move)=0
```

Example: tic-tac-toe [5/5]

- We can easily check if there is a way to reach every end state using the typical formulation:

```
LTLSPEC
    ! (F draw)
LTLSPEC
    ! (F win1)
LTLSPEC
    ! (F win2)
```

For each property, an execution satisfying the property is returned as counterexample.

Outline

1. Planning problem
2. Examples
3. Exercises

Exercises [1/3

Tower of Hanoi

Extend the tower of hanoi to handle five disks, and check that the goal state is reachable.

Exercises [2/3]

Ferryman

Another ferryman has to bring a fox, a chicken, a caterpillar and a crop of lettuce safely across a river.

- **initial state:** all goods are on the right side
- **goal state:** all goods are on the left side
- **rules:**
  - the ferryman can cross the river with at most **two** passengers on his boat
  - the fox eats the chicken if left unattended on the same side of the river
  - the chicken eats the caterpillar if left unattended on the same side of the river
  - the caterpillar eats the lettuce if left unattended on the same side of the river

Can the ferryman bring every item safely on the other side?

Exercises [3/3]

Sudoku

Encode in an SMV model the game of Sudoku, write a property so that nuXmv finds the solution.
You can find the rules on Wikipedia.

Tip

Use a `MODULE` to avoid repetitions of the same constraints.
220 lines are enough.