CSDS 452: Causality and Machine Learning

Lecture 4: Identification & Estimation

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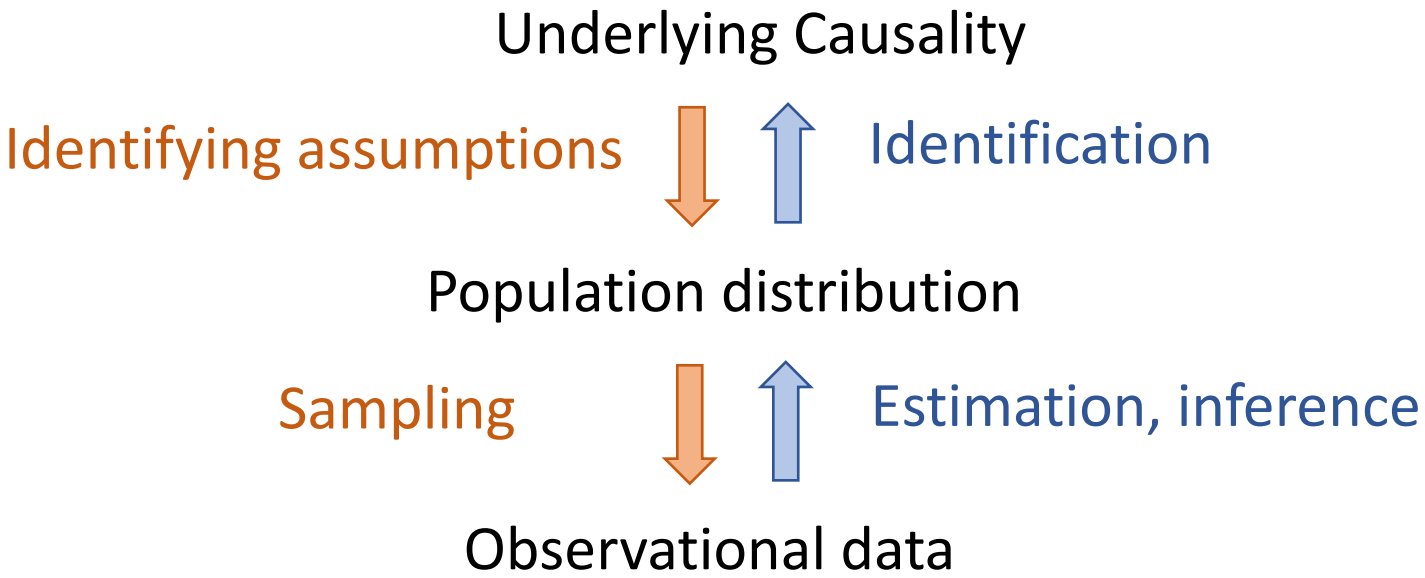
Fall 2024, CDS@CWRU

Outline

- Identifiability
  - Randomized experiments
  - Front door
  - Do-calculus
- Classical causal effect estimation methods
  - Re-weighting methods
  - Stratification methods
  - Matching methods
  - Tree-based methods

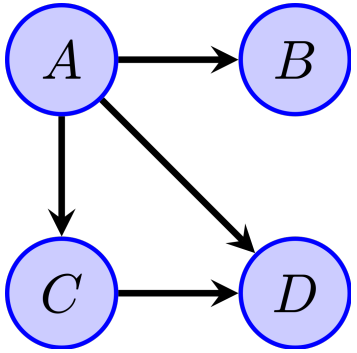
Recap: Identification and Estimation

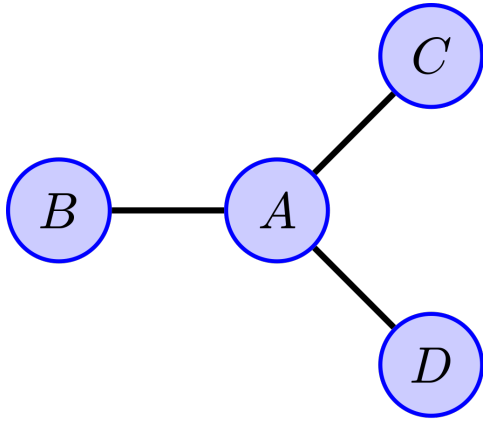
- Two components in learning causality
  - (1) Identification
  - (2) Estimation, inference



Recap: Graph Directionality

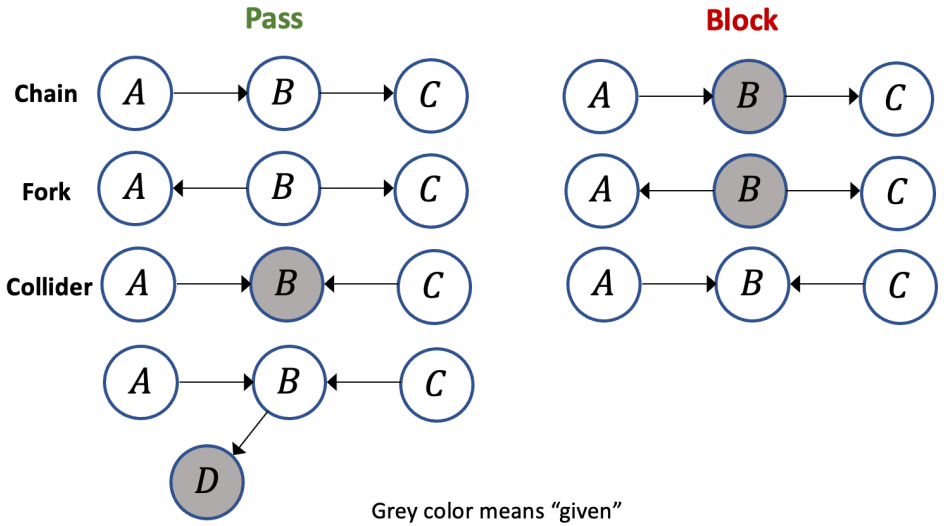
- Directed graphical models
  - Direction in edges
  - Bayesian networks
  - More popular in AI and statistics
- Undirected graphical models
  - Edges without direction
  - Markov random fields (MRFs)
    - Better suited to express soft constraints between variables
  - More popular in Vision and Physics



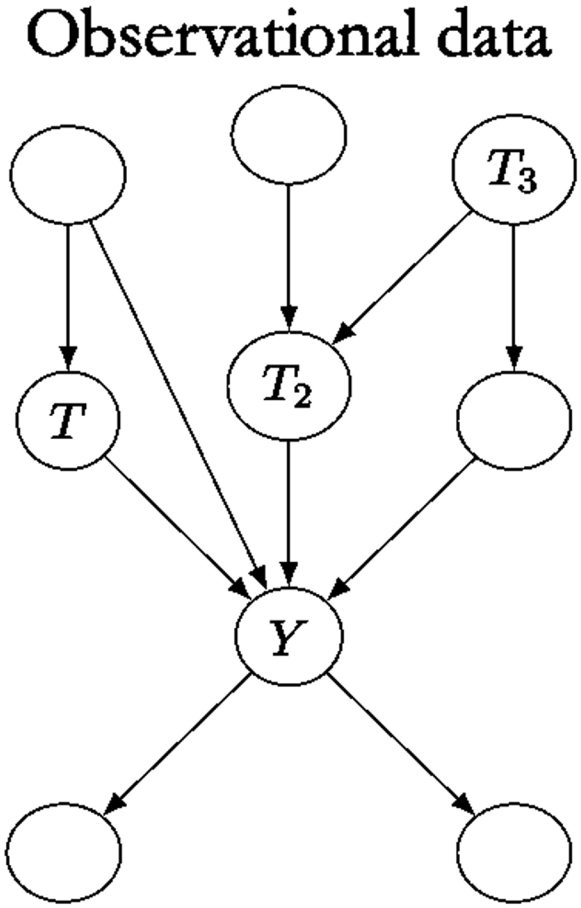


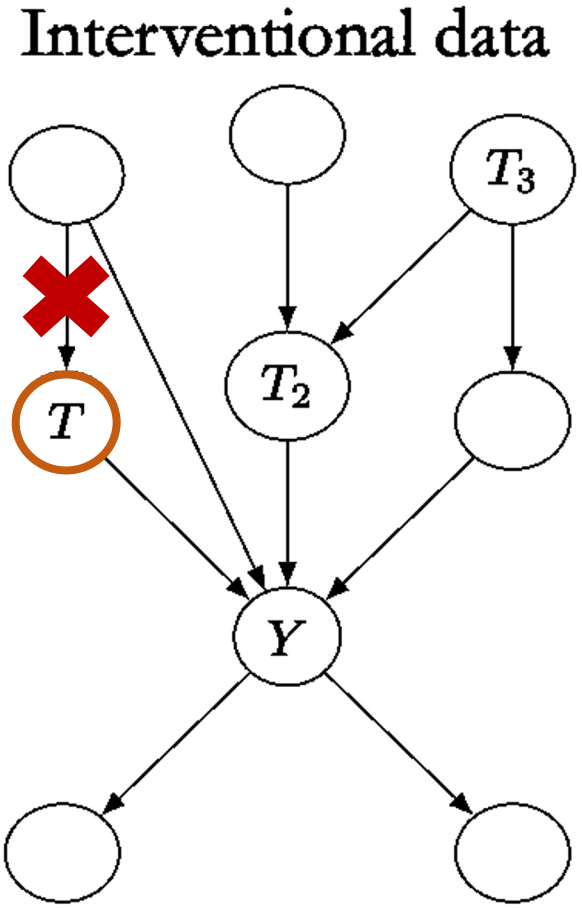
Recap: Blocked Paths

- A path between nodes X and Y is blocked by a (potentially empty) conditioning set Z if either of the following is true:
  - Along the path, there is a chain … $\rightarrow W \rightarrow \cdots$ or a fork … $\leftarrow W \rightarrow \cdots$ where W is conditioned on ($W \in Z$).
  - There is a collider W on the path that is not conditioned on ($W \notin Z$) and none of its descendants are conditioned on ($\mathrm{des}(W) \notin Z$).



Recap: Observation v.s. Intervention





$\boldsymbol{M}$: $T \coloneqq f_T(X, U_T)$

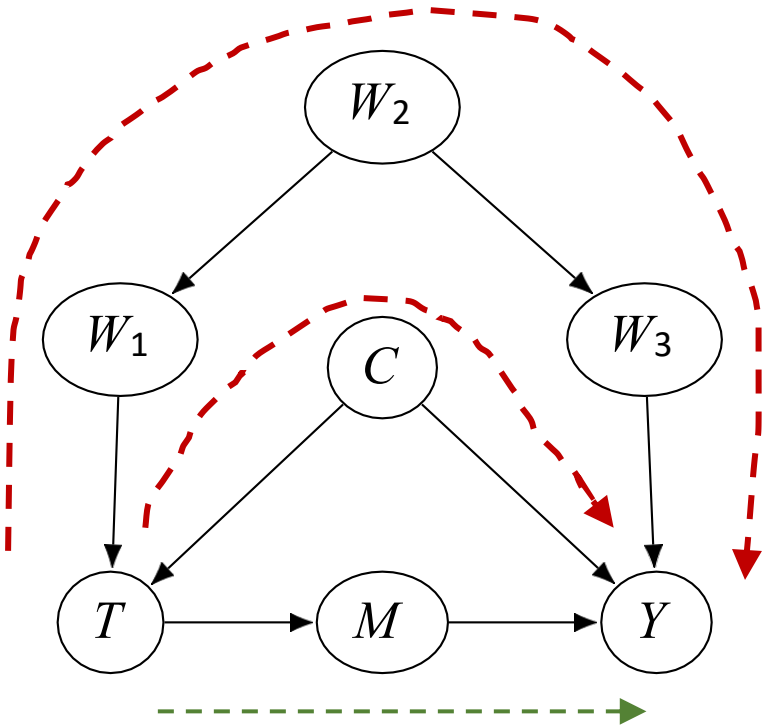
$Y \coloneqq f_Y(X, T, U_Y)$

$\boldsymbol{M_t}$: $T \coloneqq t$

$Y \coloneqq f_Y(X, T, U_Y)$

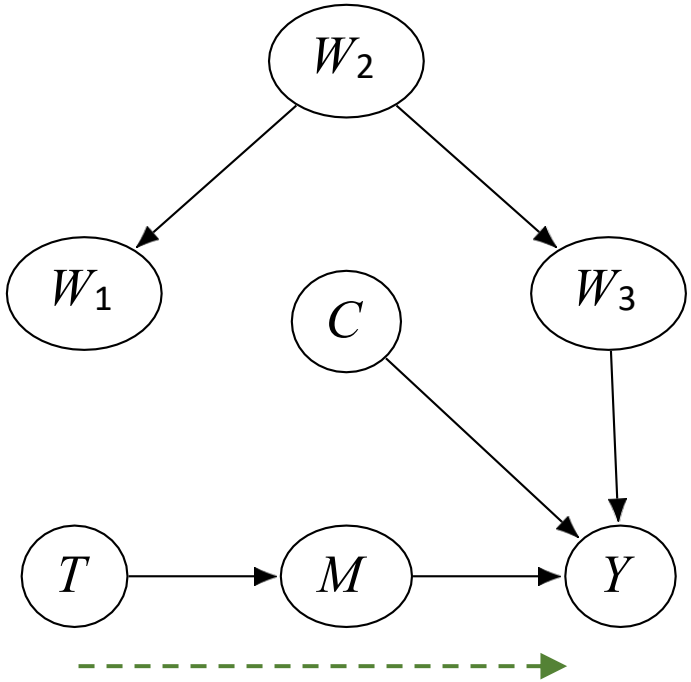
Recap: Backdoor Paths

$P(Y|t)$



Causal association

$P(Y|do(t))$



Causal association

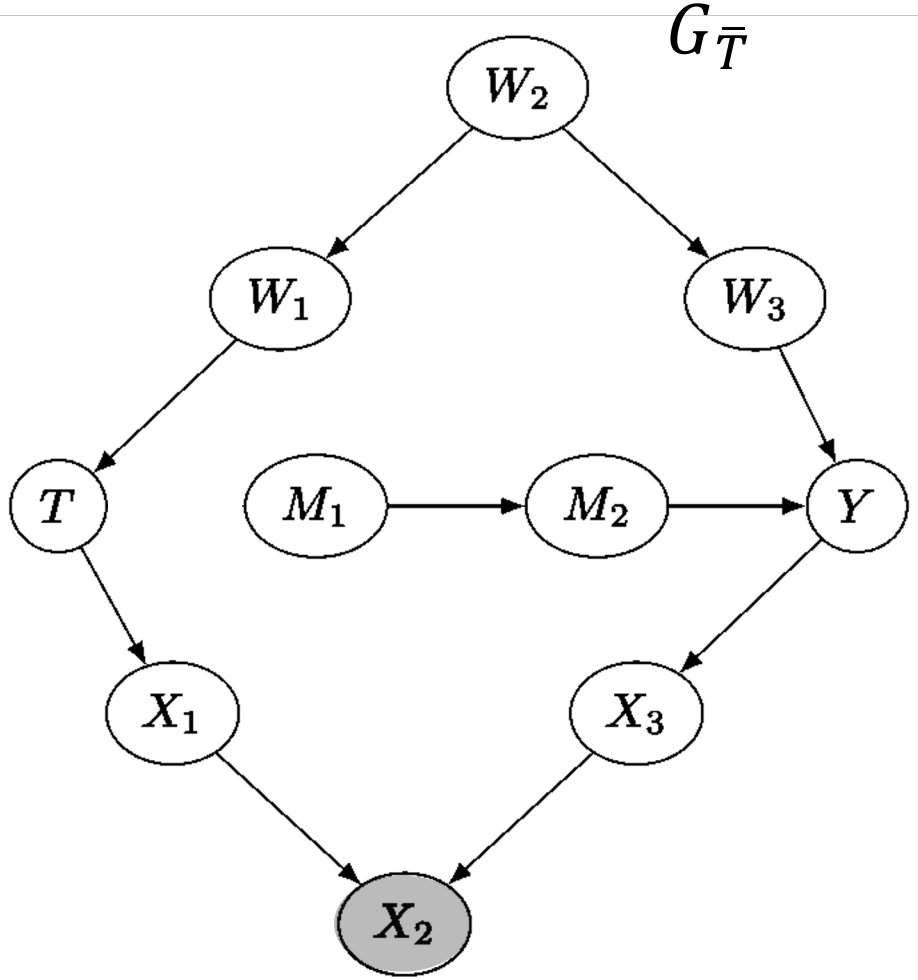
Recap: Backdoor criterion and backdoor adjustment

- A set of variables W satisfies the **backdoor criterion** relative to T and Y if the following are true:
  - W blocks all backdoor paths from T to Y
  - W does not contain any descendants of T

- Given the modularity assumption and that W satisfies the backdoor criterion, we can identify the causal effect of T on Y:

$$P(y|do(t)) = \sum_w P(w)P(y|t,w)$$

Recap: Backdoor criterion as d-separation



- W blocks all backdoor paths from T to Y
- W does not contain any descendants of T

$$Y \perp\!\!\!\perp_{G_{\bar{T}}} T \mid W$$

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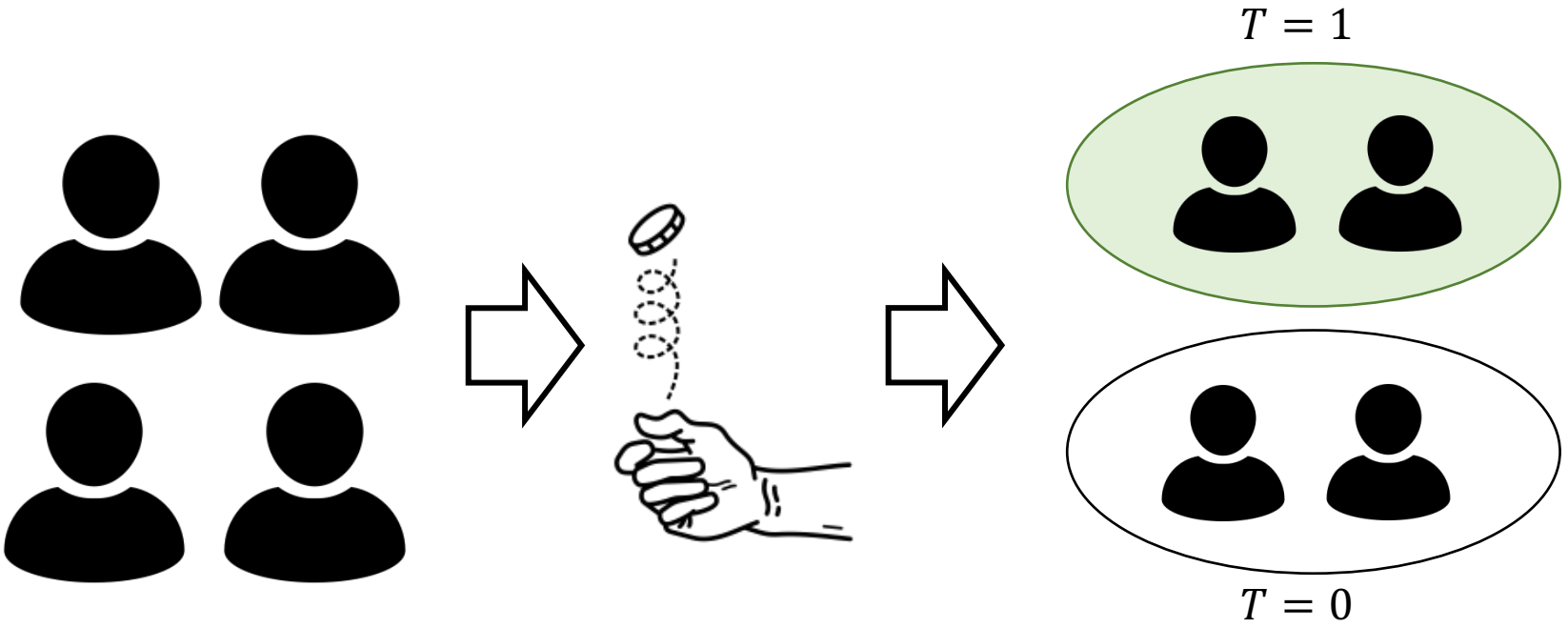
Randomized Experiments



- Gold standard to assess the causal effect.
- The allocation of the treatment is under control. The distribution of the covariates for treated and control patients is balanced.

Few different perspectives on RCT

- Comparability and covariate balance
- Exchangeability
- No backdoor paths

Comparability and covariate balance

- Treatment and control groups are the same in all aspects except treatment
- Covariate balance: the distribution of covariates X is the same across treatment groups.

$$P(X|T = 1) = P(X|T = 0)$$

Comparability and covariate balance

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Randomization => covariate balance

$$\mathrm{P}(X) = P(X|T=1) = P(X|T=0)$$

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Covariate balance => association is causation

$$P(y|do(t))$$

$$P(y|t)$$

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Covariate balance => association is causation

$$P(y|do(t)) = \sum_x P(y|t,x)P(x)$$

$P(y|t)$

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Covariate balance => association is causation

$$P(y|do(t)) = \sum_x P(y|t,x)P(x) = \sum_x \frac{P(y|t,x)P(t|x)P(x)}{P(t|x)}$$

$$P(y|t)$$

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$$= \sum_x \frac{P(y,t,x)}{P(t|x)}$$

$$P(y|t)$$

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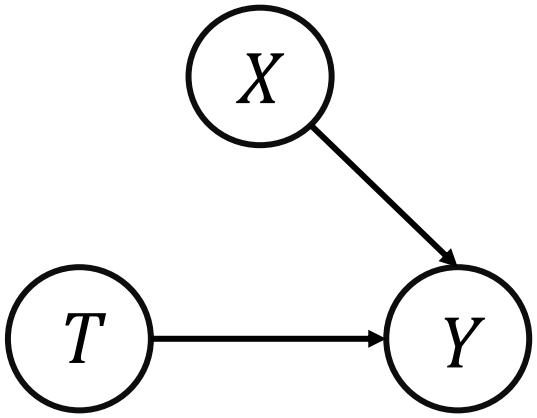
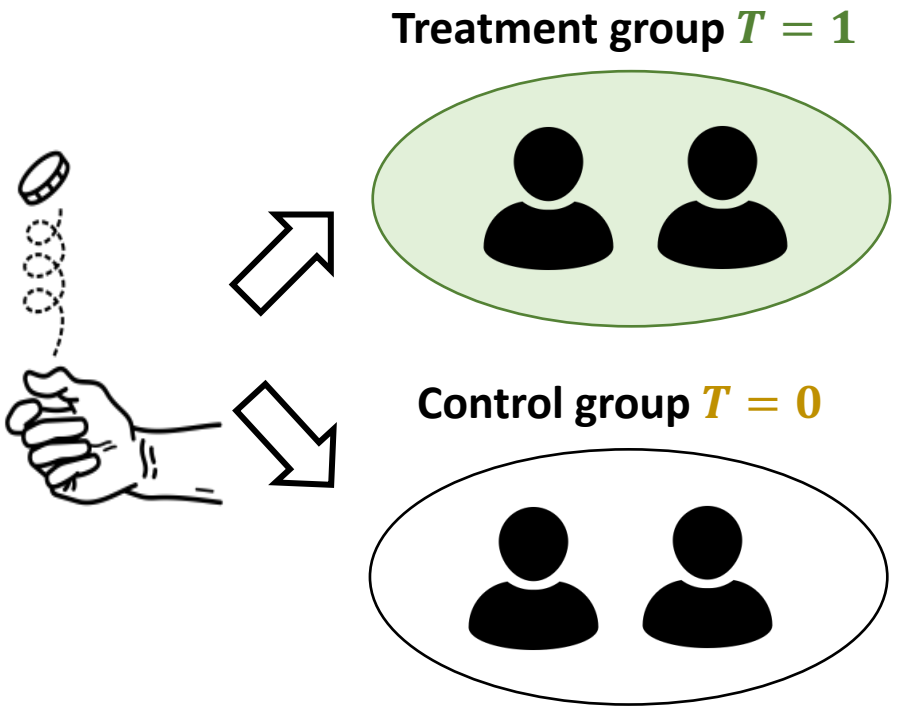
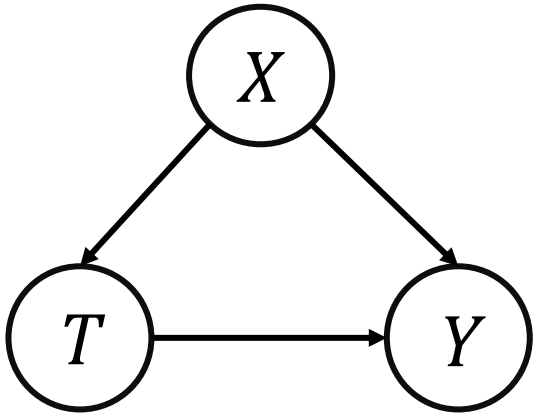
Exchangeability

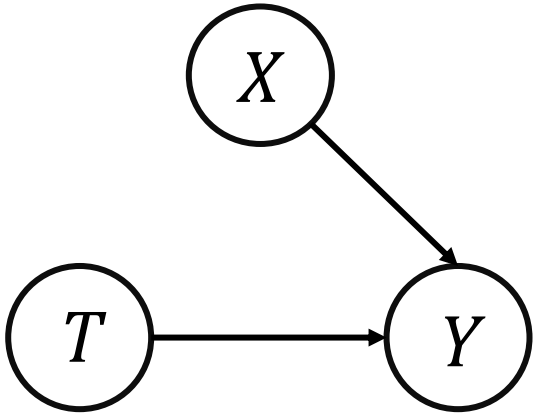
- $\big(Y(1), Y(0)\big) \perp\!\!\!\perp T$
- T is randomly assigned, will not change $Y(1), Y(0)$





No backdoor paths

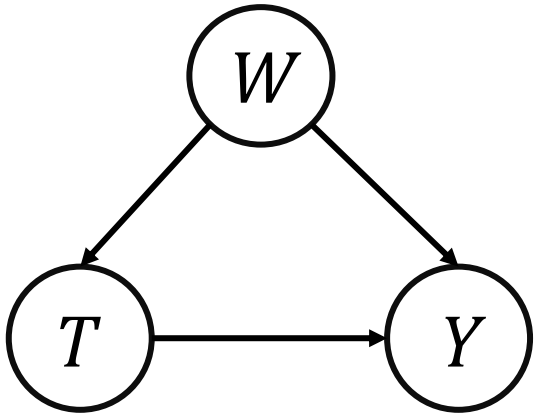




Recall Backdoor Adjustment

- Backdoor adjustment:

$$P(y|do(t)) = \sum_w P(w)P(y|t,w)$$



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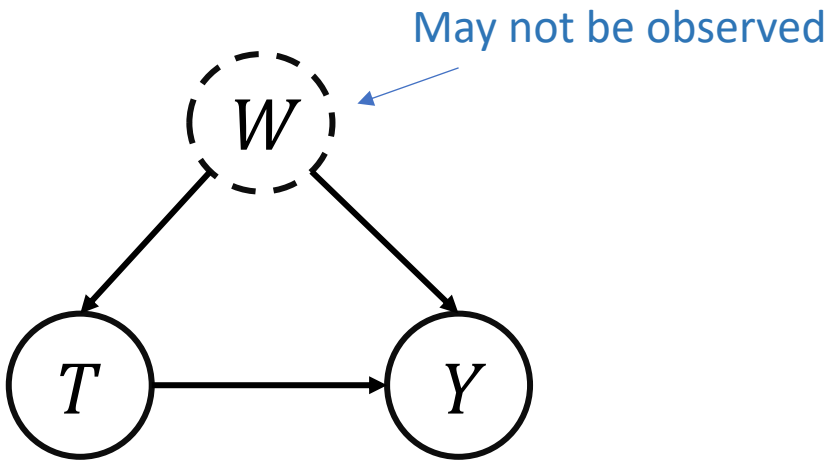

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Frontdoor Adjustment

- Backdoor adjustment:

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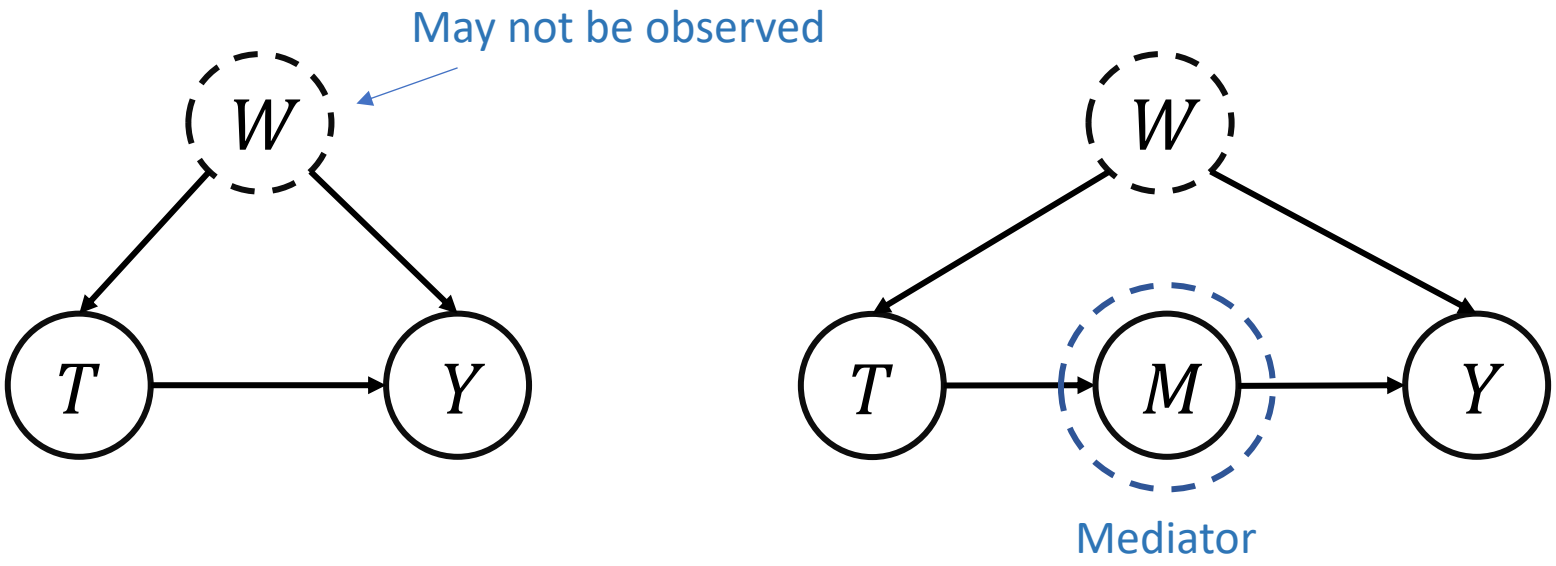
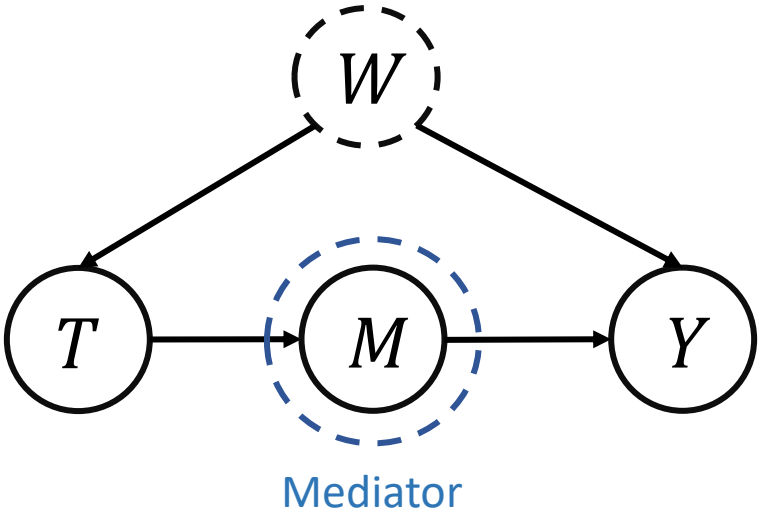


Frontdoor Adjustment

- Step 1. Identify the causal effect of T on M
- Step 2. Identify the causal effect of M on Y
- Step 3. Based on the above two steps, identify the causal effect of T on Y



Frontdoor Adjustment

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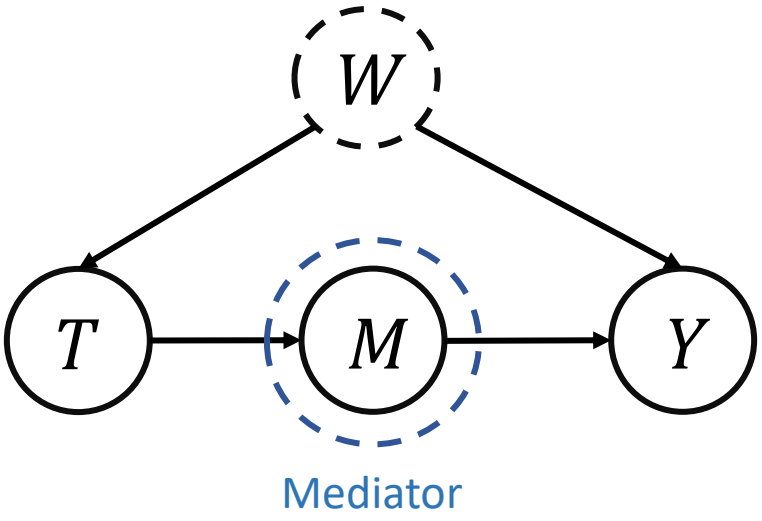
$P(m|do(t))$



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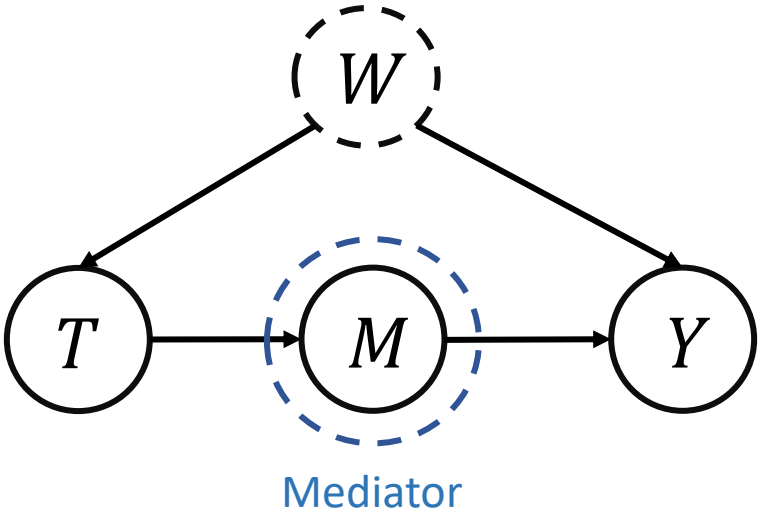
$$P(m|do(t)) = P(m|t)$$



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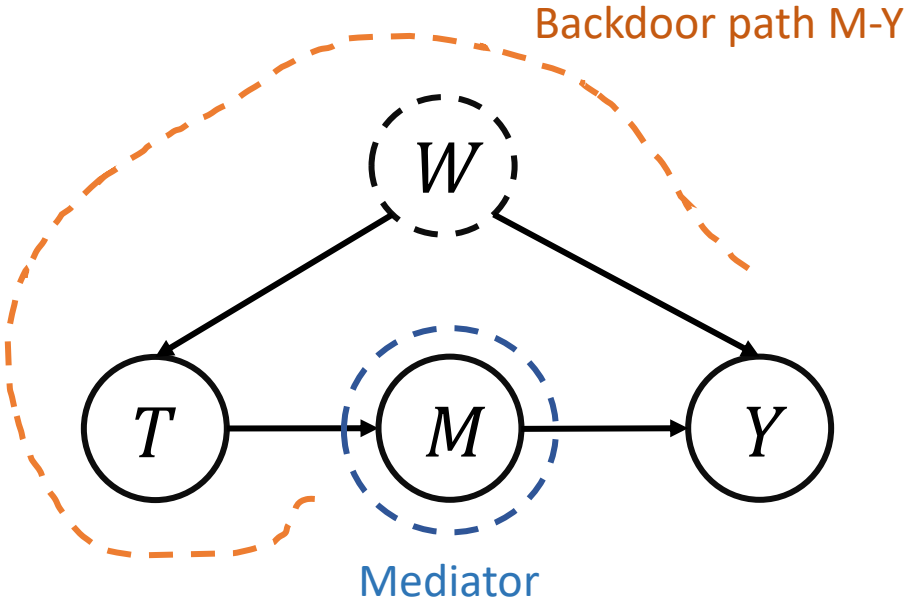
$P(y|do(m))$



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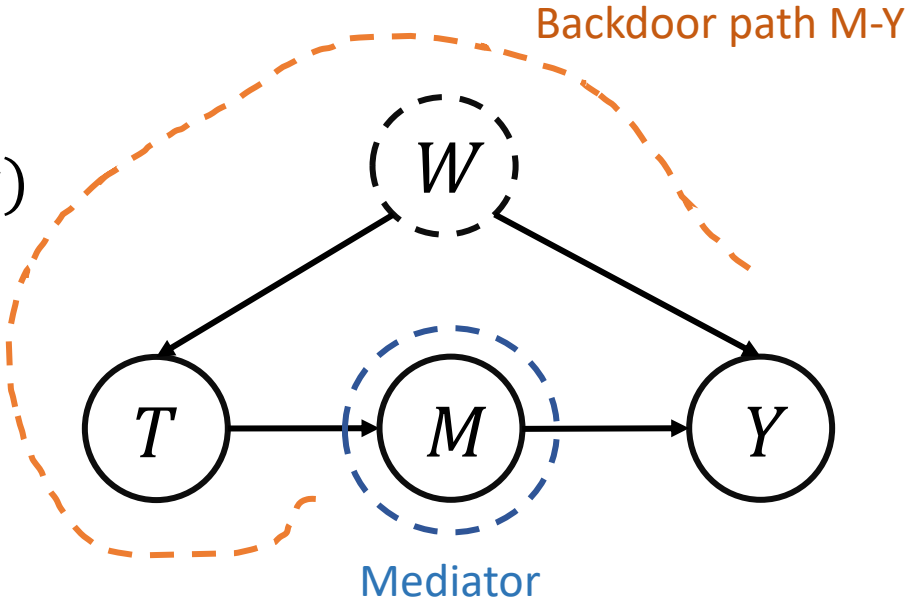

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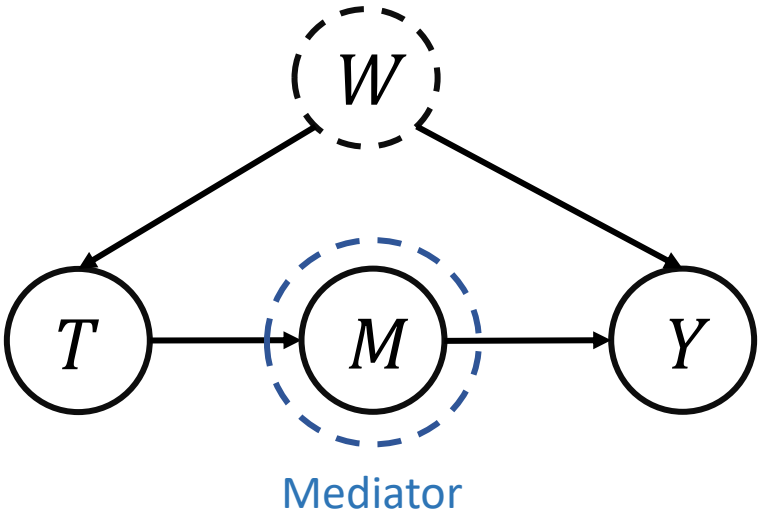
$P(y|do(t))$



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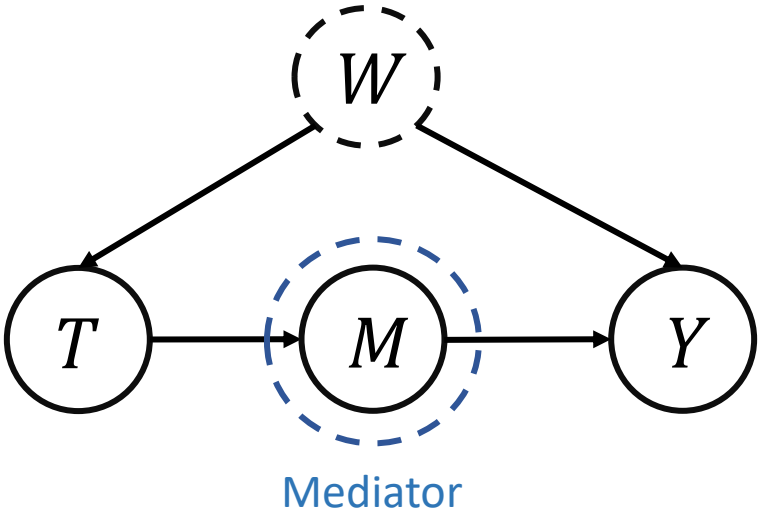
$$P(y|do(t))$$

$$P(m|do(t))P(y|do(m))$$



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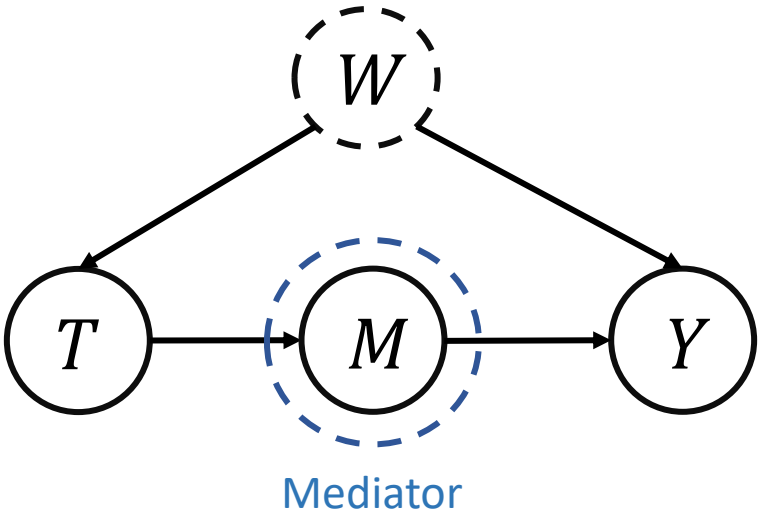
$$P(y|do(t)) = \sum_{m} P(m|do(t))P(y|do(m))$$



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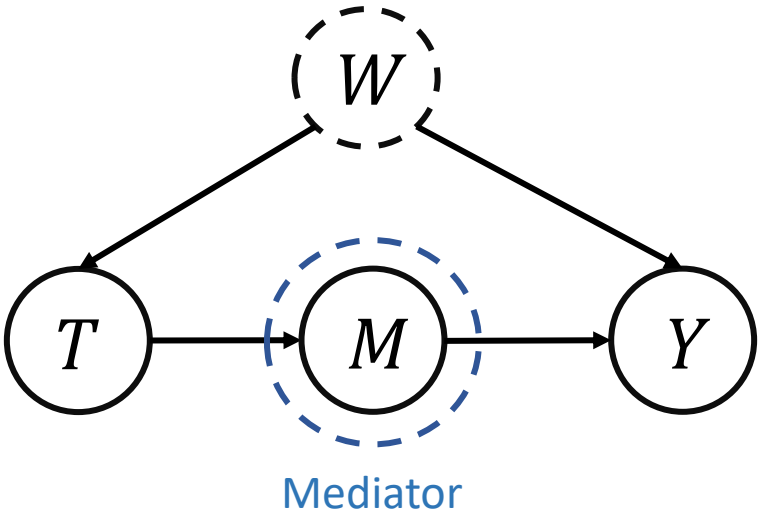
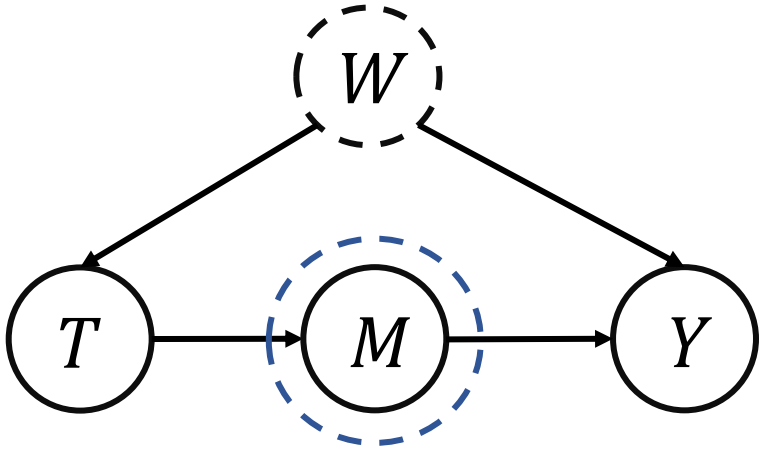
$$P(y|do(t))$$
$$= \sum_m P(m|do(t))P(y|do(m))$$
$$= \sum_m P(m|t) \sum_{t'} P(y|m,t')P(t')$$



Frontdoor Criterion

- A set of variables M satisfies the frontdoor criterion relative to T and Y if the following are true:
  - 1. M completely mediates the effect of T on Y (i.e., all causal paths from T to Y go through M).
  - There is no unblocked backdoor path from T to M.
  - All backdoor paths from M to Y are blocked by T.

$$P(y|do(t))$$
$$= \sum_m P(m|do(t))P(y|do(m))$$
$$= \sum_m P(m|t) \sum_{t'} P(y|m,t')P(t')$$



Can the causal effect be identified if neither the backdoor criterion nor the frontdoor criterion is satisfied?

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Do-calculus

- Question: how to identify causal quantities (with do operator)?
- The do-calculus is an axiomatic system for replacing probability formulas containing the do operator with ordinary conditional probabilities.
- 3 rules
  - Not limited to backdoor or frontdoor criterion, but the rules of do-calculus can be used to prove backdoor/frontdoor.

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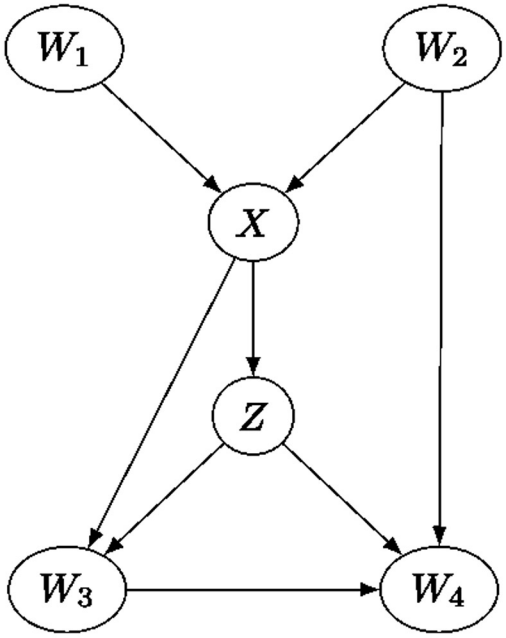
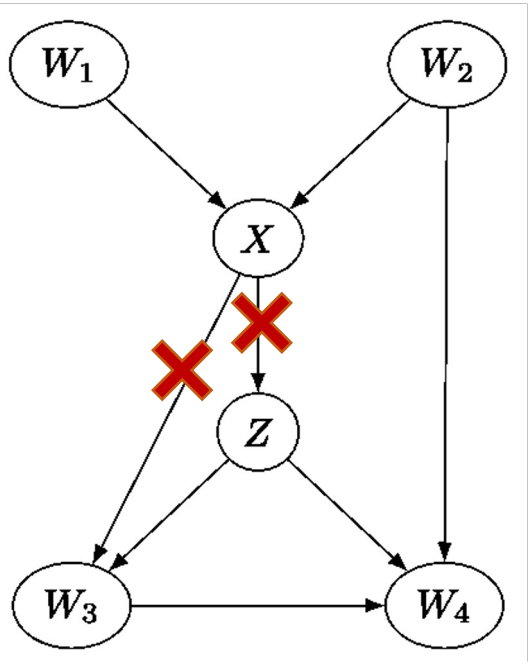
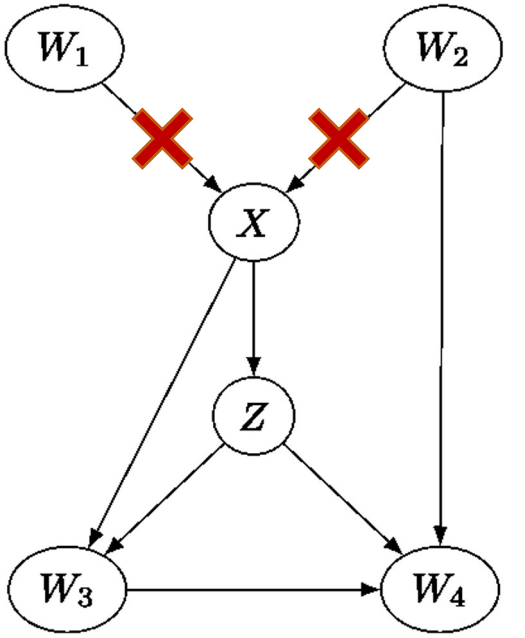


$G_{\underline{X}}$



$G_{\overline{X}}$



Do-calculus: Rule 1

- If $Y \perp\!\!\!\perp_{G_{\overline{T}}} Z | T, W$:

$$P(y|do(t), z, w) = P(y|do(t), w)$$

Do-calculus: Rule 1

- If $Y \perp\!\!\!\perp_{G_{\overline{T}}} Z \mid T, W$:

$$P(y|do(t), z, w) = P(y|do(t), w)$$

Interventional distribution w.r.t. $do(t)$

Do-calculus: Rule 1

- If $Y \perp\!\!\!\perp_{G_{\overline{T}}} Z | T, W$:

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Interventional distribution w.r.t. $do(t)$

When we remove $do(t)$:

- If $Y \perp\!\!\!\perp_{G} Z | W$:

D-separation

$$P(y|do(t), z, w) = P(y|do(t), w)$$

- Rule 1 is a generalization of d-separation to interventional distribution

Do-calculus: Rule 2

- If $Y \perp\!\!\!\perp_{G_{\overline{T},\underline{Z}}} Z | T, W$:

$$P(y|do(t), do(z), w) = P(y|do(t), z, w)$$

Do-calculus: Rule 2

- If $Y \perp\!\!\!\perp_{G_{\overline{T},\underline{Z}}} Z | T, W$:

$$P(y|do(t), do(z), w) = P(y|do(t), z, w)$$

When we remove $do(t)$:

- If $Y \perp\!\!\!\perp_{G_{\underline{Z}}} Z | W$:

$$P(y|do(t), do(z), w) = P(y|do(t), z, w)$$

- Rule 2 is a generalization of backdoor adjustment/criterion

Do-calculus: Rule 3

- If $Y \perp\!\!\!\perp_{G_{\bar{T},\overline{Z(W)}}} Z \mid T, W$

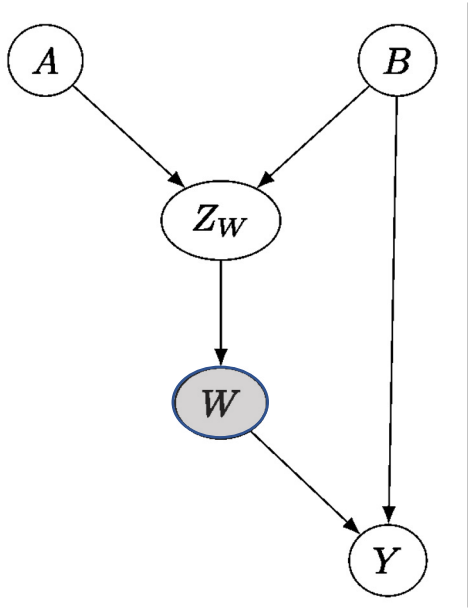
$$P(y|do(t), do(z), w) = P(y|do(t), w)$$

$Z(W)$: the set of nodes of Z that aren't ancestors of any node of W in $G_{\bar{T}}$

When we remove $do(t)$:

- If $Y \perp\!\!\!\perp_{G_{\overline{Z(W)}}} Z \mid T, W$

$$P(y|do(t), do(z), w) = P(y|do(t), w)$$



3 Rules in do-calculus

- **Rule 1:** If $Y \perp\!\!\!\perp_{G_{\overline{T}}} Z \mid T, W$:

$$P(y|do(t), z, w) = P(y|do(t), w)$$

- **Rule 2**: If $Y \perp\!\!\!\perp_{G_{\overline{T},\underline{Z}}} Z \mid T, W$:

$$P(y|do(t), do(z), w) = P(y|do(t), z, w)$$

- **Rule 3**: If $Y \perp\!\!\!\perp_{G_{\overline{T},\overline{Z(W)}}} Z \mid T, W$

$$P(y|do(t), do(z), w) = P(y|do(t), w)$$

Identification in SCM: Use the 3 rules, until there is no do-operator.

Completeness of do-calculus

- Are there identifiable causal quantities that cannot be identified with do-calculus?

Completeness of do-calculus

- Are there identifiable causal quantities that cannot be identified with do-calculus?

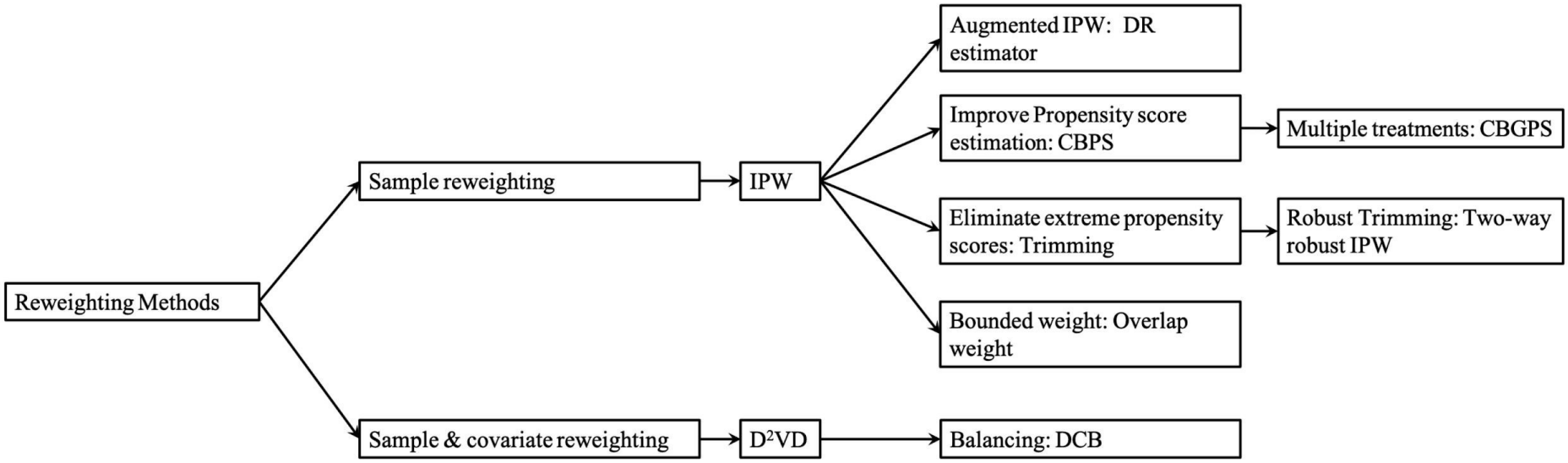
Do-calculus is complete [1]

[1] Huang Y, Valtorta M. Pearl's calculus of intervention is complete[J]. arXiv preprint arXiv:1206.6831, 2012.

Outline

- Identifiability
  - Randomized experiments
  - Front door
  - Do-calculus
  - Identification from graph
- Classical causal effect estimation methods
  - Re-weighting methods
  - Stratification methods
  - Matching methods
  - Tree-based methods

Summary of Re-weighting Methods



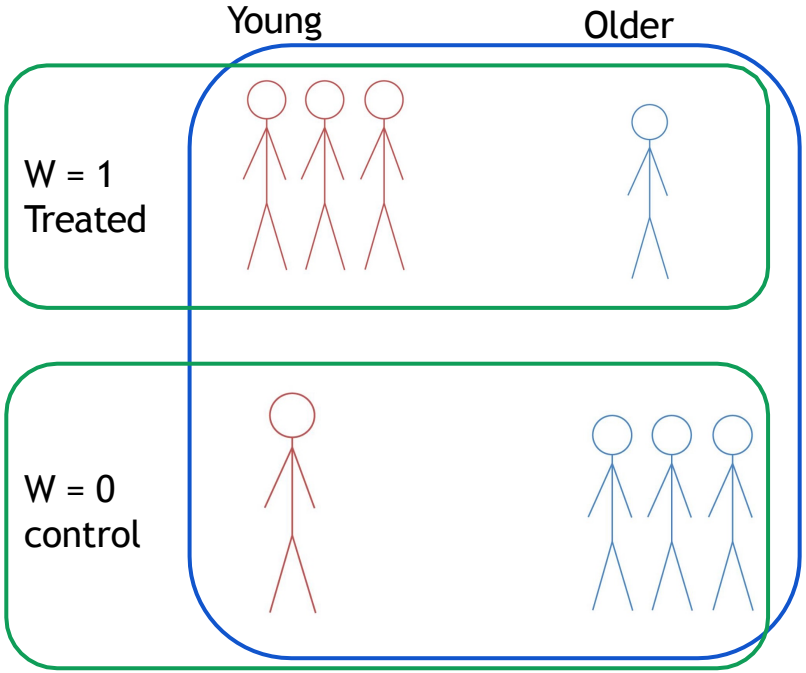
Yao L, Chu Z, Li S, et al. A survey on causal inference[J]. ACM Transactions on Knowledge Discovery from Data (TKDD), 2021, 15(5): 1-46.

Re-weighting Methods

- **Challenge of causal effect estimation**: different distributions of treated and control groups bring bias
- Sample re-weighting is a common way to overcome the selection bias problem
- **Idea**: By *assigning appropriate weight to each sample* in the observation dataset, a pseudo-population is created on which the distributions of the treated group and control group are similar

Sample Re-weighting Methods

- Intuition example: **Age** (Young/older) as the confounder



http://www.rebeccabarter.com/blog/2017-07-05-ip-weighting/

Sample Re-weighting Methods

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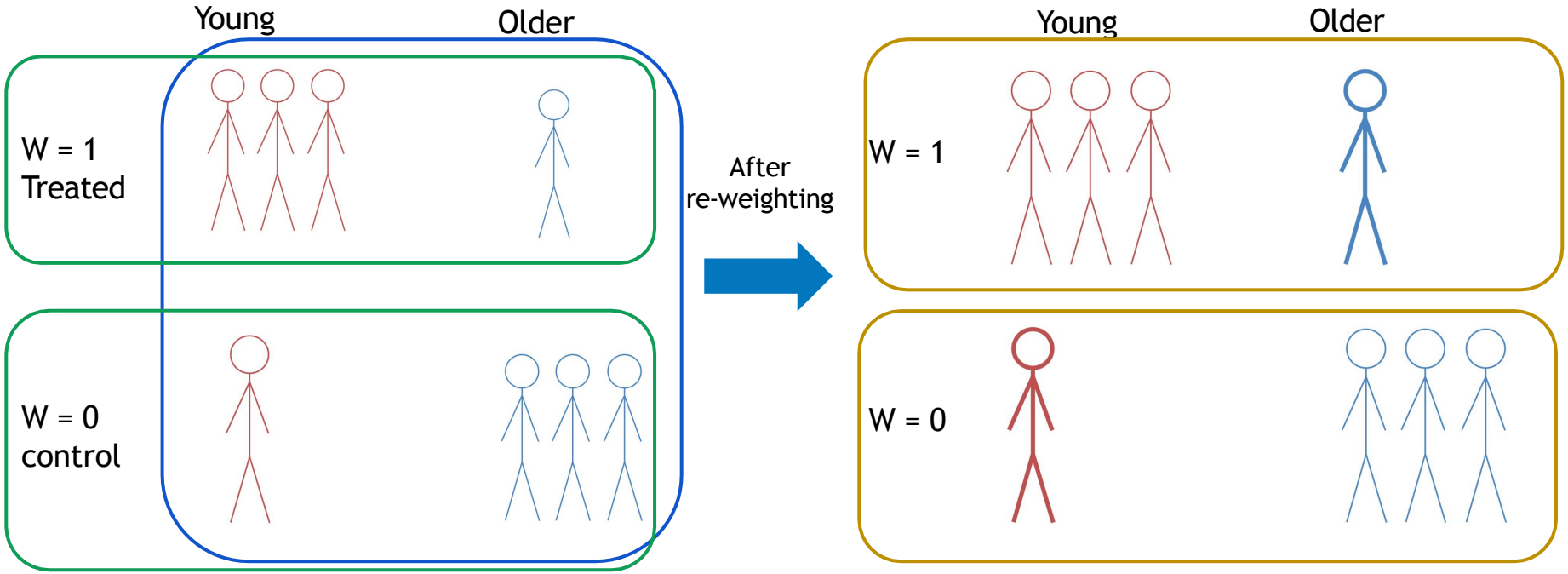

http://www.rebeccabarter.com/blog/2017-07-05-ip-weighting/

Sample Re-weighting Methods

- **Balancing Score**: Balancing score $b(\boldsymbol{X})$ is a general weighting score, which is the function of covariates $\boldsymbol{X}$ satisfying: $W \perp\!\!\!\perp \boldsymbol{x} | \, b(\boldsymbol{x})$.
  - $b(\boldsymbol{x})$ contains all information about treatment assignment
  - Able to approximate the whole population using balancing score
- One representative method: **Propensity Score based Re-weighting**
- **Propensity Score**: Conditional probability of assignment to a particular treatment given a vector of observed covariates

$$e(x) = P(W = 1 | X = x)$$

Sample Re-weighting Methods

- **Inverse propensity weighting (IPW)**
- The weight assigned for each unit is:

$$r = \frac{W}{e(x)} + \frac{1-W}{1-e(x)}$$

$W = 1 \Rightarrow r = \frac{1}{e(x)}$

$W = 0 \Rightarrow r = \frac{1}{1-e(x)}$

where W is the treatment and $e(x)$ is the propensity score

- After re-weighting, the IPW estimator of ATE is defined as:

$$\hat{\mathrm{ATE}}_{IPW} = \frac{1}{n}\sum_{i=1}^{n}\frac{W_i Y_i^F}{\hat{e}(x)} - \frac{1}{n}\sum_{i=1}^{n}\frac{(1-W_i)Y_i^F}{1-\hat{e}(x)}$$

  - Theoretical results show that adjustment for the scalar propensity score is sufficient to remove bias due to all observed covariates.
  - However, IPW highly relies on the correctness of propensity scores

J. Robins, A. Rotnitzky, and L. Zhao. "Estimation of regression coefficients when some regressors are not always observed." *Journal of the American statistical Association* 89.427 (1994): 846-866.

IPW is Unbiased

- Estimator of IPW:

$$\hat{\mathrm{ATE}}_{IPW} = \frac{1}{n}\sum_{i=1}^{n}\frac{W_i Y_i^F}{\hat{e}(x)} - \frac{1}{n}\sum_{i=1}^{n}\frac{(1-W_i)Y_i^F}{1-\hat{e}(x)}$$

$E[Y(1) - Y(0)] = E[Y(1)] - E[Y(0)]$

$E[Y(1)] =$

IPW is Unbiased

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$$E[Y(1) - Y(0)] = E[Y(1)] - E[Y(0)]$$

$$E[Y(1)] = E_X[E[Y(1)|X]]$$ law of total expectation

IPW is Unbiased

- Estimator of IPW:

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$$E[Y(1) - Y(0)] = E[Y(1)] - E[Y(0)]$$

$$E[Y(1)] = E_X[E[Y(1)|X]] = E_X\left[\frac{E[Y(1)|X]E[W|X]}{e(X)}\right]$$

Definition of propensity score

IPW is Unbiased

- Estimator of IPW:

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$$E[Y(1)] = E_X[E[Y(1)|X]] = E_X\left[\frac{E[Y(1)|X]E[W|X]}{e(X)}\right]$$

$$= E_X\left[E[\frac{Y(1)W}{e(X)}|X]\right]$$

$W \perp\!\!\!\perp Y(1) \mid X$

Assumption: no unmeasured confounders

IPW is Unbiased

- Estimator of IPW:

$$\hat{\mathrm{ATE}}_{IPW} = \frac{1}{n}\sum_{i=1}^{n}\frac{W_i Y_i^F}{\hat{e}(x)} - \frac{1}{n}\sum_{i=1}^{n}\frac{(1-W_i)Y_i^F}{1-\hat{e}(x)}$$

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$$E[Y(1)] = E_X[E[Y(1)|X]] = E_X\left[\frac{E[Y(1)|X]E[W|X]}{e(X)}\right]$$

$$= E_X\left[E[\frac{Y(1)W}{e(X)}|X]\right] = E_X\left[E\left[\frac{YW}{e(X)}\middle|X\right]\right]$$

$W = 0 \Rightarrow\ Y(1)W = YW = 0;$
$W = 1 \Rightarrow Y(1)W = YW$

IPW is Unbiased

- Estimator of IPW:

$$\hat{\mathrm{ATE}}_{IPW} = \frac{1}{n}\sum_{i=1}^{n}\frac{W_i Y_i^F}{\hat{e}(x)} - \frac{1}{n}\sum_{i=1}^{n}\frac{(1-W_i)Y_i^F}{1-\hat{e}(x)}$$

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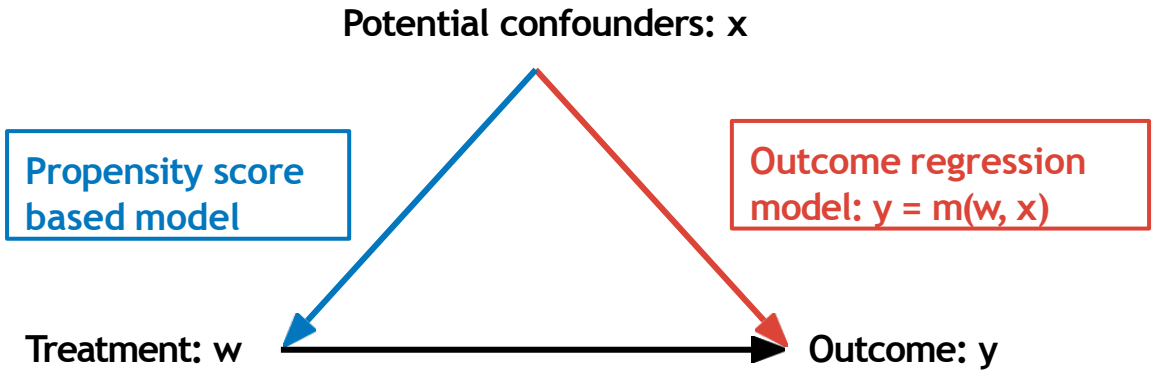
IPW

- Theoretical results show that adjustment for the scalar propensity score is sufficient to remove bias due to all observed covariates.
- However, IPW highly relies on the correctness of propensity scores

Sample Re-weighting Methods

- **Doubly Robust Estimator (DR)** or **Augmented IPW**
- It combines the propensity score weighting with the outcome regression



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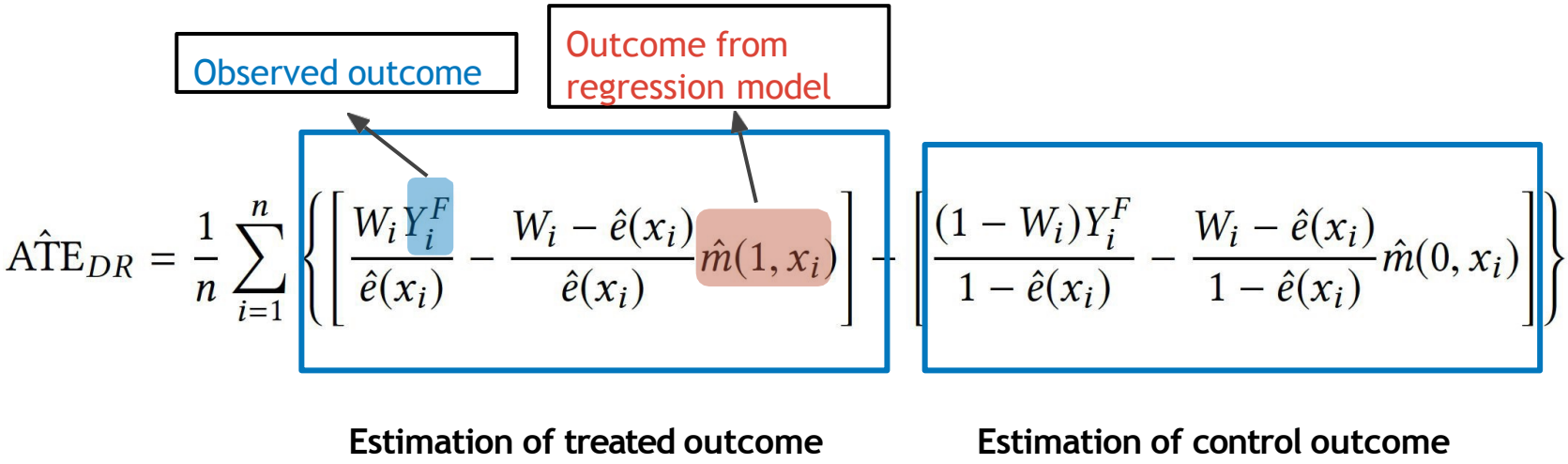
Sample Re-weighting Methods

- **Doubly Robust Estimator (DR)** or **Augmented IPW**
  - Unbiased when one of the propensity score or outcome regression is correct



Sample Re-weighting Methods

- **Doubly Robust Estimator (DR)** or **Augmented IPW**
  - Unbiased when one of the propensity score or outcome regression is correct

$$\hat{\mathrm{ATE}}_{DR} = \frac{1}{n}\sum_{i=1}^{n}\left\{\left[\frac{W_i Y_i^F}{\hat{e}(x_i)} - \frac{W_i - \hat{e}(x_i)}{\hat{e}(x_i)}\hat{m}(1,x_i)\right] - \left[\frac{(1-W_i)Y_i^F}{1-\hat{e}(x_i)} - \frac{W_i - \hat{e}(x_i)}{1-\hat{e}(x_i)}\hat{m}(0,x_i)\right]\right\}$$

IPW augmented IPW augmented

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  - Tree-based methods

Stratification

- **Stratification** adjusts the selection bias by splitting the entire group into subgroups, where within each subgroup, the treated group and the control group are similar under some measurements
- Stratification is also named as ***subclassification*** or ***blocking***
- ATE for stratification is estimated as

$$\hat{\tau}^{\text{strat}} = \sum_{j=1}^{J} q(j) \left[\bar{Y}_t(j) - \bar{Y}_c(j)\right]$$

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Matching

- Matching methods estimate the counterfactuals and meanwhile reduce the estimation bias brought by the confounders
- Potential outcomes of the i-th unit estimated by matching are:

$$\hat{Y}_i(0) = \begin{cases} Y_i & \text{if } W_i = 0, \\ \frac{1}{\#\mathcal{J}(i)} \sum_{l \in \mathcal{J}(i)} Y_l & \text{if } W_i = 1; \end{cases}$$

$$\hat{Y}_i(1) = \begin{cases} \frac{1}{\#\mathcal{J}(i)} \sum_{l \in \mathcal{J}(i)} Y_l & \text{if } W_i = 0, \\ Y_i & \text{if } W_i = 1; \end{cases}$$

Where $J(i)$ is the matched neighbors of unit i in the opposite treatment group

Matching

- **Distance Metrics for Matching**
  - *Original Data Space*
    - Euclidean distance
    - Mahalanobis distance
  - *Transformed Feature Space*
    - Propensity score based transformation
    - Other transformations (e.g., prognosis score)

Matching

- **Propensity Score Matching (PSM)**
  - Propensity scores denote conditional probability of assignment to a particular treatment given a vector of observed covariates.

$$e(x) = Pr(W = 1|X = x)$$

  - Based on propensity scores, the distance between two units is

$$D(\mathbf{x}_i, \mathbf{x}_j) = |e_i - e_j|$$

  - Alternatively, linear propensity score based distance metric

$$D(\mathbf{x}_i, \mathbf{x}_j) = |\text{logit}(e_i) - \text{logit}(e_j)|$$

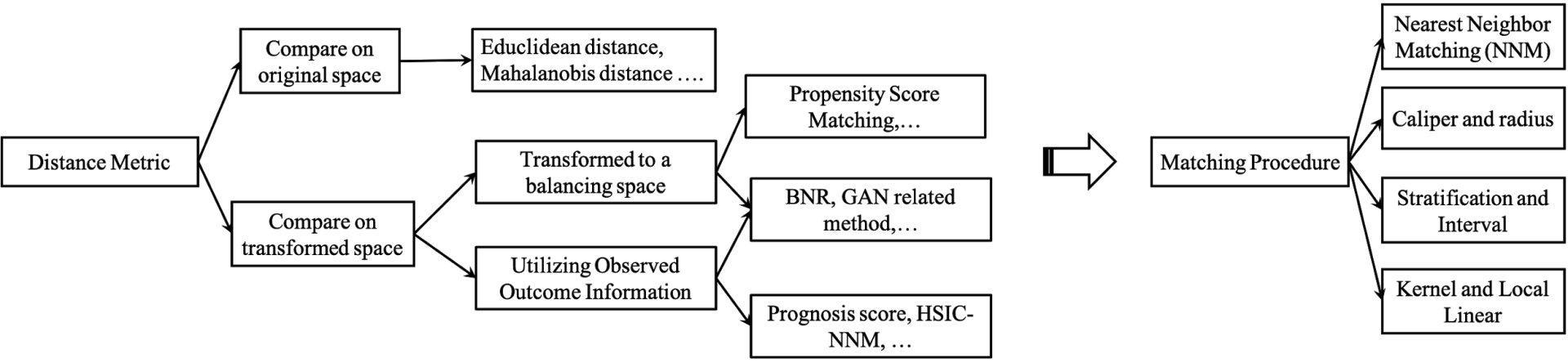
P. Rosenbaum, and D. Rubin. "The central role of the propensity score in observational studies for causal effects." *Biometrika* 70.1 (1983): 41-55.

Matching

- **Choosing Matching Algorithm**
  - Nearest Neighbors
  - Caliper
  - Stratification
  - Kernels
- **Variable Selection**
  - The more, the better?
  - Post-treatment variables

Matching

❑ Summary of Matching Methods



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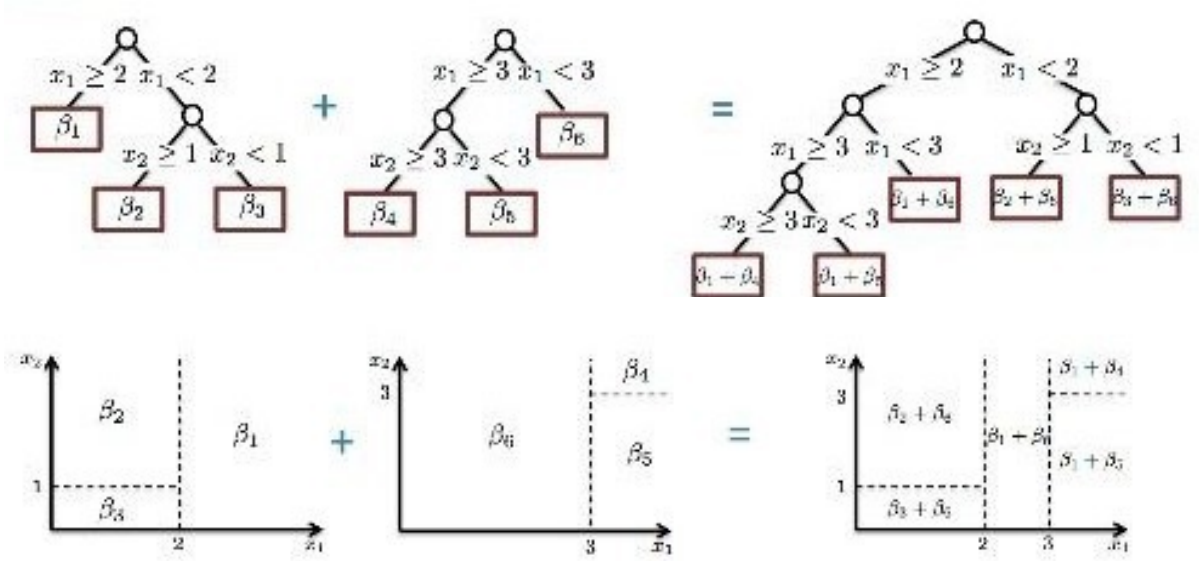
Tree-based Methods

- **Bayesian Additive Regression Trees (BART)**
  - A Bayesian "sum-of-trees" model
  - Nonparametric Bayesian regression model



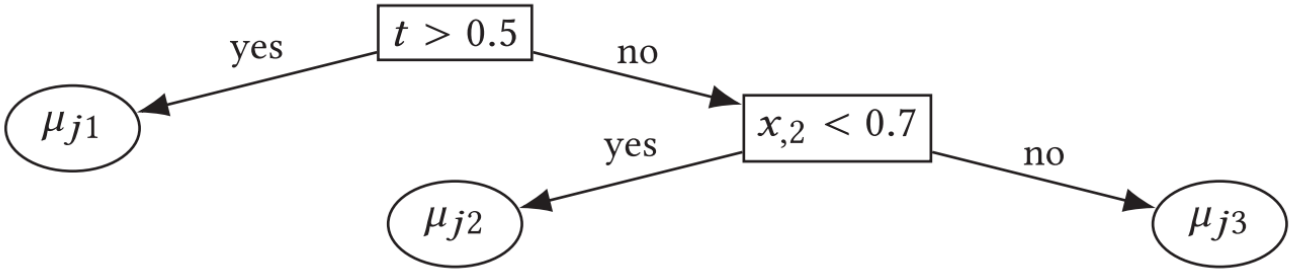
Hill, Jennifer L. "Bayesian nonparametric modeling for causal inference." *Journal of Computational and Graphical Statistics* 20.1 (2011): 217-240.
https://www.slideshare.net/SAMSI_Info/mums-bayesian-fiducial-and-frequentist-conference-multiscale-analysis-of-bart-veronika-rockova-april-29-2019

Tree-based Methods

- **Bayesian Additive Regression Trees (BART) Formulation:**
  - BART takes the features and the treatment as input and output the distribution of potential outcomes as

$$f(x,t) = \sum_j g_j(x,t) \qquad j\text{: index of Bayesian regression tree}$$

  - CATE for x can be estimated by $f(x,1) - f(x,0)$



Example of a subtree $g(x,t)$ in BART

Tree-based Methods

- Advantages of BART:
  - Easy to implement. Less requirement for parameter tuning
  - Posterior can provide uncertainty of the estimation
  - BART can deal with a mass of predictors and handle continuous treatment variables and missing data

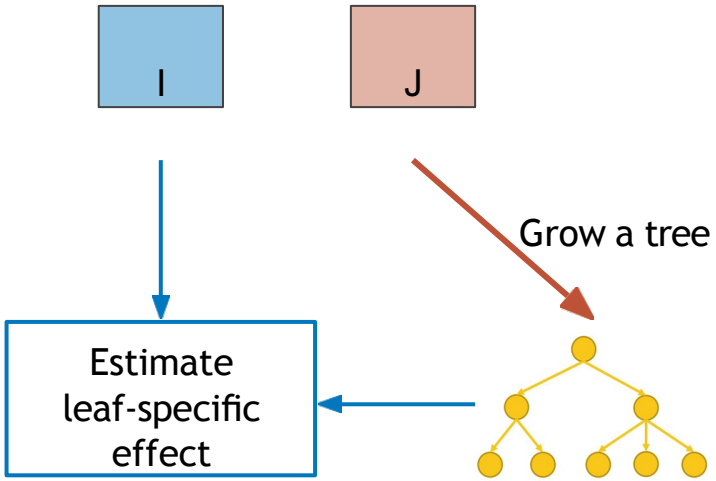
Tree-based Methods

- **Causal Forests**
  - Single tree is noisy -> using forest
  - It is based on Breiman's random forest algorithm
  - Trees and forests help find neighbors adaptively
  - Extended to multiple treatments

S. Wager, and S. Athey. "Estimation and inference of heterogeneous treatment effects using random forests." *Journal of the American Statistical Association* 113.523 (2018): 1228-1242.

Tree-based Methods

- **Causal Forest:**
- Single tree as the Double sample tree
  - Split the sample into I samples and J samples
  - **Goal**: estimate the outcome

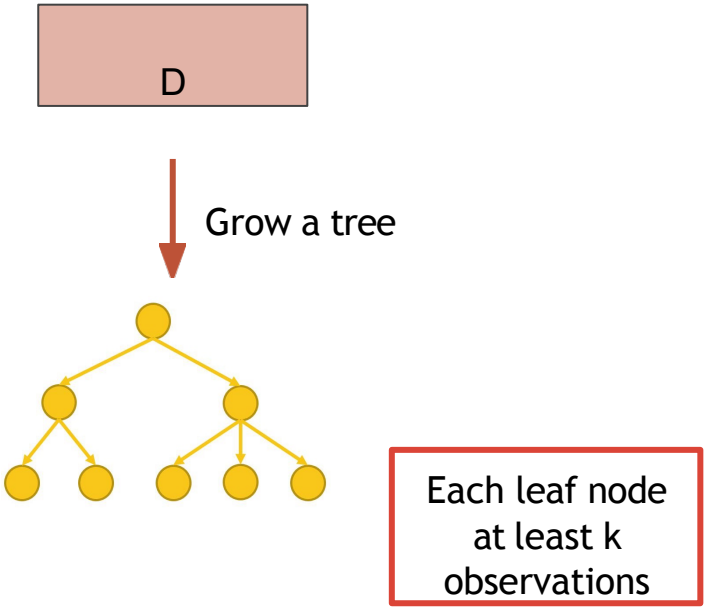


Advantage:

- Can estimate CATE
- Consistent to true CATE
- Nice Asymptotic properties

Tree-based Methods

- **Causal Forest:**
- Single tree as the propensity tree
  - **Goal**: estimate the treatment assignment W
- Use full samples



Summary

- **Classical Causal Inference Methods**
  - Simple methods with theoretical guarantee
  - May not be sufficient to handle high dimension data

Reading Materials

- Guo R, Cheng L, Li J, et al. A survey of learning causality with data: Problems and methods[J]. ACM Computing Surveys (CSUR), 2020, 53(4): 1-37.
- Yao L, Chu Z, Li S, et al. A survey on causal inference[J]. ACM Transactions on Knowledge Discovery from Data (TKDD), 2021, 15(5): 1-46.

Thank you!
Q&A

jing.ma5@case.edu