Supplementary Material: Accounting for Imputation Uncertainty During Neural Network Training

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Abstract

This is the supplementary material for the research paper: Accounting for Imputation Uncertainty During Neural Network Training. In this paper we are interested in dealing with missing values in a machine learning context, and more especially when training a neural network. We focus on improving neural network training by reducing the potential biases that can occur during the training phase on artificially imputed datasets. We do so by taking into account the between-variance that can be observed between multiple imputations. We propose two new imputation frameworks, *S-HOT* and *M-HOT*, that can be used to train neural networks on completed data in a less biased way, leading to models able of more generalization, and so, to better inference results. We perform extensive comparative experiments and statistically assess the results on both benchmark and real-world datasets. We show that our frameworks compete against and even outperform existing imputation frameworks, while being both useful in different settings. We make our entire code publicly accessible to facilitate reproduction of our experimental results.

Method	IRIS	STAT	WINE	PIMA	ABAL
MISSFOREST	1.1965%	2.8167%	1.6140%	1.8177%	0.9456%
SOFTIMPUTE	10.8808%	9.9198%	7.5060%	7.7407%	5.3656%
GAIN	1.6804%	9.3976%	1.3125%	1.3669%	1.6819%
MIDA	1.1673%	1.0666%	0.9283%	0.6446%	0.5293%
SINKHORN	1.1053%	4.3111%	0.7496%	1.2742%	1.2218%

Table 1: Average imputation standard deviation (uncertainty) for each tested imputation method on each dataset

Dataset	Pattern		*SI*		*MI*		*S-HOT*		*M-HOT*	
IRIS	MCAR	10%	0.9972681	(4)	0.9976699	(2)	0.9973865	(3)	**0.9976913**	(1)
		15%	0.9942281	(4)	0.9946880	(2)	0.9944422	(3)	**0.9947552**	(1)
		25%	0.9835323	(4)	0.9841479	(2)	0.9839692	(3)	**0.9843596**	(1)
	MAR	10%	0.9975236	(3)	0.9978336	(2)	0.9974886	(4)	**0.9978845**	(1)
		15%	0.9970188	(3)	**0.9973150**	(1)	0.9970006	(4)	0.9973097	(2)
		25%	0.9941960	(4)	0.9943643	(2)	0.9943554	(3)	**0.9943890**	(1)
	MNAR	10%	0.9972209	(4)	**0.9973548**	(1)	0.9972343	(3)	0.9973011	(2)
		15%	0.9937917	(4)	0.9941570	(2)	0.9938451	(3)	**0.9941622**	(1)
		25%	0.9891964	(4)	0.9905233	(2)	0.9902221	(3)	**0.9907251**	(1)
STAT	MCAR	10%	0.9105582	(4)	**0.9132492**	(1)	0.9106386	(3)	0.9132475	(2)
		15%	0.9060337	(4)	0.9091019	(2)	0.9066862	(3)	**0.9091584**	(1)
		25%	0.9047888	(4)	0.9077214	(2)	0.9059597	(3)	**0.9078041**	(1)
	MAR	10%	0.9127799	(3)	**0.9144097**	(1)	0.9127328	(4)	0.9142985	(2)
		15%	0.9079620	(3)	**0.9093156**	(1)	0.9078667	(4)	0.9092031	(2)
		25%	0.8959471	(4)	0.8990894	(2)	0.8968849	(3)	**0.8992881**	(1)
	MNAR	10%	0.9070192	(4)	0.9095510	(2)	0.9071004	(3)	**0.9095576**	(1)
		15%	0.9040681	(4)	**0.9061872**	(1)	0.9042151	(3)	0.9060699	(2)
		25%	0.8951658	(4)	0.8992252	(2)	0.8967363	(3)	**0.8992736**	(1)
WINE	MCAR	10%	0.9987736	(4)	0.9989672	(2)	0.9988008	(3)	**0.9989811**	(1)
		15%	0.9955454	(4)	0.9960062	(2)	0.9956407	(3)	**0.9960307**	(1)
		25%	0.9910498	(4)	0.9919927	(2)	0.9914148	(3)	**0.9923485**	(1)
	MAR	10%	0.9961058	(4)	0.9965056	(2)	0.9961142	(3)	**0.9965060**	(1)
		15%	0.9977720	(4)	**0.9982010**	(1)	0.9978677	(3)	0.9981809	(2)
		25%	0.9952116	(4)	0.9965157	(2)	0.9959576	(3)	**0.9968702**	(1)
	MNAR	10%	0.9987205	(3)	**0.9988244**	(1)	0.9987058	(4)	0.9988131	(2)
		15%	0.9974746	(4)	0.9976465	(2)	0.9974850	(3)	**0.9976683**	(1)
		25%	0.9808498	(4)	0.9841291	(2)	0.9822719	(3)	**0.9844587**	(1)
PIMA	MCAR	10%	0.8193054	(4)	**0.8211340**	(1)	0.8196586	(3)	0.8211193	(2)
		15%	0.8073739	(4)	0.8095505	(2)	0.8078378	(3)	**0.8095824**	(1)
		25%	0.8029002	(4)	0.8060367	(2)	0.8043589	(3)	**0.8065611**	(1)
	MAR	10%	0.8238900	(4)	**0.8257089**	(1)	0.8242472	(3)	0.8256414	(2)
		15%	0.8045918	(4)	0.8080830	(2)	0.8061503	(3)	**0.8083194**	(1)
		25%	0.8017568	(4)	**0.8041203**	(1)	0.8025144	(3)	0.8040062	(2)
	MNAR	10%	0.8280685	(4)	0.8302729	(2)	0.8284115	(3)	**0.8303076**	(1)
		15%	0.8279577	(4)	0.8298929	(2)	0.8283792	(3)	**0.8300464**	(1)
		25%	0.8005008	(4)	0.8043448	(2)	0.8021749	(3)	**0.8047250**	(1)
ABAL	MCAR	10%	0.8737739	(4)	0.8748059	(2)	0.8740180	(3)	**0.8749393**	(1)
		15%	0.8714861	(4)	0.8725539	(2)	0.8717831	(3)	**0.8726250**	(1)
		25%	0.8663833	(4)	0.8674332	(2)	0.8666186	(3)	**0.8675645**	(1)
	MAR	10%	0.8742551	(4)	0.8751972	(2)	0.8743399	(3)	**0.8753671**	(1)
		15%	0.8720722	(4)	0.8731502	(2)	0.8721856	(3)	**0.8731739**	(1)
		25%	0.8697060	(4)	0.8708046	(2)	0.8699619	(3)	**0.8709102**	(1)
	MNAR	10%	0.8760444	(4)	0.8768033	(2)	0.8760447	(3)	**0.8769350**	(1)
		15%	0.8753217	(4)	0.8763271	(2)	0.8754605	(3)	**0.8764456**	(1)
		25%	0.8683507	(4)	0.8696780	(2)	0.8690281	(3)	**0.8697714**	(1)
Average rank			3.8889		1.7556		3.1111		1.2444	

Table 2: Comparison of the results from the four *SI*, *MI*, *S-HOT* and *M-HOT* frameworks using the MISSFOREST imputation method.

Dataset	Pattern		*SI*		*MI*		*S-HOT*		*M-HOT*	
IRIS	MCAR	10%	0.9799937	(4)	0.9833994	(2)	0.9817654	(3)	**0.9840372**	(1)
		15%	0.9768163	(4)	0.9823092	(2)	0.9808358	(3)	**0.9831680**	(1)
		25%	0.9669967	(4)	0.9761056	(2)	0.9722641	(3)	**0.9770136**	(1)
	MAR	10%	0.9957063	(4)	0.9965493	(2)	0.9961878	(3)	**0.9966640**	(1)
		15%	0.9942239	(4)	0.9960484	(2)	0.9952689	(3)	**0.9961207**	(1)
		25%	0.9893513	(4)	0.9915209	(2)	0.9910596	(3)	**0.9920205**	(1)
	MNAR	10%	0.9936567	(4)	0.9956014	(2)	0.9950805	(3)	**0.9956776**	(1)
		15%	0.9724648	(4)	0.9802625	(2)	0.9761291	(3)	**0.9819184**	(1)
		25%	0.9412312	(4)	0.9571874	(2)	0.9464214	(3)	**0.9600941**	(1)
STAT	MCAR	10%	0.9080227	(4)	0.9134026	(2)	0.9102585	(3)	**0.9137060**	(1)
		15%	0.8998590	(4)	0.9064267	(2)	0.9028497	(3)	**0.9067930**	(1)
		25%	0.8908900	(4)	0.8980645	(2)	0.8956605	(3)	**0.8989371**	(1)
	MAR	10%	0.9084245	(4)	**0.9112693**	(1)	0.9090188	(3)	0.9111061	(2)
		15%	0.9051054	(4)	0.9078594	(2)	0.9063680	(3)	**0.9078908**	(1)
		25%	0.8991662	(4)	0.9058215	(2)	0.9027334	(3)	**0.9061238**	(1)
	MNAR	10%	0.9030340	(4)	0.9067615	(2)	0.9040986	(3)	**0.9071034**	(1)
		15%	0.9035329	(4)	0.9079736	(2)	0.9052432	(3)	**0.9082962**	(1)
		25%	0.8905727	(4)	0.8966984	(2)	0.8944715	(3)	**0.8972140**	(1)
WINE	MCAR	10%	0.9981833	(4)	0.9985619	(2)	0.9983207	(3)	**0.9985971**	(1)
		15%	0.9962062	(4)	0.9976257	(2)	0.9971960	(3)	**0.9976910**	(1)
		25%	0.9904914	(4)	**0.9943642**	(1)	0.9934990	(3)	0.9942573	(2)
	MAR	10%	0.9943647	(4)	0.9955126	(2)	0.9948008	(3)	**0.9957826**	(1)
		15%	0.9966424	(4)	**0.9974876**	(1)	0.9970709	(3)	0.9974503	(2)
		25%	0.9937316	(4)	0.9956761	(2)	0.9952213	(3)	**0.9961109**	(1)
	MNAR	10%	0.9986522	(4)	**0.9989886**	(1)	0.9989077	(3)	0.9989613	(2)
		15%	0.9970764	(4)	0.9976771	(2)	0.9974153	(3)	**0.9976784**	(1)
		25%	0.9868731	(4)	**0.9929763**	(1)	0.9913767	(3)	0.9929660	(2)
PIMA	MCAR	10%	0.8064355	(4)	0.8126446	(2)	0.8108632	(3)	**0.8147203**	(1)
		15%	0.7905122	(4)	0.7985624	(2)	0.7956762	(3)	**0.8004567**	(1)
		25%	0.7636735	(4)	0.7740310	(2)	0.7701991	(3)	**0.7771393**	(1)
	MAR	10%	0.8059250	(4)	0.8113151	(2)	0.8086973	(3)	**0.8127776**	(1)
		15%	0.7872220	(4)	0.7931241	(2)	0.7918551	(3)	**0.7958718**	(1)
		25%	0.7785481	(4)	0.7866454	(2)	0.7832708	(3)	**0.7903447**	(1)
	MNAR	10%	0.8172368	(4)	0.8234770	(2)	0.8209724	(3)	**0.8245369**	(1)
		15%	0.8045201	(4)	0.8113161	(2)	0.8093813	(3)	**0.8130785**	(1)
		25%	0.7636845	(4)	0.7734528	(2)	0.7714932	(3)	**0.7806867**	(1)
ABAL	MCAR	10%	0.8640126	(4)	0.8664509	(2)	0.8656192	(3)	**0.8665868**	(1)
		15%	0.8574107	(4)	0.8606084	(2)	0.8602595	(3)	**0.8607058**	(1)
		25%	0.8414577	(4)	0.8464660	(2)	0.8459436	(3)	**0.8465174**	(1)
	MAR	10%	0.8655842	(4)	0.8670096	(2)	0.8664084	(3)	**0.8670916**	(1)
		15%	0.8589554	(4)	0.8613812	(2)	0.8604022	(3)	**0.8614189**	(1)
		25%	0.8487023	(4)	**0.8532367**	(1)	0.8521187	(3)	0.8530732	(2)
	MNAR	10%	0.8646802	(4)	0.8670122	(2)	0.8657062	(3)	**0.8670726**	(1)
		15%	0.8592630	(4)	0.8625236	(2)	0.8613171	(3)	**0.8625561**	(1)
		25%	0.8424408	(4)	**0.8481797**	(1)	0.8449781	(3)	0.8481437	(2)
Average rank			4.0000		1.8444		3.0000		1.1556	

Table 3: Comparison of the results from the four *SI*, *MI*, *S-HOT* and *M-HOT* frameworks using the SOFTIMPUTE imputation method.

Dataset	Pattern		*SI*		*MI*		*S-HOT*		*M-HOT*	
IRIS	MCAR	10%	0.9857773	(4)	**0.9925170**	(1)	0.9895060	(3)	0.9924744	(2)
		15%	0.9875209	(4)	0.9909667	(2)	0.9899045	(3)	**0.9916338**	(1)
		25%	0.9836152	(4)	0.9881549	(2)	0.9869794	(3)	**0.9888826**	(1)
	MAR	10%	0.9964254	(4)	0.9977066	(2)	0.9972233	(3)	**0.9977583**	(1)
		15%	0.9959845	(4)	0.9971628	(2)	0.9967155	(3)	**0.9973015**	(1)
		25%	0.9941741	(4)	0.9949109	(2)	0.9948820	(3)	**0.9952507**	(1)
	MNAR	10%	0.9931132	(4)	0.9960869	(2)	0.9951754	(3)	**0.9961488**	(1)
		15%	0.9882231	(4)	0.9923640	(2)	0.9913868	(3)	**0.9925256**	(1)
		25%	0.9751727	(4)	0.9824279	(2)	0.9801958	(3)	**0.9839101**	(1)
STAT	MCAR	10%	0.9020429	(4)	0.9160128	(2)	0.9123281	(3)	**0.9161932**	(1)
		15%	0.8856269	(4)	0.9079333	(2)	0.9045924	(3)	**0.9081124**	(1)
		25%	0.8769033	(4)	0.9020794	(2)	0.8993412	(3)	**0.9025791**	(1)
	MAR	10%	0.9087615	(4)	0.9173822	(2)	0.9145394	(3)	**0.9180998**	(1)
		15%	0.8998852	(4)	**0.9114664**	(1)	0.9082186	(3)	0.9108804	(2)
		25%	0.8807372	(4)	**0.8983544**	(1)	0.8956080	(3)	0.8977701	(2)
	MNAR	10%	0.8935386	(4)	0.9037203	(2)	0.9006735	(3)	**0.9044389**	(1)
		15%	0.8939280	(4)	0.9062339	(2)	0.9030125	(3)	**0.9063437**	(1)
		25%	0.8601279	(4)	0.8908618	(2)	0.8868338	(3)	**0.8913759**	(1)
WINE	MCAR	10%	0.9983811	(4)	**0.9987367**	(1)	0.9986075	(3)	0.9987216	(2)
		15%	0.9966068	(4)	0.9976465	(3)	0.9976643	(2)	**0.9979061**	(1)
		25%	0.9915570	(4)	0.9947619	(2)	0.9946475	(3)	**0.9951303**	(1)
	MAR	10%	0.9965180	(4)	**0.9977893**	(1)	0.9975478	(3)	0.9977813	(2)
		15%	0.9970171	(4)	0.9978439	(2)	0.9975442	(3)	**0.9978859**	(1)
		25%	0.9962614	(4)	0.9977948	(2)	0.9975461	(3)	**0.9979197**	(1)
	MNAR	10%	0.9983833	(4)	0.9987648	(2)	0.9986169	(3)	**0.9987725**	(1)
		15%	0.9975738	(4)	0.9979271	(2)	0.9977293	(3)	**0.9979903**	(1)
		25%	0.9910734	(4)	0.9934225	(2)	0.9929808	(3)	**0.9936716**	(1)
PIMA	MCAR	10%	0.8127648	(4)	0.8216261	(2)	0.8193582	(3)	**0.8223632**	(1)
		15%	0.8058630	(4)	0.8131427	(2)	0.8116789	(3)	**0.8132622**	(1)
		25%	0.8003100	(4)	**0.8081799**	(1)	0.8066228	(3)	0.8080758	(2)
	MAR	10%	0.8155870	(4)	**0.8213601**	(1)	0.8189119	(3)	0.8208527	(2)
		15%	0.8036819	(4)	0.8095054	(2)	0.8082194	(3)	**0.8097639**	(1)
		25%	0.7948673	(4)	**0.8024029**	(1)	0.8006539	(3)	0.8016825	(2)
	MNAR	10%	0.8203723	(4)	**0.8253893**	(1)	0.8231400	(3)	0.8252170	(2)
		15%	0.8199156	(4)	**0.8259666**	(1)	0.8240971	(3)	0.8256314	(2)
		25%	0.7939407	(4)	**0.8020002**	(1)	0.7999276	(3)	0.8018077	(2)
ABAL	MCAR	10%	0.8648877	(4)	0.8674506	(2)	0.8670693	(3)	**0.8674665**	(1)
		15%	0.8629564	(4)	**0.8658764**	(1)	0.8653915	(3)	0.8658450	(2)
		25%	0.8567035	(4)	0.8607069	(2)	**0.8608724**	(1)	0.8606392	(3)
	MAR	10%	0.8657632	(4)	**0.8679247**	(1)	0.8673467	(3)	0.8678653	(2)
		15%	0.8644014	(4)	**0.8666815**	(1)	0.8662844	(3)	0.8666551	(2)
		25%	0.8593164	(4)	**0.8621055**	(1)	0.8617823	(3)	0.8619164	(2)
	MNAR	10%	0.8678301	(4)	0.8699069	(2)	0.8695650	(3)	**0.8699575**	(1)
		15%	0.8660991	(4)	**0.8690043**	(1)	0.8684580	(3)	0.8689099	(2)
		25%	0.8569403	(4)	**0.8600201**	(1)	0.8595820	(3)	0.8599013	(2)
Average rank			4.0000		1.6444		2.9333		1.4222	

Table 4: Comparison of the results from the four *SI*, *MI*, *S-HOT* and *M-HOT* frameworks using the GAIN imputation method.

Dataset	Pattern		*SI*	*MI*	*S-HOT*	*M-HOT*
IRIS	MCAR	10%	0.9554177 (3)	0.9580418 (2)	0.9554087 (4)	**0.9589565** (1)
		15%	0.9530381 (4)	0.9550508 (2)	0.9537060 (3)	**0.9556869** (1)
		25%	0.9453838 (4)	0.9483980 (2)	0.9467725 (3)	**0.9495824** (1)
	MAR	10%	0.9905962 (4)	0.9912635 (2)	0.9908503 (3)	**0.9913792** (1)
		15%	0.9895670 (4)	0.9906208 (2)	0.9901444 (3)	**0.9911103** (1)
		25%	0.9852364 (4)	0.9863382 (2)	0.9855719 (3)	**0.9874122** (1)
	MNAR	10%	0.9845493 (4)	0.9857842 (2)	0.9847597 (3)	**0.9862155** (1)
		15%	0.9555301 (4)	0.9571348 (2)	0.9561772 (3)	**0.9576568** (1)
		25%	0.9308975 (4)	0.9372261 (2)	0.9339321 (3)	**0.9381131** (1)
STAT	MCAR	10%	0.9077575 (4)	**0.9109495** (1)	0.9082601 (3)	0.9108722 (2)
		15%	0.9007673 (4)	**0.9037409** (1)	0.9010883 (3)	0.9035844 (2)
		25%	0.8931136 (4)	**0.8962443** (1)	0.8936269 (3)	0.8960913 (2)
	MAR	10%	0.9077300 (3)	**0.9099291** (1)	0.9076973 (4)	0.9097665 (2)
		15%	0.9034708 (4)	**0.9054812** (1)	0.9036366 (3)	0.9053064 (2)
		25%	0.8998750 (4)	**0.9030175** (1)	0.9000970 (3)	0.9029715 (2)
	MNAR	10%	0.9049275 (4)	**0.9073643** (1)	0.9052633 (3)	0.9073239 (2)
		15%	0.9028127 (4)	0.9057320 (2)	0.9029637 (3)	**0.9058654** (1)
		25%	0.8902191 (4)	**0.8927232** (1)	0.8907920 (3)	0.8926771 (2)
WINE	MCAR	10%	0.9960665 (4)	0.9966659 (2)	0.9964714 (3)	**0.9967312** (1)
		15%	0.9920738 (4)	0.9935617 (2)	0.9927122 (3)	**0.9935720** (1)
		25%	0.9851423 (4)	**0.9874086** (1)	0.9866744 (3)	0.9872919 (2)
	MAR	10%	0.9956389 (4)	0.9965696 (2)	0.9962256 (3)	**0.9965773** (1)
		15%	0.9918686 (4)	0.9929713 (2)	0.9925073 (3)	**0.9931399** (1)
		25%	0.9945250 (4)	**0.9952987** (1)	0.9948903 (3)	0.9952949 (2)
	MNAR	10%	0.9978923 (4)	0.9982540 (2)	0.9981631 (3)	**0.9982850** (1)
		15%	0.9974959 (4)	**0.9978945** (1)	0.9975922 (3)	0.9978659 (2)
		25%	0.9875513 (4)	**0.9894476** (1)	0.9883172 (3)	0.9893522 (2)
PIMA	MCAR	10%	0.8019355 (4)	0.8049024 (2)	0.8023523 (3)	**0.8057123** (1)
		15%	0.7873334 (4)	0.7913116 (2)	0.7881910 (3)	**0.7926936** (1)
		25%	0.7720143 (4)	0.7782346 (2)	0.7724599 (3)	**0.7813148** (1)
	MAR	10%	0.8005373 (4)	0.8035979 (2)	0.8008901 (3)	**0.8042388** (1)
		15%	0.7866671 (4)	0.7896857 (2)	0.7867853 (3)	**0.7906564** (1)
		25%	0.7748337 (4)	0.7792370 (2)	0.7761880 (3)	**0.7808444** (1)
	MNAR	10%	0.8129813 (4)	0.8156111 (2)	0.8135142 (3)	**0.8163256** (1)
		15%	0.7984465 (4)	0.8015427 (2)	0.7989641 (3)	**0.8024684** (1)
		25%	0.7695292 (4)	0.7753233 (2)	0.7704833 (3)	**0.7778510** (1)
ABAL	MCAR	10%	0.8468388 (3)	0.8497579 (2)	0.8465451 (4)	**0.8499176** (1)
		15%	0.8397988 (3)	0.8425319 (2)	0.8393014 (4)	**0.8427391** (1)
		25%	0.8263714 (3)	0.8295212 (2)	0.8257296 (4)	**0.8295821** (1)
	MAR	10%	0.8482142 (3)	0.8498965 (2)	0.8476160 (4)	**0.8501366** (1)
		15%	0.8410048 (3)	0.8424528 (2)	0.8408962 (4)	**0.8425750** (1)
		25%	0.8348346 (3)	0.8368085 (2)	0.8345719 (4)	**0.8369419** (1)
	MNAR	10%	0.8455356 (3)	0.8500263 (2)	0.8445578 (4)	**0.8501793** (1)
		15%	0.8443605 (3)	0.8463715 (2)	0.8437819 (4)	**0.8464826** (1)
		25%	0.8247781 (4)	0.8283600 (2)	0.8252351 (3)	**0.8283827** (1)
Average rank			3.7778	1.7333	3.2222	1.2667

Table 5: Comparison of the results from the four *SI*, *MI*, *S-HOT* and *M-HOT* frameworks using the MIDA imputation method.

Dataset	Pattern		*SI*		*MI*		*S-HOT*		*M-HOT*	
IRIS	MCAR	10%	0.9958323	(4)	**0.9973218**	(1)	0.9968544	(3)	0.9972525	(2)
		15%	0.9947010	(4)	0.9967011	(2)	0.9961452	(3)	**0.9967655**	(1)
		25%	0.9879101	(4)	0.9931490	(2)	0.9920251	(3)	**0.9933714**	(1)
	MAR	10%	0.9977831	(4)	**0.9983718**	(1)	0.9980598	(3)	0.9983512	(2)
		15%	0.9964883	(4)	0.9969672	(2)	0.9966728	(3)	**0.9970707**	(1)
		25%	0.9934700	(4)	0.9940230	(2)	0.9937325	(3)	**0.9940345**	(1)
	MNAR	10%	0.9968828	(4)	0.9975337	(2)	0.9969866	(3)	**0.9975402**	(1)
		15%	0.9922183	(4)	0.9953051	(2)	0.9939836	(3)	**0.9953053**	(1)
		25%	0.9800315	(4)	**0.9893560**	(1)	0.9851027	(3)	0.9882223	(2)
STATL	MCAR	10%	0.9004228	(4)	**0.9122272**	(1)	0.9099691	(3)	0.9119025	(2)
		15%	0.8893594	(4)	0.9048134	(2)	0.9026763	(3)	**0.9050871**	(1)
		25%	0.8766682	(4)	0.8979225	(2)	0.8956989	(3)	**0.8987079**	(1)
	MAR	10%	0.9075065	(4)	**0.9157279**	(1)	0.9135148	(3)	0.9153541	(2)
		15%	0.9002249	(4)	0.9083954	(2)	0.9072489	(3)	**0.9085098**	(1)
		25%	0.8919708	(4)	0.9031956	(2)	0.9009516	(3)	**0.9033104**	(1)
	MNAR	10%	0.8966214	(4)	0.9067652	(2)	0.9043305	(3)	**0.9076346**	(1)
		15%	0.8953022	(4)	0.9072963	(2)	0.9049765	(3)	**0.9077530**	(1)
		25%	0.8724639	(4)	0.8940123	(2)	0.8917990	(3)	**0.8946876**	(1)
WINE	MCAR	10%	0.9986694	(4)	0.9988537	(2)	0.9987253	(3)	**0.9988668**	(1)
		15%	0.9975506	(4)	**0.9979885**	(1)	0.9976749	(3)	0.9979830	(2)
		25%	0.9947814	(4)	0.9958397	(2)	0.9956436	(3)	**0.9958428**	(1)
	MAR	10%	0.9963918	(4)	0.9968344	(2)	0.9966461	(3)	**0.9969354**	(1)
		15%	0.9978826	(4)	0.9986410	(2)	0.9984737	(3)	**0.9986593**	(1)
		25%	0.9966551	(4)	0.9976426	(2)	0.9973227	(3)	**0.9977587**	(1)
	MNAR	10%	0.9990950	(4)	**0.9991486**	(1)	0.9991137	(3)	0.9991412	(2)
		15%	0.9981937	(4)	0.9984342	(2)	0.9982292	(3)	**0.9984523**	(1)
		25%	0.9901818	(4)	0.9948135	(2)	0.9940507	(3)	**0.9948292**	(1)
PIMA	MCAR	10%	0.8183348	(4)	0.8225089	(2)	0.8207717	(3)	**0.8225461**	(1)
		15%	0.8060807	(4)	**0.8123862**	(1)	0.8103984	(3)	0.8123352	(2)
		25%	0.7932394	(4)	0.8040140	(2)	0.8020177	(3)	**0.8055621**	(1)
	MAR	10%	0.8201186	(4)	**0.8235582**	(1)	0.8218285	(3)	0.8235392	(2)
		15%	0.8078931	(4)	**0.8143046**	(1)	0.8127261	(3)	0.8142421	(2)
		25%	0.7989430	(4)	**0.8072073**	(1)	0.8058399	(3)	0.8071192	(2)
	MNAR	10%	0.8250540	(4)	0.8299199	(2)	0.8278898	(3)	**0.8303720**	(1)
		15%	0.8137489	(4)	0.8211264	(2)	0.8189948	(3)	**0.8213390**	(1)
		25%	0.7916129	(4)	0.8035721	(2)	0.8011662	(3)	**0.8036587**	(1)
ABAL	MCAR	10%	0.8655977	(4)	0.8707151	(2)	**0.8707402**	(1)	0.8707068	(3)
		15%	0.8604767	(4)	0.8674447	(2)	**0.8680851**	(1)	0.8673406	(3)
		25%	0.8495787	(4)	0.8596208	(2)	**0.8621626**	(1)	0.8592872	(3)
	MAR	10%	0.8665143	(4)	0.8708308	(2)	**0.8711025**	(1)	0.8708024	(3)
		15%	0.8613696	(4)	0.8678045	(2)	**0.8689452**	(1)	0.8676311	(3)
		25%	0.8518054	(4)	0.8612268	(2)	**0.8642873**	(1)	0.8608408	(3)
	MNAR	10%	0.8697805	(4)	0.8743051	(3)	**0.8746136**	(1)	0.8743183	(2)
		15%	0.8651282	(4)	0.8716987	(2)	**0.8728212**	(1)	0.8715489	(3)
		25%	0.8518229	(4)	0.8614111	(2)	**0.8642183**	(1)	0.8609479	(3)
Average rank			4.0000		1.7778		2.6000		1.6222	

Table 6: Comparison of the results from the four *SI*, *MI*, *S-HOT* and *M-HOT* frameworks using the SINK imputation method.

Dataset	Pattern		MICE	Best Imputation Method			
				SI	*MI*	*S-HOT*	*M-HOT*
IRIS	MCAR	10%	0.9964543 (4)	0.9958323 (5)	**0.9973218** (1)	0.9968544 (3)	0.9972525 (2)
		15%	0.9916368 (5)	0.9947010 (4)	0.9967011 (2)	0.9961452 (3)	**0.9967655** (1)
		25%	0.9878213 (5)	0.9879101 (4)	0.9931490 (2)	0.9920251 (3)	**0.9933714** (1)
	MAR	10%	0.9963453 (5)	0.9977831 (4)	**0.9983718** (1)	0.9980598 (3)	0.9983512 (2)
		15%	0.9968731 (3)	0.9964883 (5)	0.9969672 (2)	0.9966728 (4)	**0.9970707** (1)
		25%	**0.9972507** (1)	0.9934700 (5)	0.9940230 (3)	0.9937325 (4)	0.9940345 (2)
	MNAR	10%	0.9973377 (3)	0.9968828 (5)	0.9975337 (2)	0.9969866 (4)	**0.9975402** (1)
		15%	0.9923837 (4)	0.9922183 (5)	0.9953051 (2)	0.9939836 (3)	**0.9953053** (1)
		25%	0.9877847 (3)	0.9800315 (5)	**0.9893560** (1)	0.9851027 (4)	0.9882223 (2)
STAT	MCAR	10%	0.9087270 (5)	0.9105582 (4)	**0.9132492** (1)	0.9106386 (3)	0.9132475 (2)
		15%	0.9038454 (5)	0.9060337 (4)	0.9091019 (2)	0.9066862 (3)	**0.9091584** (1)
		25%	0.9013979 (5)	0.9047888 (4)	0.9077214 (2)	0.9059597 (3)	**0.9078041** (1)
	MAR	10%	0.9058957 (5)	0.9127799 (3)	**0.9144097** (1)	0.9127328 (4)	0.9142985 (2)
		15%	**0.9142847** (1)	0.9079620 (4)	0.9093156 (2)	0.9078667 (5)	0.9092031 (3)
		25%	0.8976313 (3)	0.8959471 (5)	0.8990894 (2)	0.8968849 (4)	**0.8992881** (1)
	MNAR	10%	0.9085572 (3)	0.9070192 (5)	0.9095510 (2)	0.9071004 (4)	**0.9095576** (1)
		15%	0.8998035 (5)	0.9040681 (4)	**0.9061872** (1)	0.9042151 (3)	0.9060699 (2)
		25%	**0.8998697** (1)	0.8951658 (5)	0.8992252 (3)	0.8967363 (4)	0.8992736 (2)
WINE	MCAR	10%	0.9987802 (3)	0.9986694 (5)	0.9988537 (2)	0.9987253 (4)	**0.9988668** (1)
		15%	0.9959565 (5)	0.9975506 (4)	**0.9979885** (1)	0.9976749 (3)	0.9979830 (2)
		25%	0.9922771 (5)	0.9947814 (4)	0.9958397 (2)	0.9956436 (3)	**0.9958428** (1)
	MAR	10%	**0.9979961** (1)	0.9963918 (5)	0.9968344 (3)	0.9966461 (4)	0.9969354 (2)
		15%	0.9980099 (4)	0.9978826 (5)	0.9986410 (2)	0.9984737 (3)	**0.9986593** (1)
		25%	0.9950532 (5)	0.9966551 (4)	0.9976426 (2)	0.9973227 (3)	**0.9977587** (1)
	MNAR	10%	0.9972888 (5)	0.9990950 (4)	**0.9991486** (1)	0.9991137 (3)	0.9991412 (2)
		15%	**0.9991038** (1)	0.9981937 (5)	0.9984342 (3)	0.9982292 (4)	0.9984523 (2)
		25%	**0.9953236** (1)	0.9901818 (5)	0.9948135 (3)	0.9940507 (4)	0.9948292 (2)
PIMA	MCAR	10%	0.8195130 (4)	0.8183348 (5)	0.8225089 (2)	0.8207717 (3)	**0.8225461** (1)
		15%	0.8097860 (4)	0.8060807 (5)	**0.8123862** (1)	0.8103984 (3)	0.8123352 (2)
		25%	0.7993334 (4)	0.7932394 (5)	0.8040140 (2)	0.8020177 (3)	**0.8055621** (1)
	MAR	10%	**0.8252737** (1)	0.8201186 (5)	0.8235582 (2)	0.8218285 (4)	0.8235392 (3)
		15%	**0.8147294** (1)	0.8078931 (5)	0.8143046 (2)	0.8127261 (4)	0.8142421 (3)
		25%	0.8007801 (4)	0.7989430 (5)	**0.8072073** (1)	0.8058399 (3)	0.8071192 (2)
	MNAR	10%	0.8211049 (5)	0.8250540 (4)	0.8299199 (2)	0.8278898 (3)	**0.8303720** (1)
		15%	0.8157382 (4)	0.8137489 (5)	0.8211264 (2)	0.8189948 (3)	**0.8213390** (1)
		25%	0.7972827 (4)	0.7916129 (5)	0.8035721 (2)	0.8011662 (3)	**0.8036587** (1)
ABAL	MCAR	10%	0.8732051 (5)	0.8737739 (4)	0.8748059 (2)	0.8740180 (3)	**0.8749393** (1)
		15%	0.8717183 (4)	0.8714861 (5)	0.8725539 (2)	0.8717831 (3)	**0.8726250** (1)
		25%	0.8649143 (5)	0.8663833 (4)	0.8674332 (2)	0.8666186 (3)	**0.8675645** (1)
	MAR	10%	0.8752625 (2)	0.8742551 (5)	0.8751972 (3)	0.8743399 (4)	**0.8753671** (1)
		15%	0.8716980 (5)	0.8720722 (4)	0.8731502 (2)	0.8721856 (3)	**0.8731739** (1)
		25%	0.8680810 (5)	0.8697060 (4)	0.8708046 (2)	0.8699619 (3)	**0.8709102** (1)
	MNAR	10%	0.8748891 (5)	0.8760444 (4)	0.8768033 (2)	0.8760447 (3)	**0.8769350** (1)
		15%	0.8734301 (5)	0.8753217 (4)	0.8763271 (2)	0.8754605 (3)	**0.8764456** (1)
		25%	0.8678228 (5)	0.8683507 (4)	0.8696780 (2)	0.8690281 (3)	**0.8697714** (1)
Average rank			3.7333	4.5111	1.9111	3.3778	1.4667

Table 7: Comparison between MICE results and the results of the four *SI*, *MI*, *S-HOT* and *M-HOT* frameworks using the imputation method that gives the best results for each dataset.

Dataset	Metric	*SI*	*S-HOT*	*MI*	*M-HOT*
COVI	bACC	**88.6725**	88.6654	88.7365	**88.7617**
	AUC	0.9614745	**0.9620376**	0.9627426	**0.9628588**
	F1	88.7026	**88.7527**	88.7615	**88.7998**
MYOC	bACC	70.7693	**70.9092**	71.3911	**71.6052**
	AUC	0.8243456	**0.8247766**	0.8487178	**0.8489546**
	F1	**85.7213**	85.7094	**86.4856**	86.4783
NHAN	bACC	63.5868	**63.7206**	64.3151	**64.3741**
	AUC	0.7016346	**0.7023302**	**0.7172432**	0.7170467
	F1	63.3669	**63.4867**	64.0853	**64.1244**

Table 8: Comparison of the results on real-world medical datasets between *SI* and *S-HOT* on the left, and *MI* and *M-HOT* on the right using the MISSFOREST imputation method.

Dataset	Metric	*SI*	*S-HOT*	*MI*	*M-HOT*
COVI	bACC	87.8585	**88.3732**	88.1767	**88.3682**
	AUC	0.954733	**0.9575914**	0.9580374	**0.9583938**
	F1	87.9294	**88.4383**	88.2291	**88.4105**
MYOC	bACC	68.9386	**69.1718**	69.694	**69.8195**
	AUC	0.8031829	**0.8034473**	**0.8330273**	0.8321686
	F1	84.6802	**84.7812**	**85.6378**	85.6106
NHAN	bACC	62.9123	**63.1646**	63.9159	**64.0496**
	AUC	0.6919003	**0.6973462**	0.7115084	**0.7135331**
	F1	62.7138	**62.9228**	63.7059	**63.796**

Table 9: Comparison of the results on real-world medical datasets between *SI* and *S-HOT* on the left, and *MI* and *M-HOT* on the right using the SOFTIMPUTE imputation method.

Dataset	Metric	*SI*	*S-HOT*	*MI*	*M-HOT*
COVI	bACC	87.0045	**88.4374**	87.6717	**88.4832**
	AUC	0.947926	**0.9593048**	0.9542347	**0.9583125**
	F1	87.0508	**88.4577**	87.6887	**88.4995**
MYOC	bACC	68.8914	**69.518**	69.5851	**70.0704**
	AUC	0.8073393	**0.8144668**	0.8474765	**0.848819**
	F1	84.8043	**84.9436**	85.8864	**85.9842**
NHAN	bACC	62.608	**63.8408**	64.7467	**64.8621**
	AUC	0.6860431	**0.7065677**	0.7220923	**0.7268964**
	F1	62.452	**63.5959**	64.6042	**64.672**

Table 10: Comparison of the results on real-world medical datasets between *SI* and *S-HOT* on the left, and *MI* and *M-HOT* on the right using the GAIN imputation method.

Dataset	Metric	*SI*	*S-HOT*	*MI*	*M-HOT*
COVI	bACC	88.1085	**88.2243**	**88.4101**	88.3932
	AUC	0.959175	**0.9597053**	0.9606792	**0.9607641**
	F1	88.1598	**88.3403**	**88.473**	88.4681
MYOC	bACC	69.6988	**69.8614**	70.0331	**70.2846**
	AUC	0.8126983	**0.8143542**	0.8419529	**0.8424803**
	F1	84.9747	**84.9862**	85.6655	**85.7106**
NHAN	bACC	63.4962	**63.5197**	**64.2441**	64.1943
	AUC	0.6982824	**0.6984358**	**0.7141273**	0.7137177
	F1	63.2441	**63.2592**	**63.9746**	63.9232

Table 11: Comparison of the results on real-world medical datasets between *SI* and *S-HOT* on the left, and *MI* and *M-HOT* on the right using the MIDA imputation method.

Dataset	Metric	*SI*	*S-HOT*	*MI*	*M-HOT*
COVI	bACC	88.0521	**88.7577**	88.1914	**88.5625**
	AUC	0.954824	**0.9625726**	0.9624413	**0.9630812**
	F1	88.0857	**88.8315**	88.2002	**88.5704**
MYOC	bACC	69.8555	**70.0475**	70.3021	**70.4734**
	AUC	0.8158175	**0.8188904**	0.8447799	**0.8456001**
	F1	85.1389	**85.1642**	85.9508	**85.9812**
NHAN	bACC	63.1886	**63.3785**	**64.0025**	63.9914
	AUC	0.6926347	**0.6965183**	0.709166	**0.7095322**
	F1	62.9453	**63.0783**	**63.7281**	63.7012

Table 12: Comparison of the results on real-world medical datasets between *SI* and *S-HOT* on the left, and *MI* and *M-HOT* on the right using the SINKHORN imputation method.

Dataset	Method	*SI*		*MI*		*S-HOT*		*M-HOT*	
	MISS	10.61		212.30		212.30		212.30	
	SOFT	0.02		0.34		0.34		0.34	
IRIS	GAIN	17.76	0.62	355.12	12.13	355.12	0.60	355.12	12.60
	MIDA	21.02		420.40		420.40		420.40	
	SINK	43.27		865.39		865.39		865.39	
	MISS	41.09		821.84		821.84		821.84	
	SOFT	0.01		0.14		0.14		0.14	
STAT	GAIN	25.58	0.29	511.67	4.82	511.67	0.31	511.67	5.56
	MIDA	16.71		334.11		334.11		334.11	
	SINK	46.62		932.40		932.40		932.40	
	MISS	35.74		714.80		714.80		714.80	
	SOFT	0.01		0.13		0.13		0.13	
WINE	GAIN	21.48	0.58	429.58	12.30	429.58	0.60	429.58	12.51
	MIDA	16.89		337.80		337.80		337.80	
	SINK	45.74		914.75		914.75		914.75	
	MISS	21.99		439.77		439.77		439.77	
	SOFT	0.01		0.18		0.18		0.18	
ABAL	GAIN	19.48	5.98	389.58	123.53	389.58	6.45	389.58	127.83
	MIDA	17.94		358.85		358.85		358.85	
	SINK	43.89		877.82		877.82		877.82	
	MISS	24.84		496.75		496.75		496.75	
	SOFT	0.01		0.12		0.12		0.12	
PIMA	GAIN	20.87	1.31	417.40	24.38	417.40	1.33	417.40	25.44
	MIDA	19.26		385.26		385.26		385.26	
	SINK	46.61		932.21		932.21		932.21	

Table 13: Average imputation (left) and training+test (right) computational times in seconds for each framework on each dataset using each tested imputation methods. The missingness setting was arbitrarily chosen to be MNAR 15% as it does not impact running time.