# Critical Neural Networks in Atari Games

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***Abstract*—**

## I. Introduction

## II. Background

### A. Reinforcement Learning

### B. Statistical Mechanics

Criticality in statistical mechanics refers to phase transitions between states of matter. At criticality, systems exhibit "scale-free" correlations where microscopic and macroscopic scales become indistinguishable, manifesting as power-law distributions and long-range correlations that span all space and time scales of the system.

The Ising model is often used in studying criticality, where atoms in a lattice have spin states $s_i \in \{+1, -1\}$. Each atom $s_i$ couples to others with parameter $J_{ij}$ and experiences local bias from a field $h_i$. The energy of a configuration $\vec{s}$ is:

$$E(\vec{s}) = \sum_{i \neq j} J_{ij} s_i s_j + \sum_i h_i s_i \tag{1}$$

This defines a Gibbs distribution over the states $\vec{s}$:

$$\mathbb{P}(\vec{s}) = \frac{1}{Z} \exp(-\beta E(\vec{s})) \tag{2}$$

with partition function $Z$ and inverse pseudotemperature $\beta$.

For neural systems, the mean-field approximation of highly connected Ising models is particularly relevant. In this approximation, the expected value of each spin is:

$$\langle s_i \rangle = \tanh(\beta(\sum_j J_{ij} \langle s_j \rangle + h_i)) \tag{3}$$

Which can be noted to bear a striking similarity to a hyperbolic tangent artificial neuron:

$$\sigma_i = \tanh(\sum_j w_{ij} x_j + b_i) \tag{4}$$

Renormalization Group (RG) analysis allows us to understand scale-invariant behavior by coarse-graining (ie. averaging) the individual spins into blocks of size $b^d$ with block-spin variables given by:

$$S_I = \frac{1}{b^d} \sum_{i \in \text{block } I} s_i \tag{5}$$

Under this transformation, the effective parameters of the system (coupling strengths $J_{ij}$ and fields $h_i$) change according to RG flow equations $dS_I/db$. A system exhibits critical behavior when its parameters are tuned to values that remain invariant under this transformation - specifically at the non-trivial fixed point of these flow equations (ie. $dS_I/db = 0$) which form a dense subset in parameter space and construct a critical manifold.

### C. Criticality in Neural Systems

The criticality hypothesis posits that biological neural systems self-organize to operate near critical points between ordered and chaotic dynamics [1], [2]. Empirical evidence includes observations of "neuronal avalanches" in cortical tissue with size distributions following power laws with exponents of approximately -3/2, matching predictions from critical branching processes [1].

Neural networks near criticality demonstrate optimal computational properties, including maximized dynamic range [3], [4], information transmission [2], and information storage capacity [5]. Conversely, deviations from criticality correlate with neural pathologies [6], suggesting that maintaining criticality is essential for healthy brain function.

These findings motivate our approach: rather than training networks that may accidentally drift away from criticality, we leverage RG flow analysis to design networks that intrinsically maintain critical dynamics throughout operation.

## III. Methodology

### A. Critical Neural Network Architecture

The proposed architecture relies on the identification between $\tanh$ activated neurons and the mean-field Ising model approximation and leverages renormalization analysis to derive an empirical rule that dynamically biases the network towards criticality.

In renormalization analysis

### B. Enviroment Setup

### C. Experiments

## IV. Results

## V. Discussion

## VI. Conclusion

## Acknowledgment

The authors would like to thank the instructors and teaching assistants of CS637 for their guidance and support throughout this project. We extend our gratitude to our peers for their valuable feedback during presentation rehearsals and draft reviews. Special thanks to [specific professor name] for introducing us to key concepts in reinforcement learning that formed the foundation of this work. We also acknowledge the resources provided by our university's computing facilities that enabled our experimental implementations. Finally, we thank our families and friends for their unwavering support throughout our academic journey.

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## Appendix