DTU Course 02156 Logical Systems and Logic Programming

Mandatory Assignment 5 — Deadline Thursday 30/11 23:59

MUST BE SOLVED INDIVIDUALLY

You are only allowed to get help from the teacher and the teaching assistants.

You may use the notes and definitions on the first page of the sample exams.

You are allowed to use your computer and there is no 2 hours time limit.

For the whole assignment you must submit exactly 2 files on DTU Learn:

1. A single PDF file with extension `.pdf` with the report.
2. A single Isabelle theory file with extension `.thy` as explained below.

Absolutely no ZIP files, no text processing documents or any other file formats.

Important for Isabelle theory files:

- The file must have the extension `.thy` and the name of the file must be the same as the name of the theory in the file.
- The file must import the right theory as explained in the problem text.
- The file must end with the line

  ```
  end
  ```

  in order to be sure that all proofs in the file are checked by Isabelle.

And most importantly, please make sure that absolutely no errors are indicated in red to the right of the file text area in Isabelle.

Problem 1 (30%)

The details of the following answers must be provided and explained (in particular it is not sufficient to just list the result from a Prolog program).

Question 1.1

Consider the following formula: $p \to q \to p$

Use refutation and the resolution procedure. State whether this shows that the formula is valid or not.

Question 1.2

Consider the following formula: $(p \to r) \to (r \to q) \to p \to q$

Use refutation and the resolution procedure. State whether this shows that the formula is valid or not.

Question 1.3

Consider the following formula: $((p \to q) \to p) \to p$

Use refutation and the resolution procedure. State whether this shows that the formula is valid or not.

Problem 2 (40%)

The details of the following answers must be provided and explained (in particular it is not sufficient to just list the result from a Prolog program).

Question 2.1

Consider the following formula: $\exists x \forall y p(x, y) \rightarrow \forall y \exists x p(x, y)$

Prove the formula using refutation, skolemization and the general resolution procedure.

Question 2.2

Consider the following formula: $(\forall x(p(x) \rightarrow \neg \exists y q(y, x))) \rightarrow (p(a) \rightarrow \neg q(a, a))$

Prove the formula using refutation, skolemization and the general resolution procedure.

Question 2.3

Consider the following formula: $(\forall x p(x) \wedge \forall x q(x)) \rightarrow \forall x(p(x) \wedge q(x))$

Prove the formula using refutation, skolemization and the general resolution procedure.

Question 2.4

Consider the following formula: $(\forall x \exists y(p(x) \wedge \neg p(y))) \rightarrow \neg q(a)$

Prove the formula using refutation, skolemization and the general resolution procedure.

Problem 3 (30%)

Obtain the Isabelle file `Core_Logic.thy` from DTU Learn / Course Content / General Information, but do not hand in this file.

Consider the 3 proof examples in the file:

$$p \to p$$
$$p \to (p \to q) \to q$$
$$p \to q \to q \to p$$

Make a new Isabelle file `Core_Logic_Assignment.thy` starting with the line

```
theory Core_Logic_Assignment imports Core_Logic begin
```

and ending with the line

```
end
```

in order to be sure that all proofs in the file are checked by Isabelle.

Question 3.1

Construct a proof in the file `Core_Logic_Assignment.thy` of the formula:

$$(p \to q) \to p \to q$$

Use the same style as the 3 proof examples above.

Question 3.2

Construct a proof in the file `Core_Logic_Assignment.thy` of the formula:

$$p \to p \to q \to q$$

Use the same style as the 3 proof examples above.

Question 3.3

Construct a proof in the file `Core_Logic_Assignment.thy` of the formula:

$$(p \to p \to q) \to p \to q$$

Use the same style as the 3 proof examples above.