

场论与凝聚态笔记

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1 Invitation: The Cartoon of Confinement

Never see individual quarks.

For separatable particles, like electron charges, their potential is $V(r) \sim \frac{1}{r}$, thus $V(r) - V(r_0)$ is always bounded.

Two quarks forms a pion. They interacts through gluon, and forms a structure called gluon tube or string. The potential is $V(r) \sim r$ and the energy density per length is appropriately constant. To separate a quark pair, the energy inputed $V(r) - V(r_0)$ is unbounded.

Similar phenomenon appears in superconductor(type II). When electronic charge condensed, the interaction of magneticmonopole becomes $V(r) \sim r$. According to EM duality, when magneticmonopole condensed, analogy goes to its counterpart. (Perspective by t'Hooft, Polyakov and Mandlstan.)

2 Path Integral for Single Particles

From the two-slit interference, we've known the picture of wave. Whilst, the view of particle could recover the result by computing the phase e^{iS} .

For single particle mechanic, we start from the Schrödinger equation

$$i\hbar\partial_t |\psi(t)\rangle = H(p, x, t) |\psi(t)\rangle. \quad (2.1)$$

We have the time evolution operator

$$|\psi(t)\rangle = U(t, t_0) |\psi(t_0)\rangle, \quad (2.2)$$

which is unitary,

$$U^\dagger(t, t_0)U(t, t_0) = 1. \quad (2.3)$$

If H is time-dependent, we split the time interval into small slices, and we get the infinitesimal U operator as time-independent cases,

$$U(t, t_0) = \prod_{n=0}^{N-1} U(\overbrace{t_{n-1}, t_n}^{\delta t}), \quad (2.4)$$

where $t_n \equiv t + n\delta t$. Note that the product is time ordered.

Another perspective is from the Schrödinger equation, by finite differential,

$$|\psi(t + \delta t)\rangle = \left[1 - \frac{i}{\hbar} H(p, x, t + \frac{\delta t}{2}) \right] |\psi(t)\rangle \quad (2.5)$$

To the order of δt , we have

$$|\psi(t + \delta t)\rangle = e^{-\frac{i}{\hbar} H(p, x, t + \frac{\delta t}{2})} |\psi(t)\rangle \quad (2.6)$$

Next, we put the time evolution operator in spacial basis, considering

$$\langle x' | U(t + \delta t, t) | x \rangle. \quad (2.7)$$

Suppose $H = \frac{p^2}{2m} + V(x)$ for simplicity, we obtain

$$\langle x' | \left[1 - \frac{i\delta t}{\hbar} \left(\frac{p^2}{2m} + V(x, t + \frac{\delta t}{2}) \right) \right] | x \rangle. \quad (2.8)$$

Make a substitution

$$V \rightarrow \frac{V(x', t + \frac{\delta t}{2}) + V(x, t + \frac{\delta t}{2})}{2}, \quad (2.9)$$

and insert a completeness relation of p in each time slice, we arrive at

$$\langle x' | U(t + \delta t, t) | x \rangle = \int dp \frac{1}{2\pi\hbar} e^{\frac{ip(x' - x)}{\hbar}} \exp \left[-\frac{i\delta t}{\hbar} \frac{H(p, x', t + \frac{\delta t}{2}) + H(p, x, t + \frac{\delta t}{2})}{2} \right]. \quad (2.10)$$

written in a more compact form,

$$\langle x' | U(t + \delta t, t) | x \rangle \sim \int dp \frac{1}{2\pi\hbar} e^{i\frac{p\delta x}{\hbar} - i\frac{H\delta t}{\hbar}}. \quad (2.11)$$

The finite-time evolution operator,

$$U(t_N, t_0) = \prod_{n=0}^{N-1} U(t_{n+1}, t_n) \quad (2.12)$$

inserting an identity operator as x basis completeness relation, the element is

$$U(t_{n+2}, t_{n+1}) \underbrace{1}_{\int dx_n |x_n\rangle\langle x_n|} U(t_{n+1}, t_n) \quad (2.13)$$

then we obtain

$$\begin{aligned} \langle x_N | U(t_N, t_0) | x_0 \rangle &= \left(\prod_{n=1}^{N-1} \int dx_n \langle x_{n+1} | U(t_{n+1}, t_n) | x_n \rangle \right) \\ &\times \langle x_1 | U(t_1, t_0) | x_0 \rangle \end{aligned} \quad (2.14)$$

in full,

$$\begin{aligned} \langle x_N | U(t_N, t_0) | x_0 \rangle &= \left(\prod_{n=1}^{N-1} \int \frac{dp_{n+\frac{1}{2}} dx_n}{2\pi\hbar} \right) \int \frac{dp_{\frac{1}{2}}}{2\pi\hbar} \\ &\times \exp \left(\frac{i}{\hbar} \sum_{n=0}^{N-1} \left[p_{n+\frac{1}{2}} (x_{n+1} - x_n) - \delta t \frac{H(p_{n+\frac{1}{2}}, x_{n+1}, t_{n+\frac{1}{2}}) + H(p_{n+\frac{1}{2}}, x_n, t_{n+\frac{1}{2}})}{2} \right] \right) \\ &\sim \left(\prod_{n=0}^{N-1} \int \frac{dp_{n+\frac{1}{2}} dx_n}{2\pi\hbar} \right) \int \frac{dp_{\frac{1}{2}}}{2\pi\hbar} e^{\frac{i}{\hbar} \int p dx - H dt} \end{aligned} \quad (2.15)$$