Internship Report:
Emergence, control and open-ended evolution in cellular automata

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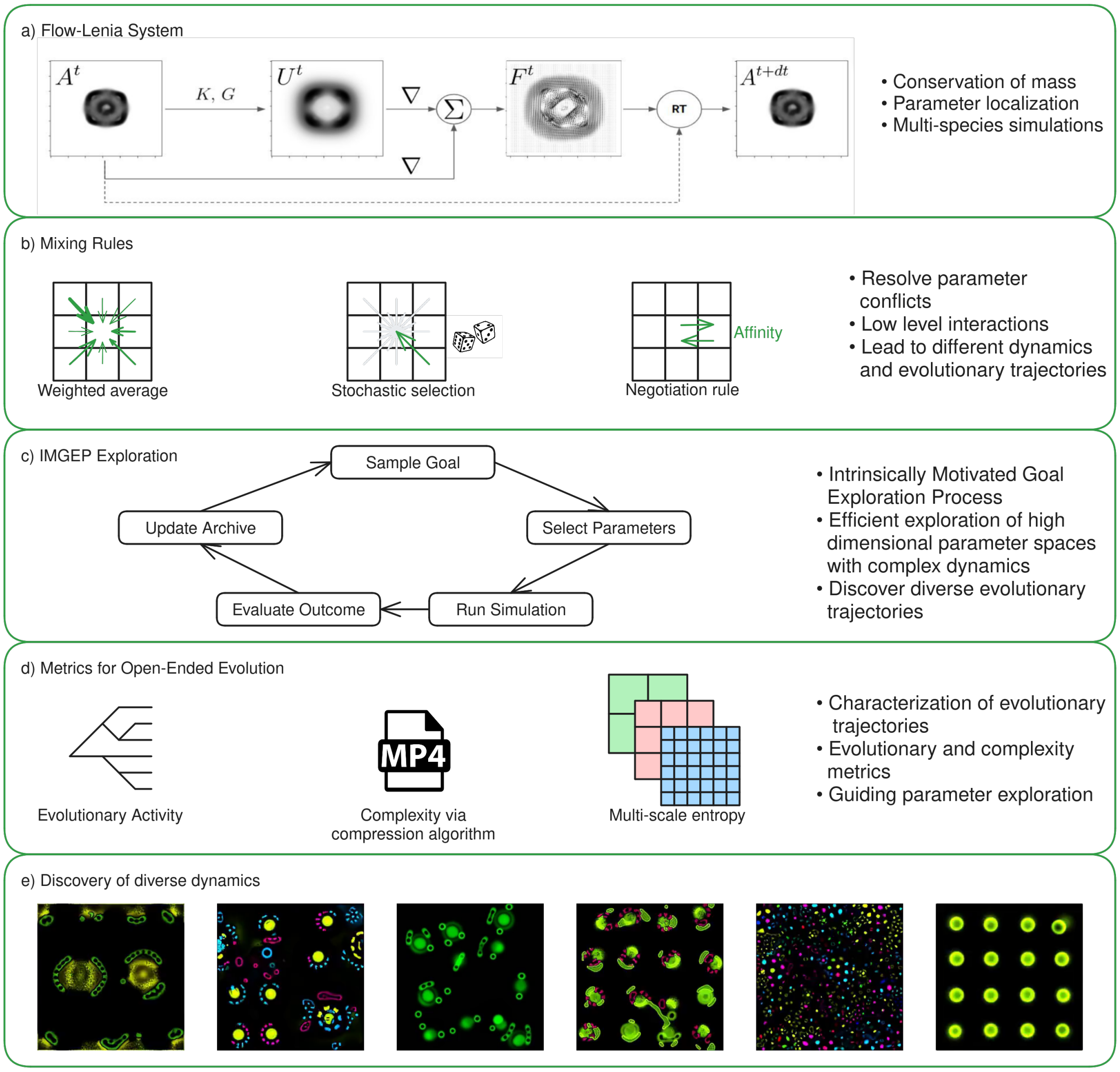


Figure 1: Flow-Lenia system and its exploration for open-ended evolution. (a) Flow-Lenia components and processes. (b) Parameter mixing rules influencing evolutionary dynamics. (c) IMGEP algorithm for efficient parameter space exploration. (d) Metrics for characterizing open-ended evolution: Evolutionary Activity, Complexity, and Multi-scale entropy. (e) Diverse patterns and behaviors discovered through IMGEP exploration. This figure illustrates our approach to studying open-ended evolution in Flow-Lenia, combining advanced exploration techniques with multi-faceted evaluation metrics.

1 Introduction

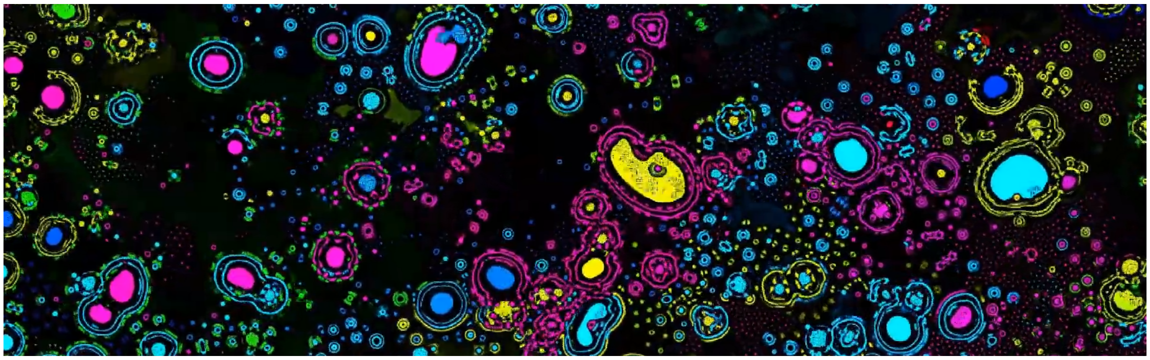


Figure 2: Snapshot of an advanced state of a multi-species Flow-Lenia simulation. Each color represents a different set of localized parameters, leading to distinct behaviors and interactions. The brightness of the cells indicates the matter density, the brighter regions indicate higher density.

Open-ended evolution, defined as the continuous generation of novelty and increasing complexity without a predetermined endpoint, is a fundamental characteristic of biological evolution that has proven challenging to replicate in artificial systems [29]. This phenomenon is characterized by the continuous emergence of novelty, the potential for unlimited growth in complexity, and the absence of a predetermined "goal" or end-state [28]. Understanding and recreating open-ended evolution in computational models is crucial for gaining deeper insights into the fundamental principles governing the emergence of life, the diversification of species, and the evolution of complex behaviors.

Natural evolution on Earth has demonstrated remarkable open-endedness, continuously producing new species, traits, and levels of complexity over billions of years. This process has led to the emergence of increasingly sophisticated organisms, from single-celled prokaryotes to complex multicellular life, and ultimately to the development of intelligence and culture. However, creating artificial systems that exhibit truly open-ended evolution remains a significant challenge, despite its potential to provide valuable tools for exploring critical questions in evolutionary biology, astrobiology, and the origins of life.

Artificial Life (ALife) research aims to recreate the features of open-ended evolution in artificial systems, not only to better understand biological evolution but also to harness the creative power of open-ended processes for technological innovation. Cellular automata (CA) have emerged as a particularly powerful and versatile tool in this pursuit due to their ability to generate complex behaviors from simple rules. Seminal works such as Conway's Game of Life [1] and Langton's self-replicating loops [21] demonstrated how basic rules could lead to complex, emergent behaviors and self-reproduction, respectively.

Recent advancements in continuous CA, particularly Flow-Lenia [25], have expanded the possibilities of ALife research. Flow-Lenia is a system that extends traditional CA by incorporating principles such as mass conservation and parameter localization. In Flow-Lenia, the state of each cell is represented as a continuous value indicating matter density, and the system's behavior is governed by localized parameters that determine how matter flows and interacts as illustrated in Figure 3. This approach creates a framework that more closely mimics certain fundamental properties of biological systems, potentially bringing us closer to realizing open-ended evolution in artificial systems.

However, the complexity of Flow-Lenia presents a significant challenge: how can we discover diverse and interesting behaviors in such a vast and complex system? The parameter space of Flow-Lenia is very large, and random exploration typically leads to a limited diversity of behaviors. This challenge mirrors a broader problem in the field of artificial life and complex systems: efficiently exploring high-dimensional spaces to uncover conditions necessary for open-ended evolution and the emergence of complex, life-like phenomena.

To address this challenge, we propose using advanced diversity search techniques, specifically the Intrinsically Motivated Goal Exploration Processes (IMGEP) framework. IMGEP is an approach for autonomous exploration of complex systems that operates by continuously generating and pursuing self-generated goals. This allows for efficient exploration of high-dimensional spaces without predefined objectives, making it particularly suited to discovering diverse behaviors in systems like Flow-Lenia.

Our work, summarized in Figure 1, aims to bridge principles from Artificial Life with methods from Artificial Intelligence to investigate how complex, life-like behaviors can emerge from the self-organization of local constituents within a Flow-Lenia environment. Specifically, we focus on:

- Developing new mechanisms to bootstrap evolutionary processes within Flow-Lenia, including novel rules for how localized parameters propagate and interact.
- Implementing IMGEP to efficiently explore Flow-Lenia's vast parameter space and uncover diverse evolutionary trajectories.
- Defining and analyzing metrics for measuring evolutionary activity and complexity in Flow-Lenia.
- Examining how environmental constraints affect pattern formation and evolutionary dynamics.

By applying these advanced exploration techniques to Flow-Lenia, we aim to uncover the conditions necessary for open-ended evolution and the emergence of complex, life-like phenomena. This approach has the potential to significantly impact our understanding of evolutionary processes, contribute to the development of more adaptable AI systems, and provide new tools for studying the origins and diversity of life.

1.1 Original Contributions during the Internship

This internship has made multiple original contributions to the study of open-ended evolution in cellular automata:

- Development of a novel "negotiation" mixing rule for Flow-Lenia, which significantly improved the system's evolutionary dynamics.
- Introduction of innovative complexity measures for high-dimensional dynamic systems and continuous cellular automata, including the use of MP4 video encoding size and multi-scale entropy as a proxy for visual complexity.
- First-time application of Intrinsically Motivated Goal Exploration Processes (IMGEP) to Flow-Lenia, enabling the discovery of diverse evolutionary trajectories through large-scale simulations.

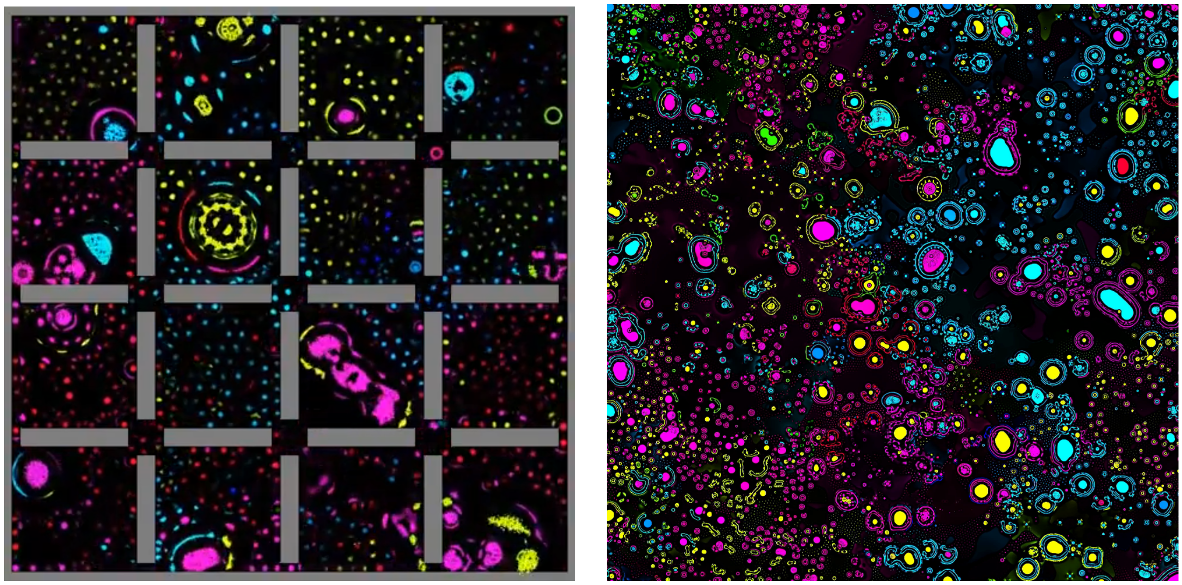


Figure 3: Examples of environments with multiple parameters co-existing and interacting in Flow-Lenia. Gray bars represent walls blocking the flow of matter.

- Creation of an interactive web-based tool for visualizing and exploring the vast dataset generated by the simulations, facilitating intuitive analysis of complex evolutionary dynamics.
- Production of a demonstration video showcasing the discovered dynamics, which was awarded at the Artificial Life Conference (ALife 2024).
- Preliminary work toward discovering emergent proto-cognitive capabilities in Flow-Lenia and extension of the model to include environmental constraints (outside the scope of this report).

2 Related Works

Our research builds upon work in open-ended evolution, artificial life, and intrinsically motivated exploration, particularly in the context of cellular automata (CA) and evolutionary systems.

Our work focuses on Flow-Lenia, an extension of the Lenia cellular automaton system. Lenia, introduced by Chan [9], is a continuous CA that can produce complex, life-like patterns. Flow-Lenia [25] further extends this by incorporating principles of mass conservation and parameter localization, creating a framework that more closely mimics certain properties of biological systems.

The complexity of Flow-Lenia presents a significant challenge: how can we discover diverse and interesting behaviors in such a vast and complex system? This mirrors a broader problem in the field of artificial life and complex systems: efficiently exploring high-dimensional spaces to uncover conditions necessary for open-ended evolution and the emergence of complex, life-like phenomena such as those shown in Figure 4.

To address this challenge, we propose using the Intrinsically Motivated Goal Exploration Processes (IMGEP) framework. IMGEP is an approach for autonomous exploration of complex systems that operates by continuously generating and pursuing self-generated goals. This allows for efficient exploration of high-dimensional spaces without predefined objectives.

2.1 Open-Ended Evolution and Complexity Measures

The challenge of creating artificial systems capable of open-ended evolution—systems that continually produce novel and increasingly complex forms—has been a focus of artificial life research. Bedau et al. [5, 4] proposed quantitative measures for evolutionary activity and complexity, providing tools to characterize open-endedness in evolving systems. These measures, such as cumulative evolutionary activity, help quantify the ongoing production of adaptive innovations in a system.

Soros and Stanley [28] and Taylor [29] identified key conditions and requirements for open-ended evolution, including large state spaces, ecological interactions, and hierarchical organization. These works provide a theoretical framework for designing systems that can exhibit open-ended behaviors, informing our approach to Flow-Lenia.

Metrics for evaluating open-ended systems have evolved from early measures like Langton's λ parameter [20], which quantifies the chaotic behavior of CA rules, and Wolfram's CA classification [31], which categorizes CA behaviors into four classes. More sophisticated measures have since been developed. The MODES toolbox [12] offers metrics for diversity, complexity, and evolvability in evolving systems, providing a comprehensive suite for analyzing evolutionary dynamics.

2.2 Evolutionary Dynamics in Artificial Systems

Understanding evolutionary dynamics in artificial systems is crucial for designing environments conducive to open-ended evolution. Hickinbotham et al. [18] demonstrated that conservation of matter increases evolutionary activity in artificial chemistries, a principle that directly influenced Flow-Lenia's design. This work highlights how physical constraints can drive evolutionary processes.

Salzberg and Sayama [27] explored genetic evolution in self-replicating CA patterns, providing insights into how information can be encoded and propagated in grid-based systems. Taylor [30] investigated ecological feedback in artificial evolution, emphasizing the importance of environmental interactions in driving evolutionary dynamics. These studies inform our approach to parameter embedding and genome propagation in Flow-Lenia.

Cisneros et al. [10, 11] investigated evolving structures in complex systems and developed visualization techniques for large-scale CA. Their work provides methods for identifying and analyzing emergent patterns in vast, complex spaces, which is particularly relevant for understanding the evolutionary dynamics in Flow-Lenia.

2.3 Intrinsically Motivated Learning and Exploration

Intrinsically motivated learning, driven by internal rewards such as novelty or learning progress [24], has emerged as a powerful approach for exploring complex dynamical systems. This methodology enables autonomous exploration without predefined external goals, which is essential for uncovering diverse behaviors in open-ended systems.

These techniques were initially conceived to model curiosity-driven learning in humans and were subsequently applied to developing versatile skill repertoires in AI and robotics. More recently, researchers have adapted these methods to tackle the challenge of discovering diverse patterns or behaviors in complex systems like Flow-Lenia. Their effectiveness is particularly notable in navigating high-dimensional parameter spaces where the relationship between parameters and resulting behaviors is highly non-linear and intricate.

Intrinsically Motivated Goal Exploration Processes (IMGEP) [16] and similar algorithms represent a class of methods designed for efficient exploration of complex simulated systems. These approaches utilize behavior descriptors - quantitative measures that characterize the outcomes of simulations. By mapping simulation results to vectors in a behavior space, these algorithms can represent and analyze diverse system outcomes efficiently.

A key innovation of these methods is their focus on exploring the behavior space rather than directly searching the parameter space. This strategy is crucial because parameter spaces often contain vast regions that yield minimal behavioral variation. In artificial life systems such as Lenia, for instance, numerous parameter configurations result in rapidly disappearing structures. Consequently, a naive uniform exploration of the parameter space, as employed in random search methods, generally proves ineffective.

The exploration process in these algorithms typically involves iterative goal-setting and pursuit. The system selects target points in the behavior space, emphasizing novel objectives that extend slightly beyond previous discoveries. It then attempts to achieve these goals by modifying parameters of previous experiments that exhibited similar behaviors. This self-directed exploration allows the system to generate and pursue its own objectives, facilitating an efficient and diverse exploration of the behavior space.

Reinke et al. [26] demonstrated the efficacy of this approach in uncovering diverse patterns in self-organizing systems. Etcheverry et al. [14] further enhanced this methodology by integrating unsupervised representation learning techniques, leading to the discovery of an even wider range of patterns.

The concept of novelty search, introduced by Lehman and Stanley [22], shares conceptual similarities with these approaches. It prioritizes the exploration of novel behaviors over the optimization of predefined objectives, proving particularly valuable in domains where optimal solutions are unclear or where a diversity of solutions is beneficial.

Recent applications of these techniques in cellular automata and artificial life systems [17, 15] have showcased their potential for revealing diverse behaviors and competencies in complex systems. However, our work marks an advancement by applying these methods to investigate long-term evolutionary dynamics in systems with localized parameter dynamics, such as Flow-Lenia. This novel application opens up new avenues for understanding the emergence of complex, life-like phenomena in computational models.

2.4 Agency, Individuality, and Cognition in Artificial Systems

As we explore the emergence of complex behaviors in Flow-Lenia, considerations of agency, individuality, and cognition become relevant. These concepts help us interpret the patterns and behaviors that emerge in our simulations.

Barandiaran et al. [3] proposed a definition of agency based on individuality, normativity, and asymmetry. This framework provides criteria for identifying agent-like behaviors in artificial systems. While promising, this approach proves challenging to define in the context of Flow-Lenia due to the continuous and highly dynamic nature of the system.

Krakauer et al. [19] developed an information-theoretic framework for quantifying individuality in complex systems. Their approach offers a way to mathematically define and measure the degree of individuality in evolving populations. Biehl et al. [7] proposed a framework for representing agents in dynamical systems based on information-theoretic measures. Both these information-theoretic frameworks seem promising for defining individuality and agency in Flow-Lenia. However, they are challenging to apply in practice due to the high dimensionality and continuous aspect of the system.

Beer's work [6] on autopoiesis and individuality in computational models provides insights into the self-maintenance of patterns within our system. This research explores how simple computational rules can give rise to self-maintaining structures, a key aspect of life-like behavior.

Baluška and Levin's [2] exploration of cognition as a fundamental property of biological systems suggests potential avenues for interpreting complex behaviors in Flow-Lenia. Their work proposes that cognitive processes may be present even in simple biological systems, offering a perspective for understanding emergent behaviors in our simulations.

2.5 Positioning of Our Work

Our research represents an advancement in the exploration of open-ended evolution in artificial systems, particularly in the context of Flow-Lenia. We extend previous work in several key ways:

- We are the first to apply intrinsically motivated exploration techniques, specifically IMGEP, to the discovery of evolutionary processes in complex systems like Flow-Lenia. While previous works have used similar techniques to find interesting patterns, our approach focuses on uncovering and analyzing long-term evolutionary dynamics linked to localized parameter dynamics.
- We develop new mechanisms for parameter localization and propagation within Flow-Lenia, inspired by genetic algorithms and ecological processes, to bootstrap evolutionary dynamics.
- We propose and implement a set of metrics for quantifying evolutionary activity and complexity in Flow-Lenia. Some of these metrics are novel, while others are adaptations of existing measures to suit the specific characteristics of our system. It's important to note that these metrics are not exhaustive, and future work should surely consider incorporating additional measures from other relevant studies to provide a more comprehensive analysis of the system's evolutionary dynamics.
- We investigate the effects of environmental constraints on pattern formation and evolutionary dynamics in Flow-Lenia, bridging concepts from ecology and artificial life.

By combining these elements, our work aims to push forward the understanding of open-ended evolution in artificial systems and provide new tools for exploring the emergence of complex, life-like phenomena in computational models. However, we acknowledge that this is an ongoing area of research, and there is still much to explore in terms of both methodologies and metrics for analyzing such complex systems.

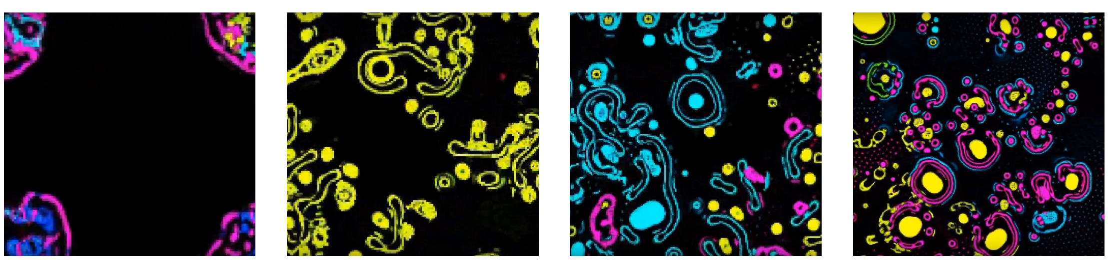
3 What is Flow-Lenia?

Building upon the foundations of cellular automata and Lenia, Flow-Lenia introduces several key innovations that make it particularly suitable for studying open-ended evolution. In this section, we detail the components and mechanisms that define the Flow-Lenia system.

Flow-Lenia [25] extends the Lenia system [9] by incorporating principles of mass conservation. The key difference of Flow-Lenia, summarized in Figure 5, lies in its interpretation of cell values as

(a) Large mass splitting into self-similar smaller patterns

(b) Merging *cells* or *predatory* behavior



(c) Timelapse of a patch of a larger scale simulation. We observe the formation of new types of structures over an extended period of time, hinting at the emergence of an evolutionary process.

Figure 4: Example of dynamics emerging in Flow-Lenia. See our video submission to the Virtual Creature Contest for additional animated examples : https://youtu.be/sSrHoe-iPiU

matter density, rather than abstract activation levels. Then, instead of computing the activation of the next step, the system computes a flow map, a 2D vector field, that will define the *flow* of matter between cells, which brings a new type of dynamic rarely considered in traditional CA. By doing so, Flow-Lenia bridges the gap between cellular automata and particle-based systems, offering a unique platform for studying complex, life-like phenomena. Indeed, mass conservation and flow not only increase the physical realism of the model but also introduces interesting effects, such as competition for resources, that are useful for modeling evolutionary processes.

Another key feature of Flow-Lenia is its capacity for parameter localization. This allows for the embedding of "genetic" information within the system itself, enabling the co-evolution of multiple "species" within a single simulation. This feature opens up new possibilities for studying evolutionary dynamics and the emergence of complex behaviors in artificial life systems.

State and Basic Components The state space of Flow-Lenia is different from Lenia. While Lenia's state space is confined to the unit range $[0, 1]$, Flow-Lenia expands this to $\mathbb{R}^C_{\geqslant 0}$, where C is the number of channels. This extension allows for unbounded positive real values, reflecting the system's interpretation of cell states as matter density.

A Flow-Lenia system is defined by the tuple $\langle K, G, A^0 \rangle$, where:

- $K = \{K_i : \mathcal{L} \to [0, 1] \mid i = 1, \dots, |K|\}$ is a set of convolution kernels, each satisfying $\int_{\mathcal{L}} K_i = 1$.
- $G = \{G_i : [0, 1] \to [-1, 1] \mid i = 1, \dots, |K|\}$ is a set of growth functions.
- $A^0 : \mathcal{L} \to \mathbb{R}^C_{\geqslant 0}$ is the initial state of the system.

Each pair (K_i, G_i) is associated with a source channel c^i_0 it senses and a target channel c^i_1 it updates. This connectivity can be represented by an adjacency matrix $M \in \mathbb{N}^{C \times C}$, where m_{ij} denotes the number of kernels sensing channel i and updating channel j.

The kernels K_i are radially symmetrical, defined as a sum of concentric Gaussian bumps:

$$K_i(x) = \sum_{j=1}^{k} b_{i,j} \exp\left(-\frac{(\frac{x}{r_i R} - a_{i,j})^2}{2w^2_{i,j}} \right) \tag{1}$$

where $a_{i,j}$, $b_{i,j}$, $w_{i,j}$, and r_i are parameters defining kernel i, k is the number of rings per kernel, and R is the maximum neighborhood radius. This formulation allows for a variety of kernel shapes while maintaining radial symmetry, enabling the modeling of complex, multi-scale interactions. The parameters provide a compact way to define these diverse kernels:

- $a_{i,j}$: Controls the distance from the center of each Gaussian bump.
- $b_{i,j}$: Defines the amplitude of each bump, influencing the strength of interactions at different distances.
- $w_{i,j}$: Sets the width of each Gaussian bump, affecting the smoothness of transitions between rings.
- r_i: Scales the overall size of the kernel relative to the maximum radius R.

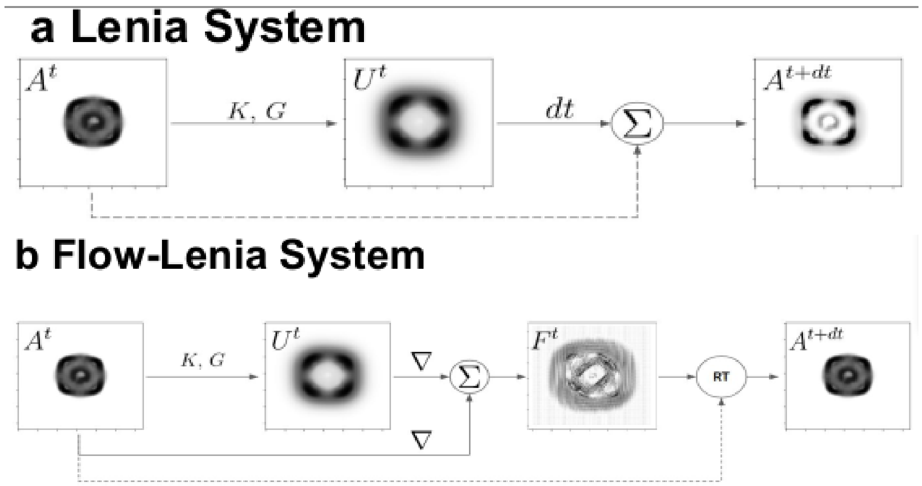


Figure 5: New state computation in Lenia (top) and Flow-Lenia (bottom). In Flow-Lenia, the flow field is computed from the affinity map, and matter is moved according to this field using reintegration tracking. Adapted from [25].

The growth functions G_i are defined as scaled Gaussian functions:

$$G_i(x) = 2\exp\left(-\frac{(\mu_i - x)^2}{2\sigma_i^2}\right) - 1 \tag{2}$$

where μ_i and σ_i are parameters of growth function i.

Affinity Map In Flow-Lenia, the movement of matter is governed by a flow field derived from the affinity map. This approach differs fundamentally from traditional cellular automata, introducing a dynamic reminiscent of fluid dynamics into the system. The affinity map is used to guide the movement of matter across the grid. Higher affinity values indicate regions where matter is likely to accumulate, while lower values suggest areas from which matter may flow away. This is different from the original Lenia implementation in which the results of this computation are directly used to derive the activation of the cells in the next step.

For each channel j, the affinity map U_j^t at time t is defined as:

$$U_j^t(x) = \sum_{i=1}^{|K|} h_i \cdot G_i(K_i * A_{c_0^i}^t)(x) \cdot [c_1^i = j] \tag{3}$$

where:

- $h_i \in \mathbb{R}$ weights the contribution of each kernel-growth function pair
- $K_i * A_{c_0^i}^t$ is the convolution of kernel K_i with its source channel c_0^i
- $[c_1^i = j]$ is the Iverson bracket, equaling 1 when $c_1^i = j$, and 0 otherwise

Flow Equation The flow field $F^t : \mathcal{L} \to (\mathbb{R}^2)^C$ is computed for each channel using a combination of the affinity gradient and a diffusion term:

$$F_i^t = (1 - \alpha^t)\nabla U_i^t - \alpha^t \nabla A_\Sigma^t \tag{4}$$

where:

- ∇U_i^t is the gradient of the affinity map for channel i
- ∇A_Σ^t is the gradient of the total mass, with $A_\Sigma^t(p) = \sum_{i=1}^C A_i^t(p)$
- $\alpha^t : \mathcal{L} \to [0, 1]$ is a weighting function defined as:

$$\alpha^t(p) = \min\left(1, \max\left(0, \left(\frac{A_\Sigma^t(p)}{\theta_A}\right)^n\right)\right) \tag{5}$$

The parameter $\theta_A > 0$ represents a critical mass threshold, and $n > 1$ controls the transition sharpness between affinity-dominated and diffusion-dominated regimes.

This formulation allows matter to flow towards regions of higher affinity while preventing excessive accumulation through the diffusion term. The α^t function ensures a smooth transition between these behaviors based on local mass concentration.

Reintegration Tracking To move matter according to the computed flow field while preserving mass conservation, Flow-Lenia employs a reintegration tracking method. This approach can be viewed as a grid-based approximation of a particle system with an infinite number of particles. Each cell send matter according to the flow field, the matter is then redistributed over the cells of the landing area in a way that ensure mass conservation. This process is illustrated in Figure 6.

The update rule for the state at the next time step is given by:

$$A_i^{t+dt}(x_{\text{dest}}) = \sum_{x_{\text{src}} \in \mathcal{L}} A_i^t(x_{\text{src}}) I_i(x_{\text{src}}, x_{\text{dest}}) \tag{6}$$

where $I_i(x_{\text{src}}, x_{\text{dest}})$ represents the proportion of matter in channel i flowing from source cell x_{src} to destination cell x_{dest}:

$$I_i(x_{\text{src}}, x_{\text{dest}}) = \int_{\Omega(x_{\text{dest}})} \mathcal{D}(x_{\text{flow}}, s) \tag{7}$$

Here, $x_{\text{flow}} = x_{\text{src}} + dt \cdot F_i^t(x_{\text{src}})$ is the target location of the flow from x_{src}, $\Omega(x_{\text{dest}})$ is the domain of the cell at location x_{dest}, and $\mathcal{D}(m, s)$ is a distribution centered at m with variance s, satisfying $\int_{\mathcal{L}} \mathcal{D}(m, s) = 1$.

In practice, $\mathcal{D}$ is implemented as a uniform distribution over a square centered at m with side length $2s$. This choice simplifies computations while still capturing the essential behavior of matter diffusion.

This method ensures that the total mass in the system remains constant over time, a crucial property of Flow-Lenia that distinguishes it from its inspiration, Lenia.

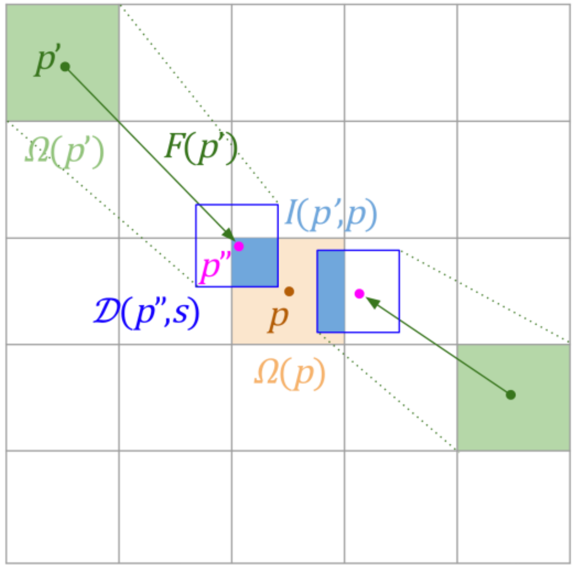


Figure 6: Computing new cell states using reintegration tracking. Matter flows from source cells to destination cells based on the computed flow field. Adapted from [25].

Parameter Embedding for Multi-Species Simulations Flow-Lenia accomodates a parameter embedding mechanism that enables the simulation of multiple "species" within a single environment. This feature attaches a vector of parameters to the matter in each cell, allowing for localized modifications of system behavior. Formally, we define a parameter map $P : \mathcal{L} \to \Theta$, where Θ is the parameter space.

In our implementation, we embed the kernel weight vector $h \in \mathbb{R}^{|K|}$, modifying the affinity map computation:

$$U_j^t(x) = \sum_{i=1}^{|K|} P_i^t(x) \cdot G_i(K_i * A_{c_0^i}^t)(x) \cdot [c_1^i = j] \tag{8}$$

As matter flows across the grid, the associated parameters must also be updated. The method for updating these parameters, referred to as the mixing rule, plays a crucial role in determining the evolutionary dynamics of the system. Various mixing rules can be implemented, each with its own properties and effects on the system's behavior.

Generically, a mixing rule can be expressed as a function $\mathcal{M}$ that determines the new parameters for a cell based on the incoming matter and parameters:

$$P^{t+dt}(x_{\text{dest}}) = \mathcal{M}((A^t(x_{\text{src}}), P^t(x_{\text{src}}), I(x_{\text{src}}, x_{\text{dest}})) \mid x_{\text{src}} \in \mathcal{L}) \tag{9}$$

The specific form of $\mathcal{M}$ can vary widely, from deterministic averages to stochastic sampling methods, each promoting different types of interactions and evolutionary dynamics. We call this function the *mixing rule*.

Mutation operator To further promote diversity and evolution, we implement a mutation mechanism. At a specified rate, we randomly select areas of the grid and apply a multivariate

Gaussian noise to the parameter map within each selected area:

$$P^{t+dt}(x) = P^t(x) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \Sigma) \tag{10}$$

where Σ is the covariance matrix of the multivariate Gaussian distribution, controlling the magnitude and correlations of mutations across different parameters. Importantly, the same noise vector ε is applied to all cells within a selected area, effectively transforming a group of cells belonging to the same "species" into a new, mutated "species". This approach allows for the emergence of spatially coherent mutations, potentially leading to the formation of new patterns or behaviors across contiguous regions of the grid.

Mixing rules The choice of mixing rule significantly influences the system's behavior, affecting aspects such as the stability of patterns, the speed of evolution, and the types of interactions that emerge between different "species". Several mixing rules have been explored in our Flow-Lenia simulations, each with unique properties and effects on system dynamics:

- Weighted Average: Computes the new parameters as an average of incoming parameters, weighted by their associated matter quantities. This rule tends to create smooth transitions but may lead to homogenization over time.
- Softmax Weighted Average: Similar to the weighted average, but uses a softmax function to determine weights. This introduces a temperature parameter that controls the influence of matter quantities on the weighting.
- Stochastic Sampling: Randomly selects one set of incoming parameters, with probabilities proportional to their associated matter quantities. This rule maintains parameter diversity but can lead to abrupt changes in local behavior.
- Stochastic Gene-wise Sampling: Independently samples each parameter (gene) from the incoming sets. This can create novel parameter combinations but may disrupt existing functional patterns.
- Softmax Stochastic Sampling: Uses a softmax function to determine sampling probabilities, introducing a temperature parameter that controls the randomness of selection.
- Dot Product-based Selection: Selects parameters based on their similarity or dissimilarity to other incoming parameters, measured by dot products. This can promote either convergence to similar parameters or maintenance of distinct "species".
- Negotiation Rule: Incorporates both the amount of incoming matter and an "affinity" for each cell to determine parameter selection. This rule aims to balance competition and cooperation between different parameter sets.

Each of these mixing rules offers different trade-offs between maintaining existing patterns, promoting diversity, and allowing for the emergence of new behaviors. The choice of mixing rule significantly impacts the evolutionary dynamics and the types of patterns that emerge in Flow-Lenia simulations.

While each of these rules have interesting or intriguing properties, in the context of exploration of evolutionary processes in Flow-Lenia, we chose to focus on the negotiation rule, which seems one of the most promising in the experiments we performed and which produces visually interesting results, with bigger and more complex patterns than what was previously observed in Flow-Lenia.

Negotiation Rule The negotiation rule is a mixing method that considers both the quantity of incoming matter and its affinity for the destination cell. This rule aims to balance competition and cooperation between different parameter sets, potentially leading to more complex evolutionary dynamics. For a destination cell x_{dest}, the probability of selecting parameters $P^t(x_{\text{src}})$ from a source cell x_{src} is given by:

$$\mathbb{P}\left[P^{t+dt}(x_{\text{dest}}) = P^t(x_{\text{src}})\right] = \frac{e^{\beta A^t(x_{\text{src}}) I(x_{\text{src}}, x_{\text{dest}}) V^t(x_{\text{src}})}}{\sum_{x \in \mathcal{L}} e^{\beta A^t(x) I(x, x_{\text{dest}}) V^t(x)}} \tag{11}$$

where:

- β is an inverse temperature parameter controlling the selectivity of the process
- $A^t(x_{\text{src}})$ is the amount of matter in the source cell x_{src} at time t
- $I(x_{\text{src}}, x_{\text{dest}})$ is the proportion of matter flowing from x_{src} to x_{dest}
- $V^t(x_{\text{src}})$ is an affinity map specifically computed for the mixing step

The affinity map V^t can be computed similarly to the flow affinity map U^t, but potentially using different parameters or kernels:

$$V_j^t(x_{\text{src}}) = \sum_{i=1}^{|K|} Q_i^t(x_{\text{src}}) \cdot G_i(K_i * A_{c_0^i}^t)(x_{\text{src}}) \cdot [c_1^i = j] \tag{12}$$

$$V^t(x_{\text{src}}) = \sum_{j=1}^{C} V_j^t(x_{\text{src}}) \tag{13}$$

where $Q_i^t(x)$ is a separate set of parameters used only for the mixing affinity computation.

This rule allows for complex interactions between different parameter sets, potentially leading to emergent behaviors such as improved defense against external invasions, matter exchange without loss of identity, and specialization for specific local environments. The local environment is the particular distribution of matter in the surrounding cells. Parameters may evolve to perform optimally in areas with certain matter densities or patterns, such as sparse regions, densely populated areas, or regions with specific multi-channel matter distributions. This environmental specialization can potentially lead to the emergence of diverse ecological niches within the Flow-Lenia grid.

4 Exploring Evolutionary Processes with IMGEP

The vast parameter space of Flow-Lenia presents a significant challenge for discovering conditions conducive to open-ended evolution. Random parameter sampling is often inefficient in exploring such high-dimensional spaces, as it tends to miss interesting regions and fails to exploit information gained from previous samples. To address this challenge, we employ the Intrinsically Motivated Goal Exploration Processes (IMGEP) algorithm.

IMGEP is particularly well-suited for our task because:

1. It aims to maximally cover a defined behavior space, which in our case corresponds to diverse evolutionary dynamics.

2. It efficiently explores high-dimensional parameter spaces by setting and pursuing goals in the behavior space, rather than blindly sampling parameters.

3. It leverages information from previous explorations to guide future sampling, increasing the likelihood of discovering novel and interesting behaviors.

The core principle of IMGEP is to iteratively set goals in the behavior space, attempt to achieve these goals by selecting or generating appropriate parameters, and use the outcomes to inform future exploration. This approach allows for a more systematic and efficient exploration of the system's potential behaviors compared to random sampling.

The pursuit of open-ended evolution in artificial systems has long been a central challenge in the field of artificial life. While previous studies using automated discovery and quality-diversity search in cellular automata primarily focused on the diversity and complexity of emergent patterns, our approach aims at exploring the diversity of evolutionary dynamics themselves to uncover the conditions that bootstrap truly open-ended evolutionary processes within the Flow-Lenia framework.

Indeed, we aim to identify parameter configurations that give rise to rich, sustained evolutionary dynamics in multi-species simulations. These dynamics should exhibit key characteristics of open-ended evolution, such as continuous novelty generation, increasing complexity, and the emergence of higher-order structures or behaviors through species interactions.

4.1 IMGEP Framework

IMGEP has proven to be particularly effective for exploring high-dimensional spaces with complex parameter-outcome relationships [16]. Our adaptation of IMGEP for Flow-Lenia exploration is outlined in Algorithm 1 and Figure 7. The framework consists of an exploration space of Flow-Lenia parameters, a goal space defined by evolutionary dynamics metrics, and strategies for goal sampling and parameter selection.

The algorithm iteratively samples goals, selects or mutates parameters based on previous explorations, runs Flow-Lenia simulations, and evaluates outcomes. This process enables systematic identification of parameter configurations that yield interesting evolutionary dynamics. The goal space metrics, which we will discuss in detail in the following subsections, guide the exploration by directing the search towards regions of the parameter space that produce diverse and complex evolutionary behaviors. This approach allows us to efficiently navigate the vast parameter space of Flow-Lenia, focusing on areas that are most likely to exhibit open-ended evolution.

4.2 Parameter Space

The parameter space for IMGEP exploration encompasses the key configurable elements of Flow-Lenia that influence its evolutionary dynamics. We focus primarily on parameters that remain constant throughout a simulation, but in some experiments, we also include initial conditions to explore their impact on long-term evolution.

Our exploration includes:

- Kernel parameters: Shape and size of interaction kernels, controlling local and long-range interactions between cells.

- Growth function parameters: Defining how cell states change based on their neighborhood.

Algorithm 1 IMGEP for Flow-Lenia Exploration

```
Initialize exploration and goal spaces
Initialize empty list of explored states
for each iteration do
    if no explored states then
        Sample random initial state
    else
        Sample random goal
        Find closest explored goal and its state
        Mutate the closest state
    end if
    Set up Flow-Lenia with the chosen state
    Run Flow-Lenia simulation
    Compute achieved goal from simulation results
    Add new state and achieved goal to explored states
    Update exploration summary
end for
```

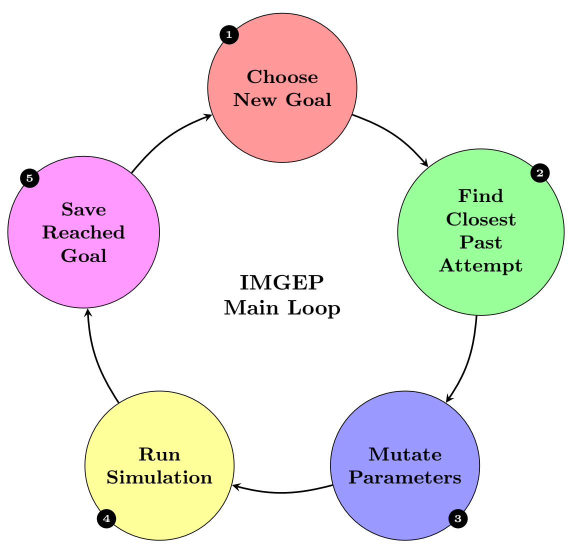


Figure 7: Summary of IMGEP's main loop.

- Mixing rule parameters: Governing how genetic information propagates and combines, crucial for multi-species dynamics.

- Environmental factors: Such as diffusion coefficients and mutation rates, affecting the spread of matter and genetic variation.

- Initial conditions: In select experiments, we explore the impact of initial matter distribution and embedded parameters on evolutionary trajectories.

This approach allows us to investigate both the underlying mechanics that drive evolutionary dynamics and the influence of starting conditions on the system's long-term behavior.

The parameter space defines the possible configurations of our Flow-Lenia system. To guide our exploration of this vast space, we need to define a goal space that captures the key aspects of evolutionary dynamics we aim to study. This goal space, comprising various metrics, allows the IMGEP algorithm to systematically search for parameter configurations that produce interesting and diverse evolutionary behaviors.

4.3 Goal Space

The goal space is designed to capture key aspects of evolutionary dynamics in Flow-Lenia simulations. We define this space using a set of metrics that quantify different characteristics of evolution in the system. The IMGEP algorithm can then explore this space by sampling goals in a vector space in which each component corresponds to a different metric.

4.3.1 Non-neutral Evolutionary Activity

In our exploration of open-ended evolution within Flow-Lenia, we require a metric that can capture the system's capacity for sustained innovation and adaptation without relying on predefined fitness functions. Evolutionary activity measures provide an excellent framework for this purpose, as they quantify the system's ability to generate and maintain novel, adaptive forms over time. Among these measures, we specifically employ the non-neutral evolutionary activity metric, introduced by Droop and Hickinbotham [13], due to its particular suitability for systems with intrinsic fitness, such as Flow-Lenia.

The non-neutral evolutionary activity metric offers several key advantages in our context. First, it allows us to measure evolutionary processes without explicitly specifying fitness criteria, which is crucial in complex systems where fitness is an emergent property rather than a predefined function. Second, it focuses on adaptive changes by considering only increases in component (species) abundance, effectively filtering out neutral drift and highlighting meaningful evolutionary events.

The non-neutral evolutionary activity for a single run is defined as:

$$A = \sum_{i} \sum_{t=1}^{T} \Delta_i(t) \tag{14}$$

where i indexes all components (species or genomes) that existed during the simulation, including extinct ones, T is the total number of time steps, and $\Delta_i(t)$ is the instantaneous activity of component i at time t.

The instantaneous activity $\Delta_i(t)$ is computed as:

$$\Delta_i(t) = \begin{cases} (p_i(t) - p_i(t-1))^2 & \text{if } p_i(t) > p_i(t-1) \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

where $p_i(t)$ represents the proportion of simulated mass attributed to component i at time t.

This formulation captures the entire evolutionary history of the system, including contributions from extinct components, providing a comprehensive view of the evolutionary dynamics. By squaring the positive changes in abundance, the metric emphasizes larger, more significant evolutionary events, aligning with our interest in substantial adaptive changes.

It is important to note that when comparing evolutionary activities between experiments, we maintain consistent simulation lengths and measurement frequencies to ensure valid comparisons.

In our implementation, we define components based on exact parameter (genome) matches to identify distinct species. While this approach provides a clear and computationally efficient method for species identification, it has limitations. Notably, it may not capture the functional similarities between closely related genomes that produce similar phenotypes.

An alternative approach would be to define species based on phenotypic characteristics or behavioral capabilities. However, this presents several challenges in the context of open-ended evolution. First, determining relevant behavioral measures in a controlled environment (without other species present) may not accurately reflect a species' fitness in the complex, dynamic ecosystem of Flow-Lenia. Second, accounting for genetic distances in a high-dimensional parameter space is non-trivial and may not correlate well with phenotypic similarities. Third, the choice of which configuration of matter to use for capability assessment is not straightforward, as species may exhibit different behaviors depending on their form and environment. Finally, using a fixed set of benchmarks to measure capabilities may be at odds with the goal of open-endedness, as it implicitly defines a limited space of possible innovations.

Despite these challenges, future work could explore more sophisticated methods of species identification that balance genetic and phenotypic factors, potentially leading to more nuanced measures of evolutionary activity in open-ended systems like Flow-Lenia.

4.3.2 Complexity Measures

Complementary to evolutionary activity, we employ complexity measures to characterize the richness and intricacy of the patterns and dynamics emerging in Flow-Lenia. Complexity measures are crucial for identifying "interesting" open-ended processes, as they can capture aspects of the system's behavior that are not fully reflected in evolutionary activity alone.

An ideal complexity measure for our purposes should possess several key properties:

- It should be applicable to the system's state, including both the distribution of matter and the associated genetic parameters.
- It should exhibit a balance between order and disorder, vanishing for both highly ordered and completely random states.
- It should be multi-scale, capable of detecting structures at various levels of organization.
- It should capture the preservation and development of structures over time.

While various approaches to measuring complexity exist, including predictability of next state, fractal analysis for images, and information-based measures, we opted for a compression-based metric due to its simplicity and effectiveness in capturing visual complexity.

Specifically, we use the size of the MP4 video file encoding the entire simulation run as a proxy for complexity. This approach is inspired by the concept of Kolmogorov complexity, which relates the complexity of a dataset to the length of its shortest description. In our case, the MP4 compression algorithm serves as an approximation of this minimal description.

The video compression captures both spatial and temporal patterns in the simulation, encoding the distribution of matter and its changes over time. We generate two versions of each video: one showing only the cell activations and another coloring the matter according to the current parameters of each cell. This allows us to capture complexity in both the physical dynamics and the genetic landscape of the system.

This compression-based metric aligns well with our intuition of complexity in cellular automata systems. Importantly, we find that the most interesting rules tend to produce videos of intermediate size, consistent with previous work suggesting that the most complex and interesting systems lie at the edge of chaos, between rigid order and complete randomness [10].

The MP4 file size metric offers several advantages:

- It provides a single, easily comparable value for each simulation run.
- It naturally incorporates both spatial and temporal aspects of complexity.
- It is computationally efficient, as video compression is a well-optimized process.

However, it's important to note some limitations of this approach. The metric is sensitive to the specific video encoding parameters used, and it may not capture all relevant aspects of complexity, particularly those not visually apparent (especially in simulations with more than 3 channels). Additionally, very large or very small file sizes may not necessarily correspond to the most evolutionarily interesting scenarios.

Despite these limitations, we find that the MP4 file size serves as a useful and intuitive proxy for complexity in Flow-Lenia, complementing our evolutionary activity measures and helping to guide our exploration of the system's parameter space towards regions of rich, complex behavior.

4.3.3 Multi-scale Matter Distribution

To capture the spatial organization and dynamics of matter in Flow-Lenia across different scales, we introduce multi-scale entropy measures. This approach allows us to quantify how matter is distributed throughout the environment, from fine-grained local patterns to large-scale global structures. The relation between the entropy at different scales have already been thought as related in previous works in particular with the concept of thermodynamic depth [23] which inspired our use of these related measures.

We compute the distribution of matter in downscaled versions of the environment at various resolutions. For a given state S of the Flow-Lenia system, we define a series of downscaled representations $S^1, S^2, ..., S^n$, where S^1 is the original state and each subsequent S^i is a coarser representation. Formally, for a downscaling factor k:

$$S^i(x, y) = \sum_{p=0}^{k-1} \sum_{q=0}^{k-1} S^{i-1}(kx + p, ky + q) \tag{16}$$

Figure 8: Multi-scale entropy analysis of a Flow-Lenia state. (a) Original state. (b-d) Downscaled representations at increasing coarseness. (e) Entropy values at each scale.

For each downscaled representation, we compute the entropy of the matter distribution. Let $p_i(x, y)$ be the proportion of matter at position (x, y) in the downscaled representation S^i:

$$p_i(x, y) = \frac{S^i(x, y)}{\sum_{x',y'} S^i(x', y')} \tag{17}$$

The entropy H_i for the i-th scale is then:

$$H_i = -\sum_{x,y} p_i(x, y) \log p_i(x, y) \tag{18}$$

This multi-scale entropy analysis provides insights into the distribution of matter at different levels of granularity:

- Low entropy at coarse scales (H_n) indicates global concentration of matter in specific regions of the environment.
- Intermediate entropy at medium scales suggests the presence of localized patterns or clusters.
- High entropy at fine scales (H_1) implies the existence of detailed, distinct patterns at the cellular level.

By comparing entropies across scales, we can characterize the hierarchical organization of matter in the system. For instance, a simulation state with high H_n, intermediate $H_{n/2}$, and low H_1 would indicate a system with a relatively uniform global distribution of matter, localized diffuse clusters, and small areas of high concentration of activity.

Figure 8 illustrates the computation of matter distribution at different scales for a sample Flow-Lenia state, showcasing how the entropy measure captures different aspects of spatial organization as the resolution changes.

This approach provides valuable information about the system's ability to form and maintain structures at various scales, offering insights into the emergence of complex, hierarchical organizations that are characteristic of many biological and ecological systems.

4.3.4 Movement and Interaction

To quantify the dynamic aspects of Flow-Lenia, particularly the movement and interaction of matter across the system, we introduce a metric based on the Wasserstein distance between states. This approach allows us to measure the amount of matter movement in the simulation without relying on the internal mechanisms of the system, making it a robust and generalizable metric.

Instead of tracking the exact movement of matter during the reintegration tracking phase, we approximate the Wasserstein distance between two states. This method provides an estimate of the "cost" of transforming one distribution of matter into another, effectively capturing the overall movement and redistribution of matter in the system. We chose this approach over the exact movements to be independent of the specific internal mechanisms of the simulation, the metric only use the observable states.

To compute this metric efficiently, we employ the sliced Wasserstein method [8], which provides a more computationally tractable approximation of the Wasserstein distance. Formally, for two states S_1 and S_2, we define our movement metric $M(S_1, S_2)$ as:

$$M(S_1, S_2) = SW(P_1, P_2) \tag{19}$$

where SW denotes the sliced Wasserstein distance, and P_1 and P_2 are the probability distributions of matter in states S_1 and S_2, respectively.

We implement this metric in two ways:

- Global movement: We compute a 2D distribution of matter by summing the matter across all channels for each state, then calculate the sliced Wasserstein distance between these aggregate distributions:

$$P_i(x, y) = \frac{\sum_c S_i(x, y, c)}{\sum_{x,y,c} S_i(x, y, c)} \tag{20}$$

- Channel-wise movement: We compute the movement within each channel independently and sum the distances:

$$M_{channel}(S_1, S_2) = \sum_c SW(P_{1,c}, P_{2,c}) \tag{21}$$

  where $P_{i,c}$ is the probability distribution of matter in channel c of state S_i.

This approach offers several advantages:

- It provides a metric related to the dynamics of matter and its movement, complementing the static measures of distribution and complexity.
- It is independent of the internal mechanisms of the simulation, relying only on observable states.
- It can be applied to non-consecutive states, allowing for analysis of movement over various time scales.
- The channel-wise computation allows for insights into the interaction and relative movement of different matter types.

By tracking this movement metric over time, we can identify periods of high activity, detect the formation and dissolution of structures, and quantify the overall dynamism of the system under different parameter configurations. This measure provides valuable insights into the system's capacity for generating and sustaining complex interactions and movements, which are crucial aspects of open-ended evolutionary systems.

Moreover, by comparing the global movement metric with the channel-wise metric, we can infer information about the coordination and interaction between different types of matter in the system. A significant difference between these two measures might indicate complex inter-channel dynamics or the emergence of specialized roles for different matter types.

For the purpose of this study, we mainly focus on the global movement metric in combination with the other evolutionary and complexity measures.

4.4 Challenges and Considerations

While the IMGEP framework offers a powerful approach to exploring open-ended evolution in Flow-Lenia, several challenges and considerations must be addressed:

- Computational Complexity: Simulating Flow-Lenia for extended periods to observe meaningful evolutionary dynamics is computationally intensive. This limits the number of iterations and the duration of individual simulations we can feasibly run, potentially missing long-term emergent behaviors.
- Parameter Sensitivity: The high-dimensional parameter space of Flow-Lenia presents a challenge in identifying which parameters are most crucial for promoting open-ended evolution. Small changes in certain parameters may lead to drastically different outcomes, making the exploration landscape highly rugged.
- Metric Limitations: While our chosen metrics are informative, they may not capture all aspects of open-ended evolution. The non-neutral evolutionary activity and complexity measures may sometimes fail to distinguish between genuinely novel innovations and mere recombinations of existing patterns. Additionally, these metrics may not fully capture qualitative shifts in the nature of evolutionary dynamics, such as the emergence of new levels of organization or fundamentally novel forms of interaction.
- Initial Condition Dependence: The strong influence of initial conditions on evolutionary trajectories makes it difficult to disentangle the effects of system parameters from those of starting configurations. This dependence may lead to false positives or negatives in our assessment of a parameter set's potential for open-ended evolution.

We have adapted our implementation of IMGEP to address these considerations and mitigate their effects, however these are not definitive solutions. These challenges still limit our ability to explore open-ended evolution in Flow-Lenia:

- To manage computational complexity, we decreased the size of the simulations and run multiple simulations in parallel, allowing us to explore a broader range of parameter configurations in a reasonable amount of time.
- To address parameter sensitivity and initial condition dependence, we employ multiple parameter initialization schemes.
- To complement our quantitative metrics and address potential blind spots, we have developed a visualization tool that allow for qualitative analysis of the evolutionary dynamics.

5 Experiments

This section presents our experimental results and analyses, structured as follows:

1. We first examine the impact of various mixing rules (defined in Section 3) on evolutionary activity in Flow-Lenia.
2. We then focus on exploring the parameter space of Flow-Lenia using the Intrinsically Motivated Goal Exploration Processes (IMGEP) algorithm, comparing its performance to random search.

3. Finally, we present visualizations and interpretations of the most interesting evolutionary dynamics discovered through our exploration.

5.1 Mixing Rules and Evolutionary Activity

We conducted a series of experiments to investigate the impact of various mixing rules on the evolutionary process in Flow-Lenia, using non-neutral evolutionary activity (EA) as our primary metric. We also examined the influence of measurement frequency and simulation length on the results.

5.1.1 Experimental Setup

Simulations were run using a 512x512 grid with 3 channels and 45 kernels, executed for 500,000 time steps. We tested multiple mixing rules, including stochastic gene-wise selection, stochastic selection with softmax probabilities, dot product-based selection, weighted average, and the novel negotiation rule introduced in Section 3.

5.1.2 Results

Our experiments revealed significant differences in evolutionary activity across the various mixing rules:

1. The negotiation rule consistently produced the highest evolutionary activity, showing rapid initial growth followed by steady increase (Figure 9).
2. Stochastic gene-wise selection performed second best, exhibiting consistent linear growth over time.
3. Rules based on averaging incoming parameters showed almost no evolutionary activity.
4. The stochastic rule used in previous Flow-Lenia studies exhibited comparatively poor evolutionary activity.

Figure 9 illustrates the evolution of EA over time for different mixing rules, while Figure 10 shows the final EA values across rules.

5.1.3 Influence of Sampling Frequency

We investigated the impact of sampling frequency on the final EA measurements. Our results, summarized in Figure 11, indicate that:

1. Larger sampling intervals result in higher measured evolutionary activity across all mixing rules.
2. The relative performance of different mixing rules remains consistent across sampling frequencies, with some minor changes in ranking.
3. Variability in EA measurements generally increases with larger sampling intervals, particularly for rules like negotiate and stoch_softmax.

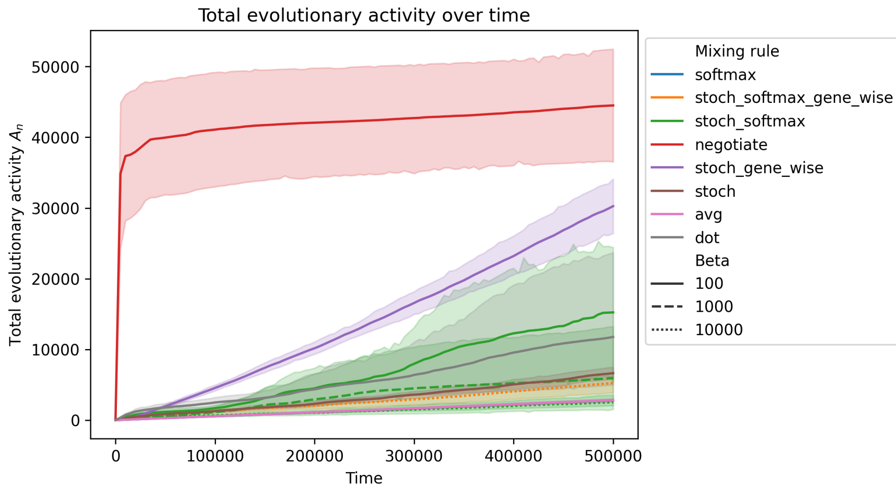


Figure 9: Evolution of evolutionary activity over time for different mixing rules.

5.1.4 Discussion

Our experiments highlight the crucial role of mixing rules in shaping the evolutionary dynamics of Flow-Lenia systems:

- The negotiation rule and stochastic gene-wise selection promote higher evolutionary activity, suggesting that maintaining genetic diversity while allowing for recombination is beneficial for sustained evolution.
- Stochastic rules generally outperform deterministic ones, indicating that probabilistic mixing of genetic information is more promising to evolutionary processes in this system.
- The choice of sampling interval significantly affects the absolute values of measured evolutionary activity, but less so the relative performance of mixing rules.
- The sensitivity of EA measurements to sampling frequency underscores the need for standardization when comparing results across different experimental setups.

While these results provide valuable insights into the evolutionary dynamics of Flow-Lenia systems, they also reveal the limitations of relying solely on evolutionary activity as a metric. The diversity of patterns and behaviors observed in the simulations suggests that evolutionary activity alone does not capture the full richness of the system's dynamics. Indeed, evolutionary activity only accounts for the propagation of specific sets of parameters inside the system and not the spatial dynamics, the complexity, and emergent behavior of the matter itself. This underscores the need for complementary metrics that can evaluate different aspects of open-ended evolution, such as complexity, novelty, and ecological diversity.

Given the superior performance of the negotiation rule in promoting evolutionary activity, the following experiments will focus on this mixing rule while exploring additional metrics to capture a more comprehensive picture of the system's evolutionary potential.

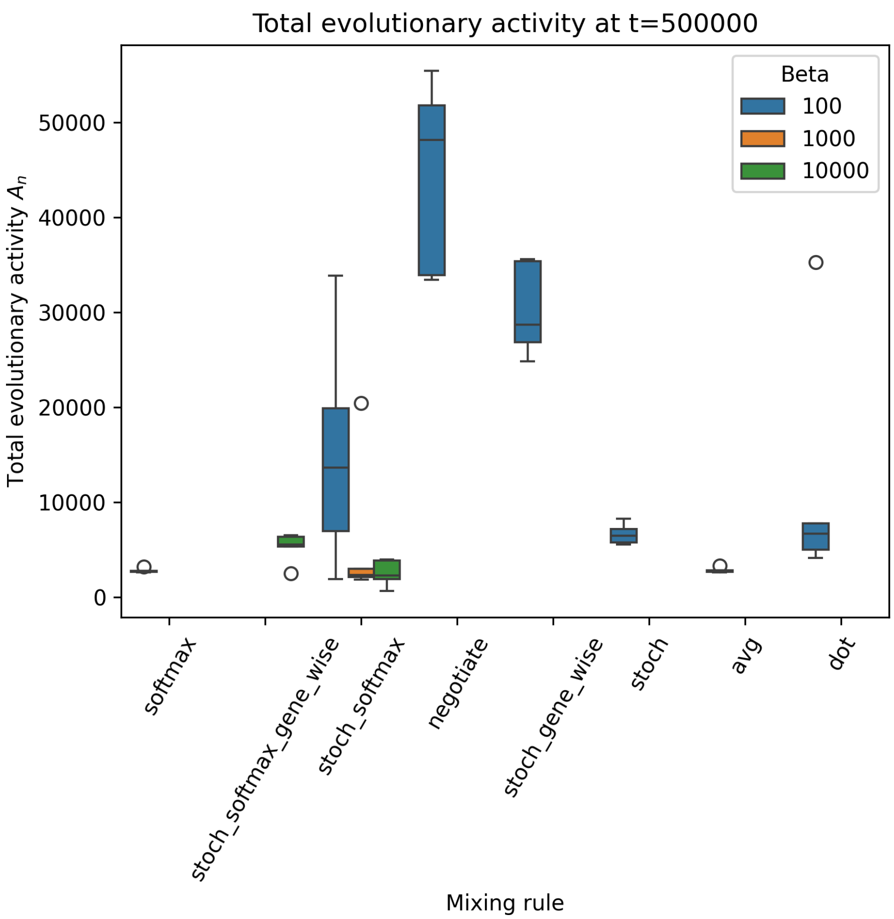


Figure 10: Final evolutionary activity values for different mixing rules and sampling intervals.

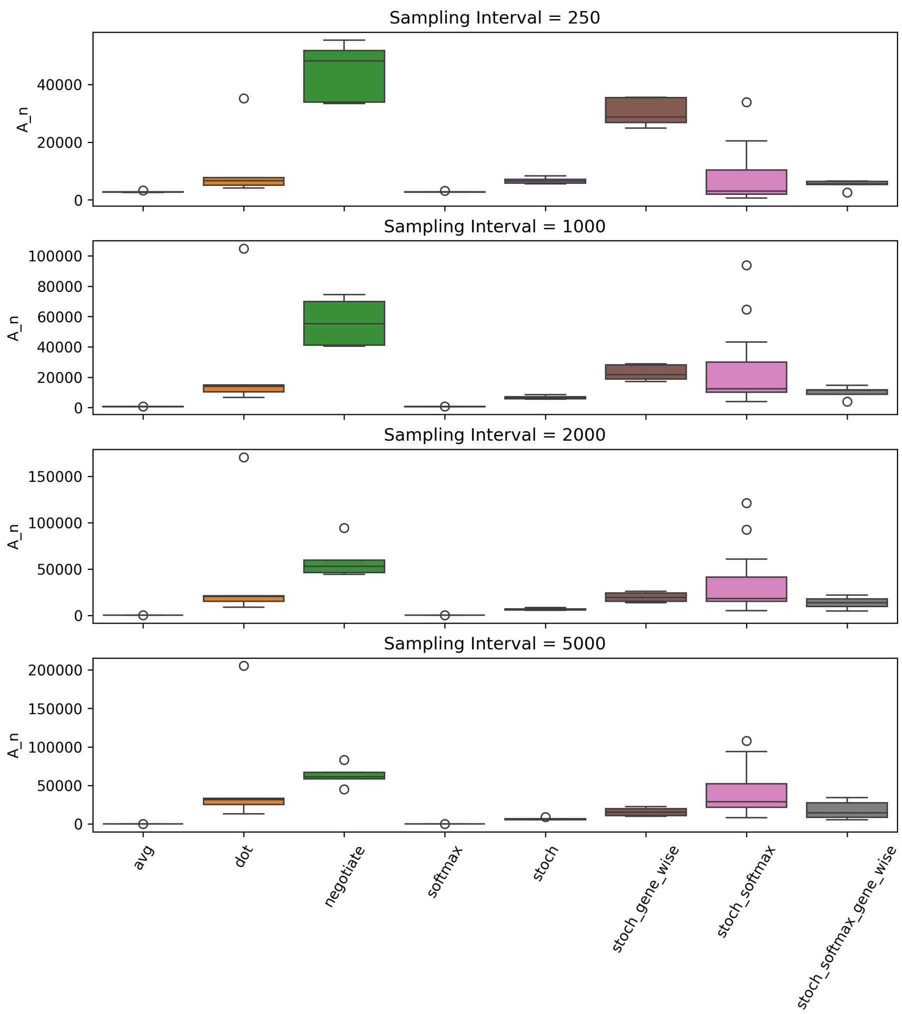


Figure 11: Impact of sampling frequency on evolutionary activity for different mixing rules.

5.2 Exploration of Flow-Lenia Parameter Space Using IMGEP

To systematically explore the vast parameter space of Flow-Lenia and identify conditions for open-ended evolution, we employed the Intrinsically Motivated Goal Exploration Processes (IMGEP) algorithm. This approach allowed us to efficiently sample diverse evolutionary trajectories and analyze their characteristics. We compared the performance of IMGEP to random search to demonstrate its effectiveness in discovering novel and interesting evolutionary dynamics.

5.2.1 Experimental Setup

We adapted the Flow-Lenia system for use with IMGEP, using a 256x256 grid with 3 channels, 15 kernels, and simulations of 10,000 time steps. States were sampled every 100 steps, with 8 parallel trials run on 4 GPUs. This configuration enabled approximately 100 trials per hour on V100 GPUs.

For the random search, we used the same setup but randomly sampled parameters from the same distribution used to initialize IMGEP, without any goal-directed exploration.

5.2.2 Goal Space and Metrics

We defined a multi-dimensional goal space to capture various aspects of the evolutionary dynamics:

- Non-neutral evolutionary activity (EA)
- Compressed video size of the simulation (as a proxy for visual complexity)
- Entropy of the final state at multiple spatial resolutions (4x4 to 64x64 grids)

5.2.3 Results and Analysis

Our results demonstrated that IMGEP effectively explored the goal space, consistently outperforming random exploration in terms of the diversity and range of outcomes achieved (Figure 12).

- IMGEP achieved higher maximum values and broader ranges for evolutionary activity (EA) and complexity (size) compared to random exploration.
- IMGEP explored more extreme values in entropy measures, particularly at lower scales, indicating its ability to discover diverse spatial organizations.
- Strong correlations between adjacent entropy scales suggest potential redundancy in these metrics.

Algorithm	Avg Pairwise Distance	Coverage	std(EA)	std(MP4)	std(H7)	std(H3)
IMGEP	**7.88e-01**	**1387**	**3.11e+03**	**7.76e+05**	**2.46e-01**	**5.43e-01**
Random Search	4.41e-01	678	1.77e+03	6.55e+05	7.65e-02	3.68e-01

Number of trials: 2140

Table 1: Summary of IMGEP and random exploration results across multiple metrics.

The results presented in Table 1 demonstrate the superiority of IMGEP over Random Search across all measured metrics. Over 2140 trials, IMGEP consistently outperformed Random Search,

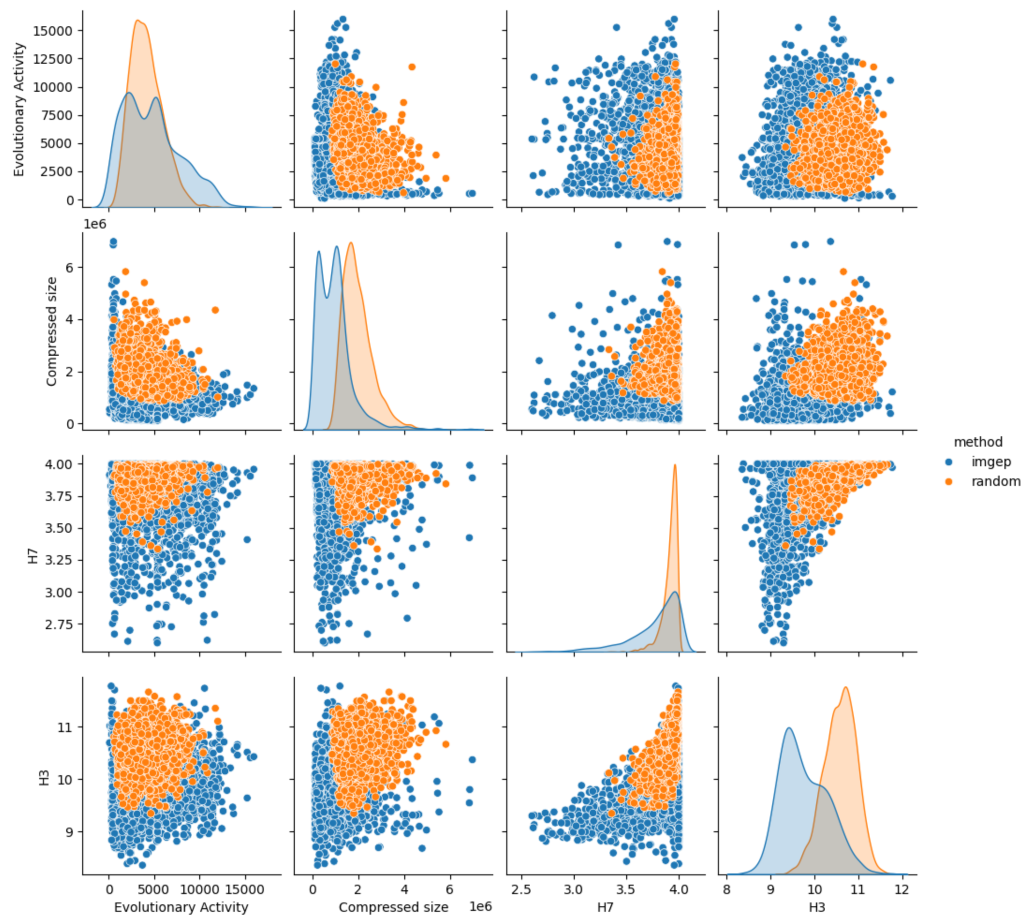


Figure 12: Comparison of goal space coverage between IMGEP and random exploration across multiple metrics. Each subplot shows the distribution of achieved goals for a specific metric. The x-axis represents the metric value, and the y-axis shows the frequency. IMGEP results are in blue, while random search results are in orange.

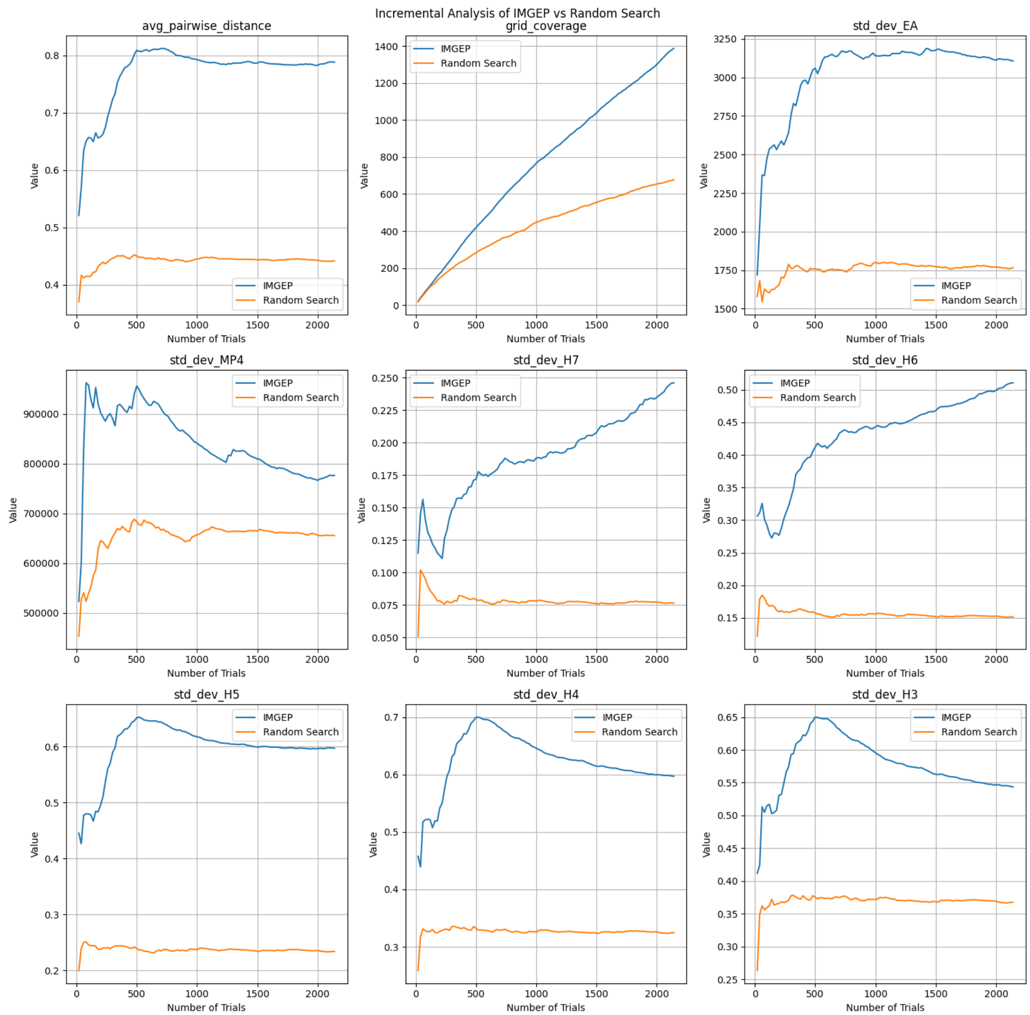


Figure 13: Evolution of the exploration metrics for IMGEP and Random Search over time. Each subplot shows a different metric, with the x-axis representing the number of trials and the y-axis showing the metric value. IMGEP results are in blue, while random search results are in orange.

showcasing its effectiveness in exploring complex goal spaces. Figure 13 shows the evolution of the different metrics over time.

The Average Pairwise Distance metric, computed using the same adjusted distance as the one used by the IMGEP algorithm, shows that the explored goals are generally more spread out over the space compared to the Random Search algorithm, indicating a more comprehensive exploration of the solution space. This is further corroborated by the Coverage metric, computed by discretizing the goal space into cells (10 along each dimension) covering a hypercube containing all the discovered goals, where IMGEP (1387) more than doubles the unique cells explored by Random Search (678).

IMGEP's advantage extends to a higher standard deviation of the explored goals in all specific goal dimensions (EA, MP4, H7, H3). These findings collectively underscore IMGEP's efficacy in guiding exploration towards diverse and potentially useful regions of the solution space. The trend of the coverage metric as well as the standard deviation of coarse-grained entropy seems to indicate that continued exploration using the IMGEP algorithm is likely to further expand the explored portion of the goal space.

5.2.4 Simulation Visualizations

The IMGEP exploration uncovered a wide variety of dynamics within the Flow-Lenia system, ranging from stable, localized patterns to complex, evolving structures and chaotic systems. These diverse behaviors demonstrate the rich potential for open-ended evolution within the Flow-Lenia framework.

Interactive Website To facilitate the exploration and analysis of the discovered dynamics, we developed an interactive website[1]. This tool allows users to visualize and interact with the simulations corresponding to all points in Figure 12, providing a comprehensive view of the diverse behaviors discovered by IMGEP.

Demo Video Our exploration of Flow-Lenia dynamics was also showcased in a demo video that was awarded at the Virtual Creature Contest (VCC) at the Artificial Life Conference[2]. The video highlights some of the most intriguing patterns and behaviors discovered through our exploration of mixing rules, including examples of symbiosis and complex multi-species interactions.

Diverse Dynamics Discovered Figure 16 presents a collection of diverse dynamics discovered by IMGEP. Each image represents a distinct simulation outcome, showcasing the wide variety of patterns and behaviors that our method was able to uncover. Many of these dynamics were not observable through random search, highlighting the effectiveness of our IMGEP-based approach in exploring the rich potential for diverse behaviors in Flow-Lenia.

The online exploration tool played a crucial role in identifying and exporting these interesting dynamics. Users can interact with the tool to explore the full range of simulations discovered by IMGEP, allowing for detailed analysis of the diverse behaviors that emerge under different parameter configurations. This visual exploration complements the quantitative metrics, providing insights into the qualitative aspects of the evolutionary dynamics that may not be fully captured by numerical measures alone.

[1]The interactive exploration tool is available at `http://flowlenia.thomichel.fr`

[2]The VCC webpage can be found at `https://sites.google.com/view/vcc-2024/2024-winner`

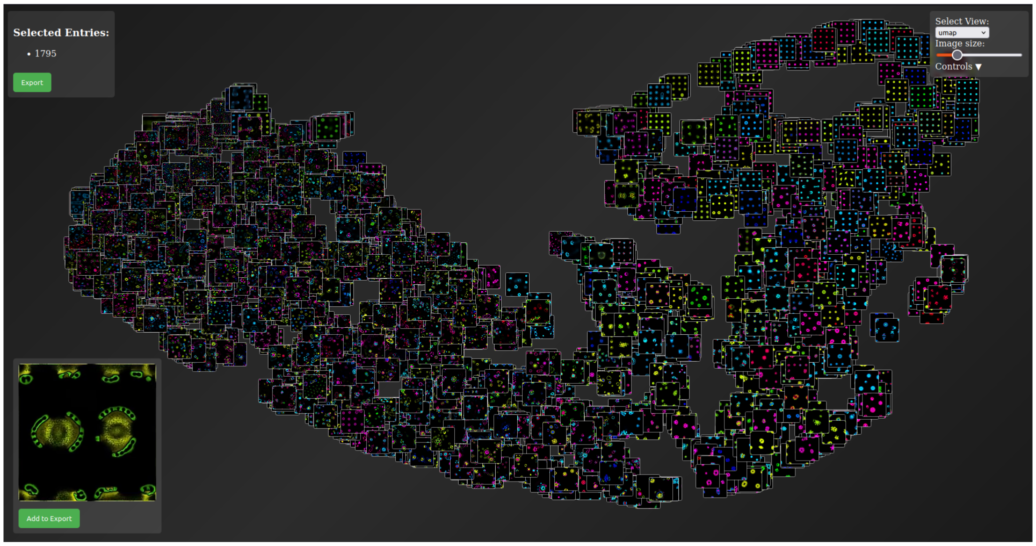


Figure 14: Snapshot of the interactive website for exploring Flow-Lenia dynamics discovered by IMGEP. The interface allows users to select specific simulations based on their metric values and visualize the corresponding evolutionary dynamics.

Figure 15: VCC demo video showcasing interesting Flow-Lenia dynamics, available at https://youtu.be/sSrHoe-iPiU

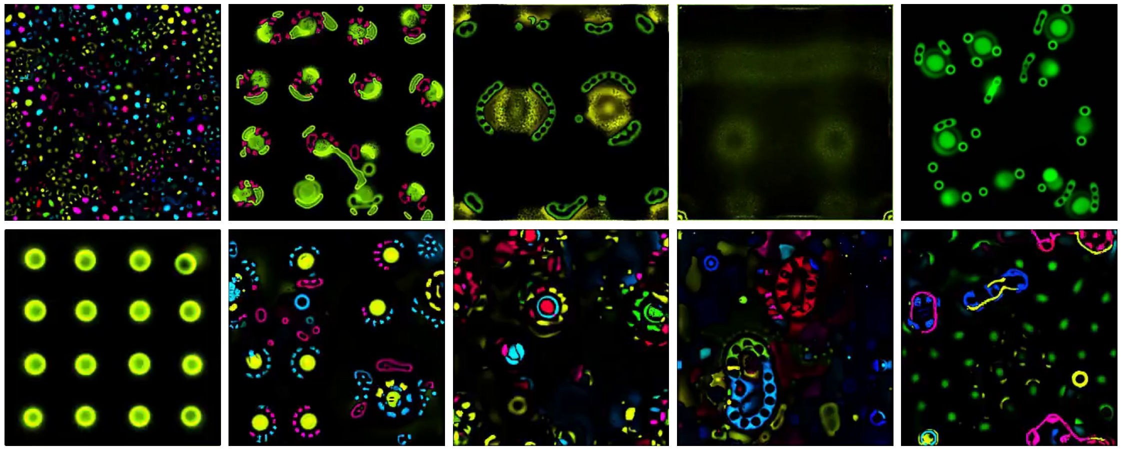


Figure 16: Diverse dynamics discovered by IMGEP. Each image represents a distinct simulation outcome, showcasing various phenomena such as pattern formation, multi-species interactions, and complex spatial organizations.

5.3 Discussion and Future Directions

Our experiments with IMGEP exploration of Flow-Lenia have revealed several important insights and areas for future investigation:

- IMGEP consistently outperforms random search in exploring the diverse dynamics of Flow-Lenia, demonstrating its effectiveness in discovering novel and complex evolutionary behaviors.
- The multi-dimensional goal space, incorporating evolutionary activity, complexity measures, and spatial entropy, provides a more comprehensive view of the system's dynamics than single metrics alone.
- Visual analysis of the discovered dynamics reveals a rich variety of behaviors, including stable patterns, complex interactions, and chaotic systems, many of which were not observed through random exploration.
- The strong performance of the negotiation rule in promoting evolutionary activity suggests its potential for fostering open-ended evolution in cellular automata systems.

However, our findings also point to several areas for refinement and future work:

- The potential impact of initial conditions on evolutionary trajectories warrants further investigation. Future studies could explore how different initial states influence the long-term dynamics and the emergence of complex behaviors.
- There is a need for metrics that better capture the full temporal dynamics of the evolutionary process. While our current metrics provide valuable insights, they often emphasize static distributions of matter or end-state characteristics. Developing measures that can quantify the temporal complexity and novelty of evolving patterns over time could provide a more nuanced understanding of open-ended evolution in Flow-Lenia.

- The observed redundancy in entropy measures across different spatial scales suggests opportunities for a more compact goal space definition. Future work could explore dimensionality reduction techniques or develop new, more orthogonal metrics to create a more efficient and informative goal space.
- Further exploration of the relationship between visually interesting dynamics and quantitative metrics could yield insights into the nature of open-ended evolution. This could involve developing new metrics inspired by human-identified interesting patterns or using machine learning techniques to automatically identify and classify novel behaviors.
- The current study focused on relatively short-term simulations due to computational constraints. Investigating longer-term evolutionary dynamics could reveal emergent behaviors and patterns that develop over extended time scales.
- While we demonstrated the effectiveness of IMGEP in exploring the Flow-Lenia parameter space, future work could compare this approach with other advanced search techniques, such as novelty search or quality diversity algorithms, to further optimize the exploration process.

6 Conclusion

This paper presents an exploratory approach to investigating open-ended evolution within the Flow-Lenia framework, utilizing AI techniques to navigate the complex parameter space of this system. By combining principles from artificial life with modern exploration methods, we have taken initial steps towards understanding some of the conditions that may promote sustained evolutionary dynamics and the emergence of complex behaviors in artificial systems.

Our work has yielded several preliminary findings:

- We developed and analyzed various mixing rules for genome propagation, offering initial insights into how different mechanisms of information transfer might affect evolutionary trajectories.
- We applied the Intrinsically Motivated Goal Exploration Processes (IMGEP) algorithm to explore Flow-Lenia's parameter space, which showed promise in discovering diverse evolutionary dynamics, though much of the space remains unexplored.
- We proposed a multi-faceted metric system for evaluating aspects of open-ended evolution, incorporating measures of evolutionary activity, complexity, and multi-scale matter distribution. While these metrics provide useful indicators, they undoubtedly capture only a fraction of the complexity inherent in open-ended systems.
- We observed some emergent phenomena arising from species interactions within the Flow-Lenia environment, hinting at the potential for complex ecological dynamics in artificial systems, though the full implications of these observations require further study.

These findings represent small but potentially useful steps in advancing our understanding of open-ended evolution in artificial systems. However, it is crucial to acknowledge that we have only scratched the surface of this immensely complex field. The integration of AI exploration techniques with artificial life systems shows promise, but significant challenges remain in uncovering the fundamental principles governing the emergence and sustenance of complex, life-like phenomena.

Our work has highlighted several substantial challenges that persist in creating and studying open-ended evolutionary systems. The high sensitivity to initial conditions, the extensive computational demands of long-term simulations slowing down the exploration, and the inherent difficulties in quantitatively capturing all aspects of open-endedness are just a few of the hurdles that our field continues to grapple with.

Looking forward, there are numerous avenues for future research that could build upon and potentially improve this work. More sophisticated methods of species identification could be explored, additional metrics for assessing cognitive or behavioral complexity could be developed, and investigations into the potential for hierarchical organization and the emergence of higher-order structures within Flow-Lenia could be pursued. Furthermore, the principles and methodologies developed in this study could be tested and refined in other artificial life systems, potentially offering broader insights into the nature of evolution and complexity.

In conclusion, while this research represents a step in our ongoing efforts to understand and recreate aspects of open-ended evolution in artificial systems, we recognize that we are still at the early stages of this journey. By attempting to bridge concepts from artificial life and artificial intelligence, we believe that progress will continue to be made in the exploration of ALife systems. As researchers continue to refine methods and deepen our collective understanding, we may gradually move closer to unraveling some of the mysteries surrounding open-ended evolution, with potential implications for fields ranging from evolutionary biology to artificial intelligence.

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