Microeconomics: Problem Set 3

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1. (a) The lagrangian for this problem:

$$\mathcal{L} = wL + rK - \lambda(L^{\frac{1}{4}}K^{\frac{1}{4}} - y)$$

gives the following first order conditions:

$$\frac{\partial \mathcal{L}}{\partial L} = w - \frac{1}{4}\lambda L^{-\frac{3}{4}}K^{\frac{1}{4}}$$
$$\frac{\partial \mathcal{L}}{\partial K} = r - \frac{1}{4}\lambda K^{-\frac{3}{4}}L^{\frac{1}{4}}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = L^{\frac{1}{4}}K^{\frac{1}{4}} - Y$$

Solving gives:

$$wL^{\frac{3}{4}}K^{-\frac{1}{4}} = rK^{\frac{3}{4}}L^{-\frac{1}{4}}$$
$$wL = rK$$
$$(\frac{rK}{w})^{\frac{1}{4}}K^{\frac{1}{4}} = Y$$
$$K^* = \sqrt{\frac{w}{r}}Y^2$$
$$L^* = \sqrt{\frac{r}{w}}Y^2$$

(b) The cost function is given by:

$$c(w, r, Y) = wL^* + rK^* = 2\sqrt{rw}Y^2$$

This is concave in w. To see this note that:

$$\frac{\partial c}{\partial w} = \sqrt{\frac{r}{w}}Y^2$$
$$\frac{\partial^2 c}{\partial w^2} = -\sqrt{\frac{r}{w^3}}Y^2$$
$$< 0$$

for all $w > 0$.

(c) By Shepard's Lemma, we have that:

$$\frac{\partial c}{\partial w_i} = x_i^*(p, w)$$

And therefore:

$$\frac{\partial^2 c}{\partial w_i^2} = \frac{\partial x_i^*}{\partial w_i}$$

which is less than 0 for ordinary goods. So the cost function is concave in the prices of inputs.

2. (a) The lagrangian for a firm's cost minimisation problem is of the form:

$$\mathcal{L} = \mathbf{w}.\mathbf{x} - \lambda(f(\mathbf{x}) - Y)$$

for given output Y. The Envelope Theorem for Constrained Optimisation states that for the optimal value function $c(\mathbf{w}, Y)$,

$$\begin{aligned} \frac{dc}{dw_i} &= \left.\frac{\partial \mathcal{L}}{\partial w_i}\right|_{\mathbf{x}=\mathbf{x}^*} \\ &= x_i \end{aligned}$$