ECS 256 - Problem set 1

Olga Prilepova, Christopher Patton, Alexander Rumbaugh, John Chen, Thomas Provan

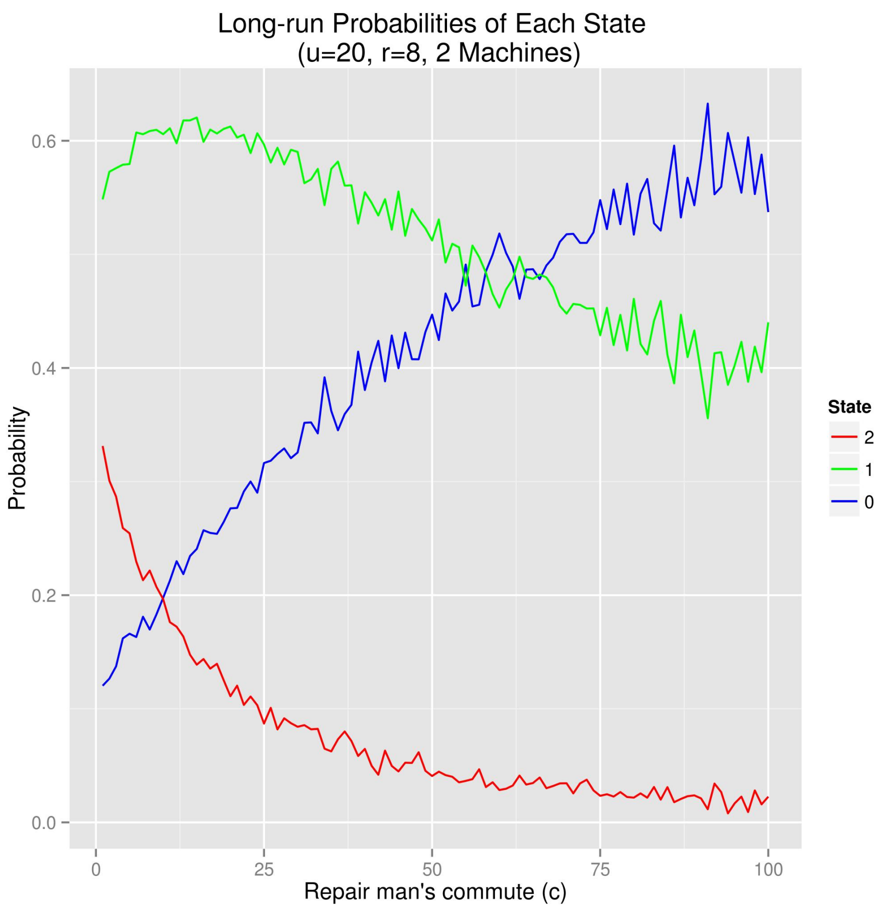
For this assignment, we adopt the following notation. Suppose W is a random variable exponentially distrubted with mean μ. Then $W \sim \mathcal{E}(\frac{1}{\mu})$.

Problem 1

(a) $w = 0 \cdot \pi_0 + 1 \cdot \pi_1 + 2 \cdot \pi_2 = \pi_1 + 2\pi_2$.

(b) Program is `1B.R`.

(c) Program is `1C.R`.



Problem 2

(a) Let $D \sim \mathcal{E}(1/d)$ be a random variable corresponding to the call druation and $R \sim \mathcal{E}(1/r)$ the time between queued calls. The balance equations for the call system:

$$\pi_{(i,j)}\lambda_{(i,j)} = \pi_{(i,j-1)}\lambda_{(i,j-1)}p_{(i,j-1)(i,j)} + \pi_{(i,j+1)}\lambda_{(i,j+1)}p_{(i,j+1)(i,j)} \quad (\forall \quad 0 < j < q, 0 < i < n)$$

$$\pi_{(i,0)}\lambda_{(i,0)} = \pi_{(i+1,0)}\lambda_{(i+1,0)}p_{(i+1,0)(i,0)} + \pi_{(i,1)}\lambda_{(i,1)}p_{(i,1)(i,0)} \quad (\forall \quad i < n)$$
$$\pi_{(i,q)}\lambda_{(i,q)} = \pi_{(i,q-1)}\lambda_{(i,q-1)}p_{(i,q-1)(i,q)} \quad (\forall \quad i > 1)$$
$$\pi_{(1,q-1)}\lambda_{(i,q-1)} = \pi_{(i-1,q)}\lambda_{(i-1,q)}p_{(i-1,q)(i,q-1)} \quad (\forall \quad i > 1)$$

$$\pi_{(n,0)}\lambda_{(n,0)} = \pi_{(n,1)}\lambda_{(n,1)}p_{(n,1)(n,0)}$$
$$\pi_{(1,q)}\lambda_{(1,q)} = \pi_{(1,q-1)}\lambda_{(1,q-1)}p_{(1,q-1)(1,q)}$$

$$\pi_{(1,0)} + \cdots + \pi_{(1,q)} + \cdots + \pi_{(n,0)} + ... + \pi_{(n,q)} = 1$$

Proportion of rejected calls. The long-run probability that the call center is in a state in which it's rejecting calls is the following quantity:

$$\sum_{i=1}^{n} \pi_{(i,q)}$$

Proportion of nurse idle time. **TODO**

Mean time spent in state $(1,1)$*.* The possible transitions from this state are $(1,0)$ (nurse finishes call, takes next on queue) and $(1,2)$ (call comes in before nurse finishes with current call). The time spent in this state is then $Z_{(1,1)} = \min\{D, R\}$. By theorem, $Z_{(1,1)} \sim \mathcal{E}(1/d + 1/r)$. Thus, the mean time spent in this state is:

$$\frac{1}{\frac{1}{d} + \frac{1}{r}}$$

Given the queue is emtpy, long-run probability that A_t is greater than 1. The sum of the stationary probabilities where $j = 0$ and $i > 1$:

$$\sum_{i=2}^{n} \pi_{(i,0)}$$