23000 11454

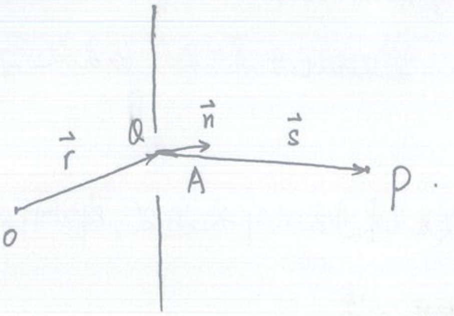
雷逸鸣

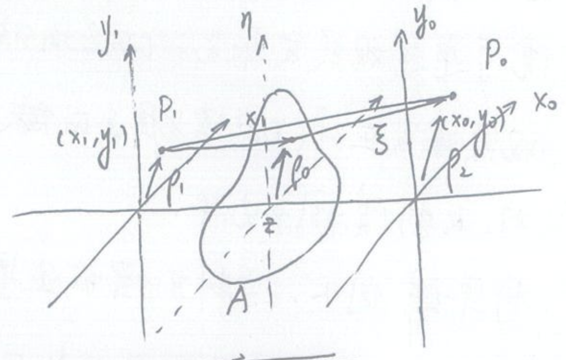
[illegible]

11.27

实验二十　光衍射的定量研究

1. $$U(P) = \frac{i\tilde{A}}{2\lambda} \iint_A \frac{e^{ik(r+s)}}{rs} [\cos(\vec{n}, \vec{r}) + \cos(\vec{n}, \vec{s})]\, ds.$$





~~取 $\cos(\vec{n}, \vec{r}) = \cos(\vec{n}, \vec{s}) \approx 2$.~~

取空间直角坐标系：

$$U(P) = \frac{i\tilde{A}}{\lambda} \iint_A d\xi\, d\eta \frac{\exp\left(ik\left[\sqrt{(x_1-\xi)^2 + (y_1-\eta)^2 + z_1^2} + \sqrt{(x_2-\xi)^2 + (y_2-\eta)^2 + z_2^2}\right]\right)}{\sqrt{(x_1-\xi)^2 + (y_1-\eta)^2 + z_1^2}\,\sqrt{(x_2-\xi)^2 + (y_2-\eta)^2 + z_2^2}}.$$

~~$\Delta\vec{\rho}_1 = \vec{\rho}_1 - \vec{\rho}_0$.　$\Delta\vec{\rho}_2 = \vec{\rho}_2 - \vec{\rho}_0$.~~

~~简~~ 上式由：

(1)
$$\begin{cases} \cos(\vec{n}, \vec{r}) = 1 - \frac{1}{2} \frac{(x_1-\xi)^2 + (y_1-\eta)^2}{z_1^2} \approx 1 & \left(~~\frac{\Delta\rho_1}{z_1} \ll 1~~\right) \\ \cos(\vec{n}, \vec{s}) = 1 - \frac{1}{2} \frac{(x_2-\xi)^2 + (y_2-\eta)^2}{z_2^2} \approx 1 & \left(~~\frac{\Delta\rho_2}{z_2} \ll 1~~\right) \end{cases}$$

(2) 注意到
$$\sqrt{(x_1-\xi)^2 + (y_1-\eta)^2 + z_1^2} = z_1\left(1 + \frac{1}{2}\frac{(x_1-\xi)^2 + (y_1-\eta)^2}{z_1^2}\right) \approx z_1$$
$$\sqrt{(x_2-\xi)^2 + (y_2-\eta)^2 + z_2^2} = z_2\left(1 + \frac{1}{2}\frac{(x_2-\xi)^2 + (y_2-\eta)^2}{z_2^2}\right) \approx z_2.$$

为使(1)(2)两式成立，要求 $\frac{\rho_1^2}{z_1^2} \ll 1$　$\frac{\rho_2^2}{z_2^2} \ll 1$　（傍轴条件）.

(3) 于是：
$$U(P) = \frac{i\tilde{A}}{\lambda} \iint_A \frac{1}{z_1 z_2} \exp\left[ik\left(z_1 + \frac{1}{2}\frac{(x_1-\xi)^2 + (y_1-\eta)^2}{z_1}\right)\right] \exp\left[ik\left(z_2 + \frac{1}{2}\frac{(x_2-\xi)^2 + (y_2-\eta)^2}{z_2}\right)\right] d\xi\, d\eta$$

$$\varphi_1 = \frac{1}{2}k \cdot \frac{x_1^2 + y_1^2 - 2x_1\xi - 2y_1\eta + \xi^2 + \eta^2}{z_1}$$　为使 ~~$\varphi_i = \frac{1}{2}\frac{k}{z_i}(\xi^2 + \eta^2)$~~

$$\varphi_2 = \frac{1}{2} \frac{x_2^2 + y_2^2 - 2x_2\xi - 2y_2\eta + \xi^2 + \eta^2}{z_2}$$　为使 $\varphi_i = -\frac{x_i\xi + y_i\xi}{z_i}$

要求：远场条件、　$z\lambda \gg \rho^2$.

综上：$\rho^2/z^2 \ll 1$

$\frac{\rho^2}{\lambda z} \ll 1$