

ECS 256: Homework 1 due 01/21/2014

Olga Prilepova, Christopher Patton, Alexander Rumbaugh, John Chen

For this assignment, we adopt the following notation. Suppose W is a random variable exponentially distributed with mean u . Then $W \sim \mathcal{E}(1/u)$.

Problem 1

(a) $w = 0 \cdot \pi_0 + 1 \cdot \pi_1 + 2 \cdot \pi_2 = \pi_1 + 2\pi_2$.

Problem 2

(a) Let $D \sim \mathcal{E}(1/d)$ be a random variable corresponding to the call duration and $R \sim \mathcal{E}(1/r)$ the time between queued calls. The balance equations for the call system:

$$\pi_{(i,j)} \lambda_{(i,j)} = \pi_{(i,j-1)} \lambda_{(i,j-1)} p_{(i,j-1)(i,j)} + \pi_{(i,j+1)} \lambda_{(i,j+1)} p_{(i,j+1)(i,j)} \quad (\forall \quad 0 < j < q, 0 < i < n)$$

$$\pi_{(i,0)} \lambda_{(i,0)} = \pi_{(i+1,0)} \lambda_{(i+1,0)} p_{(i+1,0)(i,0)} + \pi_{(i,1)} \lambda_{(i,1)} p_{(i,1)(i,0)} \quad (\forall \quad i < n)$$

$$\pi_{(i,q)} \lambda_{(i,q)} = \pi_{(i-1,q)} \lambda_{(i-1,q)} p_{(i-1,q)(i,q)} + \pi_{(i,q-1)} \lambda_{(i,q-1)} p_{(i,q-1)(i,q)} \quad (\forall \quad i > 1)$$

$$\pi_{(n,0)} \lambda_{(n,0)} = \pi_{(n,1)} \lambda_{(n,1)} p_{(n,1)(n,0)}$$

$$\pi_{(1,q)} \lambda_{(1,q)} = \pi_{(1,q-1)} \lambda_{(1,q-1)} p_{(1,q-1)(1,q)}$$

$$\pi_{(1,0)} + \cdots + \pi_{(1,q)} + \cdots + \pi_{(n,0)} + \cdots + \pi_{(n,q)} = 1$$

Proportion of rejected calls.

Proportion of nurse idle time.

Mean time spent in state (1, 1). The possible transitions from this state are (1, 0) (nurse finishes call, takes next on queue) and (1, 2) (call comes in before nurse finishes with current call). The time spent in this state is then $Z_{(1,1)} = \min\{D, R\}$. By theorem, $Z_{(1,1)} \sim \mathcal{E}(1/d + 1/r)$. Thus, the mean time spent in this state is

$$\frac{1}{\frac{1}{d} + \frac{1}{r}}$$

Given the queue is empty, long-run probability that A_t is greater than 1. The sum of the stationary probabilities where $j = 0$ and $i > 1$:

$$\sum_{i=2}^n \pi_{(i,0)}$$