Critical Neural Networks in Atari Games

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***Abstract*—**

I. Introduction

II. Background

A. Reinforcement Learning

B. Statistical Mechanics

Criticality in statistical mechanics refers to phase transitions between states of matter. At criticality, systems exhibit "scale-free" correlations where microscopic and macroscopic scales become indistinguishable, manifesting as power-law distributions and long-range correlations that span all space and time scales of the system.

The Ising model is often used in studying criticality, where atoms in a lattice have spin states $s_i \in \{+1, -1\}$. Each atom s_i couples to others with parameter J_{ij} and experiences local bias from a field h_i. The energy of a configuration $\vec{s}$ is:

$$E(\vec{s}) = \sum_{i \neq j} J_{ij} s_i s_j + \sum_i h_i s_i \tag{1}$$

This defines a Gibbs distribution over the states $\vec{s}$:

$$\mathbb{P}(\vec{s}) = \frac{1}{Z} \exp(-\beta E(\vec{s})) \tag{2}$$

with partition function Z and inverse pseudotemperature β.

For neural systems, the mean-field approximation of highly connected Ising models is particularly relevant. In this approximation, the expected value of each spin is:

$$\langle s_i \rangle = \tanh(\beta(\sum_j J_{ij} \langle s_j \rangle + h_i)) \tag{3}$$

Which can be noted to bear a striking similarity to a hyperbolic tangent artificial neuron:

$$\sigma_i = \tanh(\sum_j w_{ij} x_j + b_i) \tag{4}$$

Renormalization Group (RG) analysis allows us to understand scale-invariant behavior by coarse-graining (ie. averaging) the individual spins into blocks of size b^d with block-spin variables given by:

$$S_I = \frac{1}{b^d} \sum_{i \in \text{block } I} s_i \tag{5}$$

When short-distance degrees of freedom are integrated out, we can derive an RG flow equation dJ_{ij}/db and dh_i/db for each parameter in our model. The system is at criticality when $dJ_{ij}/db = 0$ and $dh_i/db = 0$ therefore no change is exhibited with change of scale, in fact this constructs a critical manifold in parameter space.

C. Criticality in Neural Systems

The criticality hypothesis posits that biological neural systems self-organize to operate near critical points between ordered and chaotic dynamics [1], [2]. Empirical evidence includes observations of "neuronal avalanches" in cortical tissue with size distributions following power laws with exponents of approximately -3/2, matching predictions from critical branching processes [1].

Neural networks near criticality demonstrate optimal computational properties, including maximized dynamic range [3], [4], information transmission [2], and information storage capacity [5]. Conversely, deviations from criticality correlate with neural pathologies [6], suggesting that maintaining criticality is essential for healthy brain function.

These findings motivate our approach: rather than training networks that may accidentally drift away from criticality, we leverage RG flow analysis to design networks that intrinsically maintain critical dynamics throughout operation.

III. Methodology

A. Critical Neural Network Architecture

The proposed architecture relies on the identification between $\tanh$ activated neurons and the mean-field Ising model approximation and leverages renormalization analysis to derive an empirical rule that dynamically biases the network towards criticality.

In renormalization analysis

B. Enviroment Setup

C. Experiments

IV. Results

V. Discussion

VI. Conclusion

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Appendix