

Artificial neural networks and backpropagation

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- 2 Artificial neuron
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Artificial neural networks and deep learning history

For a very complete state of the art on deep learning, see the overview by Schmidhuber [Schmidhuber, 2015].

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- 2006-: CNN implementations using Graphical Processing Units (GPU): up to a 50 speed-up factor.

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- 1980's: the backpropagation algorithm (see, for example, the work of LeCun [LeCun, 1985])
- 2006-: CNN implementations using Graphical Processing Units (GPU): up to a 50 speed-up factor.
- 2012: Imagenet image classification won by a CNN with AlexNet [Krizhevsky et al., 2012].

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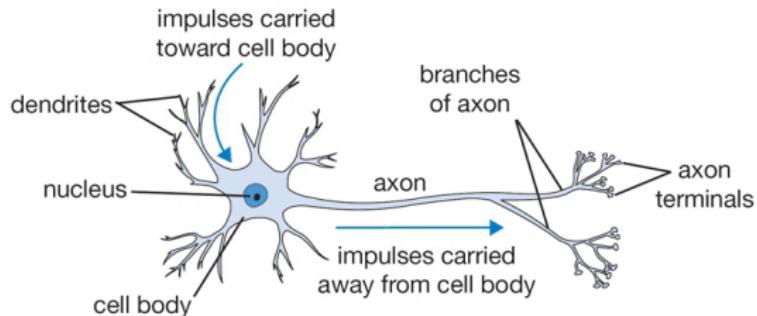
2 Artificial neuron

- Activation functions
- Artificial neuron as a classifier

3 Artificial neural networks

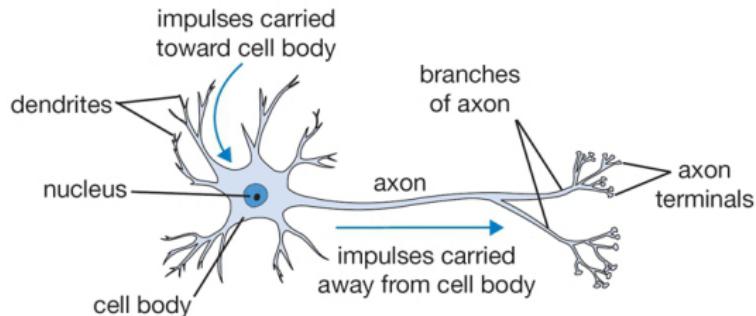
4 Training a neural network

Biological neuron



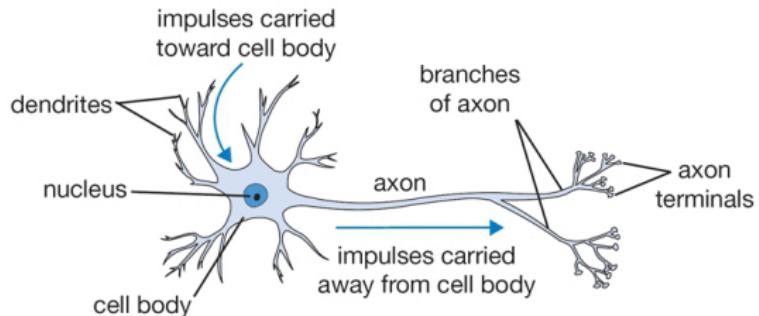
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Biological neuron



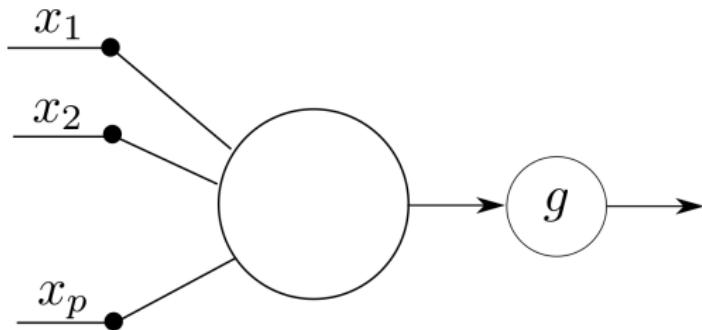
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- A human neuron can have several thousand dendrites

Biological neuron



- The human brain contains 100 billion (10^{11}) neurons
- A human neuron can have several thousand dendrites
- The neuron sends a signal through its axon if during a given interval of time the net input signal (sum of excitatory and inhibitory signals received through its dendrites) is larger than a threshold.

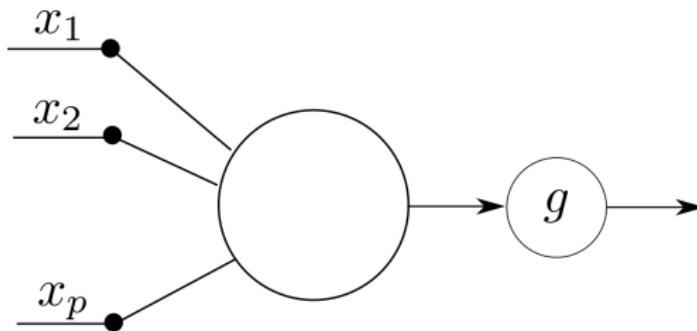
Artificial neuron



General principle

An artificial neuron takes p inputs $\{x_i\}_{1 \leq i \leq p}$, combines them to obtain a single value, and applies an **activation function** g to the result.

Artificial neuron

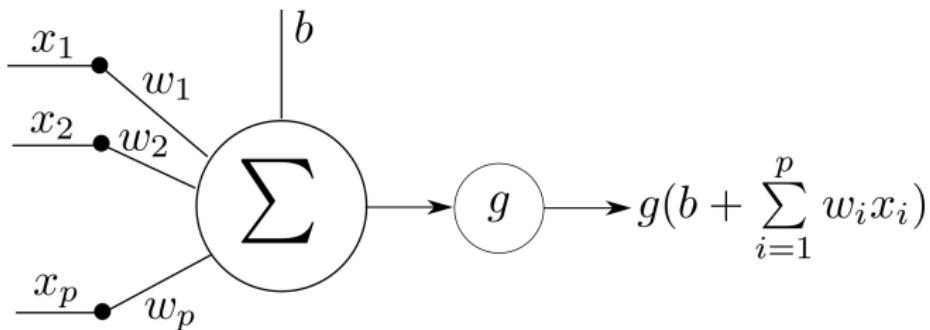


General principle

An artificial neuron takes p inputs $\{x_i\}_{1 \leq i \leq p}$, combines them to obtain a single value, and applies an **activation function** g to the result.

- The first artificial neuron model was proposed by [McCulloch and Pitts, 1943]
- Input and output signals were binary
- Input dendrites could be inhibitory or excitatory

Modern artificial neuron



- The neuron computes a linear combination of the **inputs** x_i
 - The **weights** w_i are multiplied with the inputs
 - The **bias** b can be interpreted as a threshold on the sum
- The **activation function** g somehow decides, depending on its input, if a signal (the neuron's **activation**) is produced

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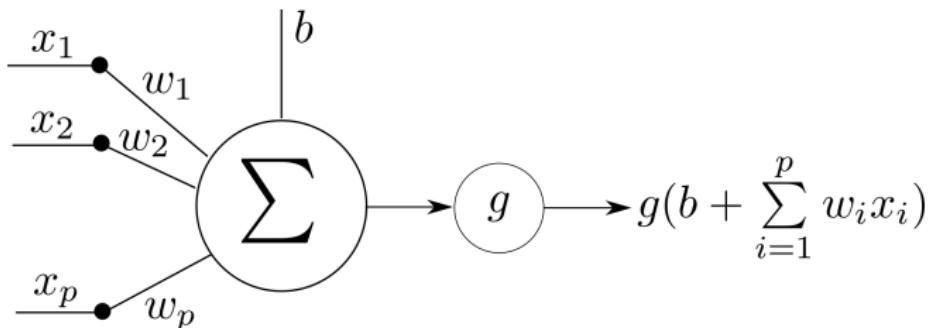
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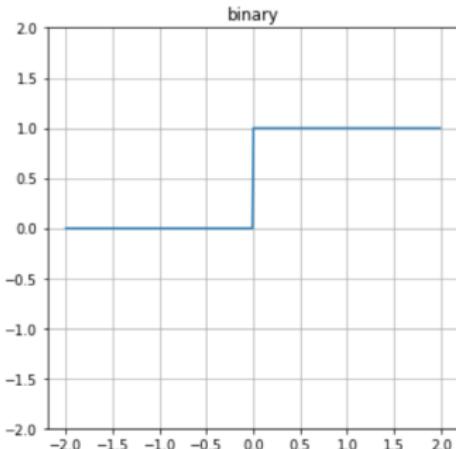
The role of the activation function



- The initial idea behind the activation function is that it works somehow as a gate
- If its input is “high enough”, then the neuron is activated, i.e. a signal (other than zero) is produced
- It can be interpreted as a source of abstraction: information considered as unimportant is ignored

Activation: binary

$$g(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

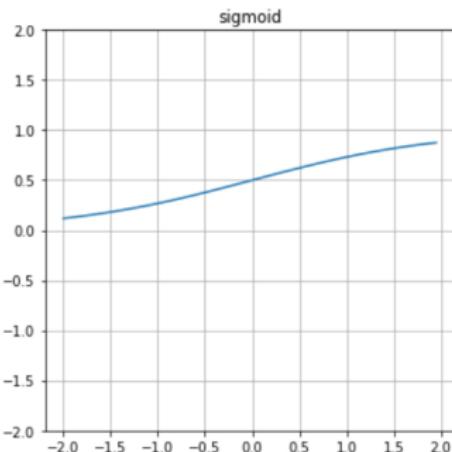


Remarks

- Biologically inspired
- + Simple to compute
- + High abstraction
- Gradient nil except on one point
- In practice, almost never used

Activation: sigmoid

$$g(x) = \frac{1}{1 + e^{-x}}$$

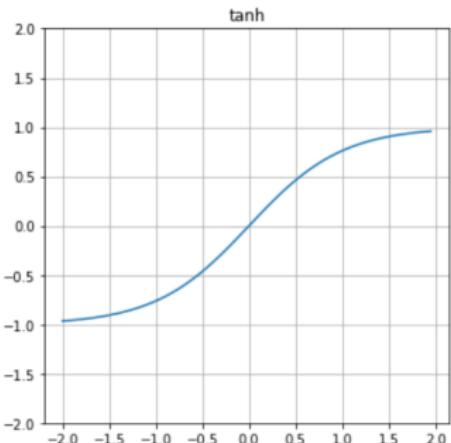


Remarks

- + Similar to binary activation, but with usable gradient
- However, gradient tends to zero when input is far from zero
- More computationally intensive

Activation: hyperbolic tangent

$$g(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

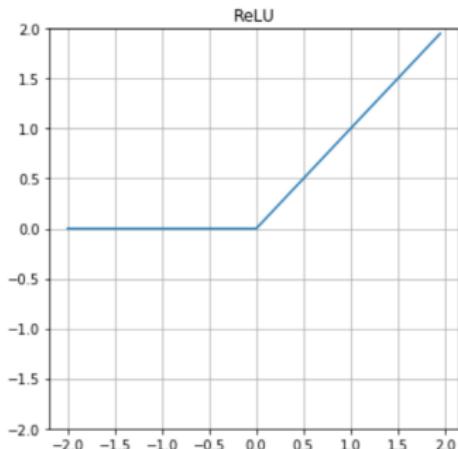


Remarks

- Similar to sigmoid

Activation: rectified linear unit (ReLU)

$$g(x) = \begin{cases} x, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

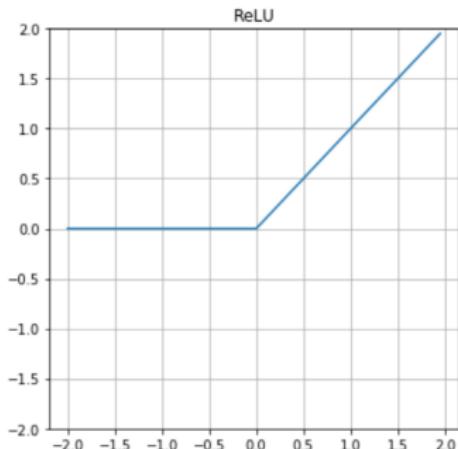


Remarks

- + Usable gradient when activated
- + Fast to compute
- + High abstraction

Activation: rectified linear unit (ReLU)

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Remarks

- + Usable gradient when activated
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ReLU is the most commonly used activation function.

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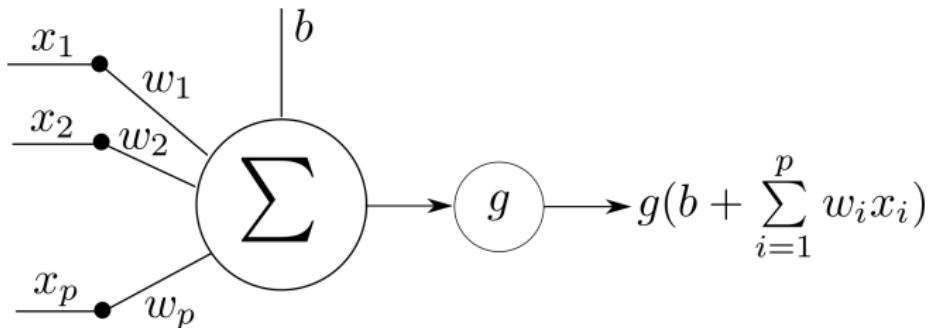
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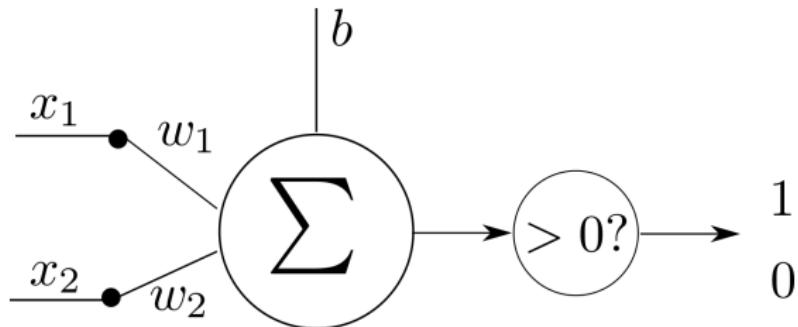
What can an artificial neuron compute?



In \mathbb{R}^p , $b + \sum_{i=0}^p w_i x_i = 0$ corresponds to a hyperplane. For a given point $\mathbf{x} = \{x_0, \dots, x_p\}$, decisions are made according to the side of the hyperplane it belongs to.

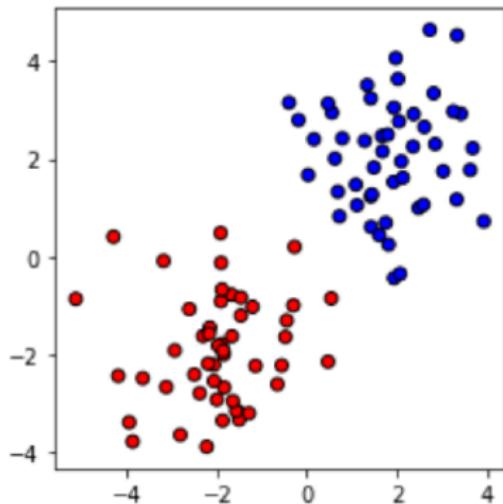
When the activation function is binary, we obtain a **perceptron**

Example of what we can do with a neuron

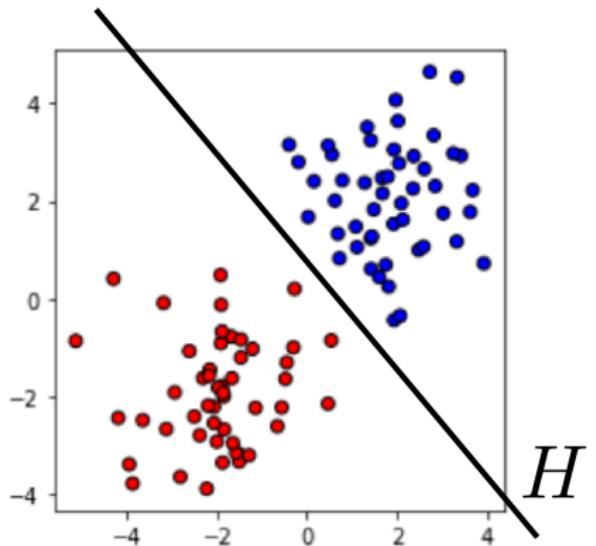


- $p = 2$: 2-dimensional inputs (can be represented on a screen!)
- Activation: binary
- Classification problem

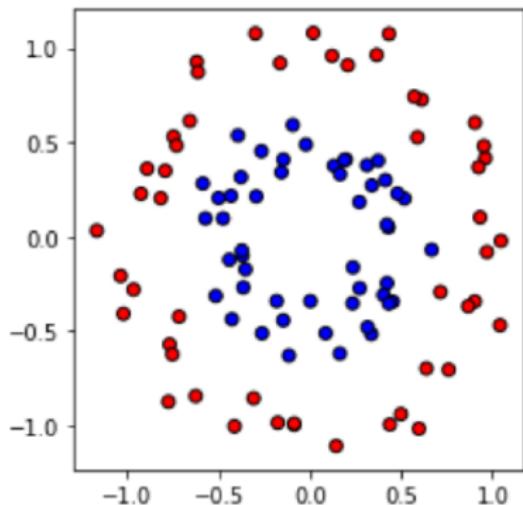
Gaussian clouds



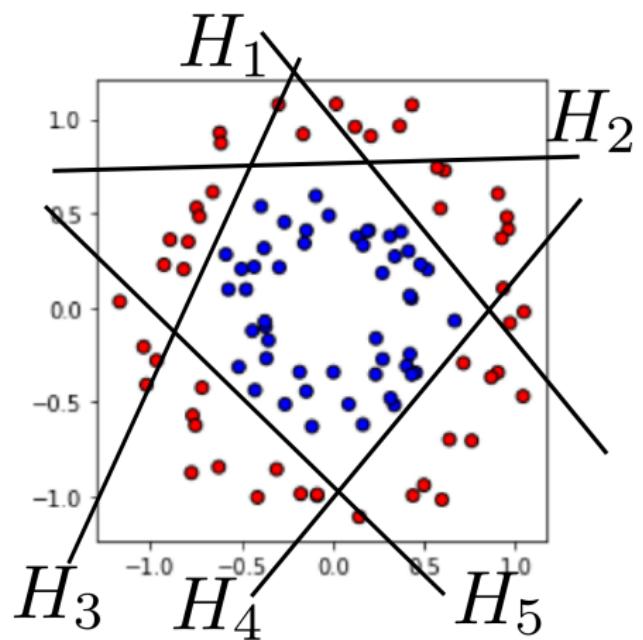
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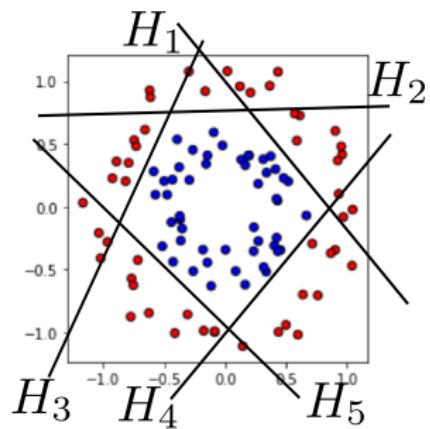
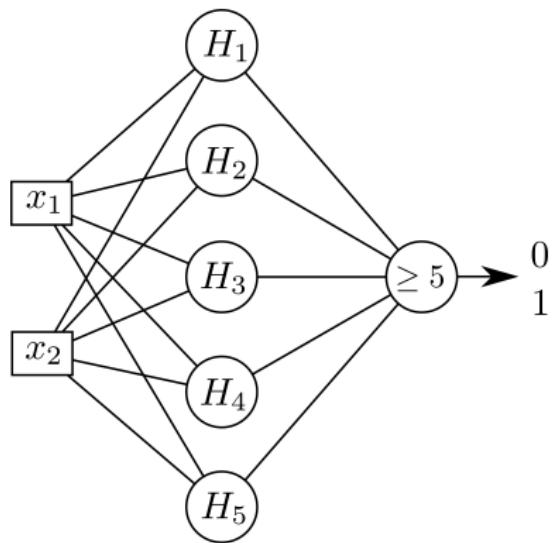
Circles



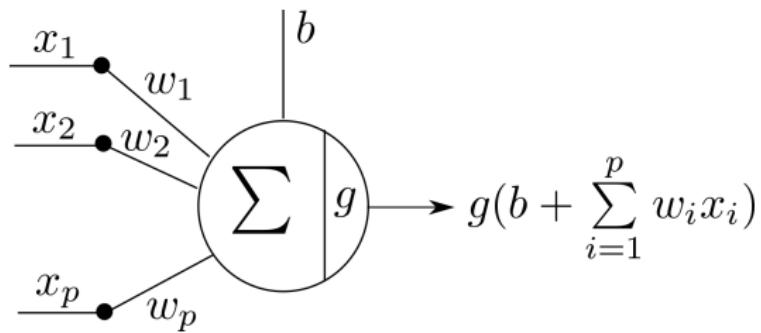
Circles



Solution



Artificial neuron compact representation



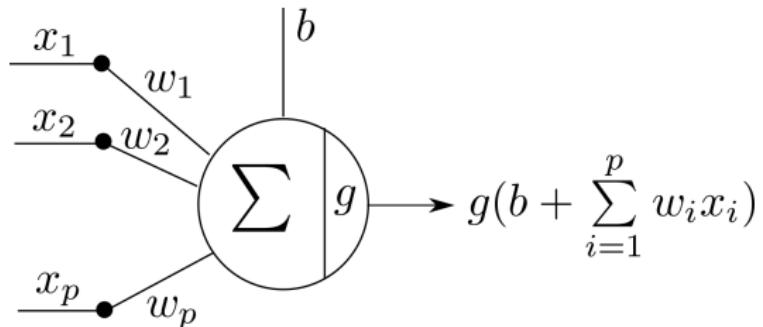
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Notations



With

$$\mathbf{w} = (w_1, \dots, w_p)^T$$

$$\mathbf{x} = (x_1, \dots, x_p)^T$$

We can simply write:

$$g(b + \sum_{i=1}^p w_i x_i) = g(b + \mathbf{w}^T \mathbf{x})$$

Computation graph

Definition

A computation graph is a directly acyclic graph such that:

- A node is a mathematical operator
- To each edge is associated a value
- Each node can compute the values of its output edges from the values of its input edges
 - Nodes without input edges are *input nodes*. They represent the input values of the graph.
 - Similarly, output values can be held in the *output nodes*.

Computing a *forward pass* through the graph means choosing its input values, and then progressively computing the values of all edges.

Computation graph example

We will compute:

$$\sigma(w_1x + w_2y + b)$$

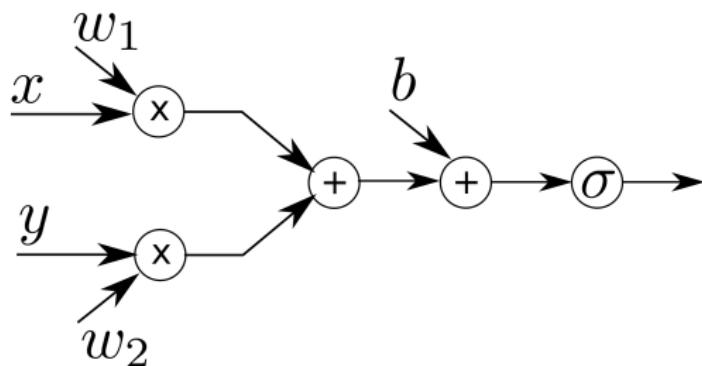
where σ is the sigmoid function: $\sigma(x) = \frac{1}{1+e^{-x}}$

Computation graph example

We will compute:

$$\sigma(w_1x + w_2y + b)$$

where σ is the sigmoid function: $\sigma(x) = \frac{1}{1+e^{-x}}$



Neural network (NN)

Definitions

- An artificial neural network is a computation graph, where the nodes are artificial neurons
- The **input layer** is the set of neurons without incoming edges.
- The **output layer** is the set of neurons without outgoing edges.

Neural network (NN)

Definitions

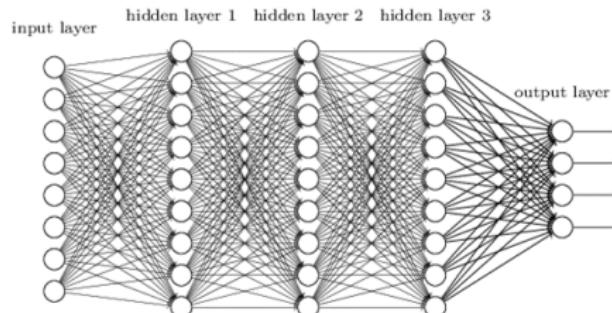
- An artificial neural network is a computation graph, where the nodes are artificial neurons
- The **input layer** is the set of neurons without incoming edges.
- The **output layer** is the set of neurons without outgoing edges.

NB: Neural networks with cycles, known as recurrent neural networks, also exist.

Feed-forward neural networks

Definition

- A feed-forward neural networks is a NN without cycles
- Neurons are organized in **layers**
 - A neuron belongs to layer q if the longest path in the graph between the input layer and the neuron is of length q .
- Any layers other than input and output layers are called **hidden layers**



(from <http://www.jtoy.net>)

Feed-forward neural networks

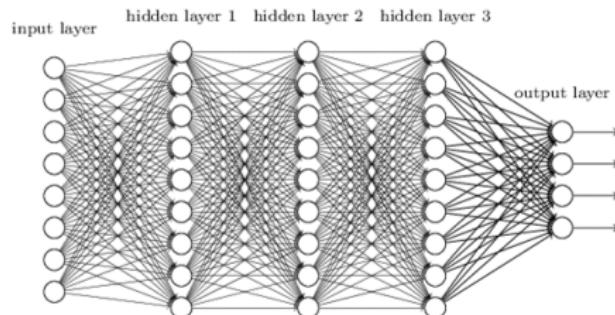
In the following of this course, except when otherwise specified, all NNs will be feed-forward. Indeed, this is the preferred type of NN for image processing.

What about other architectures?

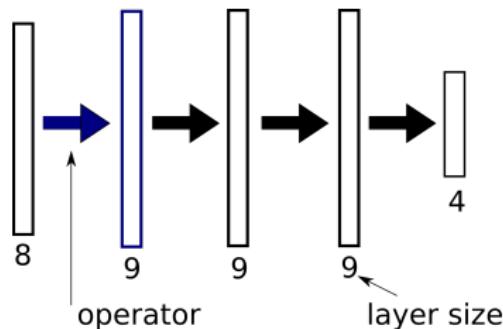
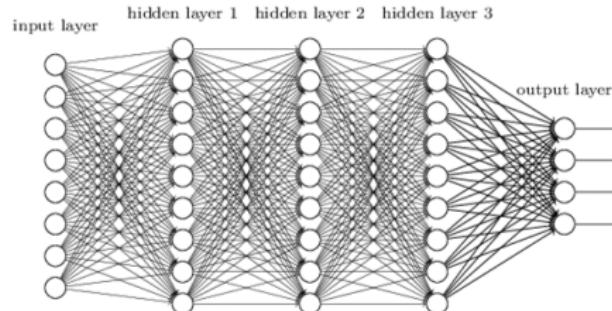
- Recurrent neural networks (RNN)
 - Long short-term memory networks (LSTM)
-
- + More powerful than feed-forward NNs
 - Complex dynamics; more difficult to train
 - Mainly used for processing temporal data

Fully-connected network

- A layer is said to be fully-connected (FC) if each of its neurons is connected to all the neurons of the previous layer
- If a FC layer contains r neurons, and the previous layer q , then its weights are 2D dimensional array (a matrix) of size $q \times r$
- A NN is said to be fully connected if all its hidden layers are fully connected

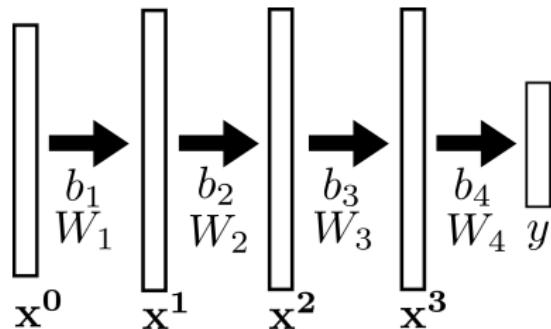


Graphical representation of NNs



- Data is organized into arrays, linked with operators
- A layer corresponds to an operator between arrays (and often an activation) as well as the resulting array.

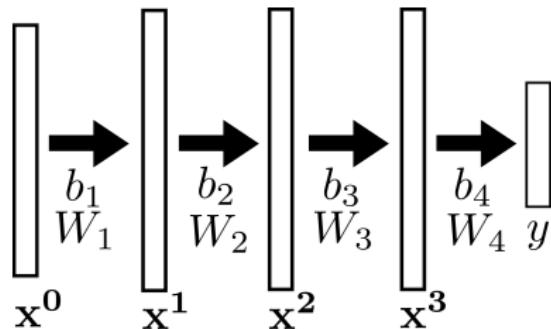
The equations of a fully connected neural network



$$\mathbf{x}^i = \mathbf{g}_i(\mathbf{x}^{i-1}\mathbf{W}_i + \mathbf{b}_i), i = 1, 2, 3$$

$$y = \mathbf{g}_4(\mathbf{x}^3\mathbf{W}_4 + \mathbf{b}_4)$$

The equations of a fully connected neural network

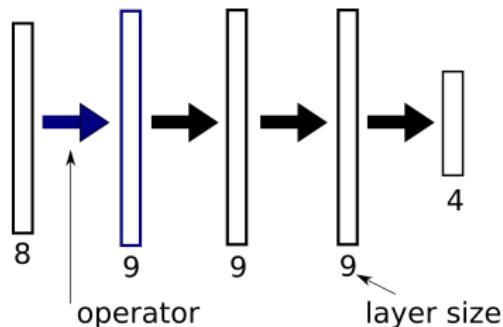


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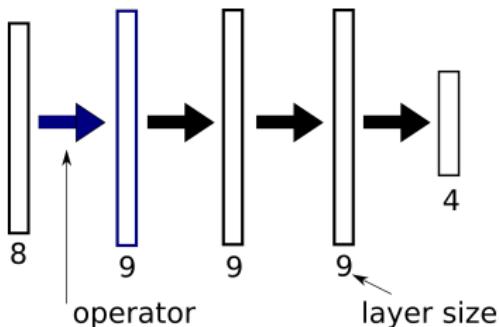
What would happen if all activation functions \mathbf{g}_i were equal to the identity function?

Number of parameters



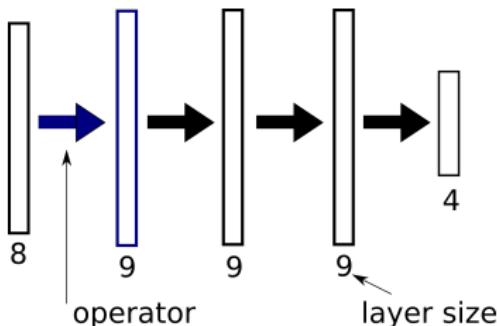
- How many parameters does the above network contain?

Number of parameters



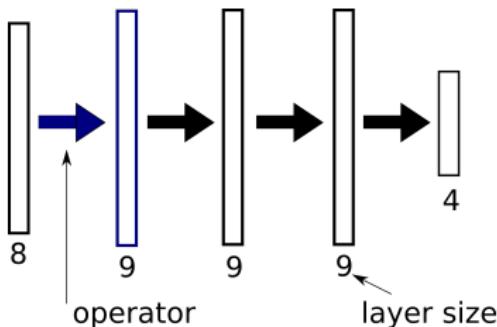
- How many parameters does the above network contain?
- First hidden layer:

Number of parameters



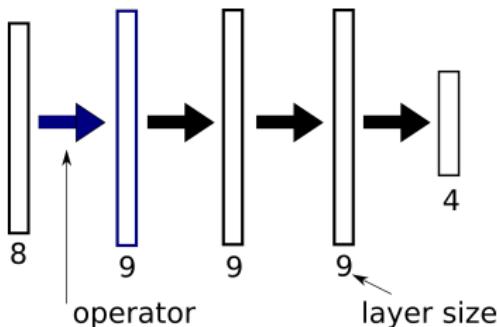
- How many parameters does the above network contain?
- First hidden layer:
 - $9 \text{ neurons} \times 8 \text{ neurons in the previous layer} + 9 \text{ biases} = 81$

Number of parameters



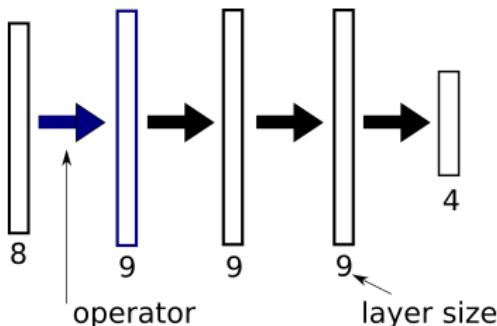
- How many parameters does the above network contain?
- First hidden layer:
 - $9 \text{ neurons} \times 8 \text{ neurons in the previous layer} + 9 \text{ biases} = 81$
- Second and third layers:

Number of parameters



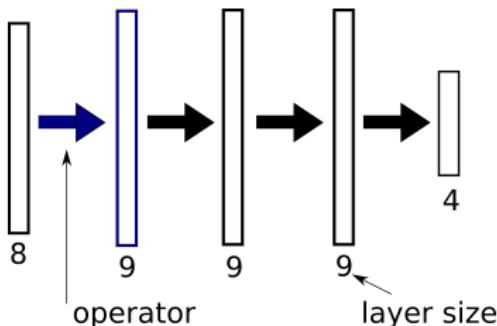
- How many parameters does the above network contain?
- First hidden layer:
 - $9 \text{ neurons} \times 8 \text{ neurons in the previous layer} + 9 \text{ biases} = 81$
- Second and third layers: $9 \times 9 + 9 = 90$

Number of parameters



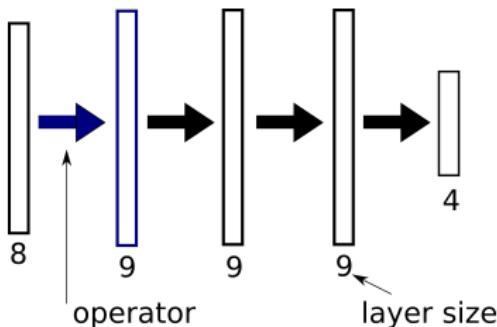
- How many parameters does the above network contain?
- First hidden layer:
 - $9 \text{ neurons} \times 8 \text{ neurons in the previous layer} + 9 \text{ biases} = 81$
- Second and third layers: $9 \times 9 + 9 = 90$
- Output layer:

Number of parameters



- How many parameters does the above network contain?
- First hidden layer:
 - $9 \text{ neurons} \times 8 \text{ neurons in the previous layer} + 9 \text{ biases} = 81$
- Second and third layers: $9 \times 9 + 9 = 90$
- Output layer: $4 \times 9 + 4$

Number of parameters



- How many parameters does the above network contain?
- First hidden layer:
 - $9 \text{ neurons} \times 8 \text{ neurons in the previous layer} + 9 \text{ biases} = 81$
- Second and third layers: $9 \times 9 + 9 = 90$
- Output layer: $4 \times 9 + 4$
- Total: 305 parameters

Batch processing

In a training context, our learning set contains n samples of vectors of length p , that can be grouped into a matrix X of size $n \times p$. The n corresponding outputs y_i can also be grouped into a vector \mathbf{y} of length n . The resulting equations are:

$$\mathbf{X}^i = g_i(\mathbf{X}^{i-1}\mathbf{W}_i + \mathbf{b}_i), \quad i = 1, 2, 3$$

$$\mathbf{y} = g_4(\mathbf{X}^4\mathbf{W}_4 + \mathbf{b}_4)$$

Mini-batch processing

- When dealing with large databases (large n and sometimes large p) for practical reasons the network cannot process the whole set in a single pass.

Mini-batch processing

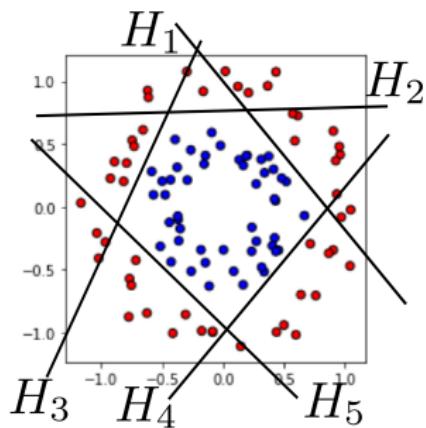
- When dealing with large databases (large n and sometimes large p) for practical reasons the network cannot process the whole set in a single pass.
- One can also separate the training databases into subsets containing m samples ($m < n$), called *mini-batches*.

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Universal approximation theorem

- We have previously seen that a neuron can be used as a linear classifier and that combining several of them one can build complex classifiers
- We will see that this observation can be generalized



Universal approximation theorem

Let f be a **continuous** real-valued function of $[0, 1]^p$ ($p \in \mathbb{N}^*$) and ϵ a strictly positive real. Let g be a non-constant, increasing, bounded real function (*the activation function*).

Then there exists an integer q , real vectors $\{\mathbf{w}_i\}_{1 \leq i \leq q}$ of \mathbb{R}^p , and reals $\{b_i\}_{1 \leq i \leq q}$ and $\{v_i\}_{1 \leq i \leq q}$ such that for all \mathbf{x} in $[0, 1]^p$:

$$\left| f(\mathbf{x}) - \sum_{i=1}^q v_i g(\mathbf{w}_i^T \mathbf{x} + b_i) \right| < \epsilon$$

A first version of this theorem, using sigmoidal activation functions, was proposed by [Cybenko, 1989]. The version above was demonstrated by [Hornik, 1991].

Universal approximation theorem: what does it mean?

$$\left| f(\mathbf{x}) - \sum_{i=1}^q v_i g(\mathbf{w}_i^T \mathbf{x} + b_i) \right| < \epsilon$$

This means that function f can be approximated with a neural network containing:

- an input layer of size p ;
- a hidden layer containing q neurons with activation function g , weights \mathbf{w}_i and biases b_i ;
- an output layer containing a single neuron, with weights v_i (and an identity activation function).

Universal approximation theorem in practice

- The number of neurons increases very rapidly with the complexity of the function
- Empirical evidence has shown that multi-layer architectures give better results

Universal approximation theorem in practice

- The number of neurons increases very rapidly with the complexity of the function
- Empirical evidence has shown that multi-layer architectures give better results

A NN can potentially have a lot of parameters. How can we set them?

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Introduction

- We have seen that NNs have a lot of potential. However, how can the parameters $\theta = (\mathbf{W}_i, \mathbf{b}_i)$ be set?
- What is our objective ?
- A very general solution, that is also the mostly used, is **gradient descent**

Learning problem

We recall that our training set contains n samples:

$$(\mathbf{x}_i, y_i) \in \mathbb{R}^p \times \mathbb{R}$$

We **choose** a family f_{θ} of functions from \mathbb{R}^p into \mathbb{R} , depending on our set of parameters θ , and **find** the value of θ that minimizes a **chosen** loss function L :

$$\theta^* = \arg \min_{\theta} (L(\theta) + \mathcal{R}(\theta))$$

where $\mathcal{R}(\theta)$ is a regularization term.

For the time being, for the sake of simplicity, we will drop the regularization term until further notice

Loss function

A general form of the loss function is:

$$L(\boldsymbol{\theta}) = \sum_{i=1}^n d(y_i, f(\mathbf{x}_i, \boldsymbol{\theta}))$$

where d is some disparity function (the more similar its parameters, the smaller its value).

Loss function: examples

Squared error

$$L(\boldsymbol{\theta}) = \sum_{i=1}^n (y_i - f(\mathbf{x}_i, \boldsymbol{\theta}))^2$$

This loss function is mainly used in regression problems.

Binary cross-entropy

In this case, $y_i \in \{0, 1\}$:

$$L(\boldsymbol{\theta}) = - \sum_{i=1}^n \left(y_i \log(f(\mathbf{x}_i, \boldsymbol{\theta})) + (1 - y_i) \log(1 - f(\mathbf{x}_i, \boldsymbol{\theta})) \right)$$

This loss function is used in binary classification problems, where the network's output can be interpreted as a probability of belonging to a class.

Gradient descent

Definition

Gradient descent is an optimization algorithm. For a derivable function L , a positive real γ (the **learning rate**) and a starting point θ_0 , it computes a sequence of values:

$$\forall e \in \mathbb{N} : \theta_{e+1} = \theta_e - \gamma \nabla L(\theta_e)$$

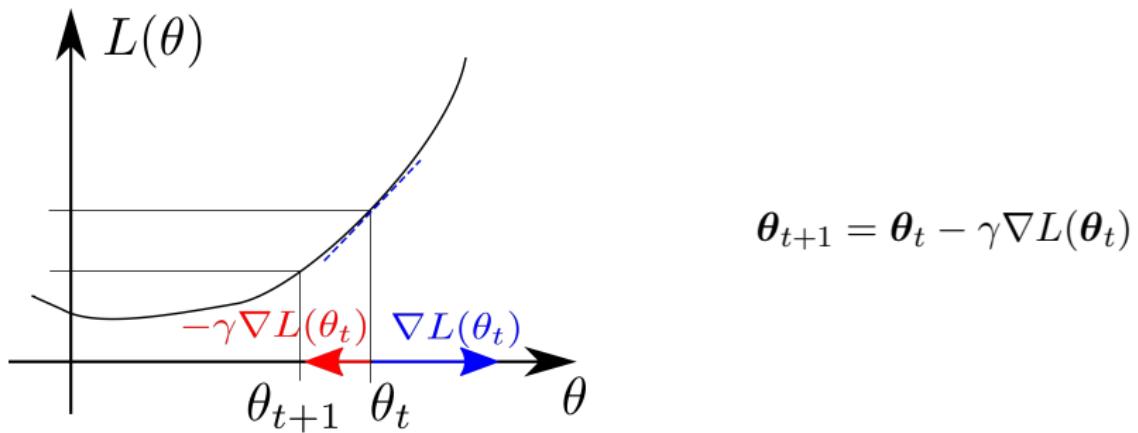
Property

If γ is small enough, then:

$$L(\theta_{i+1}) \leq L(\theta_i)$$

Gradient descent is an essential tool in optimization.

Gradient descent in the scalar case



Gradient descent: stopping criteria

In practice:

$$\forall e \in [0, \dots, E - 1] : \quad \theta_{e+1} = \theta_e - \gamma \nabla L(\theta_e)$$

- Choose E (the number of epochs) based on experience
- Track the quality of the model using a validation dataset and stop when the validation loss does not improve

Towards stochastic gradient descent

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- If n is very large, its computation is not feasible.
- A computation on the whole training set leads to a single update of the model parameters - convergence can therefore be slow.

Stochastic gradient descent

In **stochastic gradient descent**, the parameters are updated for each sample i .

- First, the loss is computed

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- Note that the learning rate γ can have a different value than in straightforward gradient descent.

Gradient descent applied to neural networks

In the case of neural networks, the loss L depends on each parameter θ_i via the composition of several simple functions. In order to compute the gradient $\nabla_{\theta} L$ we will make extensive use of the chain rule theorem.

Chain rule theorem

Let f_1 and f_2 be two derivable real functions ($\mathbb{R} \rightarrow \mathbb{R}$). Then for all x in \mathbb{R} : :

$$(f_2 \circ f_1)'(x) = f'_2(f_1(x)) \cdot f'_1(x)$$

Leibniz notation

Let us introduce variables x , y and z :

$$x \xrightarrow{f_1} y \xrightarrow{f_2} z$$

Then:

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

The backpropagation algorithm

- The backpropagation algorithm is used in a neural network to efficiently compute the partial derivatives of the loss with respect to each parameter of the network.
- One can trace the origins of the method to the sixties
- It was first applied to NN in the eighties
[Werbos, 1982, LeCun, 1985]

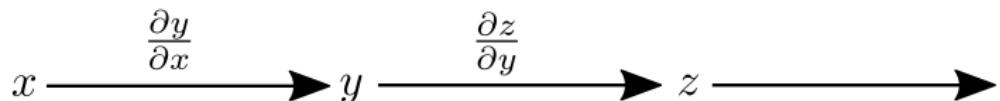
Simple backpropagation example



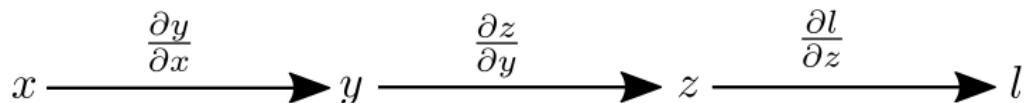
Simple backpropagation example



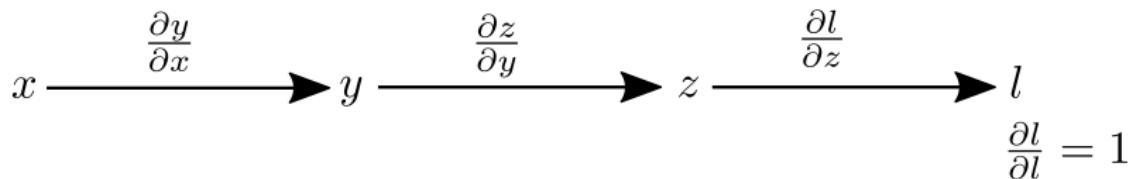
Simple backpropagation example



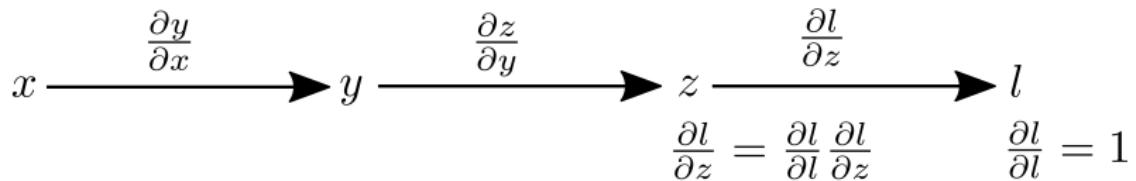
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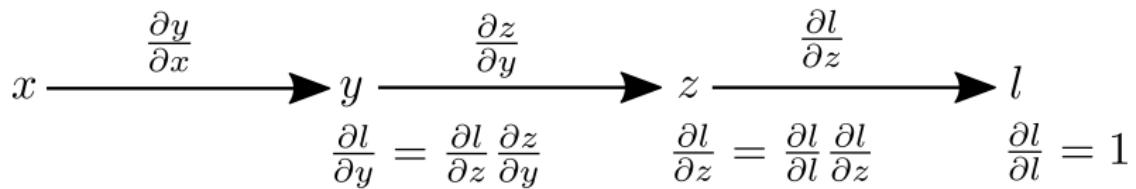
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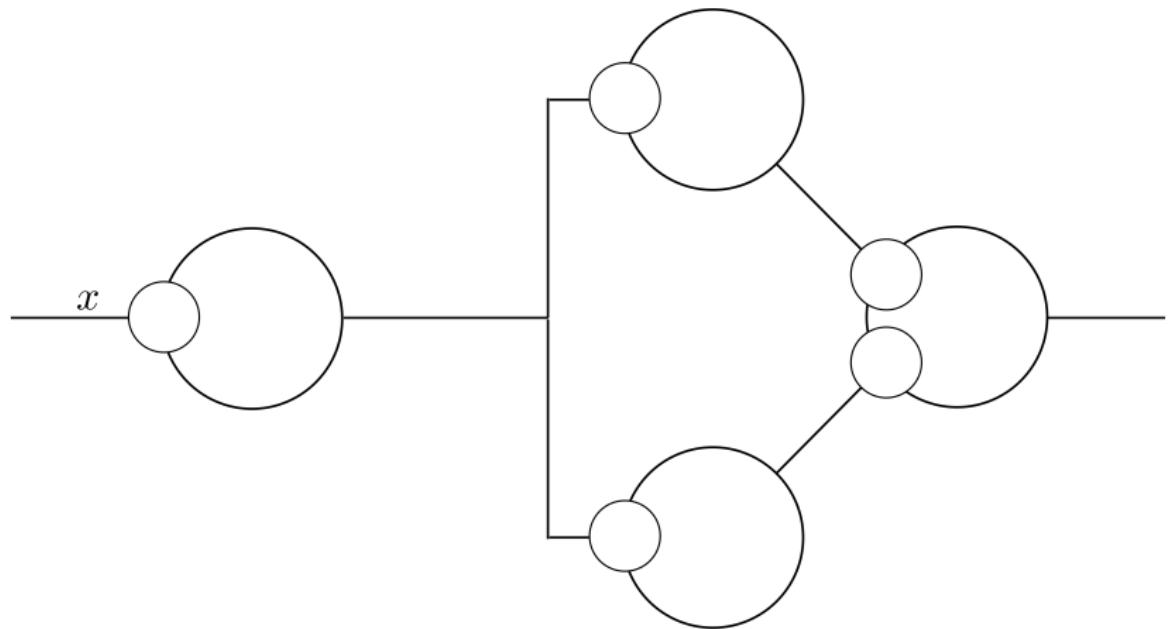
Simple backpropagation example



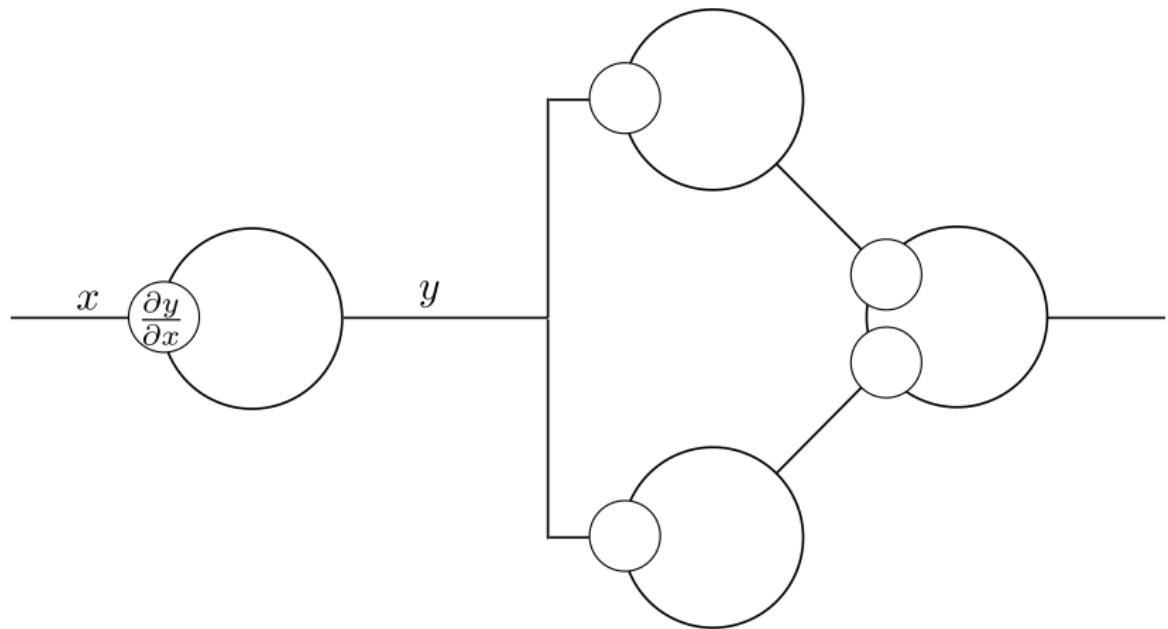
Simple backpropagation example

$$\begin{array}{ccccccc} & \frac{\partial y}{\partial x} & & \frac{\partial z}{\partial y} & & \frac{\partial l}{\partial z} & \\ x & \xrightarrow{\hspace{1cm}} & y & \xrightarrow{\hspace{1cm}} & z & \xrightarrow{\hspace{1cm}} & l \\ \frac{\partial l}{\partial x} = \frac{\partial l}{\partial y} \frac{\partial y}{\partial x} & & \frac{\partial l}{\partial y} = \frac{\partial l}{\partial z} \frac{\partial z}{\partial y} & & \frac{\partial l}{\partial z} = \frac{\partial l}{\partial l} \frac{\partial l}{\partial z} & & \frac{\partial l}{\partial l} = 1 \end{array}$$

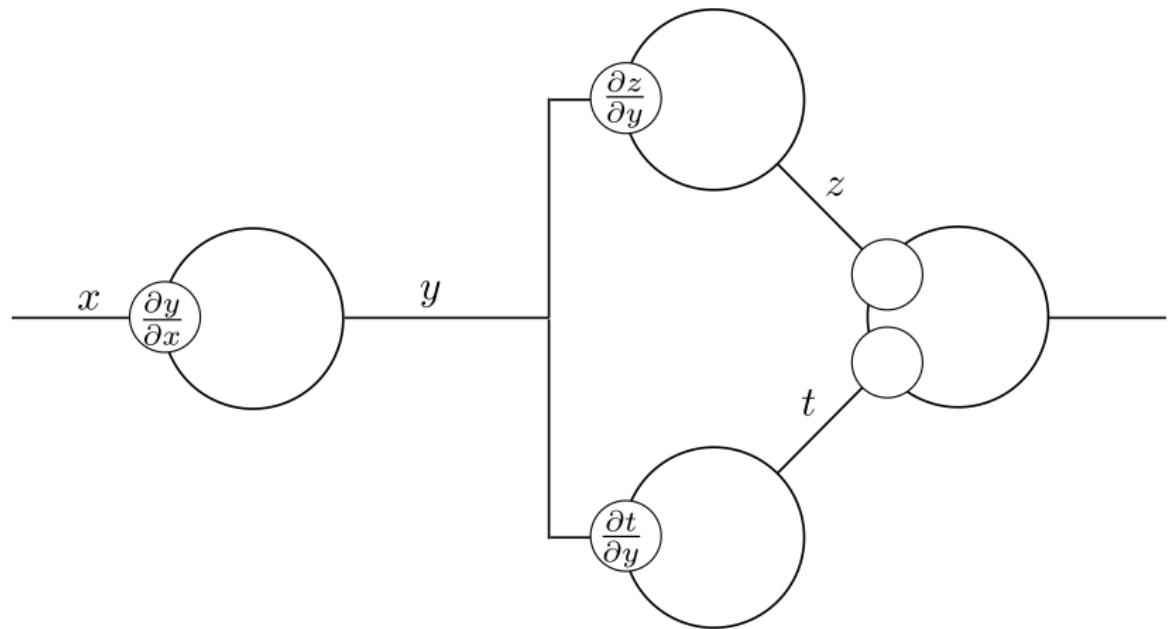
Backpropagation example



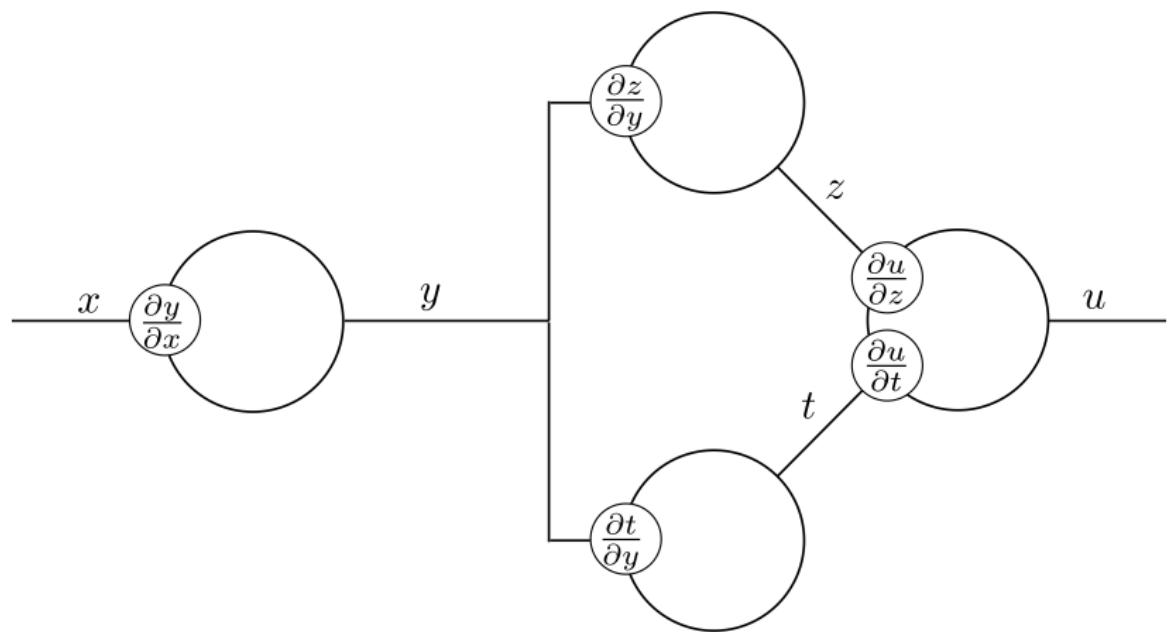
Backpropagation example



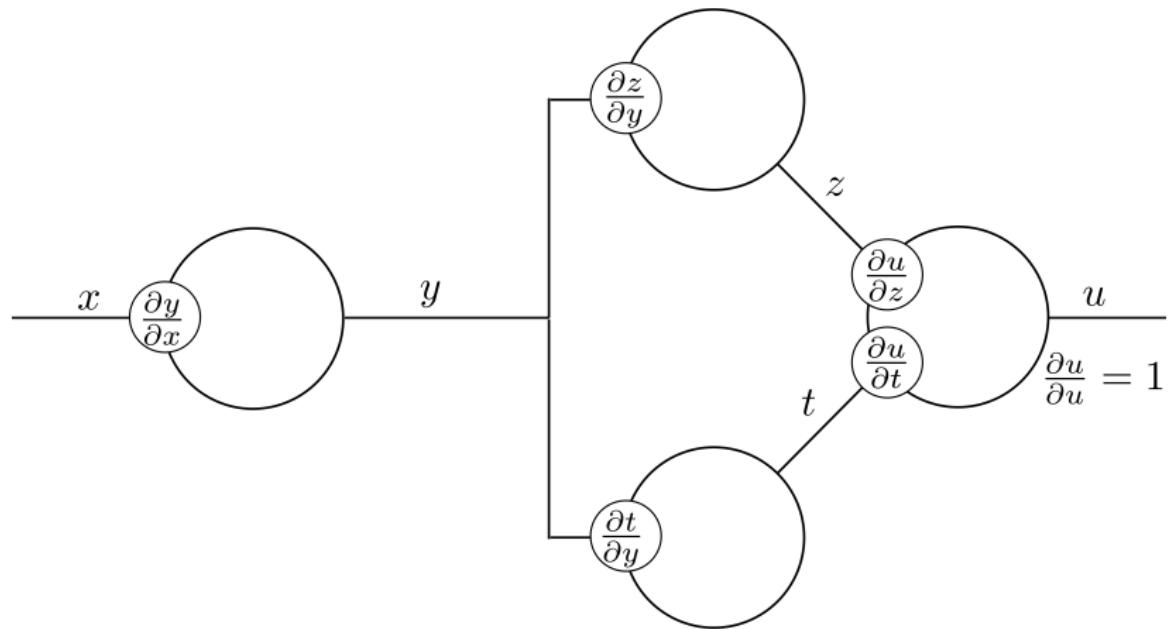
Backpropagation example



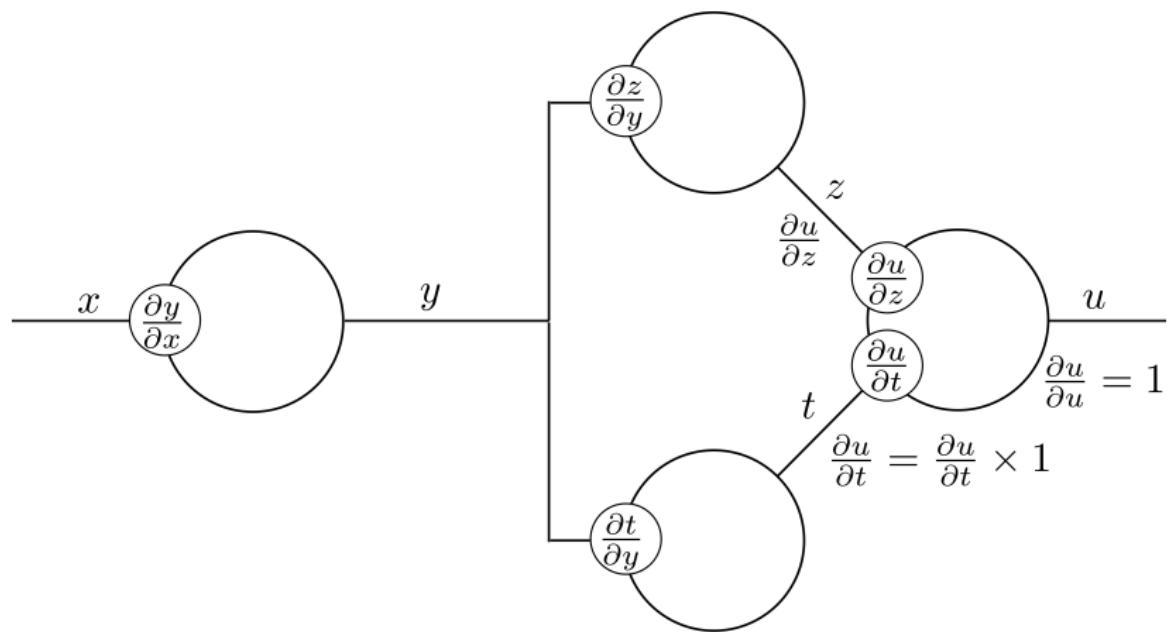
Backpropagation example



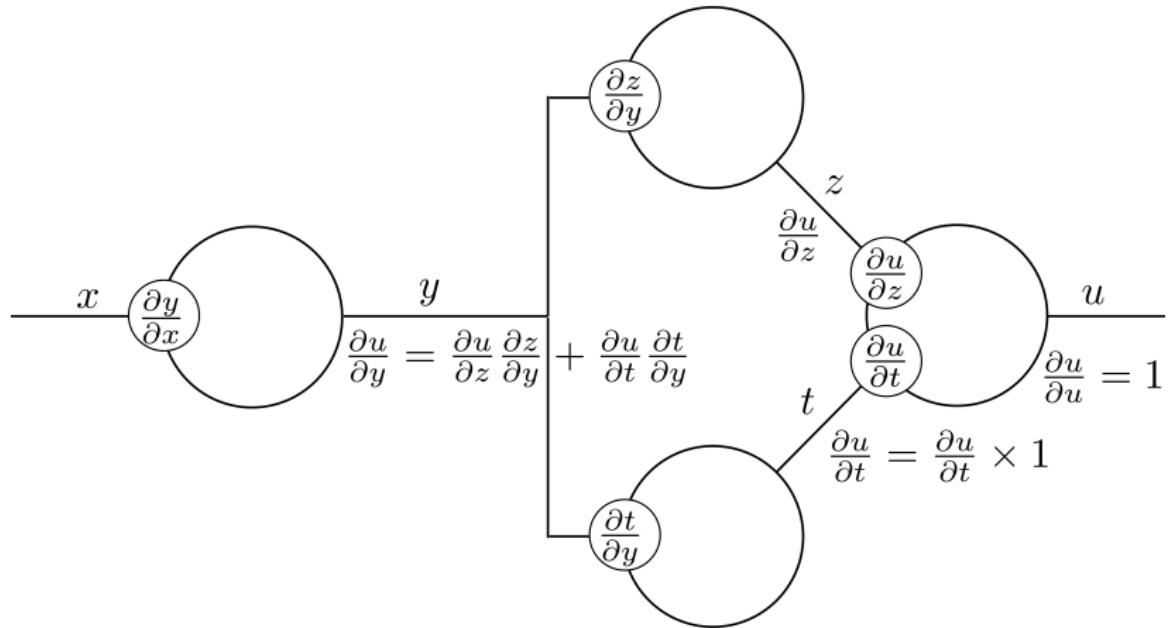
Backpropagation example



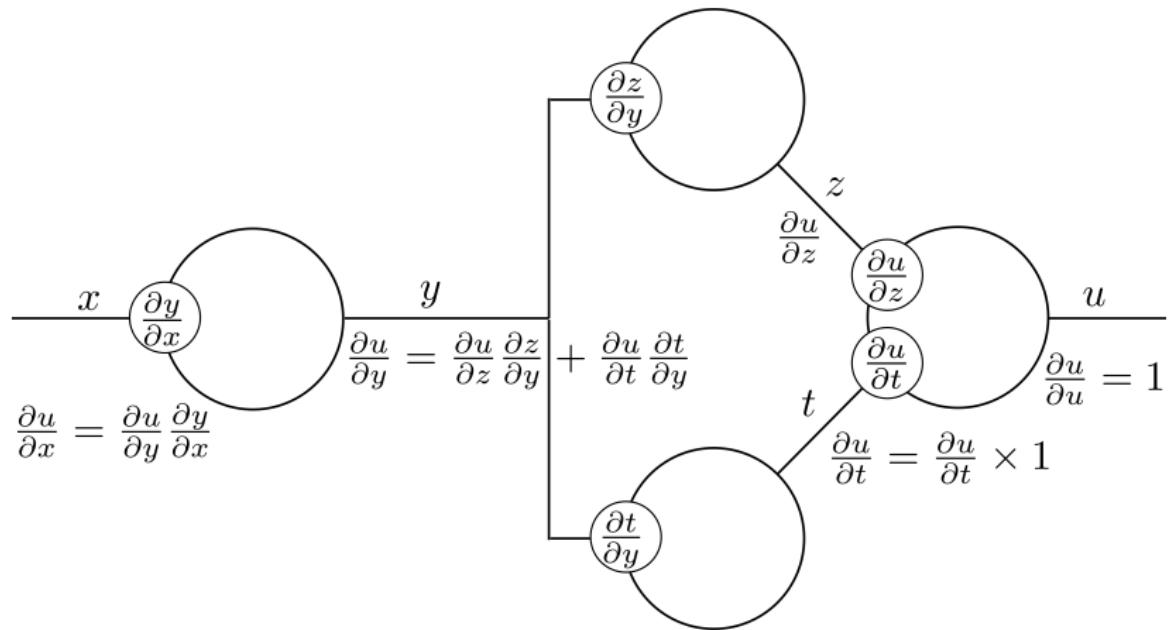
Backpropagation example



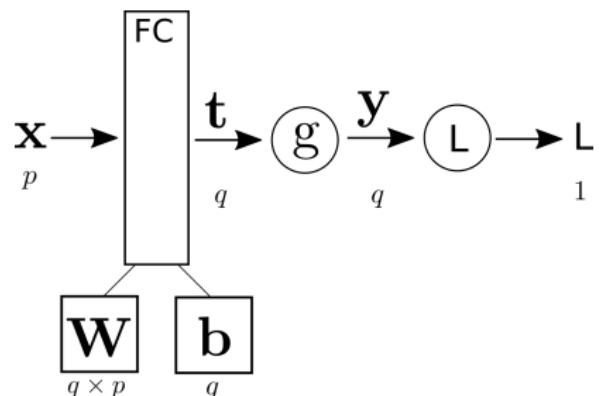
Backpropagation example



Backpropagation example



Backpropagation through a fully connected layer



Setup:

$$p, q \in \mathbb{N}^*$$

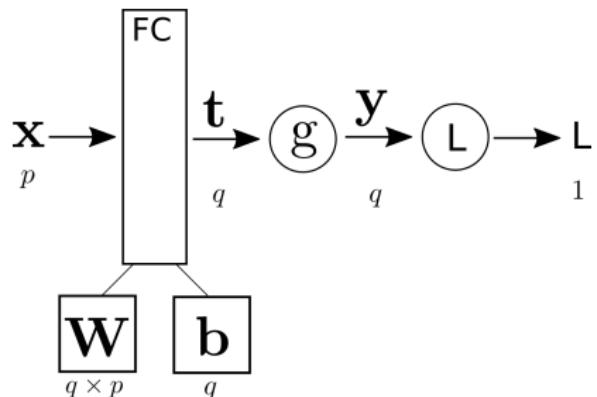
$$\mathbf{x} \in \mathbb{R}^p$$

$$\mathbf{W} \in \mathbb{R}^q \times \mathbb{R}^p$$

$$\mathbf{b}, \mathbf{t}, \mathbf{y} \in \mathbb{R}^q$$

$$L \in \mathbb{R}$$

Backpropagation through a fully connected layer



Local gradients:

Forward pass:

$$\mathbf{t} = \mathbf{W}\mathbf{x} + \mathbf{b}$$

$$\mathbf{y} = g(\mathbf{W}\mathbf{x} + \mathbf{b})$$

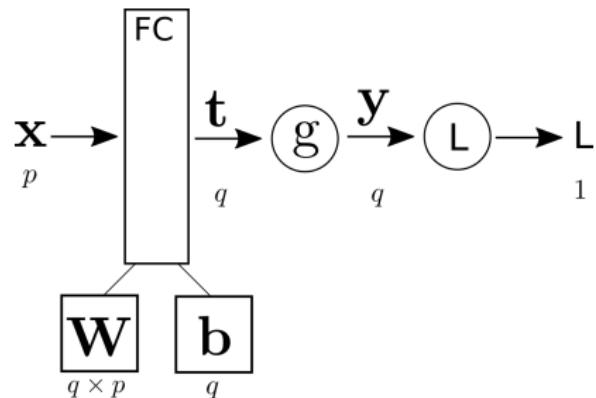
$$L = L(\mathbf{y})$$

$$\frac{\partial \mathbf{t}}{\partial \mathbf{W}} = \mathbf{x}^t$$

$$\frac{\partial \mathbf{t}}{\partial \mathbf{b}} = 1$$

$$\frac{\partial \mathbf{y}}{\partial \mathbf{t}} = g'$$

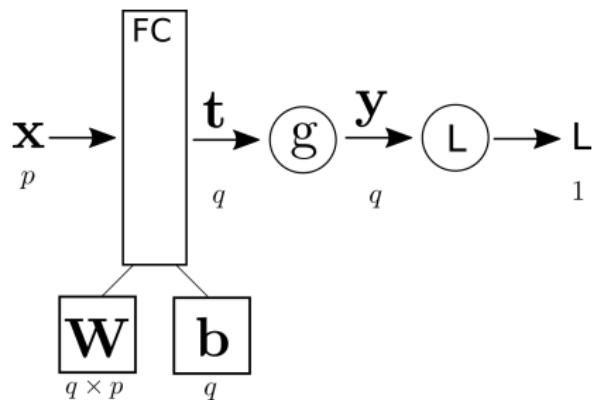
Backpropagation through a fully connected layer



Backpropagation:

$$\begin{aligned}\frac{\partial L}{\partial \mathbf{t}} &= \frac{\partial L}{\partial \mathbf{y}} \cdot \frac{\partial \mathbf{y}}{\partial \mathbf{t}} \\ &= \frac{\partial L}{\partial \mathbf{y}} \odot g'(\mathbf{t})\end{aligned}$$

Backpropagation through a fully connected layer



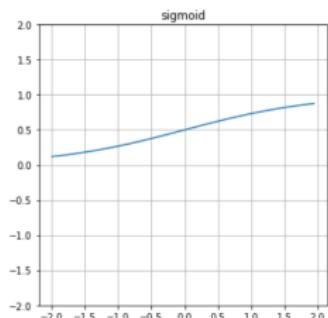
Backpropagation:

$$\begin{aligned}\frac{\partial L}{\partial \mathbf{W}} &= \frac{\partial L}{\partial \mathbf{t}} \cdot \frac{\partial \mathbf{t}}{\partial \mathbf{W}} & \frac{\partial L}{\partial \mathbf{b}} &= \frac{\partial L}{\partial \mathbf{y}} \odot g'(\mathbf{t}) \\ &= \frac{\partial L}{\partial \mathbf{y}} \odot g'(\mathbf{t}) \cdot \mathbf{x}^t\end{aligned}$$

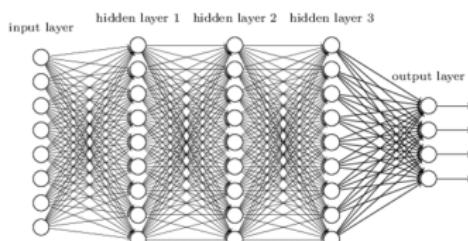
Network parameters initialization

General idea

Inputs of activation functions should be in an appropriate range (high gradient)



- If all parameters are initialized to zero, then in each layer the activations will remain equal – symmetry will never be broken
- Simple solution: random values from a normal or uniform distribution
- More advanced solutions exist:
[LeCun et al., 1998,
Glorot and Bengio, 2010,
He et al., 2015]



Conclusion

We have seen:

- What is an artificial neuron and an artificial neural network (NN)
- The (potential) power of a NN
- The backpropagation algorithm
- NN learning basics

In the following, we will see how to process images using NNs.

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