SDSC 6007 & SDSC 8006 (Semester B, 2023)
Dynamic Programming and Reinforcement Learning
Assignment 1

All questions are weighted equally.

Question 1. Ash Ketchum is preparing his next trip and packing up. His backpack has maximum weight capacity is z and he wants to fill it up with different quantities of N different items. Denote

- v_i: the value of the ith type of item
- w_i: the weight of ith item
- x_i: the number of items of type i that are loaded in the backpack

Ash Ketchum is trying to maximize the total value of all the items in his backpack, *i.e.* to maximize $\sum_{i=1}^{N} x_i v_i$ subject to the constraints $\sum_{i=1}^{N} x_i w_i \leq z$ and $x_i = 0, 1, \ldots$. Formulate this problem and state the corresponding DP algorithm.

Question 2. Ash Ketchum is currently staying in Hong Kong in order to capture a unique fierce creature, "Klint"; unfortunately, he run out of money and now he has to work in an inn to save up for the next trip. Everyday, he charges a different rate of a room as the day progresses, depending on the number of vacancies. His salary is proportional to the income during the day, and so his objective is to maximize the expected total income during the day. Denote

- x: the number of empty rooms at the start of the day
- y: the number of customers that will ask for a room during the day

After taking SDSC8004 during his free time, Ash Ketchum becomes a genius in statistics, and he now knows y with certainty. Upon arrival of a customer, Ash Ketchum quotes one of m prices r_i, $i = 1, \ldots m$, where $0 < r_1 \leq r_2 \leq \cdots \leq r_m$. A quote of a rate r_i is accepted with probability p_i and

is rejected with probability $1 - p_i$, in which case the customer departs, never to return during that day. Formulate this problem and state the corresponding DP algorithm.

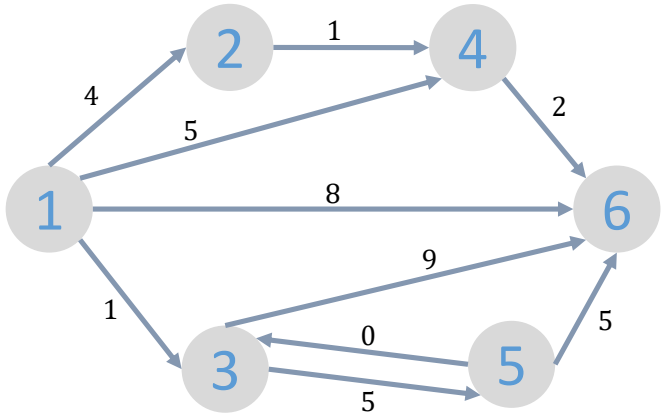

Figure 1

Question 3. Find a shortest path from each node to node 6 for the graph of Figure 1 by using the DP algorithm **manually**.

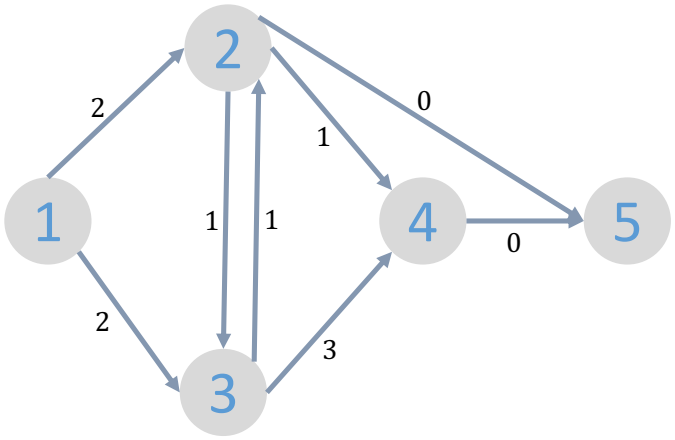

Figure 2

Question 4.

(a) Find a shortest path from node 1 to node 5 for the graph of Figure 2 by using (any) label correcting method **manually**.

(b) Confirm your answer in part (a) by using Python (with or without NetworkX) to solve the shortest path problem. Please submit your Python code as part of your submission.

Question 5. **[SDSC8006 only]** Imagine you are teaching this course 5 years later. Based on our material from Week 1 to Week 4, construct a novel question and its solution for the mid-term exam.