M.E. 530.646 Lab 2: Rigid Body Transformations

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1. See source codes

2.

(a) (b) (c)

```
Generate a random vector v:
[0.68]
[-0.21]
[0.57]

Norm of v is:0.910006
The unit vector of v is
[0.75]
[-0.23]
[0.62]

The skew of v is
[    0, -0.57, -0.21]
[ 0.57,     0, -0.68]
[ 0.21,  0.68,     0]
```

(d)

```
Compute v_skew * v
[-9.9e-10]
[-1e-08]
[-7.1e-09]
```

The result is zero, then we know v belongs to ker(v_skew).

(e)

```
The eigenvalues of v_skew are
[(-6.6e-09,0)]
[(0,0.91)]
[(0,-0.91)]

The eigenvectors of v_skew are
[      (-0.75,0),  (0.47,1.6e-07), (0.47,-1.6e-07)]
[       (0.23,0),    (0.18,-0.66),     (0.18,0.66)]
[      (-0.62,0),    (-0.5,-0.25),     (-0.5,0.25)]
```

According to the result, v is an eigenvector of v_skew.

(f) (g)

```
The exponential of v_skew is
[   0.83,   -0.56, -0.0036]
[   0.42,    0.63,   -0.65]
[   0.36,    0.53,    0.76]

The value of exp(v_skew) * v is
[0.68]
[-0.21]
[0.57]
```

According to result vector v is the same as exp(v_skew) * v.

(h)

```
The eigenvalues of v_skew exponential are
[(1,0)]
[(0.61,0.79)]
[(0.61,-0.79)]

The eigenvectors of v_skew exponential are
[    (-0.75,0),  (-0.11,0.46), (-0.11,-0.46)]
[     (0.23,0),    (0.6,0.34),   (0.6,-0.34)]
[    (-0.62,0),  (0.36,-0.42),   (0.36,0.42)]
```

According to the result, v is an eigenvector of the exponential of v_skew.

3. See source codes and plotted frames

4.

(a)

```
xf(1,1,1,1,1,1) is
[  0.29, -0.072,   0.95,      1]
[  0.45,   0.89, -0.072,      1]
[ -0.84,   0.45,   0.29,      1]
[     0,      0,      0,      1]

xf(1,1,1,1+2*PI,1,1) is
[  0.29, -0.072,   0.95,      1]
[  0.45,   0.89, -0.072,      1]
[ -0.84,   0.45,   0.29,      1]
[     0,      0,      0,      1]
```

(b) H is numerically ill defined when the rotation matrix part $H_{31} = \pm 1$. It means pitch is 90 degree or – 90 degree. In this case, roll and yaw angles will have infinite many solutions. Then H does not represent a specific transformation.

(c)

```
vector r2 is
[0]
[0]
[0]
[13]
[0]
[0]

xfinv(xf(0,0,0,4PI,0,0))is
[0]
[0]
[0]
[-6e-07]
[-0]
[0]
```

The results are different.

(d)

```
H is
[1, 0, 0, 1]
[0, 1, 0, 1]
[0, 0, 1, 1]
[0, 0, 0, 1]

xf(xfinv(H)) is
[1, 0, 0, 1]
[0, 1, 0, 1]
[0, 0, 1, 1]
[0, 0, 0, 1]
```

The results are the same.

5

(a)

$$H_3^0 = \begin{pmatrix} 1 & 0 & 0 & q1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos{(q2)} & \sin{(q2)} & 0 \\ 0 & -\sin{(q2)} & \cos{(q2)} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & 0 & q1 \\ 0 & cosq2 & sinq2 & -q3 * sinq2 \\ 0 & -sinq2 & cosq2 & q3cosq2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) See source codes and animation.

(c)

$$H_3^0 = \begin{pmatrix} 1 & 0 & 0 & \frac{5\pi}{4} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos{(\frac{5\pi}{4})} & \sin{(\frac{5\pi}{4})} & 0 \\ 0 & -\sin{(\frac{5\pi}{4})} & \cos{(\frac{5\pi}{4})} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{5\pi}{4} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 & 0 & \frac{5\pi}{4} \\ 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2\sqrt{2} \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -2\sqrt{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
The Transformation H03 is
[    1,     0,     0,   3.9]
[    0, -0.71,  0.71,   2.8]
[    0, -0.71, -0.71,  -2.8]
[    0,     0,     0,     1]
```

Results show the calculated value and experiment values are identical.