MAT370: Homework 2

Problem 1: In class, we obtained a function to carry out Gaussian Elimination. Paste the definition of that function into the code cell below and add comments which describe how the Python code corresponds to the mathematics required to use Gaussian Elimination in solving the matrix equation $A\vec{x} = \vec{b}$. After you've commented the function, add a text cell and discuss any shortcomings you see regarding our function -- specifically, are there scenarios in which you expect our function to fail?

> *Response.*

Start coding or generate with AI.

Problem 2: Repeat *Problem 1*, but with our algorithm for the Doolittle LU-Decomposition method.

> *Response.*

Start coding or generate with AI.

Problem 3: Given the LU-decomposition of A, determine A and $\det(A)$. Complete this by hand, and verify using `NUMPY` if you would like.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 5/3 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & 21 \\ 0 & 0 & 0 \end{bmatrix}$$

> *Solution.*

Problem 4: Use the LU-decompotion of the matrix A to solve the system $A\vec{x} = \vec{b}$, where

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & 1 & 0 \\ 1/2 & 11/13 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 & -1 \\ 0 & 13/2 & -7/2 \\ 0 & 0 & 32/13 \end{bmatrix}$$

and $\vec{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.

> *Solution.*

Problem 5: Solve the equations $AX = B$ using Gaussian Elimination where

$$A = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ -1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$$

> *Solution.*

Problem 6: Solve the equation $A\vec{x} = \vec{b}$ using Doolittle's decomposition method where

$$A = \begin{bmatrix} 2.34 & -4.10 & 1.78 \\ -1.98 & 2.47 & -2.22 \\ 2.36 & -15.17 & 6.18 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 0.02 \\ -0.75 \\ -6.63 \end{bmatrix}$$

> *Solution.*

Problem 7: Determine the coefficients of the polynomial $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ that passes through the points $(0, 3)$, $(1, 1)$, $(3, -27)$, $(4, -41)$.

> *Solution.*

Problem 8: Write a Python function that returns the *condition number* of a matrix based on the Euclidean norm. Test your function by computing the condition number of the ill-conditioned matrix

$$A = \begin{bmatrix} 1 & 2 & 6 & 8 \\ 2 & 4 & 11 & 16 \\ 4 & 7 & 24 & 33 \\ 8 & 16 & 41 & 63 \end{bmatrix}$$

> *Solution.*