


1.26:

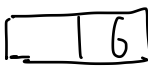
Event A: cast more than five times and 6 appears


Event B: cast less or equal to five times and 6 appears

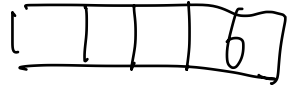
$$P(A) + P(B) = 1.$$

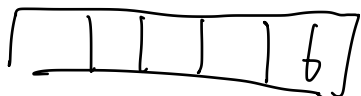
Assume the number of cast time is n ,

if $n=1$, $P(B_1) = \frac{1}{6}$ 

if $n=2$, $P(B_2) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$ 

if $n=3$, $P(B_3) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$ 

if $n=4$, $P(B_4) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{125}{1296}$ 

if $n=5$, $P(B_5) = \left(\frac{5}{6}\right)^4 \times \frac{1}{6} = \frac{625}{7776}$ 

$$P(B) = P(B_1) + P(B_2) + P(B_3) + P(B_4) + P(B_5)$$

$$P(A) = 1 - P(B) = 1 - \left(\frac{1}{6} + \frac{5}{36} + \frac{25}{216} + \frac{125}{1296} + \frac{625}{7776} \right) = \frac{3125}{7776}$$

$$\approx 0.4018776.$$

1.33 :

According to the information of question, we can get

$$P(\text{color-blind} | \text{Man}) = 5\% \quad P(\text{Man}) = 50\%$$

$$P(\text{Color-blind} | \text{Woman}) = 0.25\% \quad P(\text{Woman}) = 50\%$$

$$P(\text{Man} | \text{color-blind}) = \frac{P(\text{Man} \cap \text{color-blind})}{P(\text{color-blind})}$$

$$P(\text{Man} \cap \text{color-blind}) = P(\text{color-blind} | \text{Man}) \cdot P(\text{Man}) = 2.5\%$$

$$P(\text{Color-blind} | \text{Woman}) = \frac{P(\text{color-blind} \cap \text{Woman})}{P(\text{Woman})}$$

$$\begin{aligned} P(\text{color-blind} \cap \text{Woman}) &= P(\text{Woman}) \cdot P(\text{color-blind} | \text{Woman}) \\ &= 50\% \cdot 0.25\% \\ &= 0.125\% \end{aligned}$$

	color-blind	no color-blind	
Woman	0.125%	49.875%	50%
Man	2.5%	47.5%	50%
	2.625%	97.375%	100%

$$P(\text{Man} | \text{color-blind}) = \frac{P(\text{Man} \cap \text{Color-blind})}{P(\text{color-blind})}$$

$$= \frac{2.5\%}{2.625\%}$$

$$\approx 0.9524$$

1.35:

According to K's Axioms, We need to

Prove

$$\left. \begin{array}{l} P(C|B) \geq 0 \quad \dots \quad (1) \\ P(S|B) = 1 \quad \dots \quad (2) \\ P\left(\bigcup_{i=1}^{\infty} A_i | B\right) = \sum_{i=1}^{\infty} P(A_i | B) \quad \dots \quad (3) \end{array} \right\}$$

Prove (1):

Since $P(C \cap B) \geq 0$, and $P(B) > 0$, we can have

$$P(C|B) = \frac{P(C \cap B)}{P(B)} \geq 0.$$

Prove (2):

$$P(S|B) = \frac{P(S \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

Prove ③: Let A_1, A_2, A_3, \dots as mutually exclusive events, for $\forall i, j, i \neq j, A_i \cap A_j = \emptyset$,

$$P\left(\bigcup_{i=1}^{\infty} A_i \mid B\right) = \frac{P\left(\left(\bigcup_{i=1}^{\infty} A_i\right) \cap B\right)}{P(B)} \dots\dots (*)$$

$$\begin{aligned} \text{Since } P\left(\left(\bigcup_{i=1}^{\infty} A_i\right) \cap B\right) &= P\left(\bigcup_{i=1}^{\infty} (A_i \cap B)\right) \\ &= \sum_{i=1}^{\infty} P(A_i \cap B), \end{aligned}$$

$$\text{We can get } (*) = \frac{\sum_{i=1}^{\infty} P(A_i \cap B)}{P(B)}$$

$$= \sum_{i=1}^{\infty} \frac{P(A_i \cap B)}{P(B)}$$

$$= \sum_{i=1}^{\infty} P(A_i \mid B)$$

③ was proved.

Because , ① , ② , ③ were proved,

then $P(\cdot|B)$ also satisfies Kolmogorov's Axioms.

1.39:

(a): Prove:

If A and B are mutually exclusive,

then $A \cap B = \emptyset$, $P(A \cap B) = 0$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = 0 \dots \textcircled{1}$$

But if A and B are independent.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)} = P(A) > 0 \dots \textcircled{2}$$

Obviously, $\textcircled{1} \not\Rightarrow \textcircled{2}$.

Therefore, If A and B are mutually exclusive,
they cannot be independent.

(b): Prove:

If A and B are independent,

$$\text{then } P(A \cap B) = P(A) \cdot P(B) > 0 \dots \textcircled{2}$$

But if A and B are mutually exclusive,

$$A \cap B = \emptyset, P(A \cap B) = 0 \dots \textcircled{4}$$

Obviously, $\textcircled{2} \not\Rightarrow \textcircled{4}$.

Therefore, if A and B are independent, they cannot be mutually exclusive.

Extra problem 3:

10 mathematicians (M), 15 statisticians (S)

(a):

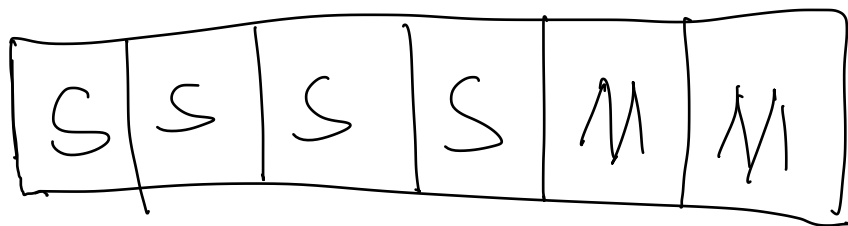
$$\binom{10}{3} \cdot \binom{15}{3} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} \times \frac{15 \times 14 \times 13}{3 \times 2 \times 1}$$

$$= 120 \times 350$$

$$= 54600$$

(b):

Case 1:

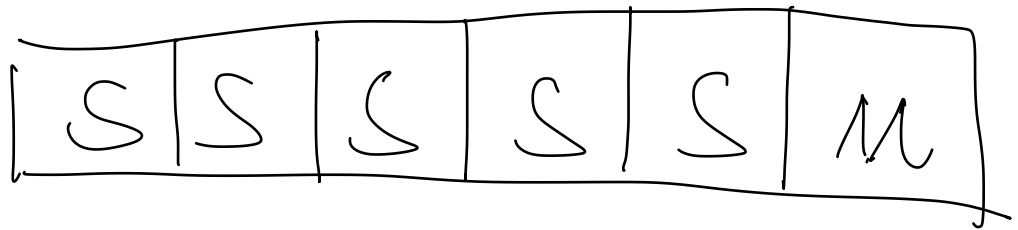


$$\binom{10}{2} \cdot \binom{15}{4} = \frac{10 \times 9}{2 \times 1} \times \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}$$

$$= 65 \times 1365 = 61425$$

..... (1)

Case 2 :

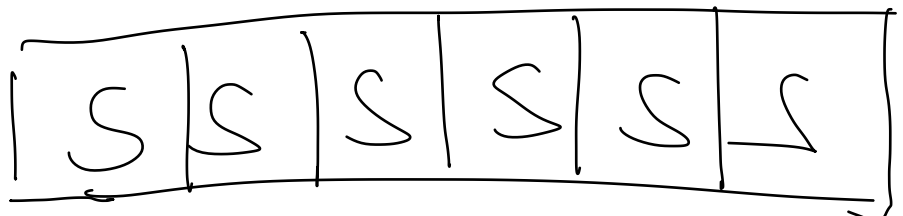


$$\begin{pmatrix} 10 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 15 \\ 5 \end{pmatrix} = 10 \times \frac{15 \times 16 \times 13 \times 12 \times 11}{5 \times 6 \times 3 \times 2 \times 1}$$

$$= 30030$$

- - - - 2

Case 3:



$$\binom{15}{6} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$$= 5005 \dots \textcircled{3}$$

$$\begin{aligned} \text{Total ways} &= \textcircled{1} + \textcircled{2} + \textcircled{3} \\ &= 61425 + 30030 + 5005 \\ &= 96460 \end{aligned}$$