1. The utility function is $u(x_1, x_2) = x_1 x_2$

   (a) Using the definition of indirect utility and duality:

$$v(p_1, p_2, m) = \frac{m^2}{4p_1p_2} \tag{1}$$
$$v(p_1, p_2, e(p_1, p_2, u)) = u \tag{2}$$
$$e(p_1, p_2, u)^2 = 4p_1p_2u \tag{3}$$
$$e(p_1, p_2, u) = \sqrt{4p_1p_2u} \tag{4}$$

   (b) CV and EV:

      i. Mr Watt's would take a £50 pay cut to achieve the same utility as he currently has.

$$e(p_1, v(p_0, m_0)) - m_0 = \sqrt{4 * \frac{1}{4} * 1 * \frac{100^2}{4 * 1 * 1}} - 100 = -50 \tag{5}$$

      ii. A £100 increase pay would be required to achieve the same level of utility as in the new town.

$$m_1 - e(p_0, v(p_1, m_1)) = 100 - \sqrt{4 * 1 * 1 * \frac{100^2}{4 * \frac{1}{4} * 1}} = -100 \tag{6}$$

   (c) The consumer surplus will be between -50 and -100.

2. (a) Hotelling's Lemma says that if y^*, x^* are the solution to the PMP for a firm then we have that:

$$\frac{\partial \pi}{\partial p} = y^*(p, \mathbf{w}) \tag{7}$$
$$\frac{\partial \pi}{\partial w_i} = -x_i^*(p, \mathbf{w}) \tag{8}$$

   To see this is true, consider the PMP:

$$\max_{y,x} py(p) - \mathbf{wx} \tag{9}$$

   Shepard's Lemma can be applied to the optimal value function $\pi(p, \mathbf{w})$ directly to give the result.

   (b) If w_i increases, then we can see that profit must decrease, as $x_i > 0$.

(c)

$$\max_L pL^{\frac{1}{2}} - wL \tag{10}$$

$$\frac{1}{2}pL^{-\frac{1}{2}} - w = 0 \tag{11}$$

$$L = (\frac{p}{2w})^2 \tag{12}$$

$$\pi = \frac{p^2}{2w} - \frac{p^2}{4w} \tag{13}$$

$$\frac{\partial \pi}{\partial w} = -\frac{p^2}{2w^2} + \frac{p^2}{4w^2} = -\frac{p^2}{4w^2} = -L \tag{14}$$

3. (a)

$$40p - 2pz - w = 0 \tag{15}$$

$$-2 \tag{16}$$

Therefore profit maximising as the second derivative is negative.

(b) ?

(c) The input demand function is $z^* = \frac{40p-w}{2p} = 20 - \frac{w}{2p}$. The output supply function is:

$$y^* = 40z^* - {z^*}^2 \tag{17}$$

$$= 40(20 - \frac{w}{2p}) - (20 - \frac{w}{2p})^2 \tag{18}$$

$$= 800 - \frac{20w}{p} - 400 + \frac{20w}{p} - \frac{w^2}{4p^2} \tag{19}$$

$$= 400 - \frac{w^2}{4p^2} \tag{20}$$

$$\pi = py(p) - wz \tag{21}$$

$$\pi = 400p - \frac{w^2}{4p} - 20w + \frac{w^2}{2p} \tag{22}$$

(d) Hotelling's Lemma:

$$\frac{\partial \pi}{\partial w} = -(20 - \frac{w}{2p}) \tag{23}$$

$$= -z^*(p, w) \tag{24}$$