M.E. 530.646 Lab 4:
Inverse Kinematics

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Put the ROS package for this assignment in the `src` directory of your catkin workspace.

To complete this assignment, you will write code in both `lab4_main.cpp` and `lab4.cpp`. You will implement a number of different functions in `lab4.cpp` (which already includes their declarations), and you will use `lab4_main.cpp` to test them and complete the rest of the assignment. You will need to use the Eigen library, and so examples of how to use the necessary functions are included in `lab4_main.cpp`. For this assignment, you will be using 3D models of the UR5 robot. To start rviz with the models imported, enter the following command in the terminal:

```
roslaunch ur5 ur5_rviz.launch
```

Assignment

Be sure to comment all of your code!

1. Using the provided solution to the inverse kinematics write a function `int inverse(Eigen::Matrix4f H0_6, double **q, double q6Des)` that accepts the homogenous transformation H_0^6 and returns the number of solutions to the inverse kinematics and writes those solutions into `q`.

   Hints:

   - Make sure that $\theta_i \in [0, 2\pi)$ for $i = 1, ..., 6$. In other words:
     - If $\theta_i < 0$ and $\theta_i \approx 0$, $\theta_i \leftarrow 0$.
     - Otherwise if $\theta_i < 0$, $\theta_i \leftarrow \theta_i + 2\pi$.
   - Remember that $\operatorname{atan2}(y, x) \neq \operatorname{atan2}(-y, -x)$

2. Write a function `Eigen::Matrix4f J(double q[6])` that accepts the vector of joint positions and returns the Jacobian matrix of the UR5.

3. For 3 different θ vectors, do the following:

   (a) Choose two different 6×1 $\delta\theta$ vectors where each is a vector containing of small joint angles (i.e. a $\pm$few degrees). For each $\delta\theta$ vector, do the following:

      i. Using `UR5::fwd(double q[])` calculate $H_6^0(\theta)$ and $H_6^0(\theta + \delta\theta)$

      ii. Calculate the Jacobian using `Eigen::Matrix4f J(double q[6])`.

      iii. Calculate $P_6^0(\theta) - P_6^0(\theta + \delta\theta)$ and $J_v\delta\theta$. Compare these two values.

      iv. Calculate $A = \left(R_6^0(\theta + \delta\theta) - R_6^0(\theta)\right) R_6^0(\theta)^T$ and $B = \widehat{J_\omega \delta\theta}$ Calculate the difference between these two matrices using the Frobenius norm $||A - B||_F^2 = \operatorname{Tr}((A - B)^T (A - B))$.

   Note: 2 tests per each vector θ results in 6 tests total.

4. What do your answers to Question 3 tell you about the accuracy of your implementation of `Eigen::Matrix4f J(double q[6])`?

Submission Guidelines

As with both previous labs, you will push your code to your git repository to submit the assignment. In addition to the code, please include a pdf titled `Lab4.pdf`. This document should include the answers to questions posed in the lab and also annotated screenshots that show you experimented with the functions you implemented.