FYS-3150 Project 3

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Abstract

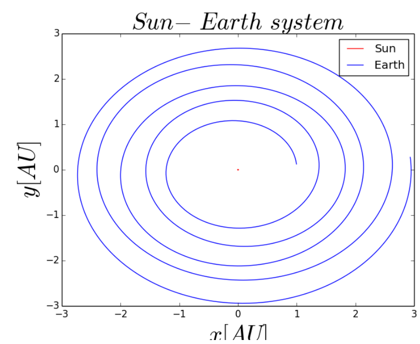
I. INTRODUCTION

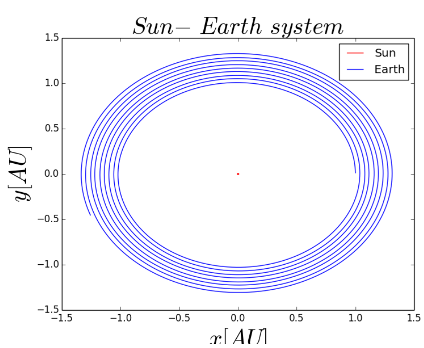
The aim of this project is to simulate the solar system with the use of object orientated programming (OOP). The main algorithm we will apply is the velocity- verlet for computing the velocities and positions of each planet. We will also use the euler-method and test the stability of both algorithms. Newton's gravitational law is also necessary for calculating the forces which we'll need for starting the velocity-verlet and euler algorithms. Object orientation comes in handy when we want to do the same operation several times. Instead of duplicating the code which often leads to inefficient and complex programs, we need only to write the code once, we can then run the same calculation on different objects i.e the force on the planets in the solar system. The structure of the program becomes more systematic because we deal with programs which have there own functionality in the code. This also leads to easier troubleshooting.

II. METHOD/APPROACH

III. RESULTS

Euler vs Verlet





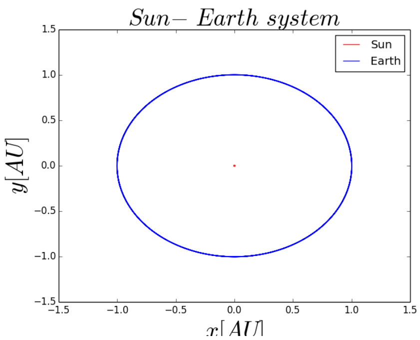
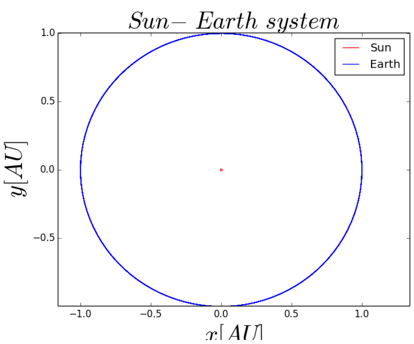
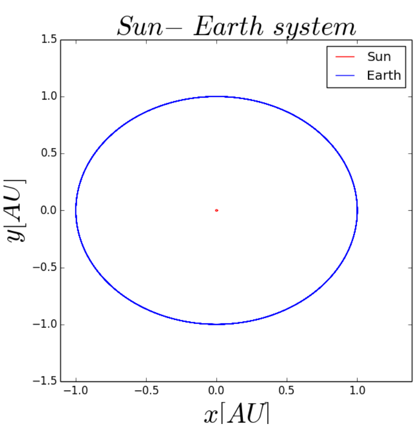


Figure 1: Euler method for 3 diffent dt's. Top has $dt = 0.01$, middle has $dt = 0.001$ and last $dt = 0.00001$





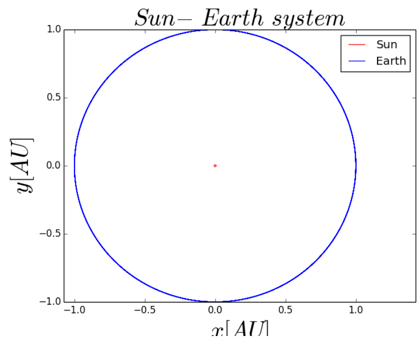
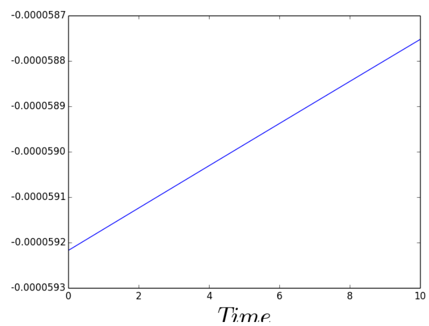


Figure 2: Verlet method for 3 different dt's. Top has $dt = 0.01$, middle has $dt = 0.001$ and last $dt = 0.00001$

Energy and angular momentum



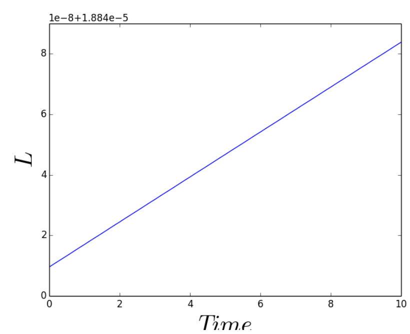


Figure 3: Energy and angular momentum using the Euler method.

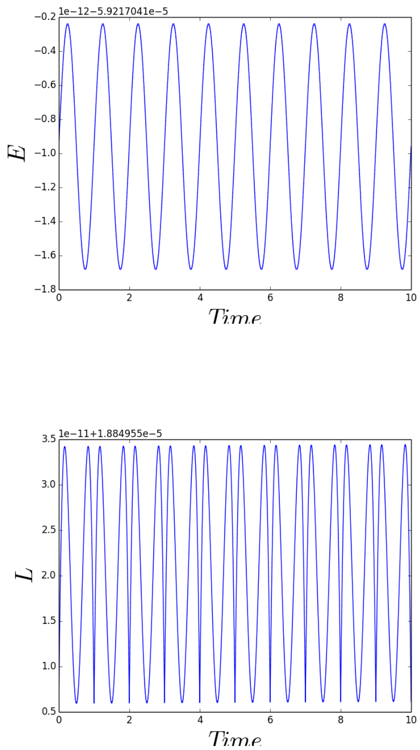


Figure 4: Energy and angular momentum using the Verlet method.

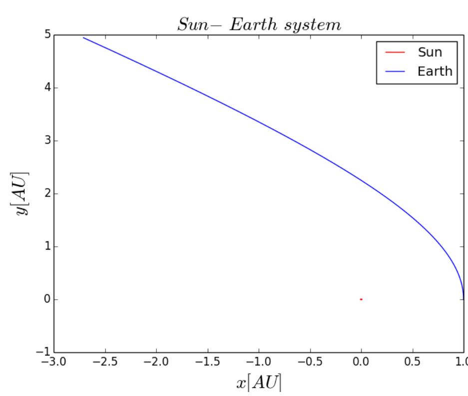


Figure 5: Plot of the earth sun system where the earth is given a initial velocity 3π. The earth overcomes the potential energy thus moving out of orbit.

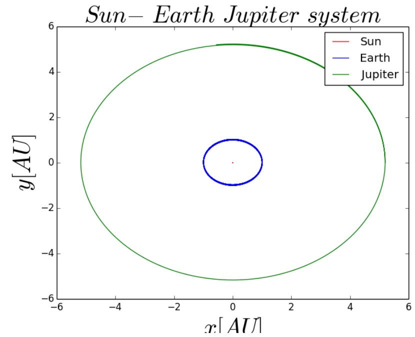


Figure 6: Three body problem. The Sun, Earth and Jupiter.

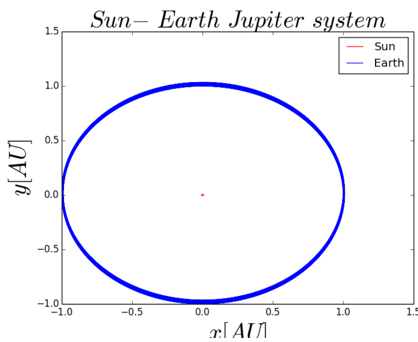


Figure 7: Increasing Jupiter by a factor 10. We are only interested how the earth behaves when Jupiter increases in size. The earth is getting larger and larger orbits.

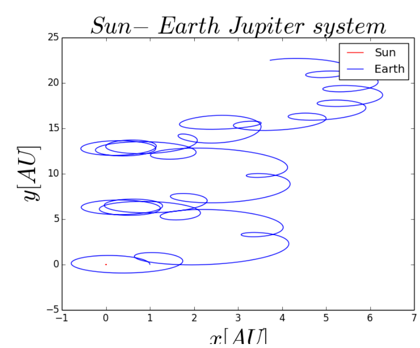


Figure 8: Jupiter is now the same mass as the sun and the earth's trajectory cannot be predicted.

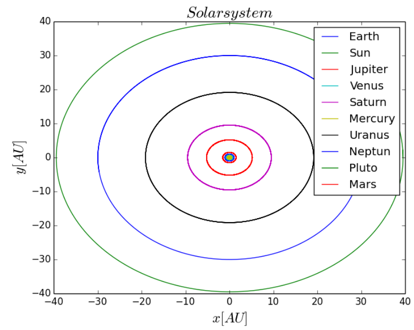


Figure 9: The complete solar system with pluto run over a time period of 10 years.

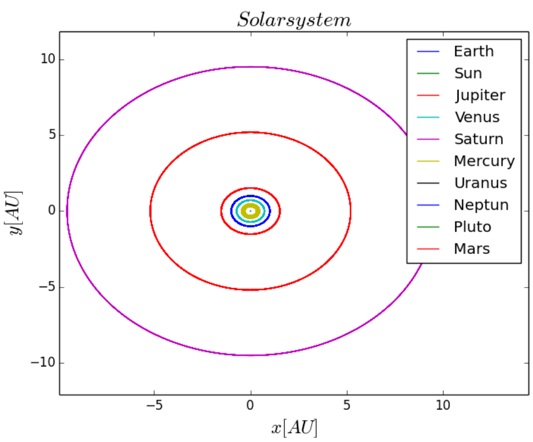


Figure 10: Zoomed inn plot of figure 6

IV. DISCUSSION

From figure 1 we see that the Euler method fails for low dt's thus making this algorithm unpractical for this use where we necessarily don't need a such fine tuned dt. A lower dt implies that we do more calculations which can be rather time consuming. The Euler method has an error that goes like $NO(dt^2) \approx O(dt)$ where $N = (t_n - t_0)/dt$. It is clear from figure 2 that Verlet does not suffer from this change in dt which has a global error that goes as $O(dt^2)$. We also see from figure 3 and 4 that the verlet algorithm conserves the energy where the euler method does not. We expect that the energy and angular momentum are conserved when no external forces are acting on the object. The reason for the oscillating energy and angular momentum is because of the circular motion of the planet which is assumed in this project. Both the energy and momentum depend on the position of the object, so when the position changes, the energy and momentum does also.

The escape velocity can derived from the total energy $E_{tot} = E_k + E_p$ where $E_k = 0.5mv^2$ and $E_p = -GMm/r$. Solving for the velocity v we find (1)$v > \sqrt{2Gm/r}$ the boundary for the escape velocity. This velocity can also give us the initial velocity of other planets. We then put the equation equal to and not greater than (see figure 5 for escape velocity plot). If we increase the power of the radial distance the velocity needed to escape the suns gravitational potential is lower than if the power is r^2. Increasing the power by one we get $E_p = -GMm/r^2$, this leads to $v > \sqrt{2Gm/r^2}$.

Making Jupiter as large as the sun made the earth's trajectory none circular, because now two objects are pulling at the earth with almost the same force, besides the distance between the objects. The Verlet algorithm is still capable of calculating the earths trajectory.

After adding all planets inn the solar system, Pluto included, the motions look reasonable where the expected orbit is circular (see figure 9 and 10). It does not look as though the center of mass correction had much effect on the whole system. The sun has a mass with magnitude of 3 larger than the next largest planet in the solar system, so it is not necessarily so odd that the center of mass contributes little comparing with the sun fixed in origo.

V. CONCLUSION

The verlet algorithm produced better results than the euler method, being able to do many calculations with a higher precision than euler, where euler suffered from error's becoming large when looking at the system for a large time period. The verlet algorithm was also capable of calculating the earth's trajectory when the mass of jupiter increased.

It was good to check the energy and angular momentum too see if these quantities were conserved, which they were. One odd notification was that the energy and angular momentum didn't always seem to be conserved for all chosen dt's, the reason for this is unknown. One possibility could be that the change in energy and angular momentum is too small that the computer suffers from numerical precision.

VI. REFERENCES

Hjort -Jensen Morten., 2015. Computational Physics Kompendiet.
http://compphysics.github.io/ComputationalPhysics/doc/pub/linalg/pdf/linalg-print.pdf