费曼物理学 (3) 笔记

mny

2023 年 9 月 20 日

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第一章　Quantum Mechanic Behaviors

1.1　Two-Slit Interference of Electrons

Both open: $N \neq N_1(x) + N_2(x)$. In reality, it turns out that the pattern is something like

$$N(x) = N_1(x) + N_2(x) + g(x) \sin\left[\omega(x)x\right], \tag{1.1.1}$$

where the last term is a interference term, which satisfies a slow changing condition

$$\frac{1}{\omega}\frac{\mathrm{d}\omega}{\mathrm{d}x} \ll \omega, \quad \frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}x} \ll \omega. \tag{1.1.2}$$

If we lower the power of electron source so that it emits each electron one by one, thus interactions between electrons will not functional, however, the result o f the experiment recovers. This shows us that the statistical pattern isn't resulted by many-body interactions. So, we come to a ridiculous conclusion, electron must interact with itself passing both hole simultaneously.

Next we build a which-way detector, using a light source for observing whether an electron has passed a hole. The pattern disappears! When we lower the power or enlarging the wavelength, the pattern re-appear gradually.

We have to admit that it is impossible to design an apparatus to determine which hole the electron passes through, that will not at the same time disturb the electrons enough to destroy the interference pattern. That is, **Heisenberg's uncertainty principle**. Hence, in quantum mechanic, we can only make predictions of probability.

Remark that,

1. The evolution of quantum states is **definite** (either by Schrödinger equation or something else).
2. The uncertainty only appears in observations.
3. Quantum mechanic is an **extremely accurate** theory.

第二章　Wave-Particle Duality

When we perform different experiments, electrons behaves differently. The word **duality** was used when we can not obtain a universally description. The concept **state** was invented and complex number was introduced.

We use ψ_1 and ψ_2 to describe the complex amplitude of hole 1 opened and hole 2 opened respectively. Add up the two terms and the tensity is

$$|\psi(x)|^2 = |\psi_1(x) + \psi_2(x)|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1\psi_2^* + \psi_1^*\psi_2. \tag{2.0.1}$$

In the formula above we have implicitly utilized the **Born rule** of probability.

All possible quantum states forms a space, in which some looks like waves and some like particles.

Plank has put forward that $E = \hbar\omega$ and he believes that this property appears only when light interacts with other materials. While Einstein supposed that this it a inner property of light when dealing with photoelectric effect.

We may notice that

$$p^\mu = (E, \vec{p}), \quad k^\mu = (\omega, \vec{k}), \tag{2.0.2}$$

are all Lorentz four vectors, thus $\vec{p} = \hbar\vec{k}$.

2.1 Fourier Transformation

A wave mode with definite $\vec{k}$, we have $\psi_{\vec{k}}(\vec{x}) \sim \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}$. For an arbitrary wave packet within some mathematical restriction, it can be written as

$$f(\vec{x}) = \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \tilde{f}(\vec{k}). \tag{2.1.1}$$

The inverse transformation is

$$\tilde{f}(\vec{k}) = \int \mathrm{d}^3\vec{x} \mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}} f(\vec{x}). \tag{2.1.2}$$

This is guaranteed by the orthogonal-normalization of plane waves,

$$\begin{aligned} \int \mathrm{d}^3\vec{x}\mathrm{e}^{-\mathrm{i}\vec{q}\cdot\vec{x}}\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} &= (2\pi)^3\delta^{(3)}(\vec{k}-\vec{q}), \\ \int \frac{\mathrm{d}^3\vec{k}}{2\pi}\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{y}} &= (2\pi)^3\delta^{(3)}(\vec{x}-\vec{y}). \end{aligned} \tag{2.1.3}$$

The Dirac δ function satisfies

$$\delta(x) = 0 \quad \text{if } x \neq 0, \tag{2.1.4}$$

and

$$\int_{-\infty}^{\infty} \mathrm{d}x\, \delta(x) = 1 \tag{2.1.5}$$

$$\Longrightarrow \forall f(x),\ \int_{-\infty}^{\infty} \mathrm{d}x\, \delta(x-y)f(x) = f(y). \tag{2.1.6}$$

2.2 Gaussian Wave Packet

For Gaussian wave packet, $\psi(\vec{x}) = \mathrm{e}^{-\frac{x^2}{4\sigma^2}}$, we would find out how many component of wave-number $\vec{k}$ does it contains.

$$\int \mathrm{d}x\, \mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}}\mathrm{e}^{-\frac{|\vec{x}|^2}{4\sigma^2}} = \int \mathrm{d}x\, \mathrm{e}^{\frac{-\left(x+\mathrm{i}2k\sigma^2\right)^2}{4\sigma^2}}\mathrm{e}^{-k^2\sigma^2}. \tag{2.2.1}$$

It's still Gaussian in frequency space, and $\sigma_k = \frac{1}{2\sigma}$, thus $\Delta x\Delta k = \frac{1}{2}$, i.e.,

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}. \tag{2.2.2}$$

2.3 Hydrogen Atom

Assuming the electron goes in a circle trajectory with diameter a, according the uncertainty principle, we can write the hamiltonian,

$$E = \frac{p^2}{2m} - \frac{e^2}{a} = \frac{\hbar^2}{2ma^2} - \frac{e^2}{a}, \tag{2.3.1}$$

thus it have a minimum at which $a \neq 0$, the result is $a = \frac{\hbar^2}{me^2} \sim 0.5\,\mathring{A}, E = -13.6eV$

For bounded states, the possible energy levels are always discrete, when it transit from a higher level E_1 to a lower E_2, it emits a photon with frequency $\omega = \frac{E_1-E_2}{\hbar}$