CS 613 - Machine Learning

Assignment 3 - Classification
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1 Theory

1. Consider the following set of training examples for an unknown target function: $(x_1, x_2) \rightarrow y$:

Y	x_1	x_2	Count
+	T	T	3
+	T	F	4
+	F	T	4
+	F	F	1
-	T	T	0
-	T	F	1
-	F	T	3
-	F	F	5

   (a) What is the sample entropy for the class label overall, $H(Y)$ from this training data (using log base 2) (3pts)?

   Number of Positive Samples: 3 + 4 + 4 + 1 = 12
   Number of Negative Samples: 0 + 1 + 3 + 5 = 9
   Total Number of Samples: 12 + 9 = 21

$$
\begin{aligned}
H(Y) = H(\frac{positive}{positive + negative}, \frac{negative}{positive + negative}) = H(\frac{12}{12+9}, \frac{9}{12+9}) \\
&= (\frac{12}{21} * log_2(\frac{12}{21})) + (\frac{9}{21} * log_2(\frac{9}{21})) \\
&= (-0.57 * log_2(0.57)) + (-0.43 * log_2(0.43)) \\
&= (-0.57 * -0.81) + (-0.43 * -1.22) \\
&= (0.46) + (0.52) \\
&= 0.46 + 0.52 \\
&= 0.98
\end{aligned}
$$

   (b) What are the weighed average entropies for branching on variables x_1 and x_2 (4pts)?

      i. Get the Total Samples = 21

ii. Get the counts of each class when split on x_1

$$\begin{aligned} Positive_T &= 3+4=7 & Positive_F &= 4+1=5 \\ Negative_T &= 0+1=1 & Negative_F &= 3+5=8 \\ Total_T &= 7+1=8 & Total_F &= 5+8=13 \end{aligned}$$

iii. Calculate Weighted Average Entropy (or Information Gain) for x_1

$$\begin{aligned} IG(x_1) &= \frac{Total_T}{Total_S} * H(\frac{Positive_T}{Total_T}, \frac{Negative_T}{Total_T}) + \frac{Total_F}{Total_S} * H(\frac{Positive_F}{Total_F}, \frac{Negative_F}{Total_F}) \\ &= \frac{8}{21} * H(\frac{7}{8}, \frac{1}{8}) + \frac{13}{21} * H(\frac{5}{13}, \frac{8}{13}) \\ &= \frac{8}{21} * (-\frac{7}{8} * log_2(\frac{7}{8}) - \frac{1}{8} * log_2(\frac{1}{8})) + \frac{13}{21} * (-\frac{5}{13} * log_2(\frac{5}{13}) - \frac{8}{13} * log_2(\frac{8}{13})) \\ &= 0.38 * ((-0.875 * -0.19) - (0.125 * 0.32)) + 0.62 * ((-0.38 * -1.4)) - (0.62 * -0.69)) \\ &= 0.38 * (0.17 + 0.4) + 0.62 * (0.53 + 0.43) \\ &= 0.38 * (0.57) + 0.62 * (0.95) \\ &= 0.21 + 0.59 = 0.8 \end{aligned}$$

Information Gain x_1 = Total Entropy - Feature Entropy = 0.98 - 0.8 = 0.13

iv. Get the counts of each class when split on x_2

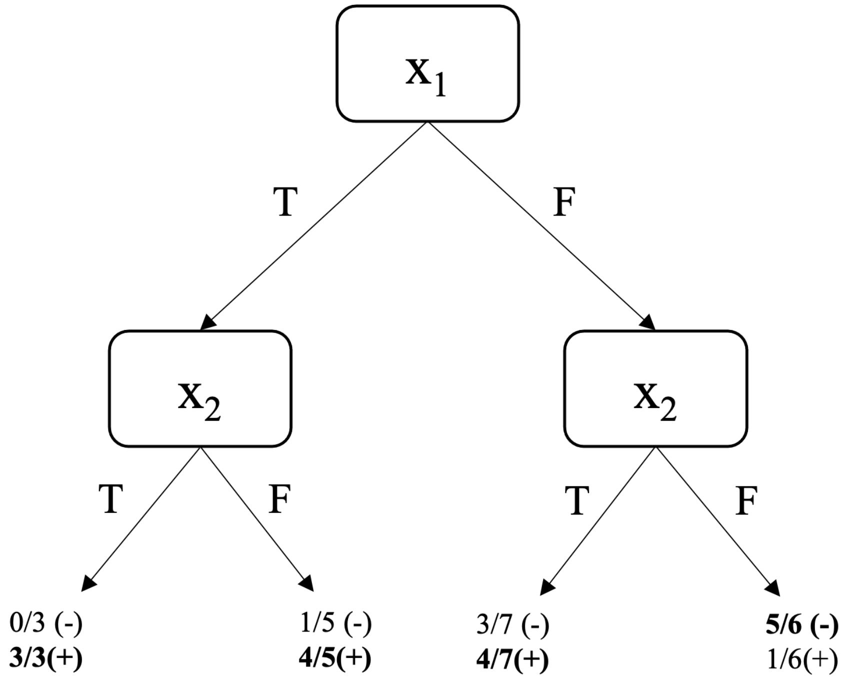
$$\begin{aligned} Positive_T &= 3+4=7 & Positive_F &= 4+1=5 \\ Negative_T &= 0+3=3 & Negative_F &= 1+5=6 \\ Total_T &= 7+3=10 & Total_F &= 5+6=11 \end{aligned}$$

v. Calculate Weighted Average Entropy (or Information Gain) for x_2

$$\begin{aligned} IG(x_2) &= \frac{Total_T}{Total_S} * H(\frac{Positive_T}{Total_T}, \frac{Negative_T}{Total_T}) + \frac{Total_F}{Total_S} * H(\frac{Positive_F}{Total_F}, \frac{Negative_F}{Total_F}) \\ &= \frac{10}{21} * H(\frac{7}{10}, \frac{3}{10}) + \frac{11}{21} * H(\frac{5}{11}, \frac{6}{11}) \\ &= \frac{10}{21} * (-\frac{7}{10} * log_2(\frac{7}{10}) - \frac{3}{10} * log_2(\frac{3}{10})) + \frac{11}{21} * (-\frac{5}{11} * log_2(\frac{5}{11}) - \frac{6}{11} * log_2(\frac{6}{11})) \\ &= 0.48 * ((-0.7 * -0.51) - (0.3 * -1.74)) + 0.52 * ((-0.45 * -1.12)) - (0.54 * -0.89)) \\ &= 0.48 * (0.36 + 0.52) + 0.52 * (0.5 + 0.48) \\ &= 0.48 * (0.88) + 0.52 * (0.98) \\ &= 0.42 + 0.51 = 0.93 \end{aligned}$$

Information Gain x_2 = Total Entropy - Feature Entropy = 0.98 - 0.93 = 0.05

(c) Draw the decision tree that would be learned by the ID3 algorithm without pruning from this training data. You may use software to draw this or draw it by hand. But either way the figure should be embedded in your PDF submission. (5pts)



2. We decided that maybe we can use the number of characters and the average word length an essay to determine if the student should get an A in a class or not. Below are five samples of this data:

# of Chars	Average Word Length	Give an A
216	5.68	Yes
69	4.78	Yes
302	2.31	No
60	3.16	Yes
393	4.2	No

(a) What are the class priors, $P(A = Yes), P(A = No)$? (3pts)

Number of Yes Samples = 3
Number of No Samples = 2
Total Number of Samples: 3 + 2 = 5

$$P(A = Yes) = P(\frac{Yes}{Yes + No}) = P(\frac{3}{3+2}) = P(\frac{3}{5}) = 0.6$$
$$P(A = No) = P(\frac{No}{Yes + No}) = P(\frac{2}{3+2}) = P(\frac{2}{5}) = 0.4$$

(b) Find the parameters of the Gaussians necessary to do Gaussian Naive Bayes classification on this decision to give an A or not. Zscore the features first over all the data together so that there is no unfair bias towards the features of different scales (5pts).

i. Defines the Training Features X = $\begin{bmatrix} 216 & 5.68 \\ 69 & 4.78 \\ 302 & 2.31 \\ 60 & 3.16 \\ 393 & 4.2 \end{bmatrix}$

ii. Calculate the Feature Means and Standard Deviations

$$\mu = \begin{bmatrix} 208 & 4.03 \end{bmatrix}$$
$$\sigma = \begin{bmatrix} 145.22 & 1.33 \end{bmatrix}$$

iii. Z-Score our Training Data

$$Features_{zscored} = \begin{bmatrix} 216 & 5.68 \\ 69 & 4.78 \\ 302 & 2.31 \\ 60 & 3.16 \\ 393 & 4.2 \end{bmatrix} - \begin{bmatrix} 208 & 4.03 \end{bmatrix} / \begin{bmatrix} 145.22 & 1.33 \end{bmatrix}$$

$$= \begin{bmatrix} 0.06 & 1.24 \\ -0.96 & 0.56 \\ 0.65 & -1.29 \\ -1.02 & -0.65 \\ 1.27 & 0.13 \end{bmatrix}$$

iv. Calculate the Mean and Standard Deviation for Class "Yes"

$$\mu_{feature1} = \frac{0.06 - 0.96 - 1.02}{3} = \frac{-1.92}{3} = -0.64$$

$$\begin{aligned} \sigma_{feature1} &= \frac{(0.06 - (-0.64))^2 + (-0.96 - (-0.64))^2 + (-1.02 - (-0.64))^2}{3-1} \\ &= \frac{(0.06 + 0.64)^2 + (-0.96 + 0.64)^2 + (-1.02 + 0.64)^2}{2} \\ &= \frac{(0.7)^2 + (-0.32)^2 + (-0.38)^2}{2} \\ &= \frac{(0.49) + (0.1) + (0.14)}{2} \\ &= \frac{0.73}{2} \\ &= 0.37 \end{aligned}$$

$$\mu_{feature2} = \frac{1.24 + 0.56 - 0.65}{3} = \frac{1.15}{3} = 0.38$$

$$
\begin{aligned}
\sigma_{feature2} &= \frac{(1.24-0.38)^2+(0.56-0.38)^2+(-0.65-0.38)^2}{3-1} \\
&= \frac{(0.86)^2+(0.18)^2+(-1.03)^2}{2} \\
&= \frac{(0.74)+(0.03)+(1.06)}{2} \\
&= \frac{1.83}{2} \\
&= 0.92
\end{aligned}
$$

v. Calculate the Mean and Standard Deviation for Class "No"

$$
\mu_{feature1} = \frac{0.65+1.27}{2} = \frac{1.92}{2} = 0.96
$$

$$
\begin{aligned}
\sigma_{feature1} &= \frac{(0.65-0.96)^2+(1.27-0.96)^2}{2-1} \\
&= \frac{(-0.31)^2+(0.31)^2}{1} \\
&= \frac{(0.1)+(0.1)}{1} \\
&= \frac{0.2}{1} \\
&= 0.2
\end{aligned}
$$

$$
\mu_{feature2} = \frac{-1.29+0.13}{2} = \frac{-1.16}{2} = -0.58
$$

$$
\begin{aligned}
\sigma_{feature2} &= \frac{(-1.29-(-0.58))^2+(0.13-(-0.58))^2}{2-1} \\
&= \frac{(-1.29+0.58)^2+(0.13+0.58)^2}{2-1} \\
&= \frac{(-0.71)^2+(0.71)^2}{1} \\
&= \frac{(0.5)+(0.5)}{1} \\
&= \frac{1}{1} \\
&= 1
\end{aligned}
$$

vi. Calculated Gaussian Parameters

$$
\mu_Y = \begin{bmatrix} -0.64 & 0.38 \end{bmatrix}
$$

$$
\sigma_Y = \begin{bmatrix} 0.37 & 0.92 \end{bmatrix}
$$

$$\mu_N = \begin{bmatrix} 0.96 & -0.58 \end{bmatrix}$$

$$\sigma_N = \begin{bmatrix} 0.2 & 1 \end{bmatrix}$$

(c) Using your response from the prior question, determine if an essay with 242 characters and an average word length of 4.56 should get an A or not. Show the computations to support your answer. (5pts).

i. Define Validation Features (X) = $\begin{bmatrix} 242 & 4.56 \end{bmatrix}$

ii. Z-Score with Feature Training Data Mean and Standard Deviation:

$$\begin{aligned} Features_{zscored} &= \begin{bmatrix} 242 & 4.56 \end{bmatrix} - \begin{bmatrix} 208 & 4.03 \end{bmatrix} / \begin{bmatrix} 145.22 & 1.33 \end{bmatrix} \\ &= \begin{bmatrix} (242 - 208) & (4.56 - 4.03) \end{bmatrix} / \begin{bmatrix} 145.22 & 1.33 \end{bmatrix} \\ &= \begin{bmatrix} \frac{34}{145.22} & \frac{0.53}{1.33} \end{bmatrix} \\ &= \begin{bmatrix} 0.23 & 0.4 \end{bmatrix} \end{aligned}$$

iii. Determine if an essay with 242 characters and an average word length of 4.56 should get an A or not:

$$\begin{aligned} P(A = Y | F_1 = 0.23, F_2 = 0.4) &= P(A = Y) \times F_1 = 0.23 \times F_2 = 0.4 \\ &= 0.6 \frac{1}{\sigma_{1Y}\sqrt{2\pi}} e^{-\frac{(F_1 - \mu_{1Y})^2}{2(\sigma_{1Y})^2}} \frac{1}{\sigma_{2Y}\sqrt{2\pi}} e^{-\frac{(F_2 - \mu_{2Y})^2}{2(\sigma_{2Y})^2}} \\ &= 0.6 \frac{1}{0.37\sqrt{2\pi}} e^{-\frac{(0.23-(-0.64))^2}{2(0.37)^2}} \frac{1}{0.92\sqrt{2\pi}} e^{-\frac{(0.4-0.38)^2}{2(0.92)^2}} \\ &= 0.6 \frac{1}{0.37\sqrt{2\pi}} e^{-\frac{(0.23+0.64)^2}{2(0.37)^2}} \frac{1}{0.92\sqrt{2\pi}} e^{-\frac{(0.4-0.38)^2}{2(0.92)^2}} \\ &= 0.6 \frac{1}{0.37\sqrt{2\pi}} e^{-\frac{(0.87)^2}{2(0.1369)}} \frac{1}{0.92\sqrt{2\pi}} e^{-\frac{(0.2)^2}{2(0.8464)}} \\ &= 0.6 \frac{1}{0.37(2.51)} e^{-\frac{0.76}{0.27}} \frac{1}{0.92(2.51)} e^{-\frac{0.02}{1.69}} \\ &= 0.6 \frac{1}{0.93}(0.05) \frac{1}{2.31}(0.98) \\ &= 0.6 \times (1.08 \times 0.05) \times (0.43 \times 0.98) \\ &= 0.6 \times 0.54 \times 0.42 \\ &= 0.13 \end{aligned}$$

iv. Conclusion - Based on the the validation data input into the Gaussian Probability Density Function, there is a 13% probability that the essay with 242 characters and an average word length of 4.56 will receive an A.

2 Naive Bayes Classifier

1. Validation Statistics

    - Precision: 0.655373831775701
    - Recall: 0.9739583333333334
    - F-Measure: 0.7835195530726258
    - Accuracy: 0.7979139504563233

3 Decision Trees

1. Validation Statistics

    - Precision: 0.8317307692307693
    - Recall: 0.9010416666666666
    - F-Measure: 0.8650000000000001
    - Accuracy: 0.894393741851369

4 Additional Evaluation

1. Multi-Class Naive Bayes

    - Accuracy: 0.8166431593794076

2. Multi-Class Decision Trees

    - Accuracy: 0.8998589562764456