1.26:

Event A: cast more than five times and 6 appears

Event B: cast less or equal to five times and 6 appears

$P(A) + P(B) = 1.$

Assume the number of cast time is n,

if $n=1$, $P(B_1) = \frac{1}{6}$ $\boxed{6}$

if $n=2$, $P(B_2) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$ $\boxed{\ }\boxed{6}$

if $n=3$. $P(B_3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$ $\boxed{\ }\boxed{\ }\boxed{6}$

if $n=4$: $P(B_4) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{125}{1296}$ $\boxed{\ }\boxed{\ }\boxed{\ }\boxed{6}$

if $n=5$: $P(B_5) = (\frac{5}{6})^4 \times \frac{1}{6} = \frac{625}{7776}$ $\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{6}$

$P(B) = P(B_1) + P(B_2) + P(B_3) + P(B_4) + P(B_5)$

$$P(A) = 1 - P(B) = 1 - \left(\frac{1}{6} + \frac{5}{36} + \frac{25}{216} + \frac{125}{1296} + \frac{625}{7776}\right) = \frac{3125}{7776}$$

$$\approx 0.4018776.$$

1.33:

According to the information of question, we can get

$P(\text{color-blind} \mid \text{Man}) = 5\%$ $\quad P(\text{Man}) = 50\%$

$P(\text{Color-blind} \mid \text{Woman}) = 0.25\%.$ $\quad P(\text{Woman}) = 50\%$

$$P(\text{Man} \mid \text{color-blind}) = \frac{P(\text{Man} \cap \text{color-blind})}{P(\text{color-blind})}$$

$$P(\text{Man} \cap \text{Color-blind}) = P(\text{Color-blind} \mid \text{Man}) \cdot P(\text{Man}) = 2.5\%$$

$$P(\text{Color-blind} \mid \text{Woman}) = \frac{P(\text{Color-blind} \cap \text{woman})}{P(\text{Woman})}$$

$$\begin{aligned} P(\text{Color-blind} \cap \text{Woman}) &= P(\text{Woman}) \cdot P(\text{Color-blind} \mid \text{Woman}) \\ &= 50\% \cdot 0.25\% \\ &= 0.125\% \end{aligned}$$

	color-blind	no color-blind	
Woman	0.125%	49.875%	50%
Man	2.5%	47.5%	50%
	2.625%	97.375%	100%

$$P(\text{Man} \mid \text{color-blind}) = \frac{P(\text{Man} \cap \text{Color-blind})}{P(\text{color-blind})}$$

$$= \frac{2.5\%}{2.625\%}$$

$$\approx 0.9524$$

1.35:

According to K's Axioms, We need to prove

$$\begin{cases} P(\cdot \mid B) \geq 0 \quad \cdots\cdots \quad ① \\ P(S \mid B) = 1 \quad \cdots\cdots \quad ② \\ P\left(\bigcup_{i=1}^{\infty} A_i \mid B\right) = \sum_{i=1}^{\infty} P(A_i \mid B) \quad \cdots \quad ③ \end{cases}$$

Prove ①:

Since $P(\cdot \cap B) \geq 0$, and $P(B) > 0$, we can have

$$P(\cdot \mid B) = \frac{P(\cdot \cap B)}{P(B)} \geq 0,$$

Prove ②:

$$P(S \mid B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

Prove ③: Let $A_1, A_2, A_3, \cdots$ as mutually exlusive events, for $\forall i,j, i \neq j$, $A_i \cap A_j = \emptyset$,

$$P\left(\bigcup_{i=1}^{\infty} A_i \mid B\right) = \frac{P\left(\left(\bigcup_{i=1}^{\infty} A_i\right) \cap B\right)}{P(B)} \quad \cdots\cdots (*)$$

Since $P\left(\left(\bigcup_{i=1}^{\infty} A_i\right) \cap B\right) = P\left(\bigcup_{i=1}^{\infty} (A_i \cap B)\right)$

$$= \sum_{i=1}^{\infty} P(A_i \cap B),$$

We can get $(*) = \dfrac{\sum_{i=1}^{\infty} P(A_i \cap B)}{P(B)}$

$$= \sum_{i=1}^{\infty} \frac{P(A_i \cap B)}{P(B)}$$

$$= \sum_{i=1}^{\infty} P(A_i \mid B)$$

③ was proved.

Because, ①, ②, ③ were proved,

then $P(\cdot \mid B)$ also satisfies Kolmogorov's Axioms.

1.39:

(a): Prove:

If A and B are mutually exclusive,

the $A \cap B = \phi$, $P(A \cap B) = 0$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 0 \cdots\cdots ①$$

But if A and B are independent.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = P(A) > 0 \cdots\cdots ②$$

Obviously, $① \not\Rightarrow ②$.

Therefore, If A and B are mutually exclusive,

they cannot be independent.

(b): Prove:

If A and B are independent,

then $P(A \cap B) = P(A) \cdot P(B) > 0 \cdots$ 3

But if A and B are mutually exclusive,

$A \cap B = \phi$, $P(A \cap B) = 0 \cdots$ 4

Obviously, 3 $\not\Rightarrow$ 4.

Therefore, if A and B are independt, they cannot be mutually exclusive.

Extra problem 3:

10 mathematicians (M), 15 statisticians (S)

(a):

$$\binom{10}{3} \cdot \binom{15}{3} = \frac{10\times 9\times 8}{3\times 2\times 1} \times \frac{15\times 14\times 13}{3\times 2\times 1}$$

$$= 120 \times 455$$

$$= 54600$$

(b):

Case 1:

S	S	S	S	M	M

$$\binom{10}{2}\cdot\binom{15}{4} = \frac{10\times 9}{2\times 1}\times\frac{15\times 14\times 13\times 12}{4\times 3\times 2\times 1}$$

$$= 45\times 1365 = 61425$$

...... ①

Case 2:

S	S	S	S	S	M

$$\binom{10}{1}\cdot\binom{15}{5} = 10\times\frac{15\times 14\times 13\times 12\times 11}{5\times 4\times 3\times 2\times 1}$$

$$= 30030$$

...... ②

Case 3:

S	S	S	S	S	S

$$\binom{15}{6} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= 5005 \quad \cdots\cdots \quad ③$$

Total ways $= ① + ② + ③$

$= 61425 + 30030 + 5005$

$= 96460$