Assignment

Modeling and Control of Hybrid Systems (SC42075)
2024-2025

Delft Center for Systems and Control, Delft University of Technology

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General remarks

- This assignment consists of two parts. In the first part, you provide your own example of a hybrid system to explain your understanding of a hybrid automaton. In the second part, you will be provided with an example of railway infrastructure maintenance scheduling, and the main goal is to develop a model predictive control strategy for condition-based railway maintenance scheduling. A roadmap is provided that should be followed to present the results in a clear and concise report.
- The deliverable of this assignment is a written report of a maximum of 50 pages (including cover pages and appendices) about the assignment. The report should also contain an *appendix* with your MATLAB files. Moreover, the report should include an evaluation and conclusions section of a maximum of 1-1.5 pages, outlining the main insights obtained through the steps of this assignment. The report should clearly explain and motivate all the choices made while solving the assignment. Make sure to mention the group number and student number(s) on the cover page.
- The *hard* deadline for submitting the report is *Monday 23rd of June at 17:00*. You will lose 0.5 point from your grade on the report for each (started) day of delay in case you exceed this deadline.
- The report is to be uploaded as a single **pdf** file to **brightspace**. Please make sure to check and verify the file you have uploaded.
- Reports are graded based on the content and discussion provided, the readability of the MATLAB files (i.e., do not forget to include explanatory comments in your MATLAB files), the originality of your answers, the correctness, and the efficiency of your answers. The assignment consists of a total of 200 points, distributed across the nine steps (1.1-2.7) as follows: 5, 5, 15, 25, 35, 40, 25, 30, and 20 points, respectively. The total number of points obtained will be scaled to a maximum of 10 for the final assignment grade.
- We suggest that you keep the computations symbolic or analytical as long as possible, and not to hard-code any of the parameters in your MATLAB programs (instead, use a separate MATLAB function or a script that defines the parameters) so that you can easily take other parameter values, longer control horizons, etc. into account.

1 Part 1: Hybrid system example

Select a system in your own field of MSc thesis or a general interest that can be considered as a hybrid system (and that has not yet been discussed in the lecture notes).

Step 1.1: Give a description of the system.

Step 1.2: Describe the system as a hybrid automaton; if needed, extend the definition of the (autonomous) hybrid automaton given in the lecture notes with inputs.

2 Part 2: Railway Condition-based Maintenance Planning

Set-up

This assignment focuses on condition-based maintenance planning in the Dutch railway network for the railway track between The Hague and Delft. The track is approximately 15 km long and divided into five equal sections of 3 km each, as shown in Figure 1. The five sections of the rail are treated as components with independent deterioration dynamics.

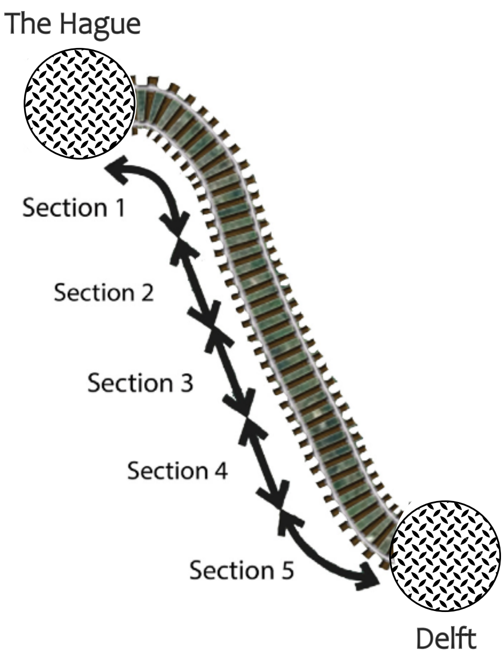
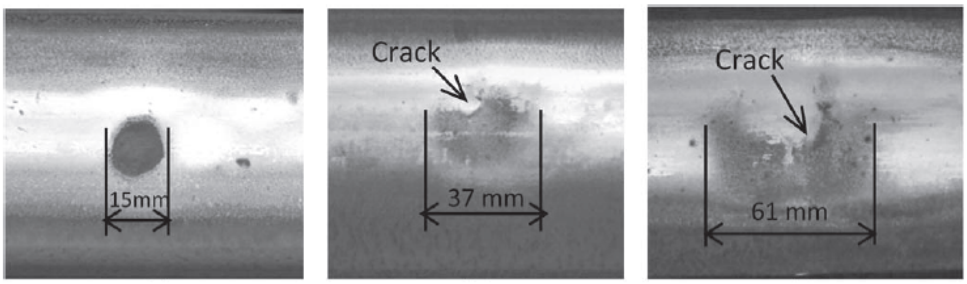


Figure 1: Railway track considered in the assignment.

Condition-Based Maintenance (CBM) planning aims to minimise maintenance interventions while ensuring safe train operations and reducing associated costs. In the assignment, CBM planning targets rail squats, a specific type of rail defect as shown in Figure 2. A rail head squat is a surface defect caused by metal fatigue, classified as rolling contact fatigue, which accelerates rail degradation. Squats contribute significantly to corrective maintenance costs, amounting to millions of euros annually. As squats grow, they can lead to speed restrictions due to the risk of rail breaks and, if left untreated, may result in system failures or derailments. We consider two maintenance interventions to address rail squats: (I) grinding, which is effective for treating early-stage squats by removing deformations and corrosion, and (II) rail replacement, the only viable solution for severe-stage squats. CBM considers squat conditions, particularly the average of lengths per section, to optimise maintenance planning and to ensure safe railway operations.



(a) (b) (c)

Figure 2: Instances of a) Light, b) Moderate, and c) Severe Rail Squats

The sampling time in this assignment is one month, and the deterioration dynamics follow a model in which at time step k, the following conditions act on the railway track:

- The rail track condition depends on the maintenance interventions (input $u(k)$) and the deterioration process of the rail segments. The condition of one section is defined as the average length of all the squats within the section. So the condition x_j^{con} of section j is defined as the average length of squats in section j. Moreover, the auxiliary variable x_j^{aux} records the number of previous grinding operations on section j since the last replacement operation. This is because grinding is not effective after $N_{\max}^{\text{G}}$ grindings, so once this number is reached, the only maintenance option is rail replacement. The dynamics of the conditions of section j can be described by the following model:

$$x_j^{\text{con}}(k+1) = f^{\text{con}}(x_j^{\text{con}}(k), u_j(k)) = \begin{cases} f_{\text{deg}}(x_j^{\text{con}}(k)) & \text{if } u_j(k) = 0 \text{ (no maintenance)} \\ f_{\text{gr}}(x_j^{\text{con}}(k)) & \text{if } u_j(k) = 1 \text{ (grinding)} \\ 0 & \text{if } u_j(k) = 2 \text{ (replacement)} \end{cases}$$

  while the dynamics of the auxiliary variable x_j^{aux} can be expressed as:

$$x_j^{\text{aux}}(k+1) = f^{\text{aux}}(x_j^{\text{aux}}(k), u_j(k)) = \begin{cases} x_j^{\text{aux}}(k) & \text{if } u_j(k) = 0 \text{ (no maintenance)} \\ x_j^{\text{aux}}(k) + 1 & \text{if } u_j(k) = 1 \text{ (grinding)} \\ 0 & \text{if } u_j(k) = 2 \text{ (replacement)} \end{cases}$$

- The MPC prediction model describes the natural degradation dynamics of each section. The prediction model f_{deg} is a piecewise affine function of the form

$$f_{\text{deg}}(x_j^{\text{con}}) = a_q x_j^{\text{con}} + b_q \quad \text{if} \quad x_j^{\text{con}} \in x_{j,q}^{\text{con}}$$

  where the condition space $X_j^{\text{con}} = [0, 70]$ for section j is partitioned into the following three intervals of $x_{j,q}^{\text{con}}$:
  $X_{j,1}^{\text{con}} = [0, 30), \quad X_{j,2}^{\text{con}} = [30, 50), \quad X_{j,3}^{\text{con}} = [50, 70)$
  and the parameters for the piecewise-affine degradation function are collected in the following two vectors:

$$A = \begin{bmatrix} 1.0075 \\ 1.0095 \\ 1.01 \end{bmatrix}, \quad B = \begin{bmatrix} 1.65 + 0.001 \cdot |\text{Group \#} - 15| \\ 2.07 + 0.001 \cdot |\text{Group \#} - 15| \\ 2.47 + 0.001 \cdot |\text{Group \#} - 15| \end{bmatrix}$$

  The function predicts the average length of squats after one month from its current value x_j^{con}.

- The function f_{gr} captures the effect of grinding, which becomes less effective when the condition deteriorates more severely. It is a piecewise-affine function of the following form:

$$f_{\text{gr}}(x_j^{\text{con}}) = \begin{cases} 0 & \text{if } x_j^{\text{con}} \leq x^{\text{eff}} \\ \psi(x_j^{\text{con}} - x^{\text{eff}}) & \text{if } x_j^{\text{con}} > x^{\text{eff}} \end{cases}$$

  where x^{eff} represents a threshold below which the grinding is not perfectly effective.

- The length of the prediction window, N_{p}, is the number of future time steps over which the controller predicts the behaviour of the system. The predicted condition of each section must be kept below a maintenance limit within the prediction window at every time step, which is expressed by the following constraint:

$$x_j^{\text{con}}(k+l) \leq x_{\max} \quad \forall j \in \{1,\dots,n\},\ \forall l \in \{1,\dots,N_{\text{p}}\}$$

- The number of grinding operations cannot exceed a maximum number $N_{\max}^{\text{gr}}$ within the prediction window:

$$x_j^{\text{aux}}(k+l) \leq N_{\max}^{\text{gr}} \quad \forall j \in \{1,\dots,n\},\ \forall l \in \{1,\dots,N_{\text{p}}\}$$

- We simulate the actual degradation process with a simulation model of the same form as the prediction model, but with different parameters. The natural evolution (without any maintenance interventions) for each segment is given by

$$f_{\text{deg}}^{\text{real}}(x_j^{\text{con}}) = a_q^{\text{real}} x_j^{\text{con}} + b_q^{\text{real}} \quad \text{if} \quad x_j^{\text{con}} \in x_{j,q}^{\text{con}}$$

  where the parameters A^{real} and B^{real} are defined as follows

$$A^{\text{real}} = \begin{bmatrix} 1.0017 \\ 1.015 \\ 1.008 \end{bmatrix}, \quad B^{\text{real}} = \begin{bmatrix} 1.6959 \\ 2.2165 \\ 2.6694 \end{bmatrix}$$

  Moreover, in the simulation, the average length of squats is reduced by the grinding operation as follows

$$f_{\text{gr}}^{\text{real}}(x_j^{\text{con}}) = \begin{cases} 0 & \text{if } x_j^{\text{con}} \leq x_{\text{eff}}^{\text{real}} \\ \psi^{\text{real}}(x_j^{\text{con}} - x_{\text{eff}}^{\text{real}}) & \text{if } x_j^{\text{con}} > x_{\text{eff}}^{\text{real}} \end{cases}$$

  The initial condition $x(0) = [(x^{\text{con}}(0))^{\text{T}}, (x^{\text{aux}}(0))^{\text{T}}]^{\text{T}}$ is given by:

$$x_{\text{con}} = \begin{bmatrix} 20 + 0.5 \cdot |\text{Group \#} - 15| \\ 22 + 0.5 \cdot |\text{Group \#} - 15| \\ 25 + 0.5 \cdot |\text{Group \#} - 15| \\ 27 + 0.5 \cdot |\text{Group \#} - 15| \\ 17 + 0.5 \cdot |\text{Group \#} - 15| \end{bmatrix}, \quad x_{\text{aux}} = \begin{bmatrix} 7 \\ 8 \\ 7 \\ 7 \\ 8 \end{bmatrix}$$

- The objective function J should be a trade-off between the condition of the infrastructure J_{deg} and the total maintenance cost J_{maint}. Define

  $$J(k) = J_{\text{deg}}(k) + \lambda J_{\text{maint}}(k)$$

  as in the MPC problem, the objective function needs to be minimised at each time step k. The parameter λ captures the trade-off between the condition of the infrastructure and the maintenance cost.

  The first part

  $$J_{\text{deg}}(k) = \sum_{j=1}^{n} \sum_{l=1}^{N_{\text{p}}} x_j^{\text{con}}(k)$$

  minimises the magnitude of the condition degradation. The second part in the objective function is the accumulated maintenance cost, which can be formulated as:

  $$J_{\text{maint}}(k) = \sum_{j=1}^{n} \sum_{l=1}^{N_{\text{p}}} \sum_{q=1}^{p} \gamma_{j,q} I_{u_j(k+l-1)=a_q}$$

  where the binary indicator function I_X takes value 1 if the statement X is true, otherwise it takes value 0, and the parameter $\gamma_{j,q}$ represents the required cost if intervention a_q is applied to component j. The values for γ_q are presented in the table 1.

Table 1 provides the parameters of the prediction models, the maximum number of grinding operations, and the cost of maintenance interventions.

Parameter	Value	Units
x^{eff}	11.9	mm
$x_{\text{eff}}^{\text{real}}$	16	mm
x^{max}	40	mm
$N_{\text{max}}^{\text{gr}}$	10	—
ψ	0.98	—
ψ^{real}	1.009	—
γ_0	0	—
γ_1	1	—
γ_2	3	—

Table 1: Parameters of the CBM problem.

Tasks & Road map

Step 2.1: Write a PWA model for the system dynamics for the prediction model that considers maintenance interventions, the maximum number of grindings, and the degradation prediction model.

Step 2.2: Transform the PWA model of Step 2.1 into an MLD model with binary and real variables only.

Step 2.3: Design an MPC controller for the MLD model using the implicit MPC approach. Write a MATLAB file that computes the optimal MPC input sequence for a given sample step k for N_{p} equal to 6 (months). Assume an arbitrary value of λ in the range of $[600, 800]$.

Hints & notes:

- The MPC optimization problem at step k can be transformed into a mixed-integer linear programming problem (MILP). In order to solve this problem, you will need an MILP solver, for which you could use one of the following options:

  - Use the `glpk` function of the MPT toolbox version 3 (for MATLAB R2011a and newer) (see `https://www.mpt3.org/`)
  - Use the Matlab interface of the Gurobi optimizer; see `https://www.gurobi.com/products/gurobi-optimizer/`
  - Use the cplex command of the TOMLAB cplex toolbox (note that you need a license, see `https://tomopt.com/tomlab/`). With this command you can solve MILP problems (see also the `milp_solve` tomlab cplex script on the course website).
  - As a last resort, you can use the function `intlinprog` of the MATLAB Optimisation Toolbox.

- As we want you to get some insight into the hybrid MPC method and the relation with integer programming, you are *not* allowed to solve the entire exercise using the `mpt_control` or related commands of the MPT toolbox or the Hybrid Toolbox (see `http://cse.lab.imtlucca.it/~bemporad/hybrid/toolbox/`). However, feel free to compare the results obtained with your own programs to those obtained with the MPT toolbox or the Hybrid Toolbox, and to discuss the differences, if any.

Step 2.4: Write a MATLAB file to simulate the closed-loop behaviour of the system using the real system model and the optimal MPC input (i.e., apply the receding horizon approach in which at each step the optimal MPC control input is recomputed and applied to the system).

Hint: for the real system model, transformation into an MLD model is not needed.

Step 2.5: Consider two values of N_{p}: first $N_{\mathrm{p},1} = 6$, and then another value $N_{\mathrm{p},2}$ that you may select yourself which $N_{\mathrm{p},2} \in \{10, 11, ..., 15\}$. For each N_{p}, run your program for the time interval $[0, T_{\mathrm{end}}]$ where $T_{\mathrm{end}} = 36, 48,$ or 60.

Make plots of the evolution of the controlled closed-loop system in the (x^{con},x^{aux}), and u over time.

Step 2.6: Define an optimal cyclic maintenance approach (i.e., a fixed cyclic maintenance plan, consisting of a regular grinding maintenance action every M months, and rail replacement after an optimal maximum number of grindings $N^{\mathrm{gr}}_{\mathrm{opt}}$). Run your simulation and make the same plots as for Step 2.5. Compare the performance of the cyclic maintenance approach with your CBM model.

Hint: Treat M and $N^{\mathrm{gr}}_{\mathrm{opt}}$ as decision variables, and use e.g. a Genetic Algorithm (GA) or grid search to minimize the same objective as in the MPC model over the planning horizon.

Step 2.7: Provide an evaluation, discussion, and conclusion in which you briefly (1–1.5 pages) outline the main insights you have obtained while doing Parts 1 and 2 of this assignment.