MATH 497 Quiz 2

Name Solutions Date:

1. Consider a Markov chain X_n with states 1, 2, and 3. The Markov chain evolves according to the following rules:

- When in state 1, the chain moves to state 2 with probability 1.
- When in state 2, the chain moves to state 3 with probability 1.
- When in state 3, the chain moves to state 1 with probability .5 and state 2 with probability .5.

a) Find the stationary distribution π.

$$[\pi_1 \quad \pi_2 \quad \pi_3]\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ .5 & .5 & 0 \end{bmatrix} = [\pi_1 \quad \pi_2 \quad \pi_3] \Rightarrow \begin{aligned} \pi_1 &= .5\pi_3 \\ \pi_2 &= \pi_1 + .5\pi_3 \\ \pi_3 &= \pi_2 \\ 1 &= \pi_1 + \pi_2 + \pi_3 \end{aligned}$$

$\pi = [1/5 \quad 2/5 \quad 2/5]$

b) Show the chain is irreducible.

Need to show there exists some n s.t. $p^n(i,j) > 0$ for all pairs i,j

$p(1,2) = 1 > 0$ $\quad$ $p(2,3) = 1 > 0$ $\quad$ $p(3,1) = .5 > 0$

$p^2(1,3) = 1 > 0$ $\quad$ $p^2(2,1) = .5 > 0$ $\quad$ $p(3,2) = .5 > 0$

so p is irreducible.

c) Show the chain is aperiodic.

Need to show all states have period 1

$p^2(3,3) \geq p(3,2)p(2,3) = .5 > 0$

$p^3(3,3) \geq p(3,1)p(1,2)p(2,3) = .5 > 0$

So state 3 has period 1 since $GCD(2,3) = 1$

Applying Lemma 1.18:

$p_{23} > 0$, $p_{32} > 0$ so state 2 has period 1

$p_{13} > 0$, $p_{31} > 0$ so state 1 has period 1

(you can also show it from the definition)

d) Suppose $f(x)$ is a function such that $f(2) = 1$, $f(3) = 2$, and $f(x) = 0$ for all other values of x. Find

$$\lim_{n\to\infty} \frac{1}{n}\sum_{m=1}^{n} f(X_m)$$

By Thm 1.23 this is

$$\sum_x f(x)\pi(x) = 0(1/5) + 1(2/5) + 2(2/5) = \frac{6}{5}$$