Some potentially useful formulas

- Binomial formula: For all $x, y \in \mathbb{R}$ and $n \in \mathbb{N}$

$$\sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i} = (x+y)^n$$

- Gamma function

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt \qquad \text{for } \alpha > 0$$

  - $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$, if n is an integer: $\Gamma(n) = (n-1)!$, and $\Gamma(0.5) = \sqrt{\pi}$

- Taylor series for e^x:

$$e^\lambda = \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$$

- Bayes Theorem: If $A_1, A_2, \ldots, A_n$ is a partition of the sample space then for any event B:

$$P(A_i \mid B) = \frac{P(B \mid A_i)P(A_i)}{\sum_{j=1}^{n} P(B \mid A_j)P(A_j)}$$

- If random variable X has pdf $f_X(x)$ that is continuous on $\mathcal{X} = \{x : f(x) > 0\}$ and $Y = g(X)$ where $g(x)$ is a monotone function, and if $g^{-1}(y)$ has a continuous derivative on $\mathcal{Y} = \{y : \exists x \in \mathcal{X} \text{ such that } y = g(x)\}$, then

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right| \qquad \text{for } \ y \in \mathcal{Y}$$

- Order Statistics: Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution, with pdf $f(x)$ and cdf $F(x)$. Then the cdf and pdf of the jth order statistic $X_{(j)}$ are

$$\begin{aligned} F_{(j)}(x) &= \sum_{k=j}^{n} \binom{n}{k} F(x)^k (1-F(x))^{n-k} \\ f_{(j)}(x) &= \frac{n!}{(j-1)!(n-j)!} f(x)\, F(x)^{j-1}\ (1-F(x))^{n-j} \end{aligned}$$

- Fisher information

$$\begin{aligned} I(\theta) &= E\left(\left(\frac{d}{d\theta}\log(f(\mathbf{X} \mid \theta))\right)^2\right) \\ &= nE\left(\left(\frac{d}{d\theta}\log(f(X \mid \theta))\right)^2\right) \quad \text{if iid} \\ &= -nE\left(\frac{d^2}{d\theta^2}\log(f(X \mid \theta))\right) \text{ if exponential family} \end{aligned}$$

Distribution	pmf/pdf	Support	$E(X)$	Var(X)	$M_X(t)$	Parameter space
Binomial(n,p)	$f(x \mid n,p) = \binom{n}{x} p^x (1-p)^{n-x}$	$x = 0,1,\dots,n$	np	$np(1-p)$	$(pe^t + (1-p))^n$	$0 \le p \le 1$, $n = 0,1,2,\dots$
DiscrUniform(N)	$f(x \mid N) = \frac{1}{N}$	$x = 1,2,\dots,N$	$\frac{N+1}{2}$	$\frac{(N+1)(N-1)}{12}$		$N \in 0,1,2,\dots$
Geometric(p)	$f(x \mid p) = p(1-p)^{x-1}$	$x = 1,2,3,\dots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$	$0 \le p \le 1$
NegBinomial(r,p)	$f(x \mid r,p) = \binom{r+x-1}{x} p^r (1-p)^x$	$x = 0,1,2,\dots$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$	$0 \le p \le 1$, $r = 1,2,3\dots$
Poisson(λ)	$f(x \mid \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$	$x = 0,1,2,\dots$	λ	λ	$e^{\lambda(e^t-1)}$	$\lambda > 0$
Uniform(a,b)	$f(x \mid a,b) = \frac{1}{b-a}$	$a \le x \le b$	$\frac{b+a}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{bt}-e^{at}}{(b-a)t}$	$a < b$
Beta(α,β)	$f(x \mid \alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{(\alpha-1)}(1-x)^{\beta-1}$	$0 \le x \le 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$		$\alpha > 0$, $\beta > 0$
Exponential(β)	$f(x \mid \beta) = \frac{1}{\beta} e^{-x/\beta}$	$x \ge 0$	β	β^2	$\frac{1}{1-\beta t}$	$\beta > 0$
	cdf: $F(x) = 1 - e^{-x/\beta}$ for $x \ge 0$					
Gamma(α,β)	$f(x \mid \alpha,\beta) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{(\alpha-1)} e^{-x/\beta}$	$x \ge 0$	$\alpha\beta$	$\alpha\beta^2$	$\left(\frac{1}{1-\beta t}\right)^\alpha$	$\alpha > 0$, $\beta > 0$
N(μ,σ^2)	$f(x \mid \mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$x \in \mathbb{R}$	μ	σ^2	$e^{\mu t + \sigma^2 t^2/2}$	$\mu \in \mathbb{R}$, $\sigma^2 > 0$