STAT 345

Theoretical Statistics I

Fall Semester 2017

Quiz 2

Name: _____

- You have 30 min to complete this quiz

- Justify your answers

- Evaluate expressions as much as you can

1. (6 points) Show that the family of Beta distributions is an exponential family. Recall that the pdf of a Beta(α, β) distribution is given by

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \, x^{\alpha-1} \, (1-x)^{\beta-1} \, \mathrm{I}_{[0,1]}(x) \qquad \forall x \in \mathbb{R}$$

2. The joint probability mass function (pmf) of two discrete random variables is given by

$$f(x, y) = \begin{cases} \frac{1}{42}(2x + y) & \text{for } x = 0, 1, 2, \text{ and } y = 0, 1, 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) (8 points) Find the marginal pmfs of X and Y

Hint: Give the value of the pmf for every possible outcome for X, and then the same for Y.

(b) (4 points) Find the conditional pmf of Y, given $X = 1$.

(c) (4 points) Find $E(X \mid Y = 2)$.

(d) (2 points) Are X and Y independent? Why / why not?

3. (6 points) Let X and Y be independent random variables and let $X \sim$ Gamma(α_1, β) and $Y \sim$ Gamma(α_2, β). What is the distribution of $X + Y$?

   Hint: The moment generating function (mgf) for Gamma(α, β) is $M(t) = \left(\frac{1}{1-\beta t}\right)^\alpha$.

Problem	1	2	3	Total
Missed				
Score				
out of	6	18	6	30