费曼物理学 (3) 笔记

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目录

第一章 Quantum Mechanic Behaviors

1.1 Two-Slit Interference of Electrons

Both open: $N \neq N_1(x) + N_2(x)$. In reality, it turns out that the pattern is something like

$$N(x) = N_1(x) + N_2(x) + g(x) \sin\left[\omega(x)x\right], \tag{1.1.1}$$

where the last term is a interference term, which satisfies a slow changing condition

$$\frac{1}{\omega}\frac{\mathrm{d}\omega}{\mathrm{d}x} \ll \omega, \quad \frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}x} \ll \omega. \tag{1.1.2}$$

If we lower the power of electron source so that it emits each electron one by one, thus interactions between electrons will not functional, however, the result o f the experiment recovers. This shows us that the statistical pattern isn't resulted by many-body interactions. So, we come to a ridiculous conclusion, electron must interact with itself passing both hole simultaneously.

Next we build a which-way detector, using a light source for observing whether an electron has passed a hole. The pattern disappears! When we lower the power or enlarging the wavelength, the pattern re-appear gradually.

We have to admit that it is impossible to design an apparatus to determine which hole the electron passes through, that will not at the same time disturb the electrons enough to destroy the interference pattern. That is, **Heisenberg's uncertainty principle**. Hence, in quantum mechanic, we can only make predictions of probability.

Remark that,

1. The evolution of quantum states is **definite** (either by Schrödinger equation or something else).

2. The uncertainty only appears in observations.

3. Quantum mechanic is an **extremely accurate** theory.

第二章　Wave-Particle Duality

When we perform different experiments, electrons behaves differently. The word **duality** was used when we can not obtain a universally description. The concept **state** was invented and complex number was introduced.

We use ψ_1 and ψ_2 to describe the complex amplitude of hole 1 opened and hole 2 opened respectively. Add up the two terms and the tensity is

$$|\psi(x)|^2 = |\psi_1(x) + \psi_2(x)|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1\psi_2^* + \psi_1^*\psi_2. \tag{2.0.1}$$

In the formula above we have implicitly utilized the **Born rule** of probability.

All possible quantum states forms a space, in which some looks like waves and some like particles.

Plank has put forward that $E = \hbar\omega$ and he believes that this property appears only when light interacts with other materials. While Einstein supposed that this it a inner property of light when dealing with photoelectric effect.

We may notice that

$$p^\mu = (E, \vec{p}), \quad k^\mu = (\omega, \vec{k}), \tag{2.0.2}$$

are all Lorentz four vectors, thus $\vec{p} = \hbar\vec{k}$.

2.1 Fourier Transformation

A wave mode with definite $\vec{k}$, we have $\psi_{\vec{k}}(\vec{x}) \sim \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}$. For an arbitrary wave packet within some mathematical restriction, it can be written as

$$f(\vec{x}) = \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \tilde{f}(\vec{k}). \tag{2.1.1}$$

The inverse transformation is

$$\tilde{f}(\vec{k}) = \int \mathrm{d}^3\vec{x}\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}} f(\vec{x}). \tag{2.1.2}$$

This is guaranteed by the orthogonal-normalization of plane waves,

$$\begin{aligned}\int \mathrm{d}^3\vec{x}\mathrm{e}^{-\mathrm{i}\vec{q}\cdot\vec{x}}\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} &= (2\pi)^3\delta^{(3)}(\vec{k}-\vec{q}),\\ \int \frac{\mathrm{d}^3\vec{k}}{2\pi}\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{y}} &= (2\pi)^3\delta^{(3)}(\vec{x}-\vec{y}).\end{aligned} \tag{2.1.3}$$

The Dirac δ function satisfies

$$\delta(x) = 0 \quad \text{if } x \neq 0, \tag{2.1.4}$$

and

$$\int_{-\infty}^{\infty} \mathrm{d}x\,\delta(x) = 1 \tag{2.1.5}$$

$$\Longrightarrow \forall f(x),\ \int_{-\infty}^{\infty} \mathrm{d}x\,\delta(x-y)f(x) = f(y). \tag{2.1.6}$$

2.2 Gaussian Wave Packet

For Gaussian wave packet, $\psi(\vec{x}) = \mathrm{e}^{-\frac{x^2}{4\sigma^2}}$, we would find out how many component of wave-number $\vec{k}$ does it contains.

$$\int \mathrm{d}x\,\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}}\mathrm{e}^{-\frac{|\vec{x}|^2}{4\sigma^2}} = \int \mathrm{d}x\,\mathrm{e}^{\frac{-\left(x+\mathrm{i}2k\sigma^2\right)^2}{4\sigma^2}}\mathrm{e}^{-k^2\sigma^2}. \tag{2.2.1}$$

It's still Gaussian in frequency space, and $\sigma_k = \frac{1}{2\sigma}$, thus $\Delta x\Delta k = \frac{1}{2}$, i.e.,

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}. \tag{2.2.2}$$

2.3 Hydrogen Atom

Assuming the electron goes in a circle trajectory with diameter a, according the uncertainty principle, we can write the hamiltonian,

$$E = \frac{p^2}{2m} - \frac{e^2}{a} = \frac{\hbar^2}{2ma^2} - \frac{e^2}{a}, \tag{2.3.1}$$

thus it have a minimum at which $a \neq 0$, the result is $a = \frac{\hbar^2}{me^2} \sim 0.5\,\mathring{A}, E = -13.6eV$

For bounded states, the possible energy levels are always discrete, when it transit from a higher level E_1 to a lower E_2, it emits a photon with frequency $\omega = \frac{E_1-E_2}{\hbar}$

2.4 The Philosophy of Quantum Mechanic

The observables are the numbers that can be measured in experiments. Physicist works for finding the numerical relations under the observables. The mission of physics is to explain the phenomena of observables qualificationally. There's no need to debate on what is the entity of something, or which conception is more fundamental.

第三章 Probability Amplitude

The superposition law of quantum mechanic imply that there lies a structure of linear algebra beneath the description of quantum mechanic, from which, Schrödinger developed the Wave Mechanic, Heisenberg the Matrix Formalism, and Feynman the Path Integral Methodology.

When we are asked about the probability of a certain process, we compute the magnitude squared of a complex number, that is, *probability amplitude*. This gives the first law

$$\text{Probability} = |\text{amplitude}|^2. \tag{3.0.1}$$

Dirac introduced his notation $\langle A|B\rangle$, which means the amplitude of transferring from the initial state A to a final process B.

The second law

$$\langle B|A\rangle = \langle B|A\rangle_{\text{path 1}} + \langle B|A\rangle_{\text{path 2}}\,. \tag{3.0.2}$$

The third law

$$\langle B|A\rangle_{\text{path 1}} = \langle B|1\rangle \langle 1|A\rangle\,. \tag{3.0.3}$$

Suppose a M-fold $\{H_i\}$ hole interference of electron, using the notation of j_i to represent the j hole of plate i.We can write the amplitude

$$\langle B|A\rangle = \sum_{j_M=1}^{H_M} \sum_{j_{M-1}=1}^{H_{M-1}} \cdots \sum_{j_1=1}^{H_1} \langle B|(j_M)_M\rangle \langle (j_M)_M|(J_{M-1})_{M-1}\rangle \times \cdots \times |(j_1)_1\rangle \langle (j_1)_1|A\rangle\,. \tag{3.0.4}$$

Or, in continuum form,

$$\langle B|A\rangle = \sum_{\text{all paths}} \langle B|A\rangle_{\text{a certain path}}\,. \tag{3.0.5}$$

To some extend, this is quite similar to the path integral, ignoring the fact that we choose $y(x)$ instead of $\vec{r}(t)$ to be the integration variable, which led us to be unable to include the paths in which a electron turns back.

Let us go back to the two-slit interference experiment and consider why observation influences the pattern. There are two detectors, D_1 and D_2, setting up closing to hole 1 and 2, using u to denote the amplitude of electrons passing hole 1 and kicking the photon into D_1, and v for hole-2-electrons kicking photons into hole 1.

Then the amplitude of electrons from A to B through hole 1 is

$$\langle B|A\rangle_{\text{photon to } D_1} = \langle B|1\rangle\, u\, \langle 1|A\rangle + \langle B|2\rangle\, v\, \langle 2|A\rangle\,. \tag{3.0.6}$$

In a valid measurement, which means $|u| \gg |v|$, there will be no interference term $\langle B|1\rangle\, v\, \langle 1|A\rangle\, \langle B|2\rangle\, v\, \langle 2|A\rangle$ in the probability.

Measurement caused the decoherence of electrons, and thus pattern disappears.

3.1 Path Integral

For non-relativistic cases, we can define synchronousness, and any particle cannot go backward in time, the amplitude can be written as

$$\langle B(t_2)|A(t_1)\rangle = \sum_{\text{all paths}} \langle B(t_2)|A(t_1)\rangle_{\text{a certain path}}\,. \tag{3.1.1}$$

Note that the amplitude is a complex function of path, when finding how to calculate its value, we need to go back to the classical situation. Feynman gives the result

$$\langle B(t_2)|A(t_1)\rangle_{\text{a certain path}} = \mathrm{e}^{\mathrm{i}S/\hbar}. \tag{3.1.2}$$

We can write in ths form,

$$\langle B(t_2)|A(t_1)\rangle = \int \mathcal{D}x(t)\, \mathrm{e}^{\mathrm{i}S[x(t)]}. \tag{3.1.3}$$

3.1.1 Single Particle Mechanics

The Lagrangian is

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 - V(x) \tag{3.1.4}$$

3.1.2 Electromagnetic Field

Any field configuration can be described by a four-vector field.

$$A_\mu(t,\vec{x}) = (\phi, \vec{A}) \tag{3.1.5}$$

The gauge invariant strength tensor is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{3.1.6}$$

The Lagrangian of EM field is

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + e j_\mu A^\mu. \tag{3.1.7}$$

We can get the Maxwell equation by variation,

$$-\partial^\nu F_{\mu\nu} + e j_\mu = 0. \tag{3.1.8}$$

3.1.3 Regularization and Renormalization

Some (perhaps most) path integrals are not well defined and we need to discretize the spacetime, or explicitly, describe the divergence in our theory. This process is called *regularization*.

However, the experiment results should have no business of the way of regularization. When we take all factors into consideration, we should get a relationship of the observables, independent of our way of regularization, which is called *Renormalization*.

3.1.4 Verify of Path Integral

Any quantum theory should satisfy two conditions, giving out the right evaluation and going back to classical mechanic when $\hbar \to 0$.

3.2 Neutron Scattering in Crystal Lattice

When we get a interference pattern on the observation screen, surprisingly we find out that except normal spiculate peaks, there are a homogeneous background in some kinds of crystal.

We may write the amplitude of this scattering process on the ith point

$$\langle B|i\rangle S \langle i|A\rangle , \tag{3.2.1}$$

where the S is the scattering amplitude.

Given that a quantum degree called *spin* exists in both neutrons and atoms, several situations are we now faced up with. Scattering by nuclear forces may cause spin flip even if energy is low.

1. All *in* and *out* neutrons are spin-paralleled with atoms.

$$\langle B|A\rangle = \sum_i \langle B|i\rangle\, S\, \langle i|A\rangle\,. \tag{3.2.2}$$

2. All *in* and *out* neutrons are spin-anti-paralleled with atoms. This is the same as former.

3. A non-trivial situation is that when a neutron and atom suffered spin flip in the process, final states are not the same and thus we need to sum up the probability **not the amplitude**.

$$P = \sum_i \left|\langle B|i\rangle\, R\, \langle i|A\rangle\right|^2. \tag{3.2.3}$$

   There's no cross term, i.e., interference disappears.

   Another explanation is, when spin flips, we can determine which lattice point the neutron passed. It becomes a which-way detector effectively, similar to electron two-slit interference!

3.3 Identical Particles

In Rutherford scatter experiment, for instance, α particle collide with oxygen atom. We use a detector which can not distinguish α particles and oxygen atoms. In the center-of-mass reference frame, the probability is

$$P = f(\theta) + f(\pi - \theta). \tag{3.3.1}$$

This result does not hold on when we do the experiment of $\alpha - \alpha$ scattering. The final state are identical, we cannot label the α particles, thus,

$$P = \left|f(\theta) + \mathrm{e}^{\mathrm{i}\delta} f(\pi - \theta)\right|^2. \tag{3.3.2}$$