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# Follow the path

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# Abstract

The first assignment of this course was about developing a path tracking algorithm for a virtual robot. We implemented two algorithms, the first one is the known ‘pure pursuit’ algorithm. The second one is a simpler and more naive algorithm that we developed ourselves. After the implementation of those two algorithms we had various ideas of improvements.

An idea was to adapt the linear speed of the robot to the computed angular speed, meaning that the robot should slow down on sharp turns and go faster whenever the path is more straight. We managed to do it by using several mathematical functions that return an adapted linear speed given an angular speed.

We also had the idea to adapt the look-ahead distance to the linear speed, to do so a very simple modification was to set the look-ahead distance to the same value as the linear speed.

After numerous tests we concluded that:

- The fastest way to follow a path is using our algorithm with a function for the linear speed and the look-ahead distance equal to the linear speed.
- The most secured way to follow the path is by using the ‘pure pursuit’ algorithm with a function for the linear speed and the look-ahead distance set to 0.7.

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# Introduction

There are more and more robots in the world nowadays, that leads the developers and scientists to develop and create new algorithms that will make the robots work more efficiently in their environments.

Many of those robots need to move in their surroundings, to make the robot able to move by itself there are many solutions, the simplest of them all is to give that robot a path and develop an algorithm that will make the robot follow it. Those kinds of algorithms are called path tracking algorithms.

A more advance type of path tracking algorithm can detect the potentials obstacles on the way in order to make the robot avoid them.

# Chapter 1

## Context

The goal of the first assignment of this course was to develop a path tracking algorithm for a virtual robot. We first choose to implement the ‘pure pursuit’ algorithm, then we also created a simpler algorithm.

### 1.1 Pure pursuit

Pure pursuit is a path tracking algorithm. [Lun03, section 1.2] It computes the angular velocity that the robot needs to follow in order to reach some look-ahead point in front of itself. The linear speed is not computed by this algorithm, it can either stays constant or be modified according to the computed angular velocity. At every new step the algorithm selects a goal point based on the selected look-ahead distance until it reaches the last point of the path. The robot will then always follow a point that stays at a look-ahead distance in front of itself. The look-ahead distance can either be constant or be modified during the running process of the algorithm.

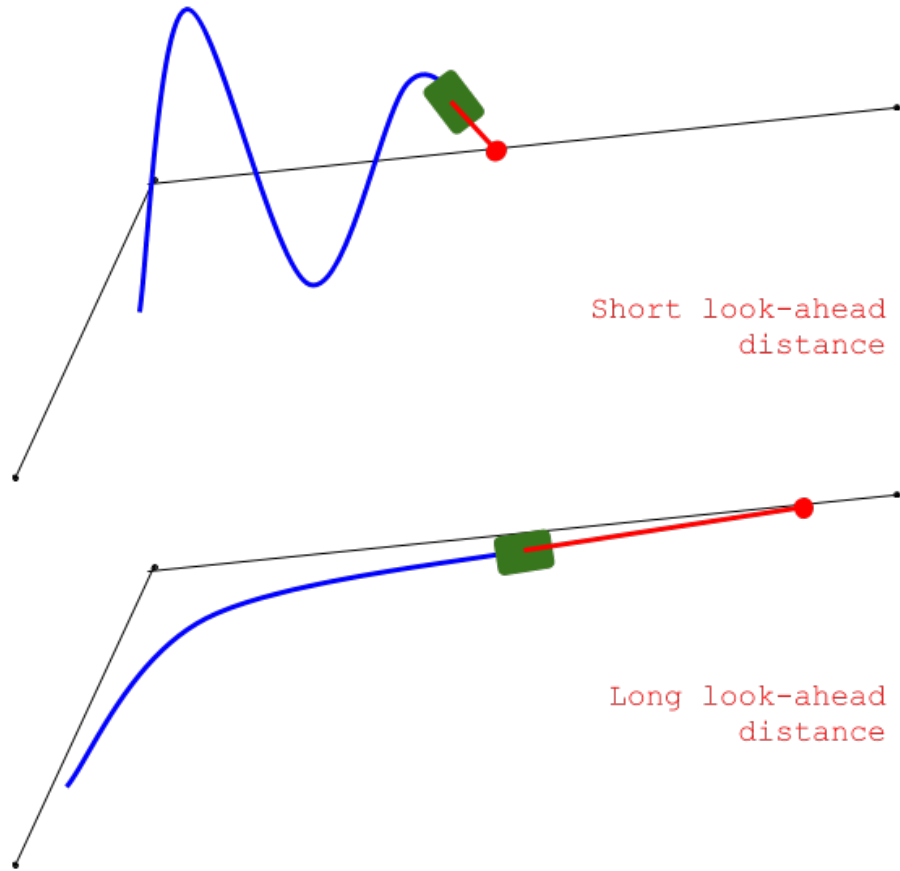
### 1.2 Look-ahead distance

The look-ahead distance is a very important value for the pure pursuit algorithm. This distance corresponds to how far along the path the robot looks from its current location. The algorithm then computes the angular velocity that the robot must have to orientate itself towards its goal point.

Changing that value changes the path that the robot will follow:

- A short look-ahead distance will make the robot stay close to the path but it will often oscillate around it.
- A long look-ahead distance will cause the robot to cut too much when the path turns a lot.

Figure 1: Look-ahead distance comparison



The look-ahead distance must be chosen so that the algorithm can follow the path as best as possible depending on the environment.

### 1.3 Our own algorithm

After the pure pursuit implementation we wondered if it was possible to create a simpler algorithm.

We came with a simple idea which is to define the robot angular speed like so:

- We compute the angle between the robot orientation and the goal point:  $\Theta$ .

- We define a trigger value:  $x$ .
- If  $abs(\Theta) \geq x$  we define  $angular\_speed$  as the  $\pm max\_angular\_speed$  corresponding to the sign of  $\Theta$ .
- Else we compute a weighted angular speed:

$$angular\_speed = \frac{max\_angular\_speed * \Theta}{x}$$

By implementing an algorithm as above and testing for numerous  $x$  values we managed to obtain results very similar to the pure pursuit algorithm and even better after more experimentation.

## Chapter 2

# Implementation on MRDS

### 2.1 Path tracking algorithms

#### 2.1.1 Pure pursuit

We implemented the pure pursuit algorithm as follows:

---

**Algorithm 1** Pure pursuit

---

```
1: procedure PURE_PURSUIT(robot_pos, goal_point)
2:   dist = compute the distance between the robot and the goal point
3:   robot_angle = compute the angle of the robot in the MRDS environment
4:   point_angle = compute the angle of the goal point using the robot as
      referential
5:   theta = point_angle - robot_angle
6:   delta_x =  $\sin(\theta)/dist$ 
7:   return  $(2 * delta\_x)/(dist * 2)$ 
8: end procedure
```

---

After implementing this algorithm we have tested it using a linear speed of 1, which is the maximum linear speed for the robot. The result was already very convincing since the robot successfully followed the path without even hitting any obstacle.

After that we wanted to do some more testing, to do so we had three main ideas:

1. Develop a more simple and naive algorithm by ourselves so we can compare the pure pursuit algorithm to another one.
2. Adapt the linear speed of the robot to the computed angular speed. Meaning that the robot should slow down when there is a sharp turn and go as fast as possible otherwise.



3. Adapt the look-ahead distance to the linear speed, so the look-ahead is shorter when the linear speed decreases and longer conversely.

### 2.1.2 Our own algorithm

We implemented our algorithm as follows:

---

#### Algorithm 2 Our algorithm

---

```

1: procedure OUR_ALGO(robot_pos, goal_point)
2:   robot_angle = compute the angle of the robot in the MRDS environment
3:   point_angle = compute the angle of the goal point with the robot as
      axis
4:    $\theta = \sin(\text{point\_angle} - \text{robot\_angle})$ 
5:    $x = 0.3$  ▷ trigger value
6:    $\text{max\_ang\_speed} = 3$ 
7:    $\text{ang\_speed} = (\text{max\_ang\_speed} * \theta) / x$ 
8:   return  $\min(\max(\text{ang\_speed}, -\text{max\_ang\_speed}), \text{max\_ang\_speed})$ 
9: end procedure

```

---

For this algorithm to work properly we have to chose a correct  $x$  value. At first we tried by setting a very low value:  $x = 0.05$ , we also computed  $\theta$  as follows:  $\theta = \text{point\_angle} - \text{robot\_angle}$ , without the sinus. The result of such settings were a very oscillating path and without the sinus  $\theta$  were going from negative to positive, which made the robot briefly lose the path.

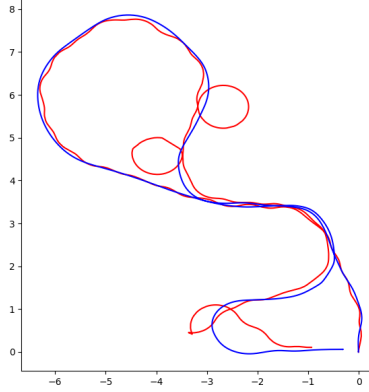


Figure 2: Our algorithm,  $x = 0.05$ ,  $\theta$  without the sinus

After seeing that result we figured out a way of finding a good  $x$  value and added the sinus to  $\theta$ .

To find the  $x$  value we have displayed the  $\theta$  value during a run and we have figured out that when the robot is on a good trajectory to follow the path the  $\text{abs}(\theta)$  value is usually below 0.5. We then tried to run our algorithm using different  $x$  values around 0.5 and the value which works the best is  $x = 0.3$ . With that value the algorithm works perfectly fine, we obtained results sensibly similar to a pure pursuit run.

## 2.2 Diverse improvements

### 2.2.1 Compute the linear speed depending on the angular speed

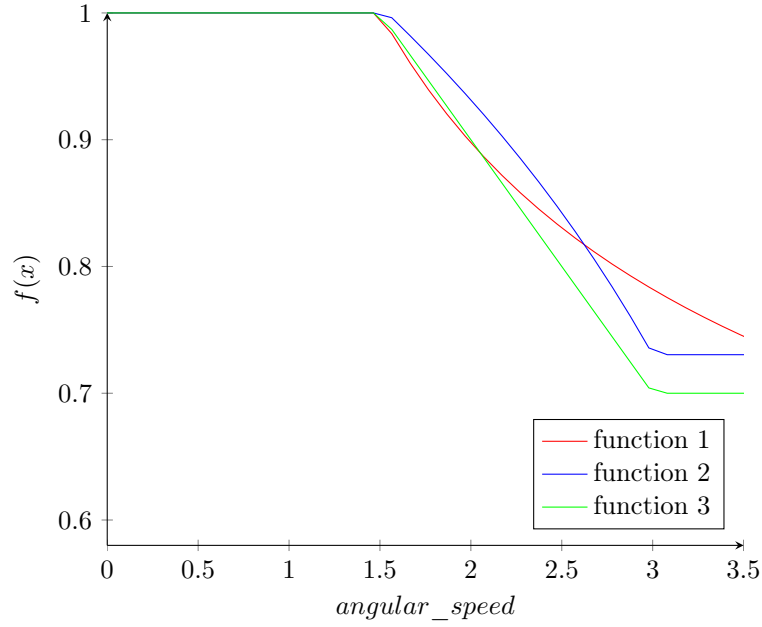
After the implementation of those two algorithms we wanted to try out to adapt the linear speed of the robot to the computed angular speed. Since the maximum linear speed of the robot is 1 we tried to find different functions that would maximize the linear speed when the angular speed is around 0 and minimize it when the angular speed is around 3.

We have found three functions that correspond to our needs:

$$\min(1, 1/\log_{10}(6 * \text{abs}(x) + 1)) \quad (1)$$

$$\min(1, \log_{10}(-(\min(3, \text{abs}(x)) - 4.7)) + 0.5) \quad (2)$$

$$\min(1, -0.2 * \min(3, \text{abs}(x)) + 1.3) \quad (3)$$



### 2.2.2 Adapt the look-ahead distance to the linear speed

After many tests with different functions to adapt the angular speed we figured out that adapting the look-ahead distance to the linear speed would probably be a good idea. The idea is to reduce the look-ahead distance when the robot slows down, and increase it conversely.

Using our functions the maximum linear speed is 1 and the minimum is around 0.7, we decided to try to set the look-ahead distance at the same value as the linear speed. By doing that we obtained some very good results using our own algorithm, on the other hand the robot always fails to follow the path using the pure pursuit algorithm.

## 2.3 How to run our program

To run our program you need to have python2 installed on your machine, then you just need to enter the following command with the parameters you want:

```
python2 path_tracking.py path.json X Y
```

$$X = \begin{cases} 1: \text{pure pursuit algorithm} \\ 2: \text{our own algorithm} \end{cases} \quad Y = \begin{cases} 1: linear\_speed = 1 \\ 2: \text{apply function 1 to } linear\_speed \\ 3: \text{apply function 2 to } linear\_speed \\ 4: \text{apply function 3 to } linear\_speed \end{cases}$$

If the optional parameter `--ahead` is set, the `look_ahead` will be equal to the `linear_speed`, 0.7 otherwise. You can also set the optional parameter `--plot` to display the path of the robot. If the ip address of the MRDS server is not `localhost`, you can set it with the `--ip` parameter.

## Chapter 3

# Experimentation and analysis

In this chapter we will present the results that we obtained after all our tests.

On the following table are the times in seconds obtained for the path ‘around the table and back’ with different settings:

Algorithm	<i>linear_speed</i>	<i>look_ahead</i> = 0.7	<i>look_ahead</i> = <i>linear_speed</i>
Pure pursuit	1	29.39	Fail
	function 1	30.62	Fail
	function 2	29.99	Fail
	function 3	32.76	Fail
Our algorithm	1	29.57	29.48
	function 1	29.20	27.54
	function 2	29.64	27.56
	function 3	29.83	27.79

We can see that with a linear speed set at 1 and a look-ahead distance at 0.7 the pure pursuit and our algorithm both have very similar times. But when we analyze the paths that the robot followed in both cases we can see that the linear speed is too high for our algorithm, causing a crash on the last sharp turn:

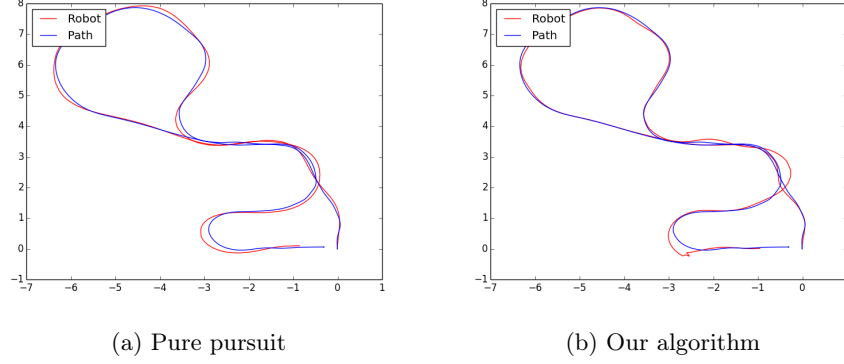


Figure 3:  $linear\_speed = 1$ ,  $look\_ahead = 0.7$

We can see that with that look ahead distance, both algorithms shifted away from the path in the sharp turns.

An interesting result is to see how the fact that the robot slows down in sharp angles changes its path, in the following image we can see that the robot stays very close to the path the entire time:

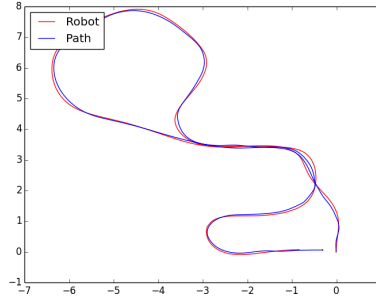


Figure 4: Pure pursuit,  $linear\_speed = \text{function 1}$ ,  $look\_ahead = 0.7$

The faster result that we obtained was using our own algorithm, with the function 2 and a look-ahead distance equals to the linear speed. With those settings the robot cuts inside every turn but stays very close to the path as soon as it's more straight. It was able to follow the path in 27.54 seconds.

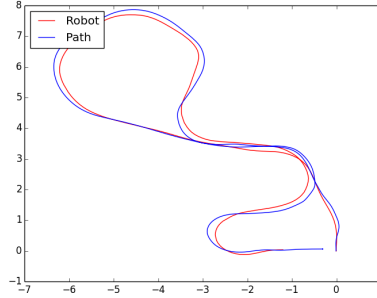


Figure 5: Our algorithm,  $linear\_speed = \text{function } 1$ ,  $look\_ahead = linear\_speed$

Of course with that kind of settings if the path was going closer to obstacles in the inside of sharp turns the robot would most likely hit them and fail to follow the path.

By analyzing those results we can conclude that:

- The safest way to follow a path is to use the pure pursuit algorithm, using the function 1 and a look-ahead distance set at 0.7.
- The fastest way to follow a path is to use our algorithm, using the function 1 and a look-ahead distance equal to the linear speed.

# Conclusion

For this assignment we have implemented a pure pursuit algorithm and developed another path tracking algorithm.

On the day of the examination we first tried a run using the fastest of our settings, meaning our algorithm with a logarithmic function on the linear speed and the look-ahead distance adapted to that linear speed. The result of that run was a crash on the turn behind the sofa, indeed as concluded in chapter 3 those settings make the robot cut the turns, since the sofa was very close in the inside of the turn the robot hit it and failed the run.

For the second run we have used our safest settings, being the pure pursuit algorithm with a logarithmic function on the linear speed and a constant look-ahead distance of 0.7. This second turn was as expected successful, the robot staying very close to the path didn't hit any obstacles. With those settings the robot was able to follow the path of the examination in 44 seconds.

To go further and minimize the time of travel we could use the 'lasers' of the robot to detect when it is too close from an obstacle and so adapt the linear speed and the look-ahead distance depending on the distance.



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# Bibliography

- [Con92] Craig Coulter. Implementation of the pure pursuit path tracking algorithm. Master's thesis, Carnegie Mellon University, 1992. [https://www.ri.cmu.edu/pub\\_files/pub3/coulter\\_r\\_craig\\_1992\\_1/coulter\\_r\\_craig\\_1992\\_1.pdf](https://www.ri.cmu.edu/pub_files/pub3/coulter_r_craig_1992_1/coulter_r_craig_1992_1.pdf).
- [Lun03] Martin Lundgren. Path tracking for a miniature robot. Master's thesis, Umeå University, 2003. <http://www8.cs.umu.se/kurser/TDBD17/VT06/utdelat/Assignment%20Papers/Path%20Tracking%20for%20a%20Miniature%20Robot.pdf>.

# Appendix

## Source code

```
1 import httpplib, json, time, sys
2 import matplotlib.pyplot as plt
3 import argparse
4 from math import *
5
6 HEADERS = {"Content-type": "application/json", "Accept": "
           text/json"}
7
8 parser = argparse.ArgumentParser()
9 parser.add_argument("path", help="The file containing the
           path (jsonFile).")
10 parser.add_argument("algo", type=int, help="id of the algo
           to use:\n\t1. Pure pursuit\n\t2. Our algo")
11 parser.add_argument("func", type=int, help="id of the
           function to use to calculate the linear speed
           :\n\t1. constant=1\n\t2. 1/log10(6*abs(angle)
           +1)\n\t3. log(-abs(angle)+0.5\n\t4. -0.2*abs(
           angle)+1.3")
12 parser.add_argument("--plot", action="store_true", help="
           display the path to follow and the robot's path
           ")
13 parser.add_argument("--ahead", action="store_true", help="
           If set, the look ahead is equal to the speed,
           0.7 otherwise")
14 parser.add_argument("--ip", default="localhost", help="Ip
           address of the MRDS server. Default : localhost
           ")
15
16 args = parser.parse_args()
17
18 MRDS_URL = args.ip + ":50000"
19
20 class UnexpectedResponse(Exception): pass
21
22 def post_speed(angular_speed, linear_speed):
```

```

23     """Sends a speed command to the MRDS server"""
24     mrds = httplib.HTTPConnection(MRDS_URL)
25     params = json.dumps({'TargetAngularSpeed':
26                          angular_speed, 'TargetLinearSpeed':
27                          linear_speed})
26     mrds.request('POST', '/lokarria/differentialdrive',
27                  params, HEADERS)
27     response = mrds.getresponse()
28     status = response.status
29     if status == 204:
30         return response
31     else:
32         raise UnexpectedResponse(response)
33
34 def get_pose():
35     """Reads the current position and orientation from the
36        MRDS"""
36     mrds = httplib.HTTPConnection(MRDS_URL)
37     mrds.request('GET', '/lokarria/localization')
38     response = mrds.getresponse()
39     if response.status == 200:
40         pose_data = response.read()
41         response.close()
42         return json.loads(pose_data)
43     else:
44         raise UnexpectedResponse(response)
45
46 def bearing(q):
47     return rotate(q, {'X': 1.0, 'Y': 0.0, "Z": 0.0})
48
49 def rotate(q, v):
50     return vector(qmult(qmult(q, quaternion(v)), conjugate(
51         q)))
52
53 def quaternion(v):
54     q = v.copy()
55     q['W'] = 0.0
56     return q
57
58 def vector(q):
59     v = {}
60     v["X"] = q["X"]
61     v["Y"] = q["Y"]
62     v["Z"] = q["Z"]
63     return v
64
65 def conjugate(q):
66     qc = q.copy()
67     qc["X"] = -q["X"]
68     qc["Y"] = -q["Y"]

```

```

68     qc["Z"] = -q["Z"]
69     return qc
70
71 def qmult(q1, q2):
72     q = {}
73     q["W"] = q1["W"] * q2["W"] - q1["X"] * q2["X"] - q1["Y"]
74     " ] * q2["Y"] - q1["Z"] * q2["Z"]
75     q["X"] = q1["W"] * q2["X"] + q1["X"] * q2["W"] + q1["Y"]
76     " ] * q2["Z"] - q1["Z"] * q2["Y"]
77     q["Y"] = q1["W"] * q2["Y"] - q1["X"] * q2["Z"] + q1["Y"]
78     " ] * q2["W"] + q1["Z"] * q2["X"]
79     q["Z"] = q1["W"] * q2["Z"] + q1["X"] * q2["Y"] - q1["Y"]
80     " ] * q2["X"] + q1["Z"] * q2["W"]
81
82     return q
83
84 def get_heading():
85     """Returns the XY Orientation as a bearing unit vector
86         """
87     return bearing(get_pose()['Pose']['Orientation'])
88
89 def get_position():
90     """Returns the XYZ position"""
91     return get_pose()['Pose']['Position']
92
93 def pythagora_hypotenus(x, y):
94     """Pythagoras theorem"""
95     return sqrt((x ** 2) + (y ** 2))
96
97 def make_path():
98     """Add all coordinates of the path to a stack"""
99     stack = []
100     with open(args.pat) as path_file:
101         json_path = json.load(path_file)
102         for i in range (len(json_path)):
103             stack.append(json_path[i]['Pose']['Position'])
104         stack.reverse()
105     return stack
106
107 def get_point(path, pos, look_ahead, update_path = True):
108     """Select the next goal point using the robot's
109         position and a look-ahead distance
110
111     Args:
112         path (array of points): The path that the robot
113             must follow
114         pos: The actual position of the robot
115         look_ahead (float): The look-ahead distance
116         update_path (boolean): At true it deletes the past
117             points, at false it doesn't

```

```

110     Return the selected goal point
111     """
112     if path:
113         for i in range(len(path)):
114             point = path[len(path) - (1 if update_path else
115                                     i)]
116             dx = point['X'] - pos['X']
117             dy = point['Y'] - pos['Y']
118
119             dist = pythagora_hypotenus(dx, dy)
120
121             if dist < look_ahead:
122                 if update_path:
123                     path.pop()
124                 else:
125                     return point
126             else:
127                 print ("Stack failed")
128
129 def pure_pursuit(robot_pos, point):
130     """compute the angular speed that the robot must have
131     to follow the path using the pure pursuit
132     algorithm
133
134     Args:
135         robot_pos: the actual position of the robot
136         point: the coordinates of the goal point
137
138     Return the computed angular speed
139     """
140     dx = point['X'] - robot_pos['X']
141     dy = point['Y'] - robot_pos['Y']
142     dist = pythagora_hypotenus(dx, dy)
143
144     robot_heading = get_heading()
145     hx = robot_heading['X']
146     hy = robot_heading['Y']
147     robot_angle = atan2(hy, hx)
148     point_angle = atan2(point['Y'] - robot_pos['Y'], point
149                         ['X'] - robot_pos['X'])
150     teta = point_angle - robot_angle
151     delta_x = sin(teta) / dist
152
153     return (2 * delta_x) / (dist ** 2)
154
155 def our_algo(robot_pos, point):
156     """compute the angular speed that the robot must have
157     to follow the path using our own algorithm
158
159     Args:

```

```

155         robot_pos: the actual position of the robot
156         point: the coordinates of the goal point
157
158     Return the computed angular speed
159     """
160     robot_heading = get_heading()
161     hx = robot_heading['X']
162     hy = robot_heading['Y']
163     robot_angle = atan2(hy, hx)
164     point_angle = atan2(point['Y'] - robot_pos['Y'], point
165                          ['X'] - robot_pos['X'])
166
167     teta = sin(point_angle - robot_angle)
168     x = 0.3
169     max_ang_speed = 3
170
171     ang_speed = (max_ang_speed * teta) / x
172
173     return min(max(ang_speed, -max_ang_speed),
174               max_ang_speed)
175
176 def run_algo(algo, pos, goal_point):
177     """Run the algorithm that the user choose
178
179     Args:
180         algo: The parameter value that the user gave
181         pos: The actual position of the robot
182         goal_point: The coordinates of the goal point
183
184     Return the angular speed returned by the corresponding
185             algorithm
186     """
187     ang_speed = 0
188     if algo == 1:
189         ang_speed = pure_pursuit(pos, goal_point)
190     else:
191         ang_speed = our_algo(pos, goal_point)
192
193     return ang_speed
194
195 def compute_linear_speed(func, ang_speed):
196     """Compute the linear speed adapted to the angular
197             speed, using the function that the user
198             picked
199
200     Args:
201         func: The parameter value that the user gave
202         ang_speed: The angular speed of the robot
203
204     Return the computed linear speed

```



```

200     """
201     max_linear_speed = 1
202     linear_speed = max_linear_speed
203
204     if func == 2:
205         linear_speed = min(max_linear_speed, 1 / log10(6 *
206             abs(ang_speed) + 1))
207     elif func == 3:
208         linear_speed = min(max_linear_speed, log10(-(min(3,
209             abs(ang_speed)) - 4.7)) + 0.5)
210     elif func == 4:
211         linear_speed = min(max_linear_speed, -0.2 * min(3,
212             abs(ang_speed)) + 1.3)
213
214     return linear_speed
215
216 if __name__ == '__main__':
217     path = make_path()
218     positions_x, positions_y, path_x, path_y = ([[] for i in
219         range(4)])
220
221     for point in path:
222         path_x.append(point['X'])
223         path_y.append(-point['Y'])
224
225     angular_constant = 0.4
226     look_ahead = 0.7
227
228     algo = args.algo
229     func = args.func
230
231     start_time = time.time()
232     while path:
233         pos = get_position()
234         positions_x.append(pos['X'])
235         positions_y.append(-pos['Y'])
236         goal_point = get_point(path, pos, look_ahead)
237         if goal_point:
238             ang_speed = run_algo(algo, pos, goal_point)
239             linear_speed = compute_linear_speed(func,
240                 ang_speed)
241             if args.ahead:
242                 look_ahead = linear_speed
243
244             response = post_speed(ang_speed *
245                 angular_constant,
246                 linear_speed)
247             time.sleep(0.01)
248         response = post_speed(0,0)
249
250     end_time = time.time()

```

```

247     run_time = end_time - start_time
248
249     if args.plot:
250         plt.plot(positions_y, positions_x, 'r', label="
                Robot")
251         plt.plot(path_y, path_x, 'b', label="Path")
252         plt.legend(loc='upper left')
253         plt.show()
254
255     print("end of run")
256     print("The robot finished the path in:", run_time, "
            seconds")

```