M.E. 530.646 Lab 4: Inverse Kinematics

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1. According to the solution to the inverse kinematics of the UR5 robot, we are able to write a function that compute the joint position of it.

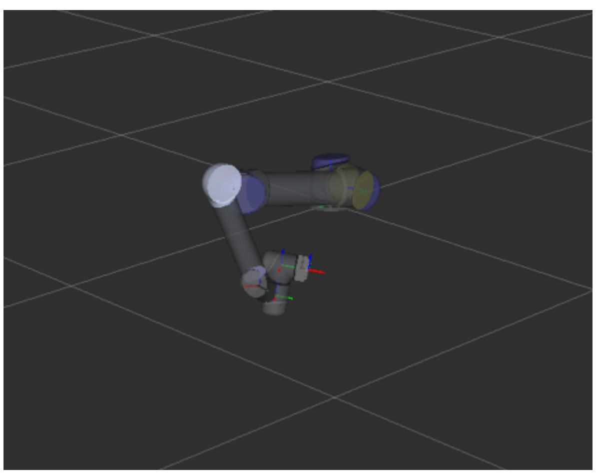
Given an end effector position in the homogenous form

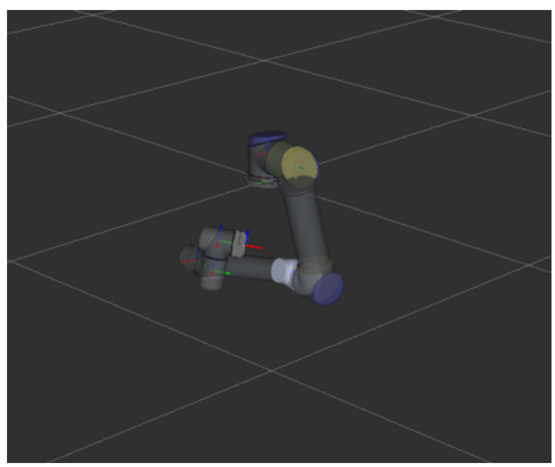
q[6] = {2.2, 3.5, 4.2, 2.9, 3.3, 1.2}

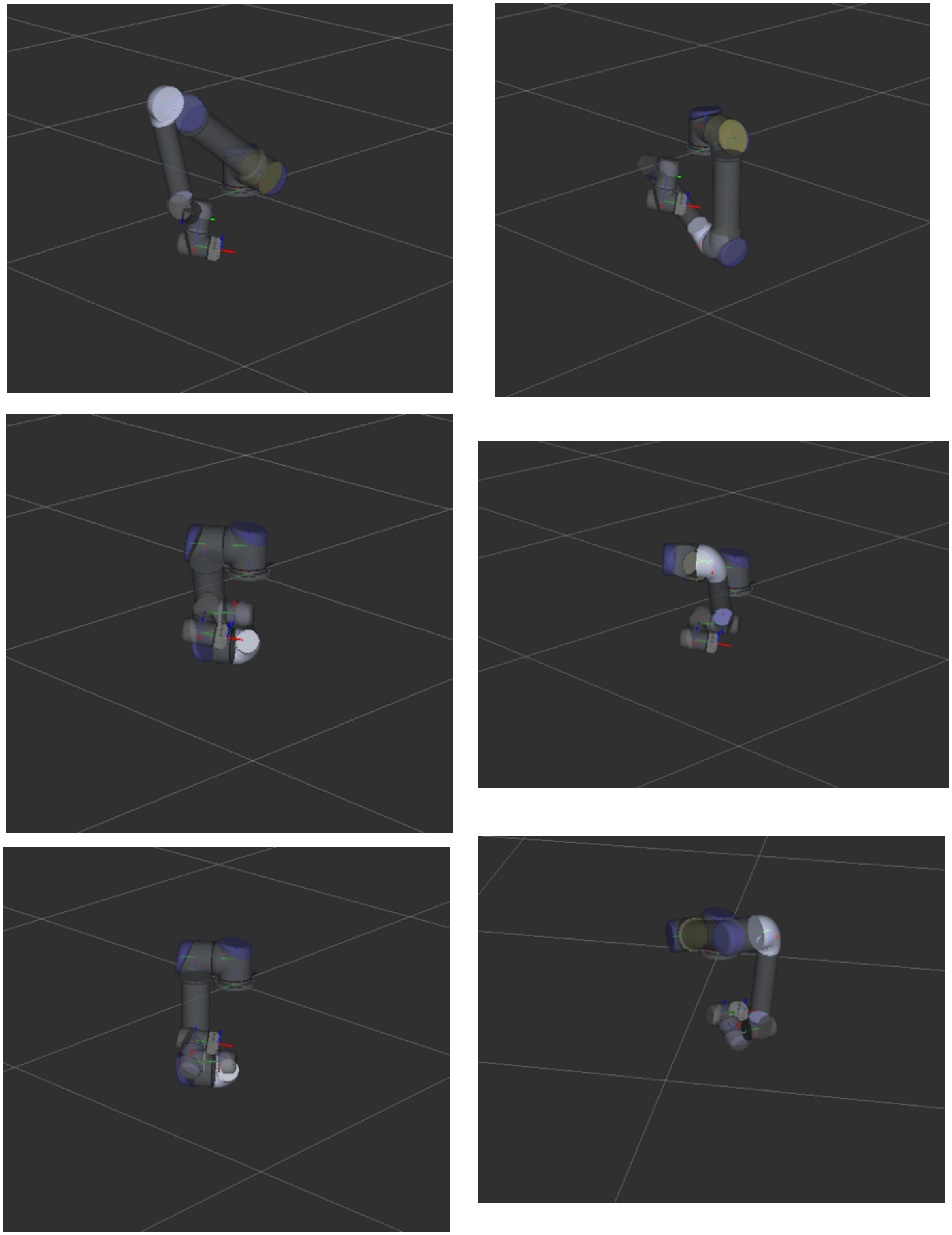
We can generate eight solutions as following.

```
For q[6] = {2.2,3.5,4.2,2.9,3.3,1.2}, the solutions of inverse kinematics are:
q_sol = [4.519726,6.125437,2.125171,0.986050,0.892060,0.148230]
q_sol = [4.519726,1.823783,4.158014,3.254861,0.892060,0.148230]
q_sol = [4.519726,5.589442,2.506466,4.282342,5.391126,3.289823]
q_sol = [4.519726,1.570081,3.776719,0.748265,5.391126,3.289823]
q_sol = [2.200000,1.199295,2.570199,3.688914,2.983185,4.341593]
q_sol = [2.200000,3.498312,3.712986,0.247110,2.983185,4.341593]
q_sol = [2.200000,1.553640,2.083185,0.679990,3.300001,1.200000]
q_sol = [2.200000,3.500000,4.200000,2.900000,3.300001,1.200000]
```

We can also plot the joint position of UR5 in Rviz.







These eight robots all have the same end-effector positions and orientations, but have eight different joint position.

2. According to the Spong book, we compute the Jacobian of linear velocity and angular velocity separately and then combine them together:

$$J_i = \begin{bmatrix} z_{i-1} \times (o_n - o_{i-1}) \\ z_{i-1} \end{bmatrix}$$

Finally we can get the combined Jacobian. We will evaluate it in the next part.

3. We choose three group of joint position and test them with different random small angle distortions. The results are followings.

```
*Joint Position0=[0.200000,3.800000,1.500000,1.500000,2.100000,5.800000]
**********************************
*Small angle0=[0.000300,0.000600,0.000700,0.000500,0.000300,0.000500]
*P0_6(theta) - P0_6(theta+delta):
-0.000341527
-1.58548e-05
-0.000101924
*J * delta:
-0.000341674
-1.57683e-05
-0.000101826
*The Frobenius difference between A and B is:4.30112e-12
**********************************
*Small angle1=[0.000600,0.000200,0.000900,0.000100,0.000200,0.000700]
*P0_6(theta) - P0_6(theta+delta):
-0.000233047
3.05846e-05
-0.000185907
*J * delta:
-0.000233207
3.07168e-05
-0.000185856
*The Frobenius difference between A and B is:1.21675e-12
```

```
*Joint Position1=[2.200000,3.500000,4.200000,2.900000,3.300000,1.200000]
***********************************
*Small angle0=[0.000000,0.000900,0.000300,0.000600,0.000000,0.000600]
*P0_6(theta) - P0_6(theta+delta):
-0.000168823
0.000231922
0.000119932
*J * delta:
-0.000168823
0.000231933
0.000119784
*The Frobenius difference between A and B is:8.69024e-13
***********************************
*Small angle1=[0.000200,0.000600,0.000100,0.000800,0.000700,0.000900]
*P0_6(theta) - P0_6(theta+delta):
-0.00010822
8.066e-05
5.68479e-06
*J * delta:
-0.000108143
8.05824e-05
5.67335e-06
*The Frobenius difference between A and B is:1.71007e-12
```

```
*Joint Position2=[2.000000,3.200000,4.100000,2.800000,3.200000,1.300000]
***********************************
*Small angle0=[0.000200,0.000000,0.000200,0.000300,0.000700,0.000500]
*P0_6(theta) - P0_6(theta+delta):
-2.17184e-05
-1.88947e-05
-0.000108689
*J * delta:
-2.17128e-05
-1.89725e-05
-0.000108676
*The Frobenius difference between A and B is:5.68248e-13
***********************************
*Small angle1=[0.000900,0.000200,0.000200,0.000800,0.000900,0.000700]
*P0_6(theta) - P0_6(theta+delta):
-0.000127062
-4.70728e-05
-0.000119507
*J * delta:
-0.00012708
-4.71711e-05
-0.000119402
*The Frobenius difference between A and B is:5.8679e-12
```

4.

According to the Spong book.

$$J_{vi} = \frac{\partial P_n^0}{\partial \theta_i}$$

$$\partial P_n^0 = J_{vi} \partial \theta_i$$

$$P_6^0(\theta + \partial\theta) - P_6^0(\theta) = J_{vi} \partial\theta$$

Therefore, the left side should match the right side if our calculation for the linear part of Jacobian is correct. According to the 6 set of data outputs from part 3. The results are almost the same, which means our linear part of Jacobian is correct.

For the angular part.

$$skew(w_n^0) = \dot{R_n^0} {R_n^0}^T$$

$$(w_n^0) = J_w \partial\theta$$

$$\dot{R_n^0} {R_n^0}^T = skew(J_w \partial\theta)$$

$$(R_6^0(\theta + \partial\dot{\theta)} - R_6^0(\theta)) {R_6^0}^T = skew(J_w \partial\theta)$$

According to the 6 set of data outputs from part 3. The Frobenius norm of A-B are almost zero for all data, which means A = B, our angular part of Jacobian is also correct.