

```
In [1]: import math
import random
import matplotlib.pyplot as plt
import numpy as np
import matplotlib.image as mpimg

from deap import base, creator, tools, gp, algorithms
import math, operator, random, numpy
import seaborn
import pandas as pd
```

Assignment 1

We have the formula

$$S_m(H) = (1 - p_m)^{d(H)}$$

For H = A1 we have:

$$S_m(\#1\#100\#\#) = (1 - 0.01)^{14} = 0.96059601$$

For H = A2 we have:

$$S_m(\#010\#011) = (1 - 0.01)^{16} = 0.9414801494$$

A1 has the highest chance of survival because $S_m(A1) > S_m(A2)$

Assignment 2

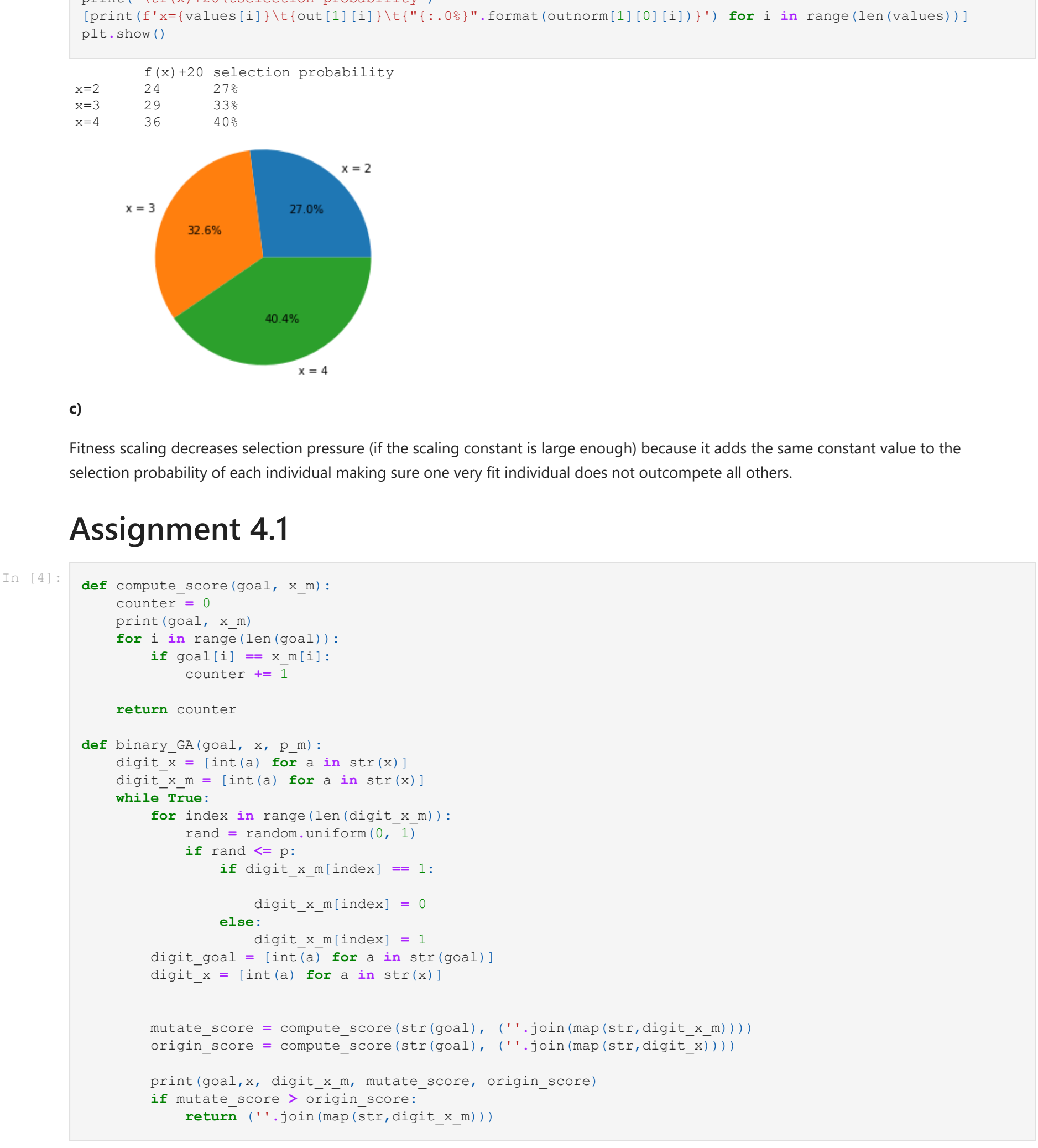
This would be the Needle in a haystack problem.

The fitness function of an EA, when modelling this problem, will for example either have a value of 0 or 1 and nothing in between. This means that there is no way to stepwise improve the fitness that will lead us from 0 to the global optimum of 1.

In other words, with the building block hypothesis we assume that a GA can create stepwise better solutions by mutating, selecting and crossing short good partial solutions(low order, high fitness, short defining length) but with this problem there are no partial solutions to combine.

Assignment 3

a)



b)

The selection pressure is high when some individuals have a higher chance of being chosen due to their fitness.

Here, function f_1 yields a lower selection pressure than $f(x)$ because the fitnesses values of the individuals are much closer to each other when using fitness function f_1 .

Thus, in f_1 the chance of being selected for the individuals are more closer to each other compared to the chance of being selected in f .



c)

Fitness scaling decreases selection pressure (if the scaling constant is large enough) because it adds the same constant value to the selection probability of each individual making sure one very fit individual does not outcompete all others.

Assignment 4.1

```
In [4]: def compute_score(goal, x_m):
    counter = 0
    print(goal, x_m)
    for i in range(len(goal)):
        if goal[i] == x_m[i]:
            counter += 1
    return counter

def binary_GA(goal, x, p_m):
    digit_x = [int(a) for a in str(x)]
    digit_x_m = [int(a) for a in str(x)]
    while True:
        for index in range(len(digit_x_m)):
            rand = random.uniform(0, 1)
            if rand <= p:
                if digit_x_m[index] == 1:
                    digit_x_m[index] = 0
                else:
                    digit_x_m[index] = 1
            digit_goal = [int(a) for a in str(goal)]
            digit_x = [int(a) for a in str(x)]

            mutate_score = compute_score(str(goal), ''.join(map(str, digit_x_m)))
            origin_score = compute_score(str(goal), ''.join(map(str, digit_x)))

            print(goal, x, digit_x_m, mutate_score, origin_score)
            if mutate_score > origin_score:
                return ''.join(map(str, digit_x_m))

In [5]: goal = '111000010'
x = '01111001'
p = 0.05

counter = 0
while not (x == goal):
    x = binary_GA(goal, x, p)
    print('iteration {0:2d} complete.\n'.format(counter))
    print(x)
    counter += 1
```

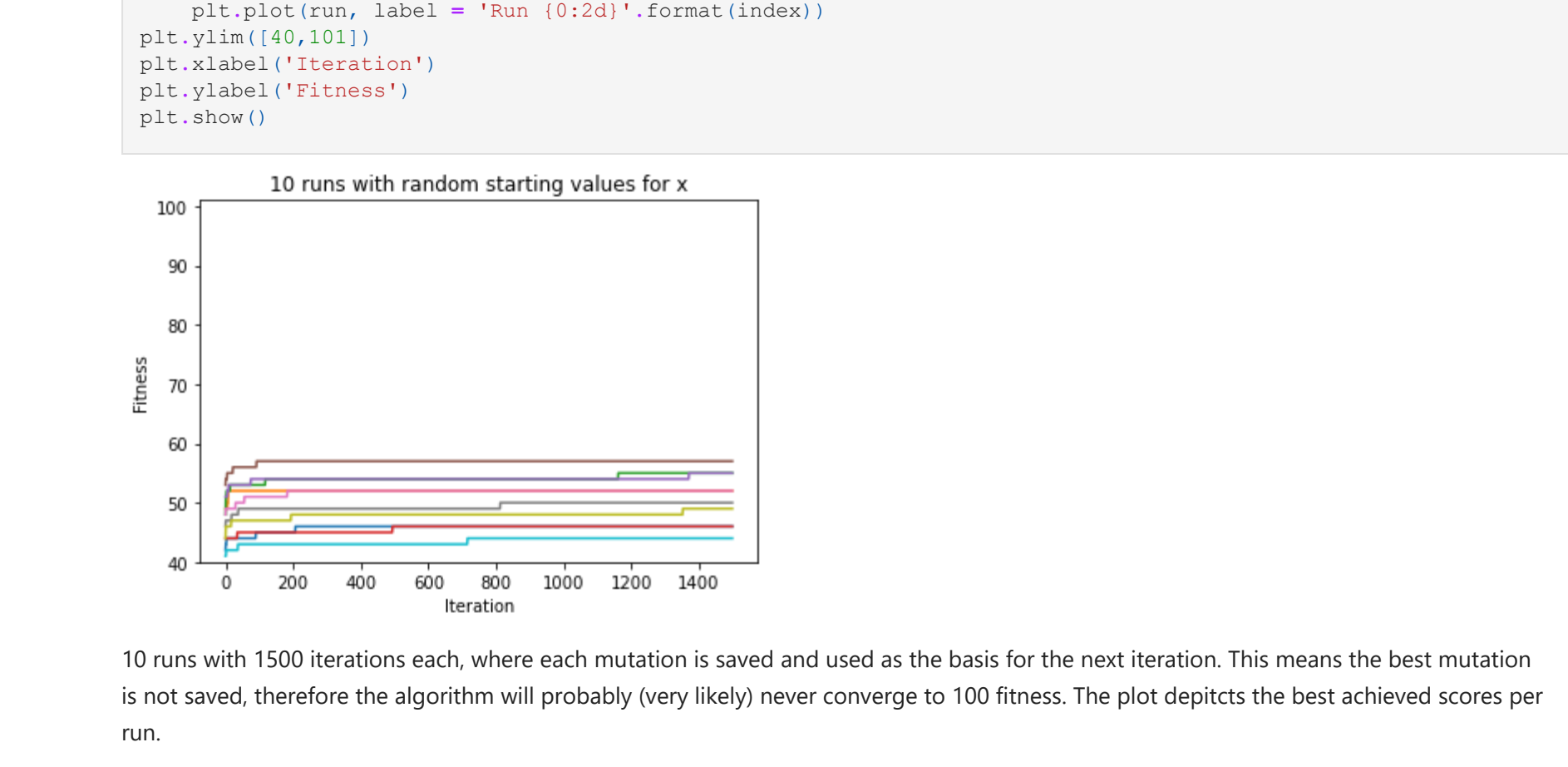

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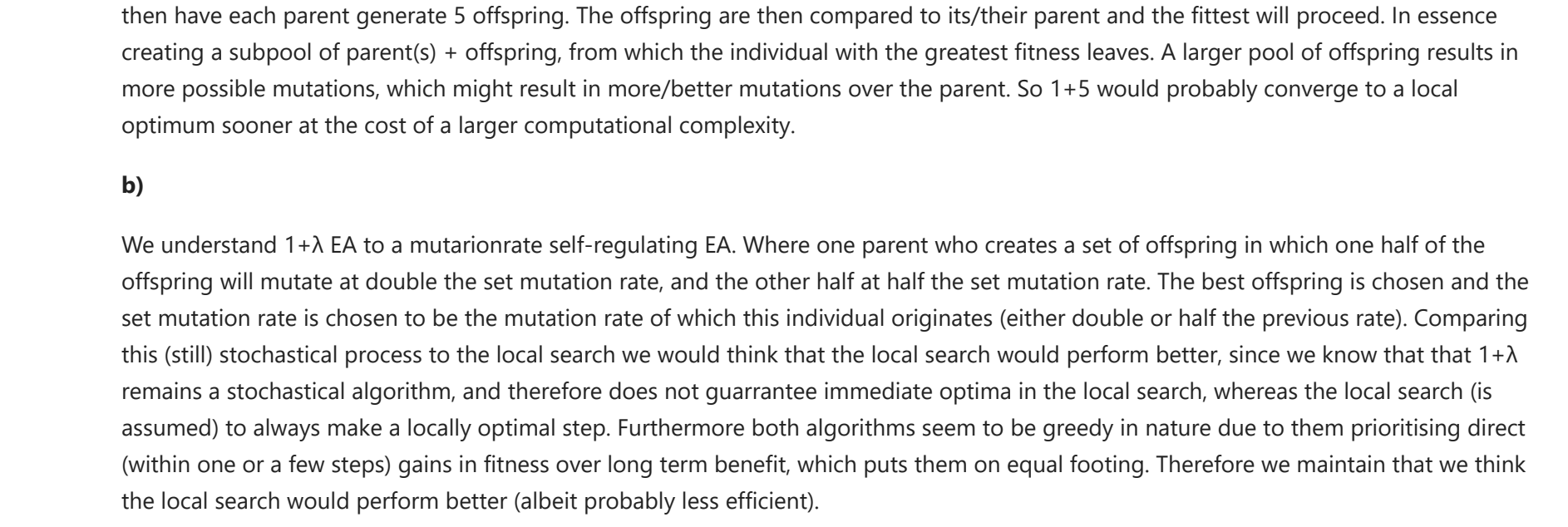

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[9]: def genbbit(n):
    number = ''
    for i in range(n):
        number += str(random.randint(0, 1))
    return number

def counting_ones(l, p, fitness, iterations, x, goal_runs):
    runs_list = []
    runs = 0
    while runs < goal_runs:
        iteration = 0
        iteration_list = []
        x = [random.randint(0, 1) for _ in range(1)]
        while len(iteration_list) < iterations:
            x_m = x.copy()
            for index in range(len(x_m)):
                rand = random.uniform(0, 1)
                if rand <= p:
                    if x_m[index] == 1:
                        x_m[index] = 0
                    else:
                        x_m[index] = 1
            mutate_score = fitness(x_m)
            origin_score = fitness(x)
            if mutate_score > origin_score:
                x = x_m
                iteration_list.append(origin_score)
            runs_list.append(iteration_list)
            runs += 1
        return runs_list

def dumb_counting_ones(l, p, fitness, iterations, x, goal_runs):
    runs_list = []
    runs = 0
    while runs < goal_runs:
        iteration = 0
        best_score = 0
        iteration_list = []
        x = [random.randint(0, 1) for _ in range(1)]
        while len(iteration_list) < iterations:
            x_m = x.copy()
            for index in range(len(x_m)):
                rand = random.uniform(0, 1)
                if rand <= p:
                    if x_m[index] == 1:
                        x_m[index] = 0
                    else:
                        x_m[index] = 1
            mutate_score = fitness(x_m)
            if mutate_score > best_score:
                best_score = mutate_score
                iteration_list.append(best_score)
            runs_list.append(iteration_list)
            runs += 1
        return runs_list
```



Plot 10 different runs of 1500 iterations, with each value having a different starting x. Pretty much all of the runs will converge, however rarely one run will be just below the optimal fitness at the 1500th iteration.



10 runs with 1500 iterations each, where each mutation is saved and used as the basis for the next iteration. This means the best mutation is not saved, therefore the algorithm will probably (very likely) never converge to 100. The plot depicts the best achieved scores per run.

Assignment 5

a)

A 1+1 evolutionary algorithm has each parent generate one offspring. A 1+5 evolutionary algorithm, to our understanding at least, would then have each parent generate 5 offspring. The offspring are then compared to its/their parent and the fittest will proceed. In essence creating a subpool of parent(s) + offspring, from which the individual with the greatest fitness leaves. A larger pool of offspring results in more possible mutations, which might result in more/better mutations over the parent. So 1+5 would probably converge to a local optimum sooner at the cost of a larger computational complexity.

b)

We understand 1+λ EA to a mutationrate self-regulating EA. Where one parent who creates a set of offspring in which one half of the offspring will mutate at double the set mutation rate, and the other half at half the set mutation rate. The best offspring is chosen and the set mutation rate is chosen to be the mutation rate of which this individual originates (either double or half the previous rate). Comparing this (still) stochastic process to the local search we would think that the local search would perform better, since we know that that 1+λ remains a stochastic algorithm, and therefore does not guarantee immediate optima in the local search, whereas the local search (is assumed) to always make a locally optimal step. Furthermore both algorithms seem to be greedy in nature due to them prioritising direct (within one or a few steps) gains in fitness over long term benefits which puts them on equal footing. Therefore we maintain that we think the local search would perform better (albeit probably less efficient).

Assignment 6

a)

b)

```
[10]: distance = lambda pos1, pos2: math.sqrt((pos2[0] - pos1[0])**2 + (pos2[1] - pos1[1])**2)

def load_file(name):
    tsp = {}
    with open(name) as f:
        lines = f.readlines()

    for line in lines:
        tsp_single = {}
        coordinates = line.split()
        tsp_single.append(float(coordinates[0]))
        tsp_single.append(float(coordinates[1]))
        tsp.append(tsp_single)

    return tsp

def total_distance(city_coords, cities):
    total_dist = 0
    for i in range(len(cities)-1):
        total_dist += distance(city_coords[cities[i]], city_coords[cities[i+1]])
    return total_dist

def two_opt(route):
    get_slice = lambda x: x if x is not None else []
    best_route = route
    best_distance = total_distance(city_coords, route)
    for i in range(len(route)-1):
        for k in range(i+2, len(route)+1):
            mid = get_slice(route[i:k])
            mid.reverse()
            new_route = get_slice(route[:i]) + mid + get_slice(route[k:])
            new_distance = total_distance(city_coords, new_route)
            if new_distance < best_distance:
                best_route = new_route
                best_distance = new_distance
    return best_route

def crossover(pool):
    m, d = random.sample(range(len(pool)), 2)
    mom = pool[m]
    dad = pool[d]
    c1 = random.randint(0, len(mom)-1)
    c2 = random.randint(c1+1, len(mom)-1)
    dadw = dad[c1:c2] + dad[c2:]
    momw = mom[c1:c2] + mom[c2:]
    rest_one = x for x in dadw if x not in mom[c1:c2]]
    rest_two = x for x in momw if x not in dad[c1:c2]]
    child_one = rest_one[len(mom)-c2:] + mom[c1:c2] + rest_one[:len(mom)-c2]
    child_two = rest_two[len(mom)-c2:] + dad[c1:c2] + rest_two[:len(mom)-c2]
    return child_one, child_two

def mutate(list, mutate_chance):
    if random.uniform(0, 1) > mutate_chance:
        return list
    i, j = np.random.choice(range(len(list)), 2, replace = False)
    list[i], list[j] = list[j], list[i]
    return list

def create_next_generation(pool, prop_fitnesses, num_start_instances, mutate_chance, use_local, num_elites):
    parents = random.choices(pool, prop_fitnesses, k=num_start_instances)
    elites = [pool[x] for x in np.argmax(partition(prop_fitnesses, num_elites){:num_elites}]
    new_generation = elites
    loop_max = int(np.ceil(len(parents)/2))
    for i in range(loop_max-num_elites):
        o1, o2 = crossover(parents)
        o1 = mutate(o1, mutate_chance)
        o2 = mutate(o2, mutate_chance)
        if use_local:
            o1 = two_opt(o1)
            o2 = two_opt(o2)
        new_generation.append(o1)
        new_generation.append(o2)

    return new_generation

def show_map(route):
    for i in range(len(route)-1):
        path_x = [city_coords[route[i]][0], city_coords[route[i+1]][0]]
        path_y = [city_coords[route[i]][1], city_coords[route[i+1]][1]]
        plt.plot(path_x, path_y, 'ro-')
    plt.show()

def mem_tsp(num_iterations, num_start_instances, mutate_chance, num_elites):
    avg_fitnesses = []
    best_fitnesses = []
    use_local = True
    pool = [np.random.permutation(len(city_coords)).tolist() for x in range(num_start_instances)]
    pool = [two_opt(r) for r in pool]
    fitnesses = [total_distance(city_coords, r) for r in pool]
    prop_fitnesses = [f/sum(fitnesses) for f in fitnesses]
    print('Memetic')
    for i in range(num_iterations):
        pool = create_next_generation(pool, prop_fitnesses, num_start_instances, mutate_chance, use_local, num_elites)
        fitnesses = [total_distance(city_coords, r) for r in pool]
        prop_fitnesses = [f/sum(fitnesses) for f in fitnesses]
        avg_fitnesses.append(sum(fitnesses)/len(fitnesses))
        best_fitnesses.append(min(fitnesses))
        print(i)
    return avg_fitnesses, best_fitnesses

def ea_tsp(num_iterations, num_start_instances, mutate_chance, num_elites):
    avg_fitnesses = []
    best_fitnesses = []
    use_local = False
    pool = [np.random.permutation(len(city_coords)).tolist() for x in range(num_start_instances)]
    fitnesses = [total_distance(city_coords, r) for r in pool]
    prop_fitnesses = [f/sum(fitnesses) for f in fitnesses]
    print('EA')
    for i in range(num_iterations):
        pool = create_next_generation(pool, prop_fitnesses, num_start_instances, mutate_chance, use_local, num_elites)
        fitnesses = [total_distance(city_coords, r) for r in pool]
        prop_fitnesses = [f/sum(fitnesses) for f in fitnesses]
        avg_fitnesses.append(sum(fitnesses)/len(fitnesses))
        best_fitnesses.append(min(fitnesses))
        print(i)
    return avg_fitnesses, best_fitnesses

city_coords = load_file('file-tsp.txt')
iterations = 1500
instances = 26
elites = 10
avg_mem, best_mem = mem_tsp(iterations, instances, 0.01, elites)
avg_ea, best_ea = ea_tsp(iterations, instances, 0.1, elites)
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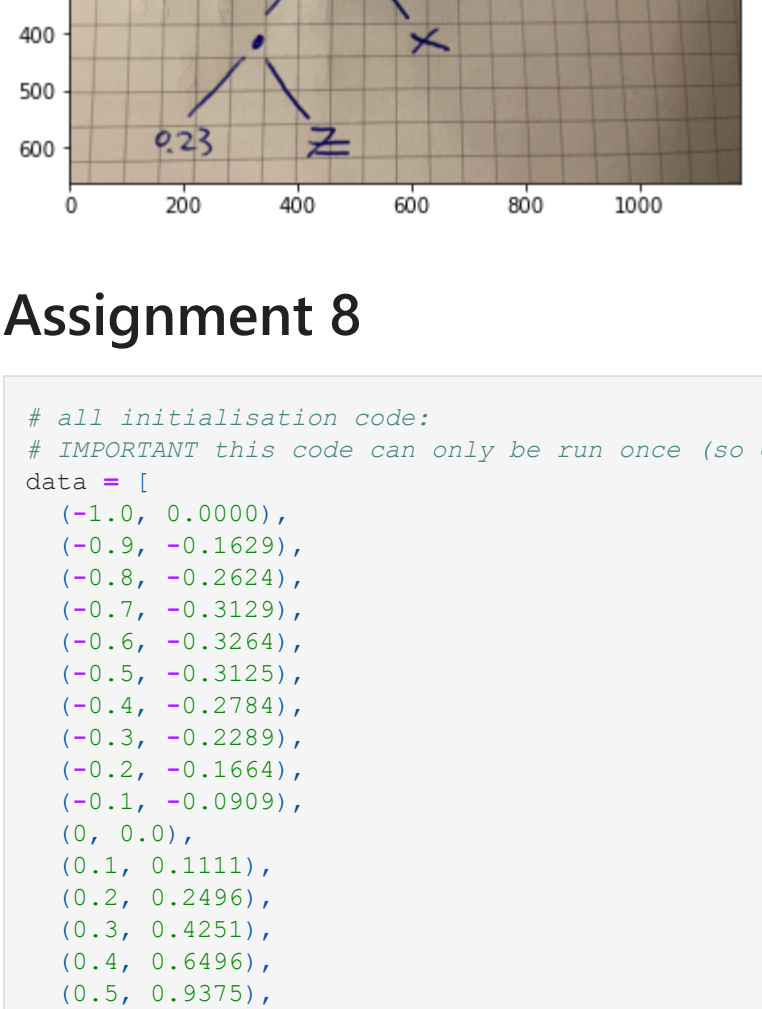
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c)

```
In [11]: plt.plot(range(iterations), avg_mem, label = 'average fitness')
plt.plot(range(iterations), best_mem, label = 'best fitness')
plt.title('memetic algorithm convergence plot')
plt.ylim(100,200)
plt.legend()
plt.ylabel('fitness')
plt.xlabel('iterations')
plt.show()

plt.plot(range(iterations), avg_ea, label = 'average fitness')
plt.plot(range(iterations), best_ea, label = 'best fitness')
plt.title('evolutionary algorithm convergence plot')
plt.ylim(100,200)
plt.legend()
plt.ylabel('fitness')
plt.xlabel('iterations')
plt.show()
```



d)

The memetic algorithm works a lot better

e)

It is not fair, because the memetic algorithm uses local search which is an additional method to improve the pool. The regular evolution algorithm doesn't have this and merely relies on the number of iterations. I think it would be more fair if we increased the number of iterations for the evolution algorithm. The evolution algorithm would otherwise be exactly the same as the memetic approach minus the local search function, and the local search function only improves the instances and never makes them worse.

f) In <https://arxiv.org/ftp/arxiv/papers/1004/1004.0574.pdf> it is found that memetic algorithms work better than genetic algorithms, it was tested on many NP-hard problems.

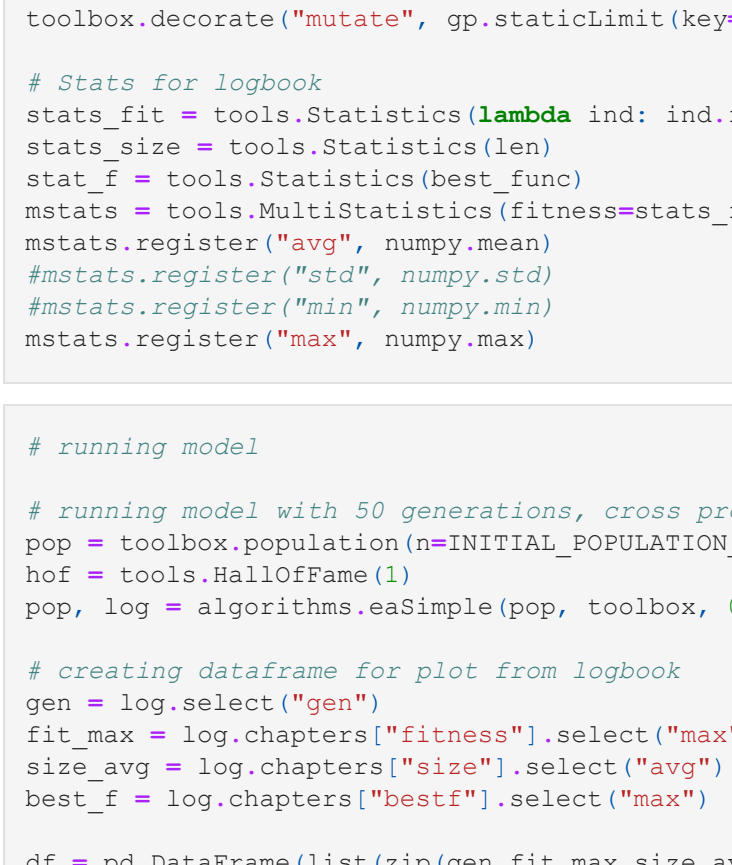
Assignment 7

a)

```
T1 = (y, True, x, z)

F1 = (f(←, x1, x2), f(Λ, x1, x2), f(v, x1, x2), f(→, x1, x2))
```

```
In [12]: plt.imshow(mpimg.imread('7a.jpeg'))
plt.show()
```

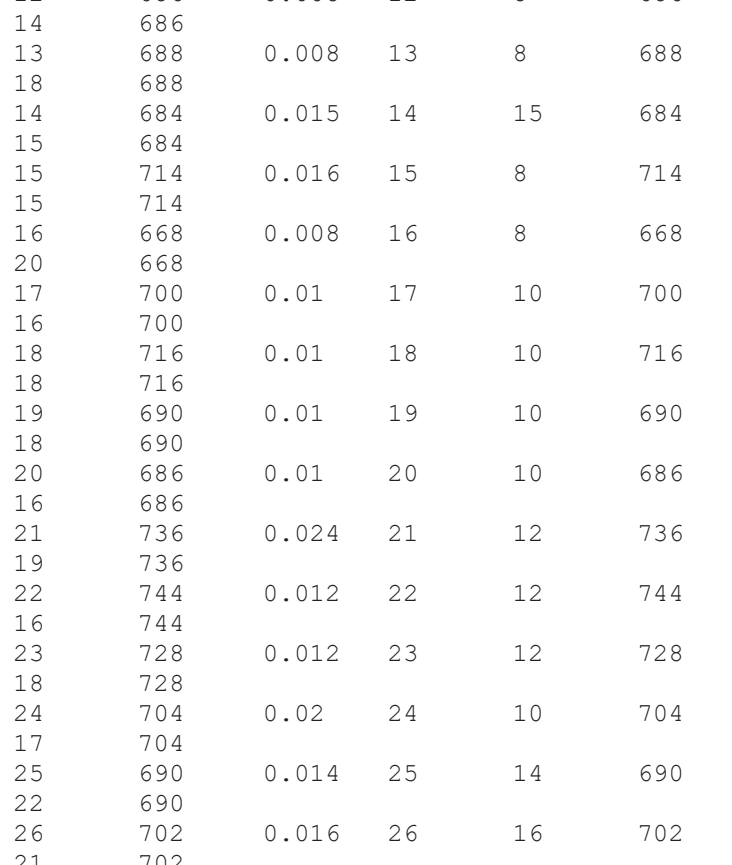


b)

T2 = (0.23, Z, X, 0.789)

```
F2 = (f(←, x1, x2), f(→, x1, x2), f(Λ, x1, x2))
```

```
In [13]: plt.imshow(mpimg.imread('7b.jpeg'))
plt.show()
```



Assignment 8

```
In [14]: # all initialisation code:
# IMPORTANT this code can only be run once (so empty kernel if you need to run it again), this is a limitation
data = [
    (-1.0, 0.0000),
    (-0.9, -0.1629),
    (-0.8, -0.2624),
    (-0.7, -0.3129),
    (-0.6, -0.3264),
    (-0.5, -0.3125),
    (-0.4, -0.2784),
    (-0.3, -0.2289),
    (-0.2, -0.1664),
    (-0.1, -0.0909),
    (0, 0.0),
    (0.1, 0.1111),
    (0.2, 0.2496),
    (0.3, 0.4251),
    (0.4, 0.6456),
    (0.5, 0.9375),
    (0.6, 1.3056),
    (0.7, 1.7733),
    (0.8, 2.3616),
    (0.9, 3.0951),
    (1.0, 4.0000)
]

# creating function set f
def f(protectedDiv(left, right):
    try:
        return left / right
    except ZeroDivisionError:
        return 1
    except ValueError:
        return 0
    except ZeroDivisionError:
        return 0

primitives = gp.PrimitiveSet("MathematicalOperators", 1)
primitives.addPrimitive(operator.add, 2)
primitives.addPrimitive(operator.sub, 2)
primitives.addPrimitive(operator.mul, 2)
primitives.addPrimitive(operator.div, 2)
primitives.addPrimitive(protectedDiv, 2)
primitives.addPrimitive(math.exp, 1)
primitives.addPrimitive(math.cos, 1)
primitives.addPrimitive(math.sin, 1)
primitives.addPhenoraConstant("k", lambda: random.uniform(-1.000000000, 1.000000000))

# create fitness and individual, which are building blocks for deap library
creator.create("FitnessMax", base.Fitness, weights=(1.0,))
creator.create("Individual", gp.PrimitiveTree, fitness=creator.FitnessMax)

INITIAL_POPULATION_SIZE = 1000

# creating initialisation
toolbox = base.Toolbox()
toolbox.register("expr", gp.genHalfAndHalf, pset=primitives, min_=1, max_=2)
toolbox.register("individual", tools.initUniform, expr=toolbox.expr, pset=primitives)
toolbox.register("population", tools.initRepeat, list, toolbox.individual)
toolbox.register("compile", gp.compile, pset=primitives)

# self made fitnessfunction for -sum of absolute value
def fitnessFunc(individual):
    func = toolbox.compile(expr=individual)
    sum = 0
    for x in data:
        sum += abs(func(x[0]) - x[1])
    return (-1*sum)

# self made func to calculate number of nodes of best individual during iteration
def best_func(ind):
    poplist = []
    for x in pop:
        poplist.append(x.fitness.values)
    bestfit = max(poplist)
    if ind.fitness.values == bestfit:
        return len(ind)
    else:
        return 0

# instantiate evaluation, selection, mate and mutate , also set max_value = 17 to avoid bloat (from
toolbox.register("evaluate", fitnessFunc)
toolbox.register("select", tools.selTournament, tournsize=3)
toolbox.register("mate", gp.cxOnePoint)
toolbox.register("expr_mut", gp.genFull, min_=0, max_=2)
toolbox.register("mutate", gp.mutUniform, expr=toolbox.expr, pset=primitives)
toolbox.decorate("mate", gp.staticLimit(key=operator.attrgetter("height"), max_value=17))
toolbox.decorate("mutate", gp.staticLimit(key=operator.attrgetter("height"), max_value=17))

# Stats for logbook
stats_fit = tools.Statistics(lambda ind: ind.fitness.values)
stats_size = tools.Statistics(lambda ind: len(ind.fitness.values))
stat_f = tools.Statistics(lambda ind: ind.fitness.values)
stats = tools.MultiStatistics(fitness=stats_fit, size=stats_size, best_fit=stat_f)
stats.register("avg", numpy.mean)
stats.register("std", numpy.std)
stats.register("min", numpy.min)
stats.register("max", numpy.max)
```

```
In [15]: # running model
# running model with 50 generations, cross prob = 0.7 , mutation prob = 0
pop = toolbox.population(n=INITIAL_POPULATION_SIZE)
logbook = tools.HallOfFame(1)
pop, log = algorithms.eaSimple(pop, toolbox, 0.7, 0, 50, stats=stats, halloffame=log, verbose=True)

# creating dataframe for plot from logbook
gen = log.select("gen")
fit_max = log.chapters["fitness"].select("max")
size_avg = log.chapters["size"].select("avg")
best_fit = log.chapters["best"].select("max")

df = pd.DataFrame(list(zip(gen, fit_max, size_avg, best_fit)), columns=['gen', 'max_fitness', 'avg_nodes', 'nodes best individual'])
```

bestfit					fitness					size				
gen	max	nevals	avg	gen	max	nevals	avg	gen	max	nevals	avg	gen	max	nevals
0	max	1000	0.005	0	5	1000	-57.1708	0	-5.45326	1000	3.843	0	max	1000
1	1	686	0.004	1	4	686	-3.37794e+13	1	-4.24698	686	3.6	1	1	686
2	2	720	0.004	2	4	720	-15.851	2	-4.24698	720	3.489	2	2	720
3	3	690	0.024	3	4	690	-14.5393	3	-4.24698	690	3.519	3	3	690
4	4	670	0.006	4	6	670	-13.9522	4	-3.9279	670	3.763	4	4	670
5	5	716	0.006	5	6	716	-12.7336	5	-3.79774	716	4.156	5	5	716
6	6	710	0.006	6	6	710	-11.9669	6	-3.04161	710	4.588	6	6	710
7	7	680	0.012	7	6	680	-11.3773	7	-3.04161	680	4.953	7	7	680
8	8	692	0.007	8	7	692	-9.99117	8	-3.04161	692	5.011	8	8	692
9	9	692	0.006	9	6	692	-10.7524	9	-2.9756	690	4.902	9	9	690
10	10	666	0.042	10	7	666	-10.0431	10	-2.63569	666	4.789	10	10	666
11	11	706	0.008	11	8	706	-13.5909	11	-2.91064	706	4.983	11	11	706
12	12	686	0.008	12	8	686	-13.0791	12	-2.92327	686	5.251	12	12	686
13	13	688	0.008	13	8	688	-10.972	13	-2.3637	688	5.654	13	13	688
14	14	684	0.015	14	8	684	-11.4923	14	-2.63569	684	5.918	14	14	684
15	15	714	0.016	15	8	714	-12.9963	15	-2.79064	714	6.327	15	15	714
16	16	668	0.008	16	8	668	-17.2702	16	-2.47565	668	6.493	16	16	668
17	17	700	0.01	17	10	700	-16.0377	17	-2.1332	700	6.666	17	17	700
18	18	716	0.01	18	10	716	-39.279	18	-2.1332	716	6.909	18	18	716
19	19	690	0.01	19	10	690	-15.6606	19	-0.990774	690	6.987	19	19	690
20	20	686	0.01	20	10	686	-19.6042	20	-0.990774	686	7.178	20	20	686
21	21	736	0.024	21	12	736	-25.5163	21	-0.993626	736	7.356	21	21	736
22	22	744	0.012	22	12	744	-24.5789	22	-0.993626	744	7.432	22	22	744
23	23	728	0.012	23	12	728	-23.1577	23	-0.993626	728	7.562	23	23	728
24	24	704	0.02	24	14	704	-27.4585	24	-1.11998	704	7.756	24	24	704
25	25	690	0.014	25	14	690	-56.6232	25	-1.02551	690	7.989	25	25	690
26	26	702	0.016	26	16	702	-18.9581	26	-1.08905	702	8.163	26	26	702
27	27	676	0.028	27	14	676	-25.3748	27	-1.05924	676	8.578	27	27	676
28	28	718	0.012	28	12	718	-35.2589	28	-0.990774	718	9.251	28	28	718
29	29	666	0.048	29	12	666	-556193	29	-0.990774	666	10.285	29	29	666
30	30	698	0.068	30	12	698	-13.3794	30	-0.990774	698	11.305	30	30	698
31	31	704	0.096	31	12	704	-18.3658	31	-0.990774	704	12.233	31	31	704
32	32	706	0.024	32	12	706	-21.9859	32	-0.990621	706	13.571	32	32	706
33	33	712	0.026	33	26	712	-30.2666	33	-0.933577	712	14.503	33	33	712
34	34	672	0.026	34	26	672	-34.3981	34	-0.933577	672	15.267	34	34	672
35	35	678	0.026	35	26	678	-29.1704	35	-0.94498	678	15.398	35	35	678
36	36	708	0.078	36	26	708	-41.1326	36	-0.94498	708	15.269	36	36	708
37	37	704	0.031	37	31	704	-1.07454e+08	37	-0.948561	704	16.262	37	37	704
38	38	700	0.03	38	30	700	-106.522	38	-0.920451	700	16.724	38	38	700
39	39	690	0.018	39	18	690	-38.08	39	-0.849018	690	17.715	39	39	690
40	40	682	0.026	40	26	682	-94.7421	40	-0.835036	682	19.109	40	40	682
41	41	690	0.052	41	26	690	-31.4626	41	-0.835036	690	21.087	41	41	690
42	42	694	0.052	42	26	694	-20.595	42	-0.835036	694	24.342	42	42	694
43	43	710	0.028	43	28	710	-48.3482	43	-0.831504	710	26.995	43	43	710
44	44	696	0.068	44	68	696	-23.8761	44	-0.800457	696	29.136	44	44	696
45	45	696	0.068	45	68	696	-26.2635	45	-0.800457	696	32.143	45	45	696
46	46	722	0.072	46	68	722	-19.2334	46	-0.800457	722	33.582	46	46	722
47	47	664	0.228	47	68	664	-4.96866	47	-0.785629	664	34.769	47	47	664
48	48	708	0.078	48	78	708	-131.61	48	-0.770763	708	35.22	48	48	708
49	49	674	0.234	49	78	674	-4.33184	49	-0.770763	674	38.018	49	49	674
50	50	706	0.072	50	72	706	-4.31858	50	-0.761353	706	42.007	50	50	706

```
Out [15]: gen max_fitness avg_nodes nodes best individual
0 0 -5.453263 3.843 5
1 1 -4.246982 3.600 4
2 2 -4.246982 3.489 4
3 3 -4.246982 3.519 4
4 4 -3.927896 3.763 6
5 5 -3.797742 4.156 6
6 6 -3.041613 4.558 6
7 7 -3.041613 4.898 6
8 8 -3.041613 5.011 7
9 9 -2.975597 4.902 6
10 10 -3.041613 4.789 7
11 11 -2.921019 4.983 8
12 12 -2.363699 5.251 8
13 13 -2.363699 5.654 8
14 14 -2.635886 5.918 8
15 15 -2.790636 6.327 15
16 16 -2.475647 6.493 8
17 17 -2.133204 6.666 10
18 18 -2.133204 6.909 10
19 19 -0.990774 6.987 10
20 20 -0.990774 7.178 10
21 21 -0.993626 7.356 12
22 22 -0.993626 7.432 12
23 23 -0.993626 7.562 12
24 24 -1.119979 7.756 10
25 25 -1.025145 7.989 14
26 26 -1.069045 8.163 16
27 27 -1.059242 8.578 14
28 28 -0.990774 9.251 12
29 29 -0.990774 10.285 12
30 30 -0.990774 11.305 12
31 31 -0.990774 12.233 12
32 32 -0.990821 13.571 24
33 33 -0.933577 14.503 26
34 34 -0.933577 15.267 26
35 35 -0.944980 15.398 26
36 36 -0.944980 15.269 26
37 37 -0.948561 16.262 31
38 38 -0.920451 16.724 30
39 39 -0.849018 17.715 18
40 40 -0.835036 19.109 26
41 41 -0.835036 21.087 26
42 42 -0.835036 24.342 26
43 43 -0.831504 26.995 28
44 44 -0.800457 29.136 68
45 45 -0.800457 32.143 68
46 46 -0.800457 33.582 68
47 47 -0.785629 34.769 28
48 48 -0.770763 35.220 78
49 49 -0.770763 38.018 78
50 50 -0.761353 42.007 72
```

8.a

```
In [16]: # question a
seaborn.lineplot(data=df, x='gen', y='max_fitness')
```

```
Out [16]: <AxesSubplot: xlabel='gen', ylabel='max_fitness'>
```



8.b

```
In [17]: # question b (BONUS plot with average nodes best individual)
# gebruik ipy vov avg_nodes de size van de beste individual (gebruik HallOfFame) ,
seaborn.lineplot(data=df, x='gen', y='avg_nodes')
```

```
Out [17]: <AxesSubplot: xlabel='gen', ylabel='avg_nodes'>
```



```
In [18]: # question b
# gebruik ipy vov avg_nodes de size van de beste individual (gebruik HallOfFame) ,
seaborn.lineplot(data=df, x='gen', y='
```