Time Series - Introduction

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Time Series Data

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Example: Johnson & Johnson Quarterly Earnings

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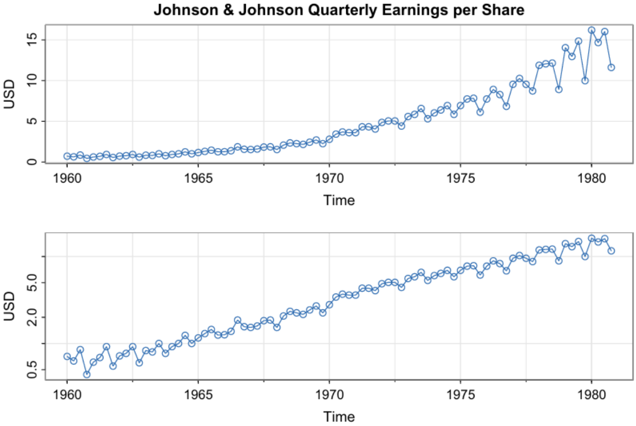

Fig. 1.1. Johnson & Johnson quarterly earnings per share in US dollars, 1960-I to 1980-IV (top). The same data on a log scale (bottom)

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Example: Global Temperature

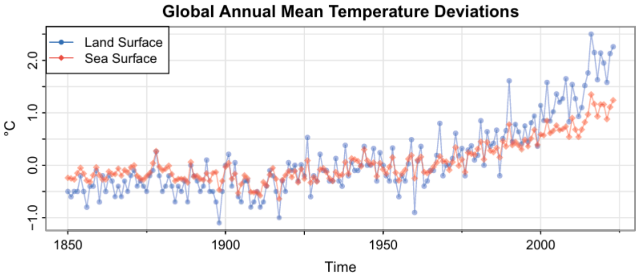

Fig. 1.2. Yearly average global temperature deviations (1850–2023) in degrees centigrade

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Example: Global Temperature

How does this differ from the Johnson & Johnson example?

Time Series Data

Example: Dow Jones Industrial Average

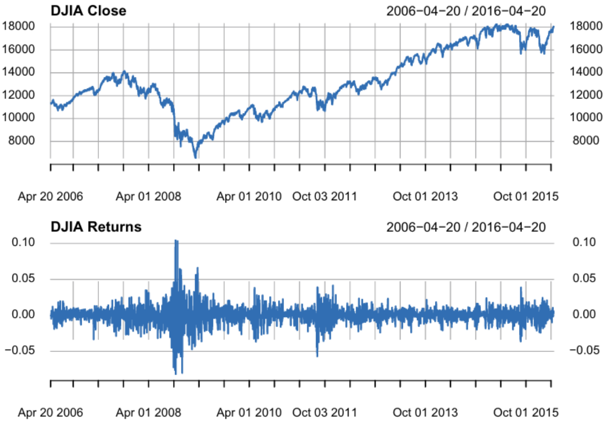

Fig. 1.4. Dow Jones Industrial Average (DJIA) trading days closings (top) and returns (bottom) from April 20, 2006 to April 20, 2016

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Example: Dow Jones Industrial Average

Let x_t denote the closing price on day t. We define the return rate on day t by

$$r_t = \frac{x_t - x_{t-1}}{x_{t-1}}$$

and we can observe

$$1 - r_t = x_t / x_{t-1}$$

and so since r_t is close to 0

$$r_t \approx \log(1 - r_t) = \log(x_t) - \log(x_{t-1})$$

We see from the graph that on average $r_t \approx 0$ but there is a lot of variation and this variation is clustered. One problem is to predict the future variation.

Time Series Data

Example: Speech Recognition

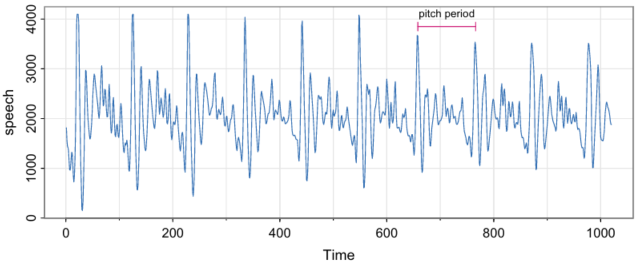

Fig. 1.3. Speech recording of the syllable *aaahhh* sampled at 10,000 points per second with $n = 1020$ points

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Example: Predator-Prey Interaction

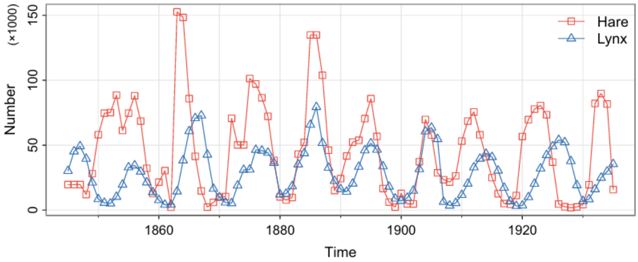

Fig. 1.6. Time series of the predator–prey interactions between the Snowshoe hare and lynx pelts purchased by the Hudson's Bay Company of Canada. It is assumed there is a direct relationship between the number of pelts collected and the number of hare and lynx in the wild

Time Series Models

We assume time series data arises from some stochastic process $\{x_t\}$ where t could be continuous or discrete.

Either way, we can only observe the process at some discrete collection of points (often equally spaced in time).

A first question we might as is whether there is some time dependence. That is does x_t depend on or correlate with $x_{t-1}, x_{t-2}, \ldots$

White Noise

A **white noise** process is a sequence of time-indexed random variables w_t such that

1. $E(w_t) = 0$ for all t
2. $Var(w_t) = \sigma_w^2$ for all t
3. The w_t are uncorrelated

We denote this by

$$w_t \sim wn(0, \sigma^w)$$

White Noise

In some cases we may additionally require that the white noise process is independent and write

$$w_t \sim iid(0, \sigma^w)$$

We could also assign some distribution such as Normal to the white noise process and write

$$w_t \sim \text{ iid } N(0, \sigma^w)$$

A white noise process itself does not have a time dependent structure, but we can construct processes that do using a white noise process.

Moving Average Model

Suppose we have a white noise process w_t and we define a process v_t by

$$v_t = (1/3)(w_{t-1} + w_t + w_{t+1})$$

v_t is a called a **moving average (MA)** model. It smooths the white noise process, which results in time dependence.

Moving Average Model

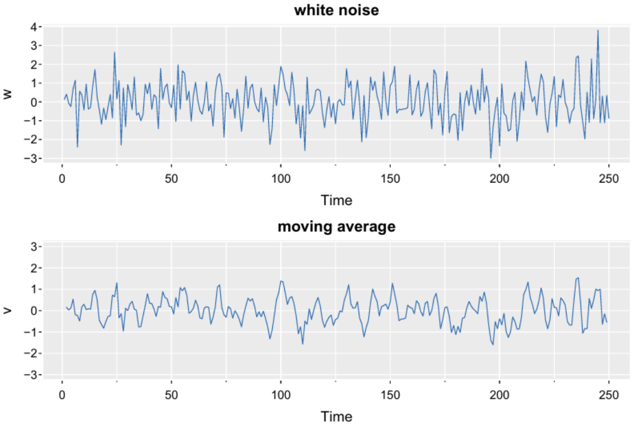

Fig. 1.9. Gaussian white noise series (top) and three-point moving average of the Gaussian white noise series (bottom)

Moving Average Model

A linear combination of values in a time series (such as our moving average model v_t) is called a **filter**.

We can observe how to generate the previous plot in R using a filter.

Autoregressive Model

Another way we can construct a dependent process from white noise w_t is to define

$$x_t = 1.5x_{t-1} + 1.75x_{t-2} + w_t$$

This is called an **autoregressive (AR)** model of order 2. (x_t depends on the two previous values and w_t).

While we might think of this model for $-\infty < t < \infty$, if we want to generate it we need some initial condition.

Let's do the example in R starting from $t = 1$ with the initial condition $x_0 = x_{-1} = 0$.

Autoregressive Model

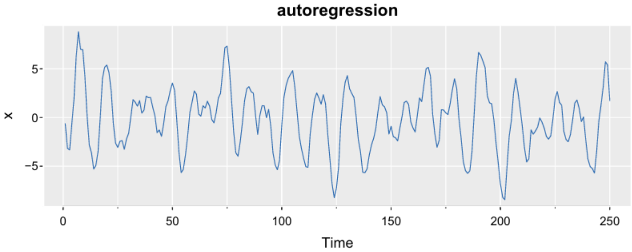

Fig. 1.10. Autoregressive series generated from model (1.2)

Random Walk with Drift

Another way we can construct a model is

$$x_t = \delta + x_{t-1} + w_t$$

Here δ is some fixed constant called the **drift**. We can also write x_t in the form

$$x_t = \delta t + \sum_{j=1}^{t} w_j$$

Let's do an example in R.

Random Walk with Drift

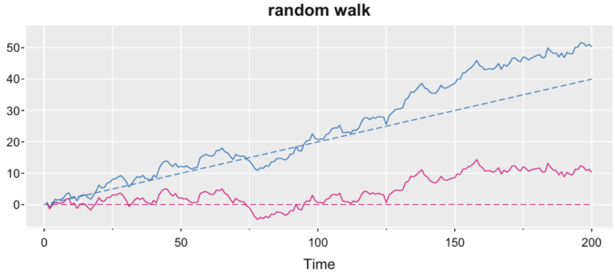

Fig. 1.11. Random walk, $\sigma_w = 1$, with drift $\delta = .2$ (upper jagged line), without drift, $\delta = 0$ (lower jagged line), and straight (dashed) lines with slope δ

Noisy Signal

w_t can also serve as noise that distorts some deterministic signal (function). For example

$$x_t = 2\cos(2\pi\frac{t+15}{50}) + w_t$$

is a periodic signal distorted by the white noise process w_t. The amount of distortion depends on σ_w^2. In this case, the problem of interest might be to estimate the deterministic part of the signal from the noise data.

Noisy Signal

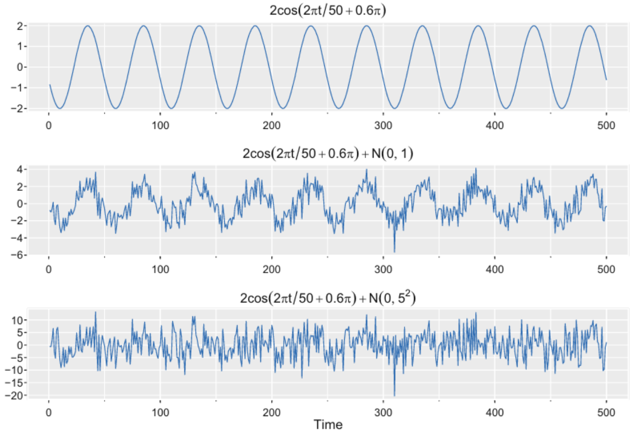

Fig. 1.12. Cosine wave with period 50 points (top panel) compared with the cosine wave contaminated with additive white Gaussian noise, $\sigma_w = 1$ (middle panel) and $\sigma_w = 5$ (bottom panel); see (1.5)