Summary of material for Final Exam

Material:

- Chapters 1, 2, 3, 4 and Sections 5.1, 5.2, 5.3, and 5.4
  - No question will be asked from Section 2.4
  - We only covered the Chebychev inequality from Section 3.6
  - We skipped Section 4.7

A formula sheet with geometric sums, Binomial formula etc will be provided. Also a table with the pdfs of most common distributions

Chapter 1

- Know the Axioms of Kolmogorov and how to use them to prove basic probability rules
- Counting methods
- Conditional Probability, Bayes Theorem, Independent Events
- Definitions and properties of pdfs, pmfs and cdfs

Chapter 2

- Univariate transformations - both continuous and discrete!
  - Discrete: Find pmf by adding up the probabilities of outcomes in the inverse map
  - Continuous:
    - if $g(x)$ is monotone: pdf method
    - If not, revert to "cdf-method"
- $E(X)$, $V(X)$, and $M_X(t)$: Properties and know how to derive them for both continuous and discrete distributions
- Know how to derive moments using $M_X(t)$

Discrete distributions

- Understand what situations lead to Discrete Uniform, Bernoulli, Binomial, Hypergeometric, Geometric, and Negative Binomial distributions
- Know how to derive $E(X)$, $V(X)$, and $M(t)$ for the following cases
  - Discrete Uniform (no simplification available for $M(t)$)
  - Bernoulli - $E(X)$ and $V(X)$: directly and via $M(t)$
  - Binomial - $E(X)$ and $V(X)$: directly and via $M(t)$
  - Poisson - $E(X)$ and $V(X)$: directly and via $M(t)$
- STAT 445 only: Know how to derive the $M(t)$ for the Negative Binomial distribution and to use it to derive $E(X)$ and $V(X)$.

Continuous distributions

- Know how to derive $E(X)$, $V(X)$, and $M(t)$ for the following cases
  - Uniform
  - Beta - $E(X^n)$ instead of $M(t)$
  - Gamma - $E(X)$ and $V(X)$: directly and via $M(t)$
  - Exponential
    - Know the cdf and the memoryless property
  - Normal - $E(X)$ and $V(X)$: only via $M(t)$

Families of distributions, Inequalities

- Know how to show whether a distribution belongs to the exponential family
- Be able to use the Chebychev inequality to provide bounds on probability of distance from the mean
- STAT 445 only: Know how to derive the Chebychev inequality

Chapter 4: Multivariate distributions

- Joint pmfs/pdfs, marginal pmfs/pdfs, conditional pmfs/pdfs
  - Always check the support!
- $E(g(X, Y))$, marginal and conditional means and variances
- Independent random variables, how to show or disprove independence
  - STAT 445 only: Know how to prove the lemma on slide 12 and Theorem on slide 14 from Lecture 14.
- Understand the situation that leads to the multinomial distribution and be able to derive marginal and conditional distributions from it
- Bivariate transformations - both discrete and continuous
- Hierarchical models - finding the marginal distributions
  - Iterative expectation and variance

Chapter 5

- Random samples and statistics
  - Using mgfs to find the distribution of a sample mean or a sum of a random sample
  - Mean and variance of sums (or mean) of a random sample
  - Mean of the sample variance of a random sample
- Random sample from a normal population
  - Independence of $\overline{X}$ and S^2
  - Distribution of $\overline{X}$ and $(n-1)S^2/\sigma^2$
  - zero correlation $\Leftrightarrow$ independence
  - Student's t-distribution - how defined
  - F distribution - how defined
- Order Statistics - find the pdf of $X_{(j)}$