

# Critical Neural Networks in Atari Games

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*Abstract—*

## I. INTRODUCTION

## II. BACKGROUND

### A. Reinforcement Learning

### B. Statistical Mechanics

Criticality in statistical mechanics refers to phase transitions between states of matter. At criticality, systems exhibit "scale-free" correlations where microscopic and macroscopic scales become indistinguishable, manifesting as power-law distributions and long-range correlations that span all space and time scales of the system.

The Ising model is often used in studying criticality, where atoms in a lattice have spin states  $s_i \in \{+1, -1\}$ . Each atom  $s_i$  couples to others with parameter  $J_{ij}$  and experiences local bias from a field  $h_i$ . The energy of a configuration  $\vec{s}$  is:

$$E(\vec{s}) = \sum_{i \neq j} J_{ij} s_i s_j + \sum_i h_i s_i \quad (1)$$

This defines a Gibbs distribution over the states  $\vec{s}$ :

$$\mathbb{P}(\vec{s}) = \frac{1}{Z} \exp(-\beta E(\vec{s})) \quad (2)$$

with partition function  $Z$  and inverse pseudotemperature  $\beta$ .

For neural systems, the mean-field approximation of highly connected Ising models is particularly relevant. In this approximation, the expected value of each spin is:

$$\langle s_i \rangle = \tanh(\beta(\sum_j J_{ij} \langle s_j \rangle + h_i)) \quad (3)$$

Which can be noted to bear a striking similarity to a hyperbolic tangent artificial neuron:

$$\sigma_i = \tanh(\sum_j w_{ij} x_j + b_i) \quad (4)$$

Renormalization Group (RG) analysis allows us to understand scale-invariant behavior by coarse-graining (ie. averaging) the individual spins into blocks of size  $b^d$  with block-spin variables given by:

$$S_I = \frac{1}{b^d} \sum_{i \in \text{block } I} s_i \quad (5)$$

Under this transformation, the effective parameters of the system (coupling strengths  $J_{ij}$  and fields  $h_i$ ) change according to RG flow equations  $dS_I/db$ . A system exhibits critical

behavior when its parameters are tuned to values that remain invariant under this transformation - specifically at the non-trivial fixed point of these flow equations (ie.  $dS_I/db = 0$ ). The fixed points are being dense construct critical manifold, which manifest as phase boundaries in condensed matter physics.

### C. Criticality in Neural Systems

The criticality hypothesis posits that biological neural systems self-organize to operate near critical points between ordered and chaotic dynamics [1], [2]. Empirical evidence includes observations of "neuronal avalanches" in cortical tissue with size distributions following power laws with exponents of approximately  $-3/2$ , matching predictions from critical branching processes [1].

Neural networks near criticality demonstrate optimal computational properties, including maximized dynamic range [3], [4], information transmission [2], and information storage capacity [5]. Conversely, deviations from criticality correlate with neural pathologies [6], suggesting that maintaining criticality is essential for healthy brain function.

These findings motivate our approach: rather than training networks that may accidentally drift away from criticality, we leverage RG flow analysis to design networks that intrinsically maintain critical dynamics throughout operation.

## III. METHODOLOGY

### A. Critical Neural Network Architecture

The proposed architecture relies on the identification between tanh activated neurons and the mean-field Ising model approximation and leverages renormalization analysis to derive an empirical rule that dynamically biases the network towards criticality.

In renormalization analysis

### B. Environment Setup

### C. Experiments

## IV. RESULTS

## V. DISCUSSION

## VI. CONCLUSION

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#### APPENDIX