

ECS 256: Homework 1 due 01/21/2014

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For this assignment, we adopt the following notation. Suppose W is a random variable exponentially distributed with mean μ . Then $W \sim \mathcal{E}(\frac{1}{\mu})$.

Problem 1

(a) $w = 0 \cdot \pi_0 + 1 \cdot \pi_1 + 2 \cdot \pi_2 = \pi_1 + 2\pi_2$.

Problem 2

(a) Let $D \sim \mathcal{E}(1/d)$ be a random variable corresponding to the call duration and $R \sim \mathcal{E}(1/r)$ the time between queued calls. The balance equations for the call system:

$$\pi_{(i,j)} \lambda_{(i,j)} = \pi_{(i,j-1)} \lambda_{(i,j-1)} p_{(i,j-1)(i,j)} + \pi_{(i,j+1)} \lambda_{(i,j+1)} p_{(i,j+1)(i,j)} \quad (\forall \quad 0 < j < q, 0 < i < n)$$

$$\pi_{(i,0)} \lambda_{(i,0)} = \pi_{(i+1,0)} \lambda_{(i+1,0)} p_{(i+1,0)(i,0)} + \pi_{(i,1)} \lambda_{(i,1)} p_{(i,1)(i,0)} \quad (\forall \quad i < n)$$

$$\pi_{(i,q)} \lambda_{(i,q)} = \pi_{(i-1,q)} \lambda_{(i-1,q)} p_{(i-1,q)(i,q)} + \pi_{(i,q-1)} \lambda_{(i,q-1)} p_{(i,q-1)(i,q)} \quad (\forall \quad i > 1)$$

$$\pi_{(n,0)} \lambda_{(n,0)} = \pi_{(n,1)} \lambda_{(n,1)} p_{(n,1)(n,0)}$$

$$\pi_{(1,q)} \lambda_{(1,q)} = \pi_{(1,q-1)} \lambda_{(1,q-1)} p_{(1,q-1)(1,q)}$$

$$\pi_{(1,0)} + \cdots + \pi_{(1,q)} + \cdots + \pi_{(n,0)} + \cdots + \pi_{(n,q)} = 1$$

Proportion of rejected calls. The long-run probability that the call center is in a state in which it's rejecting calls is the following quantity:

$$\sum_{i=1}^n \pi_{(i,q)}$$

Proportion of nurse idle time. **TODO**

Mean time spent in state $(1, 1)$. The possible transitions from this state are $(1, 0)$ (nurse finishes call, takes next on queue) and $(1, 2)$ (call comes in before nurse finishes with current call). The time spent in this state is then $Z_{(1,1)} = \min\{D, R\}$. By theorem, $Z_{(1,1)} \sim \mathcal{E}(1/d + 1/r)$. Thus, the mean time spent in this state is:

$$\frac{1}{\frac{1}{d} + \frac{1}{r}}$$

Given the queue is empty, long-run probability that A_t is greater than 1. The sum of the stationary probabilities where $j = 0$ and $i > 1$:

$$\sum_{i=2}^n \pi_{(i,0)}$$