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# Impact of storm risk on Faustmann rotation

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ABSTRACT

Global warming may induce in Western Europe an increase in storms. Hence the forest managers will have to take into account the risk increase. We study the impact of storm risk at the stand level. From the analytical expressions of the Faustmann criterion and the Expected Long-Run Average Yield, we deduce in presence of storm risk the influence of criteria and of discount rate in terms of optimal thinnings and cutting age. We discuss the validity of using a risk adjusted discount rate (a rate of storm risk added to the discount rate) without risk to mimic the storm risk case in terms of optimal thinnings.



## 1. Introduction

Global warming may induce in Western Europe an increase in storms (Haarsma et al., in press) and also a modification of the distributions governing their frequency and severity. Wind storms will induce high yield losses for forest managers in terms of timber losses and clearing costs (Hanewinkel et al., 2013). Moreover, after a storm, due to an influx of wood on the market, the timber price and hence the future value of forest will decrease. So the forest managers will have to take into account the risk increase and its consequences for stand forest. Therefore they will probably have to modify the rotation period and more generally the silviculture.

In the absence of risk, Faustmann (1849) proposed a formalism, based on the expected discounted income, which allows to determine the optimal rotation period. Many authors have been studying (Clark, 1976; Haight et al., 1992; Kao and Brodie, 1980; Näslund, 1969; Schreuder, 1971) the determination of optimal thinning and cutting age at the stand level. In parallel, empirical studies examined the economic impact on optimal silviculture: Brodie et al. (1978) analyzed the impact of discount rate on optimal thinnings by simulation for Douglas stand, Hyytiainen and Tahvonen (2003) studied the joint influence of the rate of interest and the initial state on the rotation period for Spruce and Scots Pine sites.

The risk of destruction has been introduced for forest stands by Martell (1980) and Routledge (1980) in discrete time. Thereafter, Reed (1984) has studied the optimal forest rotation in continuous time with the risk of fire. Thorsen and Helles (1998) analyzed endogenous risk. More recently concerning natural risk, Staupendahl and Möhring (2011) studied the impact of risk on the expected value of a Spruce stand for various hazard rate functions. Loisel (2011) examined the impact of density dependence growth on optimal cutting age. Price (2011) focused on the validity of using the rate of physical risk, added to the discount rate as a new adjusted discount rate.

More precisely, concerning the risk of storm, Schmidt et al. (2010) studied the impact of storm on the stand forest. Holecy and Hanewinkel (2006) analyzed model insurance. But few works in the literature were focused on the impact of storm risk on forest rotation: Haight et al. (1995) studied the impact of storm on the expected present value without taking thinnings into account. Meilby et al. (2001) focalized their analysis on shelter effect to prevent windthrow in multiple-stand model but they considered an exogeneous land value. Moreover, using empirical material Deegen and Matolepszy (2012) studied the combined effects if storm survival probabilities and site productivities change simultaneously. Susaeta et al. (2012) evaluated the impact of hurricane-related production risk in Pine plantations using a generalized Reed model. In all these numerical studies concerning the storm risk, thinnings are either fixed or not taken into account. There is a lack of studies permitting the analysis and the prediction of the modification induced by storm risk on optimal thinnings and cutting age. In contrast with the previous studies, our analysis is generic and based on the analytic expressions of the criteria. More precisely, it uses the relative contribution of thinnings income in case of storm risk. In this paper, the analytical nature of the proposed methology is a novelty in contrast with the previous works available in the literature, which were based on empirical/numerical techniques.

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In the present work, we model the impact of storm risk on optimal silviculture at the stand level. We consider two criteria which permit to evaluate scenario with various thinnings and cutting age: the Expected Discounted Value (the Faustmann criterion) and the Expected Long-Run Average Yield. We optimize these criteria in presence of storm risk. We consider an adaptation of the model of Reed (1984) toward a forest stand to take into account thinnings for the specific storm risk case. Moreover, the depreciation of timber price due to an influx of wood on the market after a storm is considered. The goal is to determine the joined influence of the presence of storm risk and of the chosen maximized criteria on optimal thinnings and rotation period.

In the first section, we consider the reference case without the storm risk, which will be used for comparison. In the second section, we present the different models in case of storm risk: the storm risk process, the impact of the storm on the stand forest, the timber price depreciation. Assuming we know the expected thinning incomes, final income at cutting age and the various costs link to storm risk, we develop the obtaining of analytical expressions of the Faustmann Value and the Expected Long-Run Averaged Yield in the presence of storm risk taking the depreciation of timber price reference into account. From these analytic expressions, we highlight in presence of storm risk the influence of criteria and of discount rate in terms of optimal thinnings and cutting age. Even if, the influence of the discount rate or the level of risk has been previously observed empirically in specific forest stands, the originality of our work is to provide an explanation of generic properties: the obtained results do not depend on the species or the forest growth. Moreover we discuss the validity of using a risk adjusted discount rate (a rate of storm risk added to the discount rate) without risk to mimic the storm risk case in terms of optimal thinnings. Due to the fact that, if a storm occurs at a date anterior to a fixed time limit, the storm has no impact, we infer that the time limit is an important threshold. The relative positions in time of the optimal thinning without risk and of the time limit allow to deduce the behavior of the optimal thinning with respect to the risk. Finally we illustrate and confirm the obtained conjectures by considering a beech stand.

## 2. In absence of storm risk

We first consider a forest stand in the absence of storm risk. The analysis of this case will allow us to give the notations and to define a benchmark management of the stand, useful for comparison with the more complex case in presence of storm risk.

### 2.1. The Faustmann Value

For a cutting age $T$, a discount rate $\delta$, the Faustmann Value $J_0$ taking into account thinning incomes (up to the constant cost of regeneration $c_1$) of a stand is the discounted value of cutting incomes minus cost of regeneration $c_1$:

$$J_0 = \sum_{i=1}^{+\infty} (W(0,T) - c_1) e^{-i\delta T} = \frac{(W(0,T) - c_1) e^{-\delta T}}{1 - e^{-\delta T}} = \frac{W(0,T) - c_1}{e^{\delta T} - 1} \tag{1}$$

where $W(0,T)$ is the total income on $[0,T]$ composed of the sum of thinning incomes summed on $[0,T]$ actualized at time $T$ and the final income.

We express the thinning income and the final income. Let $N$ the number of thinning dates, let the thinning dates $(u_k)_{k=1..N}$ such that $0 < u_1 < u_2 < .. < u_N < T$ and $h_k$ the vector of the corresponding rate of thinnings at these dates. Let a timber price reference $p_0$, which can depend on the economic conditions, $R(p_0,t)$ the vector of potential income at time $t$. We denote $R_k = R(p_0,u_k)$. Hence $\mathcal{H}_0(t_1,t_2)$ the thinning income on period $[t_1,t_2]$ actualized to time $t_2$ is given by: $\mathcal{H}_0(t_1,t_2) = \sum_{t_1 < u_k \le t_2}^{N} {}^{\mathbf{t}}\mathbf{R_k}.\mathbf{h_k} e^{\delta(t_2 - u_k)}$ and the final income: $V(T) = {}^{\mathbf{t}}\mathbf{R}(\mathbf{p_0}, \mathbf{T}).1$. Thus the total income is:

$$W(0,T) = \mathcal{H}_0(0,T) + V(T) = \sum_{k=1}^{N} {}^{\mathbf{t}}\mathbf{R_k}.\mathbf{h_k} e^{\delta(T-u_k)} + V(T). \tag{2}$$

The expression of income $R(p_0,t)$ depends on the type of the model chosen for forest growth: $R(p_0,t)$. For a stand model Clark (1976) or an average tree model, $h_k$ is reduced to scalar. For a size-structured model or a tree-individualized model, the components of $h_k$ are related to the tree-sizes, classically the tree-basal area. We assume that income $R$ is linear in its first argument.

### 2.2. The Long Run Average Yield

The Long-Run Average Yield is defined by the ratio of the averaged net economic return (stumpage minus regeneration cost) and the cutting age:

$$\overline{Y}_0 = \frac{W_0(0,T) - c_1}{T} \quad \text{wher } W_0(0,T) = \sum_{k=1}^{N} {}^{\mathbf{t}}\mathbf{R_k}.\mathbf{h_k} + V(T). \tag{3}$$

### 2.3. Maximization of the Faustmann Value

We consider the maximization of the Faustmann Value with respect to thinnings and cutting age: $\max_{(h_k)_k,(u_k)_k,T} J_0$. For fixed cutting age $T$, the maximization of the Faustmann Value $J_0$ is equivalent to the maximization of $W(0,T)$. Hence the maximization with respect to thinnings and cutting age can be decomposed into two levels: $\max_T [W^*(0,T) - c_1]/(e^{\delta T} - 1)$ where $W^*(0,T)$ is the maximum of $W(0,T)$ with respect to thinnings: $\max_{(h_k)_k,(u_k)_k} W(0,T)$.

For a fixed cutting age $T$, the behavior of the coefficient relative to the thinning income $\beta_{\delta,0}^k = e^{\delta(T-u_k)}$ in the income $W(0,T)$ be studied in order to deduce the influence of the discount rate in terms of optimal thinnings.

#### 2.3.1. Dependence of optimal thinning with respect to the discount rate

The derivative of $\beta_{\delta,0}^k$ with respect to the discount rate $\delta$ normalized by $\beta_{0,0}^k$ is:

$$\frac{1}{\beta_{0,0}^k} \frac{\partial \beta_{\delta,0}^k}{\partial \delta} |_{\delta=0} = T - u_k \tag{4}$$

We deduce that the relative additional contribution of $\beta_{\delta,0}^k$ decreases as $k$ increases. Hence for a fixed cutting age $T$, the greater the discount rate $\delta$, the earlier the optimal thinnings. Moreover, the optimal cutting age decreases with respect to the discount rate (Appendix A). Hence, also for the optimal cutting age, by considering a fixed final tree-density and a fixed number of thinning dates (only thinning dates can be changed), the greater the discount rate $\delta$, the earlier the optimal thinnings.

This result is not surprising and well known: the greater the discount rate, the lower the actualized income for the last thinnings. In earlier works such as in Brodie et al. (1978), a similar result has been checked by simulation for specific Douglas stand. In contrast to this study, the obtained result is generic and does not depend on the forest growth.

## 3. In the presence of storm risk

In order to deduce the behavior of a forest in presence of storm risk, we present the risk model, then obtain analytical expressions of the criteria that permit a comparison of the optimal silviculture without and with risk.

### 3.1. The storm risk modeling

The storm risk modeling is based on various models which interact: the model of the risk of storm, the model of the impact of a storm on a stand forest and finally the model of timber price depreciation following a storm.

#### 3.1.1. Model of the storm risk

As in Reed (1984), we assume that storms occur in a Poisson process i.e. that storms occur independently of one another, and randomly in time. Thus the distribution of the times between successive storms $\tau$ is an exponential with mean $1/\lambda$: $F(x) = 1 - e^{-\lambda x}$ where $\lambda$ is the expected number of storms per unit time. The severity of the storm is given by the random variable $\mathcal{A}$. The storm risk is described by the couple of random variables $(\tau, \mathcal{A})$.

#### 3.1.2. Model of the impact of a storm on the stand forest

The consequences of storm are characterized by the fact that the impacts are very differentiated according to the stand age at which the storm occurs. The storms have a low impact on young stands (Schmidt et al., 2010): for a tree-height $H$ less or equal than $H_L$, there is no damage for the trees. The height $H_L$ is reached at a time $t_L$, the time $t_L$ explicitly depends on the height limit $H_L$ and can be deduced using the time dependent function $H$: $H(t_L) = H_L$. Therefore, in case of storm at time $\tau \le t_L$ the storm has no impact, for a storm occuring at time $\tau > t_L$ the proportion of damaged trees $\theta_t$ is positively correlated with the severity $\mathcal{A}$ of the storm. Moreover, following Schmidt et al. (2010), the proportion of damaged trees $\theta_t$ depends on the state of the stand at the date $\tau$ the storm occurs: height $H(\tau)$, tree-diameter $d(\tau)$. Let $\alpha(t)$ the expectation of the proportion of surviving trees $1 - \theta_t$. It is possible to integrate specific behavior for $\theta$ hence for $\alpha(\tau)$, such as the dependence on the height $H(\tau)$ and the $H(\tau)/d(\tau)$ ratio or the time spent since last thinning, relative thinning volume in the latest thinning (Lohmander and Helles, 1987). Tree damages increase with respect to the $H/d$ ratio. In order to prevent tree-damages, forest managers make earlier thinnings.

#### 3.1.3. Model of the timber price depreciation

Due to a timber influx on the market following the storm, we observe a variable timber price depreciation. $\xi_t$ the rate depreciation of timber price is positively correlated with the severity $\mathcal{A}$. Hence, generally $\theta$ and $\xi$ are positively correlated random variables. We denote $\alpha_d$ the expectation of $\xi$. We also denote $\alpha_p(t)$ the expectation of $(1 - \theta_t)(1 - \xi_t)$. To simplify, we assume here $\xi$ constant, so $\xi \equiv \alpha_d$, hence: $\alpha_p(t) = (1 - \alpha_d)\alpha(t)$. We take into account a lasting impact of the last storm, occuring before time $t_L$, on the timber price. Let $\tau_t^- < t_L$ the date of the last storm before $t$ and $\rho_{\tau_t^-}(t)$ the rate depreciation of timber price at time $t$. The timber price reference $p_0$ in the case without risk is then replaced by $p_t = \left(1 - \rho_{\tau_t^-}(t)\right)p_0$ in the presence of storm risk. The rate depreciation $\rho_{\tau_t^-}(t)$ decreases with respect to time $t$ and only depends on the elapsed time since the last storm: $t - \tau_t^-$, hence it is sufficient to define $\rho_0(t)$ and then deduce $\rho_{\tau_t^-}(t) = \rho_0\left(t - \tau_t^-\right)$. $\rho_0(.)$ decreases with respect to the elapsed time since the last storm with $\rho_0(0) = \alpha_d$. The support of $\rho_0(.)$ is $[0, \Delta]$ where $\Delta$ is the perturbation time of impact for the rate depreciation of the timber price. Finally, if the last storm before $t$ occurs at time $\tau_t^-$, the timber price reference $p_t$ is given by:

$$p_t = (1 - \rho_0(t - \tau_t^-))p_0 \tag{5}$$

For example, with a linear increasing of the timber price: $\rho_0(t) = \alpha_d (1 - \frac{t}{\Delta})$ for $0 \le t < \Delta$. We notice that the date of the last storm $\tau_t^-$ is a random variable, hence the rate depreciation and the reference price are random variables.

### 3.2. The Faustmann Value

Let $\tau$ the spending time between the beginning of the stand and the first event of the stand after $t_L$, either by storm or by logging at time $T$. The distribution of the spending time $\tau$ is:

$$F_\tau(t) = F(t - t_L) = 1 - e^{-\lambda(t - t_L)} \quad \text{for} \quad t_L < t \le T. \tag{6}$$

We consider an adaptation of the simplified scenario of Reed (1984) to our case of storm risk:

- if a storm occurs at time $\tau \le t_L$, the storm has no impact, the stand continues to grow.
- if a storm occurs at time $\tau$ with $t_L < \tau \le T$, the proportion of damaged trees $\theta_t$ is non neglectable, a clearcutting and a regeneration of the stand is made at time $\tau$.
- if no storm occurs before time $T$, a clearcutting and a regeneration of the stand is made at time $T$.

We assume that perturbation time $\Delta$ is sufficiently small such that $\Delta < \min(t_L, T - t_L)$ and the timber depreciation have no impact on the thinning incomes before time $\Delta$: for small tree-diameter the conjuncture does not modify timber price. These conditions are commonly checked and avoid a link (in terms of the price reference) between successive rotations.

Let $\tau_i$ the spending time between the beginning of the stand and the first event of the stand after $t_L$, either by storm or by logging at time $T$ for the $i$th rotation. The net economic return $\mathcal{Y}_i$ (thinning incomes, final income minus costs) actualized at final time $\tau_i$ is given by:

$$\mathcal{Y}_i = \begin{cases} \mathcal{H}(0, \tau_i, \sigma) + \mathcal{V}(\theta_\tau, \xi_\tau, \tau_i) - c_1 - C_n(\theta_\tau, \tau_i) & \text{if } t_L < \tau_i < T \\ \mathcal{H}(0, T, \sigma) + V(T) - c_1 & \text{if } \tau_i = T \end{cases} \tag{7}$$

where $\mathcal{H}(0, \tau_i, \sigma)$ is the thinning income (taking into account the previous storm $\sigma$, see Appendix B.1), $\mathcal{V}(\theta_\tau, \xi_\tau, \tau)$ is the final income, $C_n(\theta_\tau, \tau)$ are the clearing costs for a storm occuring at time $\tau < T$, composed of a constant cost $c_0$ and variable costs $C_v(\theta_\tau, \tau)$ depending on the volume of damaged trees.

The major differences with the model of Reed (1984) are located in the expression of the net economic return $\mathcal{Y}$, more precisely in the thinning income and the final income in case of storm. In agreement with what Reed (1984) rightly provided, if we do not take into account thinning incomes, the salvageable income in case of event is proportional to the total income in the absence of event. At the opposite in this paper, it is important to note that thinning incomes are included in total income. Thus, even if the final income in case of storm is proportional to the final income in the absence of risk, due the contribution of the thinning income, the total income in case of storm may not be proportional to the total income in the absence of storm.

As the storm occurs independently of one another and randomly in time, we deduce the following expression for the Faustmann Value:

$$J_\lambda = E\left(\sum_{i=1}^{\infty} e^{-\delta(\tau_1 + \tau_2 + \ldots + \tau_i)} \mathcal{Y}_i\right) = \sum_{i=1}^{\infty} \prod_{j=1}^{i-1} E\left(e^{-\delta \tau_j}\right) . E\left(e^{-\delta \tau_i} \mathcal{Y}_i\right) = \frac{E\left(e^{-\delta \tau} \mathcal{Y}\right)}{1 - E\left(e^{-\delta \tau}\right)}. \tag{8}$$

Hence, the Faustmann Value is the expected value of the actualized sum at the initial time of the net economic return $\mathcal{Y}$ and the Faustmann

Value:

$$J_\lambda = E\left[e^{-\delta\tau}(\mathcal{Y} + J_\lambda)\right]. \tag{9}$$

This recursive formulation shows that, contrary to Meilby et al. (2001), the land value is endogenous in our framework. Moreover this formulation can be used to facilitate the obtention of the Faustmann Value for more complex scenario.

Let $E_\mathcal{V}(\tau) = E[\mathcal{V}(\theta_\tau, \xi_\tau, \tau)]$ the expected final income with respect to $\theta_\tau$ and $\xi_\tau$ and $E_{C_v}(\tau) = E(C_v(\theta_\tau, \tau))$ the expected variable clearing costs in case of a storm at time $\tau, E_{R_k} = E\left[R\left(p_{u_k}, u_k\right)\right]$ is the expected potential thinning income at time $u_k$. Then, from Eq. (8), we can deduce the Faustmann Value (Appendix B.2–B.4):

$$J_\lambda = \frac{E_W(0,T) - c_1 - (c_0 + c_1)a(T)}{b(T)} \tag{10}$$

where $(\delta + \lambda)a(T) = \lambda\left(e^{(\delta+\lambda)(T-t_L)} - 1\right), b(T) = e^{(\delta+\lambda)T-\lambda t_L} - a(T) - 1$ and $E_W(0,T)$ is the following modified expected income:

$$E_W(0,T) = \sum_{k=1}^{N} \beta^k_{\delta,\lambda}\ {}^{\mathbf{t}}\mathbf{E}_{\mathbf{R_k}}.\mathbf{h_k} + \lambda \int_{t_L}^{T} \left[E_\mathcal{V}(\tau) - E_{C_v}(\tau)\right] e^{(\delta+\lambda)(T-\tau)} d\tau + V(T) \tag{11}$$

with $\beta^k_{\delta,\lambda} = e^{(\delta+\lambda)(T-u_k)+\lambda(u_k-t_L)_-}$ the coefficient relative to the thinning income at time $u_k$.

As in the case without risk, the Faustmann Value can be deduced from an income $E_W(0,T)$. $E_W(0,T)$ is a modified linear combination of thinning incomes and final income.

The major differences are the following: first the discount rate $\delta$ is replaced by the addition of the discount rate and expected number of storms $\delta + \lambda$ (as previously noticed (Reed, 1984) for the case without thinnings), secondly the total income $W(0,T)$ is replaced by the modified expected income $E_W(0,T)$. More precisely, the thinning incomes $R_k$ are replaced by the expected thinning incomes $E_{R_k}$, the coefficients of the thinning income $\beta^k_{\delta,0}$ are replaced by $\beta^k_{\delta,\lambda}$ and finally an integral relative to the final cutting, minus clearing costs in case of a storm before the cutting age $T$ is added. We notice that the coefficients of the thinning incomes $\beta^k$ only depend on risk and economic parameters but are independent of the forest stand.

Moreover, if we assume that the potential income $R(p,t)$ is linear in its first argument $p$, we deduce (see Appendix B.5): $E_{R_k} = R((1 - E_{\rho_k})p_0, u_k)$ where $E_{\rho_k}$ is the expectation of $\rho_{\sigma_k}(u_k)$ with respect to the date of the last storm $\sigma_k$ before $u_k$. In this case, the impact of risk of storm results in the substitution of the reference price $p_0$ by the modified value $(1 - E_{\rho_k})p_0$ for thinning time $u_k$.

The influence of forest growth and storm damage appears in $E_{R_k}, E_\mathcal{V}(\tau), E_{C_v}(\tau)$ and $V(T)$. This structure does not depend on the tree species.

### 3.3. The Expected Long-Run Average Yield

The Expected Long-Run Average Yield is the expected net economic return per unit of time $\left(\lim_{t\to\infty} E\left(\sum_{i=1}^{N_t} \mathcal{Y}_i\right)/t\right.$ where $N_t$ is the number of rotations before time $t$). Following the reasoning of Reed (1984), the Expected Long-Run Average Yield is equal to the ratio of the averaged net economic return (stumpage minus regeneration and the clearing costs in presence of risk) and the averaged effective cutting age: $\overline{Y}_\lambda = E(\mathcal{Y})/E(\tau)$, hence:

$$\overline{Y}_\lambda = \lambda \frac{E_{W_0}(0,T) - c_1 - (c_0 + c_1)\left(e^{\lambda(T-t_L)} - 1\right)}{(1 + \lambda t_L)e^{\lambda(T-t_L)} - 1} \tag{12}$$

where:

$$E_{W_0}(0,T) = \sum_{k=1}^{N} \beta^k_{0,\lambda}\ {}^{\mathbf{t}}\mathbf{E}_{\mathbf{R_k}}.\mathbf{h_k} + \lambda \int_{t_L}^{T} \left[E_\mathcal{V}(\tau) - E_{C_v}(\tau)\right] e^{\lambda(T-\tau)} d\tau + V(T).$$

### 3.4. Maximization of the criteria

We consider the maximization of the Faustmann Value with respect to thinnings and cutting age: $\max_{(h_k)_k,(u_k)_k,T} J_\lambda$. As in the case without risk, for fixed cutting age $T$, the maximization of the Faustmann Value $J_\lambda$ is equivalent t o the maximization of $E_W(0,T)$. Hence the maximization of the Faustmann Value with respect to thinnings and cutting age can be decomposed into two levels: $\max_T \left[E^*_W(0,T) - c_1 - (c_0 + c_1)a(T)\right]/b(T)$ where $E^*_W(0,T)$ is the maximum of $E_W(0,T)$ with respect to thinnings: $\max_{(h_k)_k,(u_k)_k} E_W(0,T)$.

We notice that for fixed thinnings $u_k$, $h_k$ and cutting age $T$, knowing potential thinning income $R_k, E_{R_k}$, final income $V(T), E_\mathcal{V}(t)$ and costs $E_{C_v}(t)$ for $t_L < t < T$ is sufficient to express the Faustmann Value without or with risk. Hence, from a numerical point of view, thanks to the analytical obtained expressions of the criteria, a unique simulation of the stand growth is needed to express the Faustmann Value and the Expected Long-Run Average Yield without or with risk, for various discount rates or function prices.

For fixed cutting age $T$, the behavior of the thinning income $\beta^k$ coefficients will be studied to deduce the influence of discount rate, risk and considered criteria (Faustmann Value, Expected Long Run Average Value) in terms of optimal thinnings.

#### 3.4.1. Dependence of optimal thinning with respect to discount rate

The derivative of $\beta^k_{\delta,\lambda}$ with respect to the discount rate $\delta$ normalized by $\beta^k_{0,\lambda}$ is:

$$\frac{1}{\beta^k_{0,\lambda}} \frac{\partial \beta^k_{\delta,\lambda}}{\partial \delta}\Big|_{\delta=0} = T - u_k. \tag{13}$$

We then deduce that the relative additional contribution of $\beta^k_{\delta,\lambda}$ decreases as $k$ increases. Hence as in the case without risk, for fixed cutting age $T$, the greater the discount rate $\delta$, the earlier the optimal thinnings. Moreover, for a sufficiently small risk rate $\lambda$, by continuity with respect to $\lambda$ we deduce that for fixed thinnings, the optimal cutting age decreases with respect to discount rate. Hence, also for the optimal cutting age, as in the case without risk, the greater the discount rate $\delta$, the earlier the optimal thinnings.

#### 3.4.2. Dependence of optimal thinning with respect to risk

The derivative of $\beta^k_{\delta,\lambda}$ with respect to $\lambda$ normalized by $\beta^k_{\delta,0}$ is:

$$\frac{1}{\beta^k_{\delta,0}} \frac{\partial \beta^k_{\delta,\lambda}}{\partial \lambda}\Big|_{\lambda=0} = \begin{cases} T - t_L & \text{if } u_k < t_L \\ T - u_k & \text{if } u_k > t_L \end{cases}. \tag{14}$$

We then deduce that the relative additional contribution of $\beta^k_{\delta,\lambda}$ decreases as $k$ increases. These results imply that the thinnings at dates $u_k$ prior to $t_L$ have a greater influence and the last thinnings a lower influence in case of risk. It is not surprising because after $t_L$ the dates $u_k$ are less and less reached as $k$ increases.

Hence, assuming that the integral term in case of risk is neglectable, we deduce that, in presence of storm risk, the optimal thinnings are done earlier than in the case without risk for fixed cutting age. Moreover, for sufficiently small risk rate $\lambda$, it is possible, as for the $\delta$ dependence, to deduce that for fixed thinnings, the optimal cutting age decreases with respect to the risk rate $\lambda$. So, for the optimal cutting age, the greater the risk rate $\lambda$, the earlier the optimal thinnings. The obtained result is generic and does not depend on the forest growth nor on the species. Our result is in accordance with forest managers decisions: shorter cutting age and earlier thinnings. Concerning the thinnings, our study enhances the previous argument concerning the $H/d$ ratio.

*3.4.3. An adjusted discount rate without risk compared to the risk case*

Reed (1984) showed that in case of risk, without taking thinnings into account, the optimal cutting age can be achieved by replacing $\delta$ by $\delta+\lambda$ in the case without risk. What happens if we take thinnings into account? Without risk for an adjusted discount rate $\delta+\lambda$, the derivative with respect to $\lambda$ normalized by $\beta^k_{\delta,0}$ is:

$$\frac{1}{\beta^k_{\delta,0}}\frac{\partial\beta^k_{\delta+\lambda,0}}{\partial\lambda}|_{\lambda=0}=T-u_k. \tag{15}$$

From the comparison of Eqs. (14) and (15) we first deduce that for $u_k<t_L$ the normalized derivatives of the coefficient are greater for an adjusted discount rate without risk than for the risk case: with the adjusted discount rate, the first thinnings are favored and have a greater influence than in the risk case. Hence, assuming that the integral term in case of risk is neglectable, the optimal thinnings would be earlier for the adjusted discount rate without risk than for the real discount rate with risk. Secondly for $u_k>t_L$ the derivatives are equal. Hence, the optimal thinnings would be close to each other in this case. Moreover, for a sufficiently large value of the discount rate, the first optimal thinnings should satisfy $u_k<t_L$. At the opposite for sufficiently small value of discount rate the first optimal thinnings should satisfy $u_k>t_L$. We suggest the following conjectures:

- C1 If the first optimal thinnings for an adjusted discount rate $\delta+\lambda$ without risk are smaller than $t_L$ (potentially satisfied if $\delta+\lambda$ is not too small) then the optimal thinnings in an ajusted $\delta+\lambda$ without risk occur earlier than in the case with risk and discount rate $\delta$.
- C2 If the first optimal thinnings with an adjusted discount rate $\delta+\lambda$ without risk are greater than $t_L$ (satisfied if $\delta+\lambda$ sufficiently small) then the optimal thinnings with an adjusted discount rate $\delta+\lambda$ without risk and with discount rate $\delta$ with risk are close to each other.

Taking $\delta=0$ in Conjecture $C_2$, we obtain that, for $\lambda$ not too high, the optimal thinnings for the Faustmann Value with discount rate $\lambda$ without risk and for the Expected Long-Run Average Yield with risk are close to each other.

Some information on the optimal thinnings dates with risk can be deduced from the comparison of the first optimal thinnings dates in the without risk case, using an adjusted discount rate, with $t_L$. The date $t_L$ is a threshold which permits to predict the behavior of the optimal thinnings with respect to the criteria. This conjecture has to be supported by numerical optimization in concrete cases.

## 4. Application for a beech stand

We now apply the analytic expressions given above to evaluate and compare the criteria without or with risk for a forest stand using a specific growth model. Maximization of the various criteria will permit in a first step to confirm or not the previous properties and conjectures. In a second step we analyze the impact of the rate depreciation of timber price on silviculture.

*4.1. Growth model*

We consider an average tree model, the state variables are the tree-density $n$ per hectar and the tree-basal area $s$ measured at breast 1.30 m from the ground. For initial tree-number $n(0)$ and tree-basal area $s(0)$ set, the evolutions of $n$ and $s$ are governed by the following system of equations:

$$(\mathcal{S}_0)\begin{cases}\dfrac{dn(t)}{dt}=-m(t)n(t)\\ \dfrac{dn(t)}{dt}=\dfrac{g(n(t),s(t))}{n(t)}\Gamma(t)\end{cases}$$

at the thinning times $u_k: n\left(u_k^+\right)=(1-h_k)n(u_k^-), 0\le h_k<\overline{h}$

where $m(.)$ is the natural mortality and $g(n,s)\Gamma(t)$ is the instantaneous increase of the stand basal area: $\Gamma(t)$ is the potential increase at its peaks of tree-density and $g(n,s)$ a reductor depending on the tree-density.

The thinning rates are limited to an upper value $\overline{h}$ to insure regular income.

In case of storm risk, the system $(\mathcal{S}_1)$ to be considered consists of equations of system $(\mathcal{S}_0)$, plus Eq. (5). To complete the model, the tree-height $H$ is expressed via a time-dependent function: $H(t)$ and the tree-volume $v(s,H)$ is a combination of the tree-basal area and the tree height.

The expressions of the expected thinnings income and clearing costs if a storm occurs are given in Appendix C.

*4.2. Optimization*

We are interested in the silviculture of a beech stand: we give in Appendix D the related parameters and functions used for the evaluation of the criteria. As usual, the silviculture is decomposed into two steps: a first step of respacings and a second step of thinnings. We assume two respacings at time 10 years and $u_0=20$ years, the tree number is significantly reduced, the fixed rate is $h_0=29/30$. For the second step, the thinning dates are separated by a minimum of $\Delta t$ years: $u_k-u_{k-1}\ge\Delta t$ years. We choose $N=5, \Delta t=8$ years, $\overline{h}=.25$ year $^{-1}$ and we set the final tree-number $n_T=125$ stems/ha in order to facilitate the comparison for various criteria. To summarize we determine, by optimization of the various criteria ($J_0,\overline{Y}_0$ without and $J_\lambda,\overline{Y}_\lambda$ with risk), the optimal thinning dates $u_1,..., u_N$ and thinning rates $h_1,..., h_N$ subject to the constraints:

$$\begin{array}{ll} u_k-u_{k-1}\ge\Delta t, & k=1..N\\ 0\le h_k\le\overline{h}, & k=1..N\\ n(T)=n_T. & \end{array}$$

**Table 1**
Optimal Faustmann Value with respect to thinning rates $h_k$, dates $u_k$ and cutting age $T$.

| Scenario discount rate | Optimal thinnings (year) | | | | | Cutting age (year) | Diameter (cm) | Faustmann Value (Euro/ha) |
|---|---|---|---|---|---|---|---|---|
| *Without risk* | | | | | | | | |
| $\delta$ | | | 71.8<br>.186 | 79.8<br>.247 | 87.9<br>.234 | 105.3 | 56.4 | 16515 |
| $\delta+\lambda$ | 50.9<br>.246 | 58.9<br>.222 | 67.0<br>.200 | | | 84.0 | 53.2 | 4725 |
| *With risk* | | | | | | | | |
| $\delta$ | | 59.1<br>.240 | 67.2<br>.250 | 75.3<br>.177 | | 83.3(81.2) | 50.6 | 13994 |

**Table 2**
Optimal LRAY with respect to thinning rates $h_k$, dates $u_k$ and cutting age $T$.

| Scenario | Thinnings (year) | | | | | | Cutting age (year) | Diameter (cm) | LRAY (Euro/ha) |
|---|---|---|---|---|---|---|---|---|---|
| *Without risk* | | | | | | | | | |
| | | | | 138.3<br>.167 | 146.4<br>.250 | 154.5<br>.250 | 162.8 | 61.7 | 312.8 |
| *With risk* | | | | | | | | | |
| | 71.5<br>.167 | 79.5<br>.250 | 87.6<br>.250 | | | | 95.7(90.5) | 52.6 | 219.8 |

**Table 3**
Optimal Faustmann Value for two values of rate depreciation.

| Scenario | Thinnings (year) | | | Cutting age (year) | Diameter (cm) | Faustmann Value (Euro/ha) |
|---|---|---|---|---|---|---|
| $\alpha_d = .5$ | 59.1 | 67.2 | 75.3 | 83.3(81.2) | 50.6 | 13994 |
| | .240 | .250 | .177 | | | |
| $\alpha_d = .9$ | 56.3 | 64.3 | 72.3 | 80.3(78.3) | 49.5 | 13318 |
| | .186 | .250 | .231 | | | |

**Table 4**
Optimal LRAY for two values of rate depreciation.

| Scenario | Thinnings (year) | | | Cutting age (year) | Diameter (cm) | LRAY (Euro/ha) |
|---|---|---|---|---|---|---|
| $\alpha_d = .5$ | 71.5 | 79.5 | 87.6 | 95.7(90.5) | 52.6 | 219.8 |
| | .167 | .250 | .250 | | | |
| $\alpha_d = .9$ | 65.6 | 73.6 | 81.7 | 89.7(86.1) | 51.9 | 206.5 |
| | .249 | .235 | .182 | | | |

The presented results are obtained by using the classical Nelder Mead algorithm to optimize the various considered criteria. As this algorithm does not manage constraints on control variables, the constraints on thinnings dates ($u_k - u_{k-1} \geq \Delta t$ years) and final tree number ($n(T) = n_T$) are managed by using artificial variables.

## 5. Results and discussion

We analyze the optimal silviculture for the two criteria: Faustmann Value (Table 1) and the Expected Long-Run Average Yield (Table 2). For the two criteria, we give the optimal thinnings and cutting age without and with risk: the thinnings dates $u_k$ in the first row and the thinnings rate $h_k$ in the second row. For the Faustmann Value in Table 1, we also give the scenario without risk, with a adjusted discount rate $\delta + \lambda$ instead of $\delta$.

Finally in each table, the scenario with risk is given in the last rows. In all cases, we specify the expected effective values of the final cutting age in parentheses (given by $E(\tau) = t_L + F(T - t_L)/\lambda$).

Concerning the dependence of optimal thinning with respect to risk, for the Faustmann value in Table 1, the first thinnings occur at 71.8 years without risk and 59.1 years with risk. For the Expected Long-Run Average Yield in Table 2, the difference in the first thinnings dates is greater: 138.3 years without risk and 71.5 years with risk. Hence for the two criteria, the optimal thinnings occur earlier in the case with risk. Moreover for the two criteria, the presence of risk implies earlier thinnings and a shorter cutting age $T$. We therefore make the risk endogenous through optimization. An earlier cutting age with risk implies a lower final diameter at the cutting age.

Concerning the conjecture $(\mathcal{C}_1)$, we compare the results without risk obtained with $\delta + \lambda$ in the second scenario and the result with risk obtained with $\delta$ in the third scenario of Table 1. The results show that, in case of storm risk, by taking thinnings into account, the optimal cutting ages are identical: 83.3 years. It is in accordance with the previous result by Price (2011). The first thinning date with the adjusted discount rate 50.9 years is smaller than $t_L = 61.5$ years. In accordance with our conjecture, the thinnings occur slightly earlier for the adjusted discount rate without risk: 50.9 years with the adjusted discount rate and 59.1 years with the risk case for the respective first thinning dates. Replacing $\delta$ by $\delta + \lambda$ in the case without risk to mimic the case with risk is valid to predict the optimal age, we do not obtain the optimal thinning dates but may predict the behavior of predicted thinning dates. Considering the conjecture $(\mathcal{C}_2)$ with $\delta = 0$, first of all the cutting ages for $J_0$ and $\overline{Y}_\lambda$ are respectively 105.3 and 95.7 years and are of the same order of magnitude. Secondly, the first thinning date, with discount rate $\lambda$, 71.8 years is later then $t_L = 61.5$ years. In accordance with this conjecture, the dates of the first thinnings are very similar: 71.8 years for $J_0$ and 71.5 years for $\overline{Y}_\lambda$. A similar result is found for the magnitudes.

In Tables 3 and 4, we present the optimized silviculture for two values of rate depreciation of timber price $\alpha_d = .5,.9$. The real value of $\alpha_d$ is stochastic but in the range [.5,.9]. From the results we deduce that, the greater the rate depreciation $\alpha_d$, the earlier the optimal thinnings dates and cutting age. Moreover there is no significant difference for the tree-diameter. Finally, we remark that, in all optimizations, after the first thinning, the other thinnings are separated by the miminal authorized delay: 8 years.

## 6. Conclusion

We have studied the management of a stand in the presence of risk of storm. From the analytic expressions of the criteria, we highlighted the influence of the presence of storm risk on optimal thinning and cutting age: in presence of storm risk, the optimal thinnings occur earlier than in the case without risk. Moreover, we highlight the role of a time threshold in the behavior of optimal thinnings with respect to the criteria. In particular we determine in which case it is valid or not to use an adjusted discount rate without risk to mimic the storm risk case. Simulations based on an average tree model for a beech stand validate the conjectures. More precisely, if the first optimal thinnings with and adjusted discount rate and without risk occur before the time threshold, the optimal thinnings with an adjusted discount rate without risk are done earlier than in the case with risk for the Faustmann Value. At the opposite, if the first optimal thinnings with and adjusted discount rate and without risk are beyond the time threshold, the optimal thinnings and the optimal rotation period for the Faustmann Value $J_0$ calculated with discount rate $\lambda$ and for the Expected Long-Run Average Yield are close to each other.

It can be observed that some forest managers want to drastically reduce the cutting ages. Considering the second conjecture with $\delta = 0$, this can be interpreted in two ways. This desire can be interpreted as an adaptation to climate change by taking into account an increased risk of storm related to climate change. But it can also be explained by the desire to change the criterion to maximize: maximizing the Faustmann criterion may be preferred to maximizing the Expected Long-Run Average Yield.

We showed that the obtention of analytic expressions for the criteria opens the way to the analysis of the qualitative behavior of the optimal thinnings and cutting age. It would be interesting to apply this technique in a more complex risk framework especially for the combination of consecutively various types of risks such as storm followed by an epidemic event.

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## Appendix A

For fixed thinnings, omitting $T$ dependency in $\mathcal{H}_0(0, T)$, differentiating $J_0$ with respect to $T$ gives the following first-order condition:

$$J_T = \frac{\left(\delta\mathcal{H}_0 + V'(T)\right)\left(e^{\delta T} - 1\right) - \delta e^{\delta T}(H_0 + V(T) - c_1)}{\left(e^{\delta T} - 1\right)^2} = 0.$$

Differentiating $J_T = 0$ with respect to $\delta$ yields: $J_{TT}T_\delta + J_{T\delta} = 0$. Hence, from $J_{TT} < 0$, we deduce that $T_\delta$ and $J_{T\delta}$ have the same sign.

If we let $\mathcal{H}_1=\frac{\partial\mathcal{H}_0}{\partial\delta}=\sum_{k=1}^{N}{}^{\mathbf{t}}\mathbf{R_k}.\mathbf{h_k}(T-u_k)e^{\delta(T-u_k)}$, $J_{T\delta}$ is proportional (with the same sign) to:

$$(\mathcal{H}_0+\delta\mathcal{H}_1)\left(e^{\delta T}-1\right)+\left(\delta\mathcal{H}_0+V'(T)\right)Te^{\delta T}-(1+\delta T)e^{\delta T}(\mathcal{H}_0+V(T)-c_1)$$
$$-\delta e^{\delta T}\mathcal{H}_1\ \text{(then using } J_T=0)$$
$$=(\mathcal{H}_0+\delta\mathcal{H}_1)\left(e^{\delta T}-1\right)-\delta e^{\delta T}\mathcal{H}_1+\left[\frac{\delta Te^{\delta T}}{e^{\delta T}-1}-(1+\delta T)\right](\mathcal{H}_0+V(T)-c_1)e^{\delta T}$$
$$=\mathcal{H}_0\left(e^{\delta T}-1\right)-\delta\mathcal{H}_1+\left(1+\delta T-e^{\delta T}\right)\frac{e^{\delta T}}{e^{\delta T}-1}(\mathcal{H}_0+V(T)-c_1)$$

which is proportional, with the same sign, to:

$$=\left((\delta T-1)e^{\delta T}+1\right)\mathcal{H}_0-\delta\left(e^{\delta T}-1\right)\mathcal{H}_1-\left(e^{\delta T}-1-\delta T\right)e^{\delta T}(V(T)-c_1)$$
$$\text{(then, from the expressions of } \mathcal{H}_0,\ \mathcal{H}_1):$$
$$=\sum_{k=1}^{N}R_k.h_k\left[\left((\delta T-1)e^{\delta T}+1\right)-\delta(T-u_k)\left(e^{\delta T}-1\right)\right]e^{\delta(T-u_k)}$$
$$-\left(e^{\delta T}-1-\delta T\right)e^{\delta T}(V(T)-c_1)$$
$$=\sum_{k=1}^{N}R_k.h_k\left[\delta u_k\left(e^{\delta T}-1\right)-\left(e^{\delta T}-1-\delta T\right)\right]e^{\delta(T-u_k)}$$
$$-\left(e^{\delta T}-1-\delta T\right)e^{\delta T}(V(T)-c_1).$$

We remark that $R_k$ is increasing with respect to $k$, hence the major contribution to the sum is the last thinning. Moreover for small value of $\delta$, the coefficients of $R_kh_k$ for $u_k\sim T$ is closed to $e^{\delta T}-1-\delta T$, hence for sufficiently small value of $h_k$ and $c_1$, we deduce that $J_{T\delta}$ is negative.

## Appendix B

### B.1. Evaluation of $\mathcal{H}(t_1,t_2,\sigma)$

Let $\sigma=(\sigma_k)_{k\in K}$ the vector composed of $\sigma_k=\tau_{u_k}^-$ the date of the last storm before $u_k$ for each $k\in K=\{k|u_k<t_L+\Delta\}$. We denote $\mathcal{H}(t_1,t_2,\sigma)$ the thinning income between $t_1$ and $t_2$, taking into account the dates of the last storm before thinning times. Hence the modified value of the timber price reference is $p_{u_k}=\left(1-\rho_{\sigma_k}(u_k)\right)p_0$, and $\mathcal{H}(t_1,t_2,\sigma)$ can be deduced:

$$\mathcal{H}(t_1,t_2,\sigma)=\sum_{t_1<u_k\le t_2}{}^{\mathbf{t}}\mathbf{R}\left(\left(1-\rho_{\sigma_\mathbf{k}}(\mathbf{u_k})\right)\mathbf{p}_0,\mathbf{u_k}\right).\mathbf{h_k}\ e^{\delta(t_2-u_k)}$$

### B.2. Evaluation of $E(e^{-\delta\tau})$

We express $E(e^{-\delta\tau})=\int_{t_L}^{T}e^{-\delta\tau}dF(\tau-t_L)+e^{-\delta T}(1-F(T-t_L))=\frac{\lambda+\delta e^{-(\delta+\lambda)(T-t_L)}}{\delta+\lambda}e^{-\delta t_L}$

### B.3. Evaluation of $E(e^{-\delta\tau}\mathcal{Y})$

Let $E_{R_k}=E\left[R\left(\left(1-\rho_{\sigma_k}(u_k)\right)p_0,u_k\right)\right]$ the expected potential thinning income at time $u_k$, then the expected thinning income between $t_1$ and $t_2$ is::

$$E_{\mathcal{H}}(t_1,t_2)=E_\sigma[\mathcal{H}(t_1,t_2,\sigma)]=\sum_{t_1<u_k\le t_2}{}^{\mathbf{t}}\mathbf{E_{R_k}}.\mathbf{h_k}e^{\delta(t_2-u_k)}.$$

Knowing the expressions of the expected incomes and costs:

$$E\left(e^{-\delta\tau}\mathcal{Y}\right)=E_{\mathcal{H}}(0,t_L)e^{-\delta t_L}+\int_{t_L}^{T}\left(E_{\mathcal{H}}(t_L,\tau)+E_{\mathcal{V}}(\tau)-c_1-c_0-E_{C_v}(\tau)\right)e^{-\delta\tau}dF(\tau-t_L)$$
$$+(E_{\mathcal{H}}(t_L,T)+V(T)-c_1)e^{-\delta T}(1-F(T-t_L)).$$

The part relative to the thinning after $t_L$ in $E(e^{-\delta\tau}\mathcal{Y})$, by inverting the integral and the summation, is:

$$\int_{t_L}^{T}E_{\mathcal{H}}(t_L,\tau)e^{-\delta\tau}dF(\tau-t_L)+E_{\mathcal{H}}(t_L,T)e^{-\delta T}(1-F(T-t_L))$$
$$=\sum\nolimits_{t_L<u_k<T}{}^{\mathbf{t}}\mathbf{E_{R_k}}.\mathbf{h_k}e^{-\delta u_k-\lambda(u_k-t_L)}$$

The part in $E(e^{-\delta\tau}\mathcal{Y})$ relative to the clearing costs is:

$$\int_{t_L}^{T}\left(c_0+E_{C_v}(\tau)\right)e^{-\delta\tau}dF(\tau-t_L)=c_0\frac{\lambda}{\delta+\lambda}\left(1-e^{-(\delta+\lambda)(T-t_L)}\right)e^{-\delta t_L}$$
$$+\int_{t_L}^{T}E_{C_v}(\tau)e^{-\delta\tau}dF(\tau-t_L)$$

Finally:

$$E\left(e^{-\delta\tau}\mathcal{Y}\right)=\sum_{k=1}^{N}{}^{\mathbf{t}}\mathbf{E_{R_k}}.\mathbf{h_k}e^{-\delta u_k-\lambda(u_k-t_L)_+}+\lambda\int_{t_L}^{T}\left[E_{\mathcal{V}}(\tau)-E_{C_v}(\tau)\right]e^{-\delta\tau-\lambda(\tau-t_L)}d\tau$$
$$+(V(T)-c_1)e^{-\delta T-\lambda(T-t_L)}-(c_0+c_1)\frac{\lambda}{\delta+\lambda}\left(1-e^{-(\delta+\lambda)(T-t_L)}\right)e^{-\delta t_L}.$$

### B.4. The Faustmann Value

By using the expression of the expectations $E(e^{\delta\tau}), E(e^{\delta\tau}\mathcal{Y})$, multiplying the numerator and the denominator of Eq. (8) by $e^{(\delta+\lambda)(T-t_L)+\delta t_L}$ and reordering the terms, we obtain the Faustmann Value $J_\lambda$:

$$J_\lambda=\frac{E_W(0,T)-c_1-(c_0+c_1)\frac{\lambda}{\delta+\lambda}\left(e^{(\delta+\lambda)(T-t_L)}-1\right)}{e^{(\delta+\lambda)(T-t_L)+\delta t_L}-\frac{\lambda}{\delta+\lambda}e^{(\delta+\lambda)(T-t_L)}-\frac{\delta}{\delta+\lambda}}$$

where $E_W(0,T)$ is the modified expected income (Eq. (11)).

### B.5. Evaluation of $E_{R_k}$

From the assumption that $R$ is linear in its first argument: $E_{R_k}=E\left(R\left(\left(1-\rho_{\sigma_k}(u_k)\right)p_0,u_k\right)\right)=R((1-E_{\rho_k})p_0,u_k)$ where $E_{\rho_k}=E_{\sigma_k}\left[\rho_{\sigma_k}(u_k)\right]$.

For a sufficiently small $\Delta$, we obtain:

$$E_{\rho_k}=I_{u_k>\Delta}E_{\sigma_k}\left[\rho_{\sigma_k}(u_k)I_{u_k-\Delta<\sigma_k<\min(u_k,t_L)}\right]$$
$$=I_{u_k>\Delta}E_{\sigma_k}\left[\rho_0(u_k-\sigma_k)I_{\max(u_k-t_L,0)<u_k-\sigma_k<\Delta}\right]$$
$$=I_{\Delta<u_k<t_L+\Delta}\int_{\max(u_k-t_L,0)}^{\Delta}\rho_0(x)dF(x).$$

## Appendix C. Expected incomes and costs: $E_{\mathcal{V}}$ and $E_{C_v}$

Assuming that the timber price per unit of volume is the product of the timber price reference $p_0$ and a quality related function $q$ depending on the tree basal-area $s$, the income function $R$: $R(p_0,t)=[p_0q(s(t))-e_0]v_T(t)$ where $v_T(t)$ is the total volume $v_T(t)=v(s(t),H(t))n(t)$ and $e_0$ is the operating cost per unit of volume. The quality return function $q$ is assumed increasing with respect to the tree-basal area $s$. Due to the assumptions made on $\theta$ and $\xi$, the final income $\mathcal{V}$ is given by $\mathcal{V}(\theta,\xi,\tau)=[(1-\theta)(1-\xi)p_0q(s(\tau))-(1-\theta)e_0]v_T(\tau)$. We assume that the variable clearing costs $C_v$ linearly depend on the volume of damaged trees $\theta_\tau v_T(\tau)$ after a storm, hence $C_v(\theta_\tau,\tau)=c_v\theta_\tau v_T(\tau)$. We notice that, if $e_0>0$, the final income in case of storm is proportional to the final income without risk. From the definition of $\alpha$ and $\alpha_p$, we deduce the expected final income $E_V(\tau)=[\alpha_p(\tau)p_0q(s(\tau))-\alpha(\tau)e_0]v_T(\tau)$ and the expected variable clearing costs $E_{C_v}(\tau)=c_v(1-\alpha(\tau))v_T(\tau)$.

## Appendix D. Functions and parameters of the models

### D.1. The growth model

The reductor is $g(n,s)=1-e^{-m_1 n\sqrt{s}}$ and the potential increase $\Gamma(t)=m_2+m_3 dH(t)/dt$ where the tree-height $H(t)$ at time $t$ given by $H(t)=H_0(1-e^{-m_4 t})$ (Dhôte, 1995) and $H_0$ the limited tree-height which depends mainly on soil fertility. The time unit for the case study is the year. We assume usual values for the initial tree number $n(0)=$ 8000 stems/ha and the initial tree basal area $s(0)=.000125\ \mathrm{m}^2$.

### D.2. Risk modeling

The expected number of storms per year is: $\lambda=0.01$ year$^{-1}$. We assume no damage under a height $H_L=21$ m, a height limit $H_0=40$ m, so from the expression of $H(t)$ we deduce $t_L=-\log(1-H_L/H_0)/m_4=$ 61.5 years. From data provided in Schmidt et al. (2010), we infer that the expected proportion of surviving trees is: $\alpha(t)=1-\phi((H(t)-H_L)/(H_0-H_L))$, with $\phi$ an increasing function in $x$, (we choose $\phi(x)=\min(.1+1.5x,1)$). Due to the lack of a precise model for beech stand, we do not take into account the $H/d$ dependence.

### D.3. Economic parameters and functions

The timber price reference is $p_0=57$ Euros/m$^3$. The timber price quality related function depending on the tree basal area is $q(s)=12/57+45/57\left(1-\left(1-\sqrt{s/0.3318}\right)^{1.8}\right)$ for $0.0038<s<0.3318\ \mathrm{m}^2$ which has been fitted from data in Tarp et al. (2000). The operating cost per unit of volume is $e_0=10$ Euros/m$^3$ and the regeneration cost is $c_1=1000$ Euros/ha.

Criteria are evaluated with the discount rate $\delta=0.01$ year$^{-1}$. The rate depreciation of timber price following a storm is $\alpha_d=.5$, the perturbation time is $\Delta=5$ years and $\rho_0(t)=\alpha_d\left(1-t^4/\Delta^4\right)$ for $0<t<\Delta$.

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