场论与凝聚态笔记

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1 Invitation: The Cartoon of Confinement

Never see individual quarks.

For separatable particles, like electron charges, their potential is $V(r) \sim \frac{1}{r}$, thus $V(r) - V(r_0)$ is always bounded.

Two quarks forms a pion. They interacts through gluon, and forms a structure called gluon tube or string. The potential is $V(r) \sim r$ and the energy density per length is appropriately constant. To separate a quark pair, the energy inputed $V(r) - V(r_0)$ is unbounded.

Similar phenomenon appears in superconductor(type II). When electronic charge condensed, the interaction of magneticmonopole becomes $V(r) \sim r$. According to EM duality, when magneticmonopole condensed, analogy goes to its counterpart. (Perspective by t'Hooft, Polyakov and Manldstan.)

2 Path Integral for Single Particles

From the two-slit interference, we've known the picture of wave. Whilst, the view of particle could recover the result by computing the phase $\mathrm{e}^{\mathrm{i}S}$.

For single particle mechanic, we start from the Schrödinger equation

$$\mathrm{i}\hbar\partial_t \left|\psi(t)\right\rangle = H(p, x, t) \left|\psi(t)\right\rangle . \tag{2.1}$$

We have the time evolution operator

$$\left|\psi(t)\right\rangle = U(t, t_0) \left|\psi(t)\right\rangle , \tag{2.2}$$

which is unitary,

$$U^\dagger(t, t_0) U(t, t_0) = 1. \tag{2.3}$$

If H is time-dependent, we split the time interval into small slices, and we get the infinitesimal U operator as time-independent cases,

$$U(t,t_0) = \prod_{n=0}^{N-1} U(\overbrace{t_{n-1},t_n}^{\delta t}), \tag{2.4}$$

where $t_n \equiv t + n\delta t$. Note that the product is time ordered.

Another perspective is from the Schrödinger equation, by finite differential,

$$|\psi(t+\delta t)\rangle = \left[1 - \frac{\mathrm{i}}{\hbar} H(p,x,t+\frac{\delta t}{2})\right] |\psi(t)\rangle \tag{2.5}$$

To the order of δt, we have

$$|\psi(t+\delta t)\rangle = \mathrm{e}^{-\frac{\mathrm{i}}{\hbar} H(p,x,t+\frac{\delta t}{2})} |\psi(t)\rangle \tag{2.6}$$

Next, we put the time evolution operator in spacial basis, considering

$$\langle x'| U(t+\delta t, t) |x\rangle . \tag{2.7}$$

Suppose $H = \frac{p^2}{2m} + V(x)$ for simplicity, we obtain

$$\langle x'| \left[1 - \frac{\mathrm{i}\delta t}{\hbar}\left(\frac{p^2}{2m} + V(x,t+\frac{\delta t}{2})\right)\right] |x\rangle . \tag{2.8}$$

Make a substitution

$$V \to \frac{V(x',t+\frac{\delta t}{2}) + V(x,t+\frac{\delta t}{2})}{2} 1, \tag{2.9}$$

and insert a completeness relation of p in each time slice, we arrive at

$$\langle x'| U(t+\delta t, t) |x\rangle = \int \mathrm{d}p \, \frac{1}{2\pi\hbar} \mathrm{e}^{\frac{\mathrm{i}p(x'-x)}{\hbar}} \exp\left[-\frac{i\delta t}{\hbar} \frac{H(p,x',t+\frac{\delta t}{2}) + H(p,x,t+\frac{\delta t}{2})}{2}\right] . \tag{2.10}$$

written in a more compact form,

$$\langle x'| U(t+\delta t, t) |x\rangle \sim \int \mathrm{d}p \, \frac{1}{2\pi\hbar} \mathrm{e}^{\mathrm{i}\frac{p\delta x}{\hbar} - \mathrm{i}\frac{H\delta t}{\hbar}} . \tag{2.11}$$

The finite-time evolution operator,

$$U(t_N,t_0) = \prod_{n=0}^{N-1} U(t_{n-1},t_n) \tag{2.12}$$

inserting an identity operator as x basis completeness relation, the element is

$$U(t_{n+2},t_{n+1}) \underbrace{1}_{\int \mathrm{d}x_n |x_n\rangle\langle x_n|} U(t_{n+1},t_n) \tag{2.13}$$

then we obtain

$$\begin{aligned}\langle x_N|\, U(t_N,t_0)\,|x_0\rangle &= \left(\prod_{n=1}^{N-1}\int \mathrm{d}x_n\ \langle x_{n+1}|\, U(t_{n+1},t_n)\,|x_n\rangle\right)\\ &\quad\times \langle x_1|\, U(t_1,t_0)\,|x_0\rangle\end{aligned} \tag{2.14}$$

in full,

$$\begin{aligned}\langle x_N|\, U(t_N,t_0)\,|x_0\rangle &= \left(\prod_{n=1}^{N-1}\int \frac{\mathrm{d}p_{n+\frac{1}{2}}\,\mathrm{d}x_n}{2\pi\hbar}\right)\int\frac{\mathrm{d}p_{\frac{1}{2}}}{2\pi\hbar}\\ \times\exp\left(\frac{\mathrm{i}}{\hbar}\sum_{n=0}^{N-1}\left[p_{n+\frac{1}{2}}(x_{n+1}-x_n)-\delta t\frac{H(p_{n+\frac{1}{2}},x_{n+1},t_{n+\frac{1}{2}})+H(p_{n+\frac{1}{2}},x_n,t_{n+\frac{1}{2}})}{2}\right]\right)&\\ &\sim\left(\prod_{n=0}^{N-1}\int\frac{\mathrm{d}p_{n+\frac{1}{2}}\,\mathrm{d}x_n}{2\pi\hbar}\right)\int\frac{\mathrm{d}p_{\frac{1}{2}}}{2\pi\hbar}\mathrm{e}^{\frac{\mathrm{i}}{\hbar}\int p\,\mathrm{d}x-H\,\mathrm{d}t}\end{aligned} \tag{2.15}$$