In this short essay we will discuss the difference between the first event study (with the different assets) and the third event study (with the portfolio) as it seems according to the graphics, that the third event study is way more volatile, despite it seems, using similar parameters. Please note that this discussion also applies for the first and third event study with the weight of the minimum variance portfolio.

We denote X_A the group of the different assets (first event study) and X_P the portfolio (third event study). How does X_P differs from X_A ?

Both seem to be the same object as both are a group of assets returns with a specific weight all combined together. The answer is in the code, it seems like in the first case, the five assets are being taken in count where in the second it looks like the portfolio is just characterised as a bigger asset made of its components. We can denote both as follows:

1) $X_A = \{X_1, X_2, X_3, X_4, X_5\}$

2) $X_P = \{\sum_1^5 X_i\}$

Logically, we show that they both have the same expected returns.

1) $E(X_A) = E(X_1) + E(X_2) + E(X_3) + E(X_4) + E(X_5)$

2) $E(X_P) = E(\sum_1^5 X_i) = E(X_1 + X_2 + X_3 + X_4 + X_5) = E(X_1) + E(X_2) + E(X_3) + E(X_4) + E(X_5)$

What about the Variance ? For the sake of simplicity, we will use only 2 assets, however please note that these results are valid for any finite number of assets. We denote $\rho(X_1, X_2)$ the coefficient of correlation between asset 1 and 2, σ_{Xi} the standard deviation of asset i and $Cov(X_1, X_2)$ the covariance between both assets.

We have $X_A = \{X_1, X_2\}$ and $X_P = \{\sum_1^2 X_i\}$, therefore:

1) $V(X_A) = V(X_1) + V(X_2) + 2Cov(X_1, X_2)$

$Cov(X_1, X_2) = \rho(X_1, X_2) \times \sigma_{X1} \times \sigma_{X2}$

Hence $V(X_A) = V(X_1) + V(X_2) + 2\rho(X_1, X_2) \times \sigma_{X1} \times \sigma_{X2}$.

2) $V(X_P) = V(X_P)$ because the portfolio is considered as a single asset! Even if it is composed of assets, we do not take into account the correlation between them, we can always split the portfolio into different parts, for example:

$$\begin{aligned} V(0,5X_P + 0,5X_P) &= V(0,5X_P) + V(0,5X_P) + 2\,\rho(0,5X_P, 0,5X_P) \times \sqrt{V(0,5X_P) \times V(0,5X_P)} \\ &= 0,25V(X_P) + 0,25V(X_P) + 2 \times 0,25V(X_P) \\ &= V(X_P) \end{aligned}$$

We can consider the splits as shares, as weights, so we can split our portfolio according to the weights of the assets.Here we give a weight of ½ to the two assets in the portfolio, we find:

1) $V(X_A) = V(0,5X_1 + 0,5X_2) = 0,25V(X_1) + 0,25V(X_2) + 2\rho(X_1, X_2) \times \sigma_{X1} \times \sigma_{X2}$

If we take the extreme case in which the variances of the two assets are equal, we show that:

$$\begin{aligned} V(0,5X_1 + 0,5X_2) &= 0,25V(X_1) + 0,25V(X_2) + 2\rho(X_1, X_2) \times \sqrt{V(0,5X_1) \times V(0,5X_2)} \\ &= 0,5V(X_1) + \rho(X_1, X_2) \times 2 \times 0,25V(X_1) \\ &= 0,5V(X_1) \times (1 + \rho(X_1, X_2)) \end{aligned}$$

Even with equal variances, we have the same variance, $V(X_A) = V(X_P)$ only if the correlation is 1!

The portfolio (X_P) because it is considered like a big asset, as a correlation with himself that is one, the portfolio (X_A) however takes in count that it is composed of five assets all possibly correlated. Hence the volatility of (X_P) will always be (for $\rho(X_1, X_2) < 1$) greater than the volatility of (X_A). This is why the third event study graph shows cumulative returns more volatile than the first one.