DTU Course 02156 Logical Systems and Logic Programming

Mandatory Assignment 3 — Deadline Sunday 29/10 23:59

MUST BE SOLVED INDIVIDUALLY

You are only allowed to get help from the teacher and the teaching assistants.

You may use the notes and definitions on the first page of the sample exams.

You are allowed to use your computer and there is no 2 hours time limit.

For the whole assignment you must submit exactly 2 files on DTU Learn:

1. A single PDF file with extension `.pdf` with the report.
2. A single Prolog file with extension `.pl` with the programs.

Absolutely no ZIP files, no text processing documents or any other file formats.

The report should not contain any program listings (just refer to the Prolog file).

The programs should load in SWI-Prolog without any errors or warnings.

The programs must be properly documented with comments.

If possible use only the ISO Prolog features of SWI-Prolog covered in the course.

All programs must be tested and the tests must be included in the report.

In particular:

- Test a few normal cases.
- Test the special cases.
- Test with variables only (if the instantiation pattern allows variables).

Show the Prolog queries and the corresponding answers — and keep explanations short.

Problem 1 (40%)

The aim is to write programs for the many-valued logic introduced in Assignment 1.

The starting point is the file `boolean.pl` available as `boolean.txt` on DTU Learn — the file must be modified — in the end delete everything unused.

Again let $n = 3$ in the following questions.

Question 1.1

This question does not consider the many-valued logic but rather the two-valued "boolean" propositional logic in the textbook.

The file contains the following Prolog code.

```
:- op(650,xfy,eqv).  /* equivalence */
:- op(640,xfy,imp).  /* implication */
:- op(630,xfy,dis).  /* disjunction */
:- op(620,xfy,con).  /* conjunction */
:- op(610,fy, neg).  /* negation    */

tt(neg A,  V,TV) :- tt(A,V,TVA), negate(TVA,TV).
tt(A con B,V,TV) :- tt(A,V,TVA), tt(B,V,TVB), opr(con,TVA,TVB,TV).
tt(A dis B,V,TV) :- tt(A,V,TVA), tt(B,V,TVB), opr(dis,TVA,TVB,TV).
tt(A eqv B,V,TV) :- tt(A,V,TVA), tt(B,V,TVB), opr(eqv,TVA,TVB,TV).
tt(A imp B,V,TV) :- tt(A,V,TVA), tt(B,V,TVB), opr(imp,TVA,TVB,TV).
tt(A,      V,TV) :- member((A,TV),V).

/*

negate(t,f). negate(f,t).

opr(con,t,t,t). opr(con,t,f,f). opr(con,f,t,f). opr(con,f,f,f).
opr(dis,t,t,t). opr(dis,t,f,t). opr(dis,f,t,t). opr(dis,f,f,f).
opr(eqv,t,t,t). opr(eqv,t,f,f). opr(eqv,f,t,f). opr(eqv,f,f,t).
opr(imp,t,t,t). opr(imp,t,f,f). opr(imp,f,t,t). opr(imp,f,f,t).

*/

negate(P,Q) :- P = t -> Q = f ; Q = t.

opr(con,P,Q,R) :- P = t, Q = t -> R = t ; R = f.
opr(dis,P,Q,R) :- P = f, Q = f -> R = f ; R = t.
opr(eqv,P,Q,R) :- P = Q -> R = t ; R = f.
opr(imp,P,Q,R) :- negate(P,N), opr(dis,N,Q,R).
```

The problem continues on the next page

What does the following query show about the formula $p \rightarrow q$.

```
?- tt(p imp q,[(p,t),(q,f)],f).

Yes
```

Question 1.2

This question also does not consider the many-valued logic.

Write a Prolog program `boolean/0` that checks that the predicates `negate/2` and `opr/4` work as required for the two-valued "boolean" propositional logic in the textbook (notice the code shown as a comment).

Hence `boolean` must succeeed but it is not necessary to document this in the report.

Question 1.3

The file also contains a comment with the following code that prints a truth table for negation in the many-valued logic.

```
values([t,f,x]).

negate(P,Q) :-
  P = t -> Q = f ;
  P = f -> Q = t ;
  Q = P.

negate :-
  values(L), write(neg), nl,
  member(P,L), negate(P,Q),
  write(P), write(' '), write(Q), nl,
  fail.
negate.
```

Write similar Prolog programs `opr/4` and `opr/0` that prints the truth tables for conjunction, disjunction, bi-implication (equivalence) and implication in the many-valued logic.

Question 1.4

Does `boolean/0` succeed for the many-valued logic?

It is optional to comment on this in the report.

Problem 2 (20%)

Let A be the following formula in propositional logic:

$$((p \land q) \to r) \leftrightarrow (p \to (q \to r))$$

The aim is to decide whether A is a valid formula or not.

Consider the following incomplete proof in the Gentzen system $\mathcal{G}$ for propositional logic:

1.	$\vdash \neg r, \neg p, \neg q, r$	Axiom
2.	$\vdash q, \neg p, \neg q, r$	Axiom
3.	$\vdash \boxed{1}$	$\beta\to$,2,1
4.	$\vdash p, \neg p, \neg q, r$	Axiom
5.	$\vdash \neg p, \neg q, r, \neg(p \to (q \to r))$	$\beta\to$,4,3
6.	$\vdash \neg(p \land q), r, \neg(p \to (q \to r))$	$\boxed{2}$
7.	$\vdash \neg(p \to (q \to r)), (p \land q) \to r$	$\alpha\to$,6
8.	$\vdash (p \to (q \to r)) \to ((p \land q) \to r)$	$\alpha\to$,7
9.	$\vdash \neg r, \neg q, r, \neg p$	$\boxed{3}$
10.	$\vdash q, \neg q, r, \neg p$	Axiom
11.	$\vdash p, \neg q, r, \neg p$	Axiom
12.	$\vdash p \land q, \neg q, r, \neg p$	$\beta\land$,11,10
13.	$\vdash \neg q, r, \neg p, \neg((p \land q) \to r)$	$\beta\to$,12,9
14.	$\vdash \neg p, q \to r, \neg((p \land q) \to r)$	$\alpha\to$,13
15.	$\vdash \boxed{4}$	$\alpha\to$,14
16.	$\vdash ((p \land q) \to r) \to (p \to (q \to r))$	$\alpha\to$,15

Question 2.1

Complete the first part of the proof by giving the content of the boxes $\boxed{1}$ and $\boxed{2}$.

Question 2.2

Complete the second part of the proof by giving the content of the boxes $\boxed{3}$ and $\boxed{4}$.

Question 2.3

Recall that the aim is to decide whether A is a valid formula or not.

Is it possible to complete the proof, and if so, what is the missing line at the end?

Problem 3 (40%)

The details of the following answers must be provided and explained (in particular it is not sufficient to just list the result from a Prolog program).

Question 3.1

Consider the following formula: $\exists x \forall y p(x, y) \rightarrow \forall y \exists x p(x, y)$

Use refutation and the systematic construction of a semantic tableau. State whether this shows that the formula is valid or not.

Question 3.2

Consider the following formula: $(\forall x(p(x) \rightarrow \neg \exists y q(y, x))) \rightarrow (p(a) \rightarrow \neg q(a, a))$

Use refutation and the systematic construction of a semantic tableau. State whether this shows that the formula is valid or not.

Question 3.3

Consider the following formula: $(\forall x p(x) \wedge \forall x q(x)) \rightarrow \forall x(p(x) \wedge q(x))$

Use refutation and the systematic construction of a semantic tableau. State whether this shows that the formula is valid or not.

Question 3.4

Consider the following formula: $(\forall x \exists y(p(x) \wedge \neg p(y))) \rightarrow \neg q(a)$

Use refutation and the systematic construction of a semantic tableau. State whether this shows that the formula is valid or not.