THE 2017 HIMCM

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The High School Mathematical Contest in Modeling (HiMCM) continues to be an amazing and rewarding experience for students, advisors, schools, and judges. A total of 938 teams, with up to 4 students each, from 256 schools competed. Congratulations to our 2017 Outstanding team winners and to all teams participating in our twentieth year of HiMCM!

Outstanding Teams

Capital High School, Helena, MT, USA Advisor: Dennis Peterson

Columbus North High School, Columbus, IN, USA Advisor: Mike Spock

North Carolina School of Science and Mathematics, Durham, NC, USA Advisor: Daniel Teague (2 Teams)

Raffles Institution, Singapore Advisor: Haojin Yang

Shenzhen College of International Education, Shenzhen, Guangdong, China Advisor: Dan Li

Shenzhen Vanke Meisha Academy, Shenshen, Guangdong, China Advisor: Wentao Zhang

St. Mark's School, Southboro, MA, USA Advisor: Allyson Brown

YK Pao School, Shanghai, China Advisor: Jianping Yang

The 2017 Contest

The solutions we received were truly impressive. Our participating teams accomplished the vision of our founders by providing unique and creative mathematical solutions to complex open-ended real-world problems. As in the past, students chose from two problems, both representing "real-world issues." This year's problems allowed students to either plan a drone aerial light show in *Drone Clusters as Sky Light Displays* Problem A, or design a ski resort in our *Ski Slope* Problem B. The final judging results and 2017 HiMCM statistics are shown in Figure 1.

Overview

The HiMCM, now in its twentieth year, continues to grow. As more high schools engage their students in mathematical modeling, we hope participation in the contest will follow. Figure 2 shows a plot of the growth over time. The trend continues to reflect an increasing engagement of high school students in mathematical modeling.

The 2017 contest had 1044 registered teams resulting in 938 total submissions (89.8%), a registration increase of about 10% over last year. In total, 3922 students competed in the contest, representing an increase of 10%. A wide range of schools/teams competed including teams from Australia, Canada, Chile, China, Costa Rica, Germany, Hong Kong (SAG), Korea, the Philippines, Singapore, Thailand, Turkey, United Arab Emirates, the United States of America, and the United Kingdom. The 359 teams from the United States represented 28 states. This number was up 8.8% from last year. Submissions included 579 foreign teams, representing a 13.5% growth. China represented about 91% of the foreign entries.

Of the 3922 student participants this year, 1390 or about 37.4% were female, 2429 were male, and 103 did not specify gender. Since the start of HiMCM in 1999, females represent 36.4% (10,985) of the 30,097 total participants. This participation has been fairly consistent over a number of years. We hope that all competing students will enjoy their contest experience and continue to pursue further STEM education.

Problem Choice

We congratulate all students and advisors for the creativity and ingenuity of their mathematical efforts. Again this year, it appears that teams are enjoying developing solutions to their chosen problem. We continue to encourage all registered teams to submit a solution in order to experience the learning impact and satisfaction of fully completing this

Problem	Outstanding	%	Finalist	%	Meritorious	%	Honorable Mention	%	Successful Participant	%	Unsuccessful	%	Total
A	6	1%	40	7%	77	13%	181	32%	255	45%	10	2%	569
B	3	1%	25	7%	46	12%	113	31%	173	47%	9	2%	369
Total	9	1%	65	7%	123	13%	294	31%	428	46%	19	2%	938

Figure 1: *2017 HiMCM Statistics*

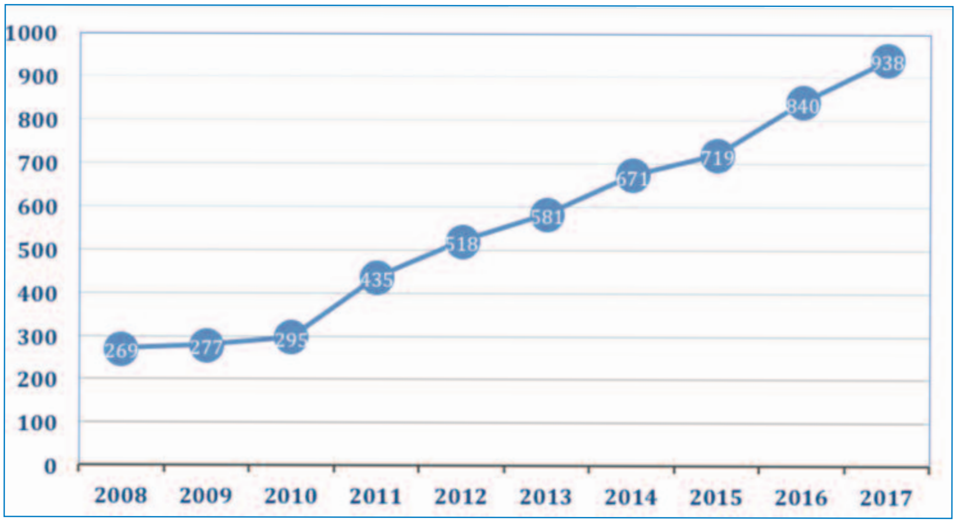

Figure 2: *Number of HiMCM teams vs. contest year from 2008-2017*

challenging contest. Of the 938 submissions, 569 completed Problem A: *Drone Clusters as Sky Light Displays*, and 369 completed Problem B: *Ski Slope*. The 36 continuous hours to work on the problem provided for high quality papers for both problems.

Judging

The mathematical modeling ability of participating students and their faculty advisors continues to be evident in the problem solutions and professional submissions we receive. As you and your students engage in mathematical modeling at a higher level, we are happy and excited to assist your efforts. Let us know how we might support your modeling activities.

All submissions this year were electronic which allowed us to expand our judging pool. In December 2017, we used two regional judging sites and a third group of remote judges. The regional sites were located at Francis Marion University in Florence, SC and Carroll College in Helena, MT. Remote judges were located in Alabama, California, Colorado, Georgia, Illinois, Massachusetts, New York, Ohio, Pennsylvania, South Carolina, and Virginia.

All judging was blind with respect to any identifying information about the participants or their schools. Judges ranked papers as Finalist, Meritorious, Honorable Mention, and Successful Participant. Each site judged papers for both problems A and B. Judges sent all papers ranked as "Finalist" to the National Judging in San Diego, California. This year, 74 papers were forwarded to San Diego for eleven judges, from both academia and industry, to consider. As these 74 papers were the best submissions from the triage round, at final judging the judges chose the *"best of the best"* as Outstanding papers. Nine papers earned the Outstanding award. The national judges commended the regional judges for their efforts in selecting the high quality Finalist papers forwarded to San Diego. We feel that the structure of Regional and National Judging provides a good process for identifying the winning papers.

The Future

The HiMCM attempts to give all high school students an opportunity to compete and achieve success in mathematics. Our efforts are always toward meeting this important goal.

Mathematical modeling is growing within the curricula at the high school level. This contest provides a vehicle for using mathematics to build models that allow students to represent and to understand real world behavior in a quantitative way. It enables student teams to look for patterns and think logically about mathematics and its role in our lives. We continue to strive to improve the contest, and want the contest accessible to all students. Any school and team can enter, as there are no restrictions on the number of schools or the numbers of teams from a school. A regional judging structure offers flexibility to accommodate the number of teams.

Mathematics continues to be more than just learning skills and operations. Mathematics is a language that involves our daily lives. Applying the mathematical principles and concepts that one learns is key to individual and societal future success. The ability to recognize problems, formulate a mathematical model, use technology, and communicate and reflect on one's work are important skills to develop. Students gain confidence by tackling ill-defined problems and working together to generate a solution. We are excited that in our contest applying mathematics is a team sport!

Mathematical modeling is an art and a science. Advisors need only be motivators and facilitators to encourage students to be creative and imaginative. Through modeling, students learn to think critically, communicate effectively, and be confident, competent problem solvers. Success is not only about the procedural technique used, but the conceptual understanding in discovering the role of assumptions and model development in driving those techniques to a valid solution and conclusion. We encourage all high school mathematics faculty to get involved, encourage your students to

Awards

The award format has changed slightly in the past two years as the contest continues to grow internationally.

After final judging, HiMCM papers are designated in the categories below. The top approximately 21% of submitted papers receives a designation of Meritorious or above. The top approximately 1% is designated as Outstanding.

Successful Participant

Honorable Mention

Meritorious

Finalist

Outstanding

be problem solvers, make mathematics relevant, and open the doors to future success.

2018 Contest Dates

Mark your calendars for the next HiMCM to be held November 9 – 19, 2018. Registration for the 2018 HiMCM will open in September. A rule change in 2018 eliminates the requirement for teams to restrict contest work to a consecutive 36-hour block. Teams will have an 11-day window to download and choose their problem, complete their modeling solution, and electronically submit their solution document. Also new in 2018 will be awards for the top Outstanding teams selected at final judging. Teams can learn more and register via the Internet at www.himcmcontest.com.

MathModels.org

Powered by COMAP content, Mathmodels.org is a wonderful resource for students and teachers to make math modeling a year round activity. Teachers and students may use the materials found on this site to enrich their classes and help prepare students for mathematical modeling competitions. We encourage you to visit www.mathmodels.org.

The International Mathematical Modeling Challenge, IM2C

We are several years into the newest international secondary school mathematical modeling competition: The International Mathematical Modeling Challenge. The purpose of the IM^{2}C is to promote the teaching of mathematical modeling and applications at all educational levels for all students internationally. It is based on the firm belief that students and teachers need to experience the power of mathematics to help better understand, analyze and solve real world problems outside of mathematics itself – and to do so in realistic contexts. Each country administers the contest for its students and then sends its top two teams to the international final judging.

All USA teams that successfully compete in the HiMCM contest and are awarded a designation of Meritorious or above (Meritorious, Finalist, and Outstanding) are invited to compete in IM^{2}C. From these participants, our USA IM^{2}C judges will select the two top teams to represent the USA in the IM^{2}C international round. The fourth annual IM^{2}C is set to take place March 12th through May 7th, 2018. To learn more visit: www.immchallenge.org.

2017 Problem Discussions and Judges' Commentary

Regardless of the problem chosen, competitive papers include a comprehensive summary, address all requirements with reasonable responses within 30 pages, and write a clear letter or memo. Better papers do all of the above in an articulate, well-supported, well-organized, and well-presented manner. The best papers combine complete mathematical and logical analysis in an organized presentation beyond simply addressing the requirements. These papers are easy to read, flow logically, and they include sections that addressed assumptions with justifications, the modeling process(es), results of modeling and analysis, strengths and weaknesses, conclusions, and references.

The specific problem discussions below provide comments on how teams addressed each problem.

Following this section we break down the various parts of a submission with examples, and provide judges' comments about the solutions and the presentation of the solutions. To view the complete problems visit www.mathmodels.org.

Problem A: Drone Clusters as Sky Light Displays

In Problem A, teams worked with their town's Mayor to design an aerial light show using clusters of drones. The Mayor asked the teams to investigate the idea of using drones to create three possible sky light displays for the city's annual festival: a Ferris wheel, a dragon, and an image of their choosing.

The problem materials pointed teams to the video of a 2016 light show created by a cluster of 500 drones, controlled by a single laptop and one pilot. The problem also offered the Intel® Shooting Star™ drone as an example of a drone used for aerial light shows.

For each of the three required sky light displays, teams had to determine the number of drones required and mathematically describe the initial location for each drone device. They then determined the flight paths of each drone, or set of drones, that would animate each of their images, and describe their animations. Teams established and discussed the requirements for their 3-display light show to include required launch area, air space, safety, and duration of the show. Finally, teams wrote a two-page memo to the Mayor to report the results of their investigation and make a recommendation as to whether or not to do the aerial light show.

Most teams conducted initial research on drones and the use of drones in aerial light displays. This research provided teams specifications of example drones, particularly the Shooting Star™ drone, to include size, speed, life of battery, and safe separation distance. In addressing all requirements, competitive papers designed the drone positions and animation actions for each of the three displays, determined and discussed requirements for the light show, and wrote an articulate and clear memo to the Mayor.

In designing the drone positions and actions, many teams first sketched each image using a "continuous" picture and then began to discretize it to account for the individual drones. Some teams used various graphing programs such as DESMOS™, while others simply sketched their design on graph paper. Either method was fine if it resulted in an illustrative diagram. The best papers placed the drones in the sky and determined an appropriate number of drones by explaining and incorporating a scale that accounted for the distance from ground level (noting that the lights will be a distance away from the viewer). These papers offered clear mathematical descriptions of locations and animation paths. Additionally, these best papers addressed nuances such as the audience viewing angle, 3D space perspective, local or governmental restrictions on drone usage, weather conditions, and the geography of the display and viewing areas. The best papers used more sophisticated mathematical descriptions, were more creative in their images and animations, and incorporated more of the actual drone capabilities in their analysis.

Problem B: Ski Slope

By Pudelek (Marcin Szala) (Own work) [CC BY-SA 3.0 (https://creativecommons.org/licenses/by-sa/3.0)], via Wikimedia Commons

Problem B had the teams working for the representative of a group of wealthy winter sports fans interested in finding a new mountain to develop as a ski resort. Wasatch Peaks Ranch in Peterson, Utah, USA is for sale and Ms. Mogul, the representative for the investors, asked the teams to identify potential ski slopes and trails on the property in order to develop it as a top ski resort and a potential future Winter Olympics location.

The problem materials included a brochure for Wasatch Peaks Ranch, a topographic map of the area, and a partial list of North American ski slopes with comparison data. The problem statement also included a discussion of how to measure and determine ski trail levels.

The problem required that teams design the new Wasatch Peaks Ranch ski area to have main slopes of varying lengths, plenty of trails, at least a total of 160 km of slopes and trails, and a distribution of 20% beginner, 40% intermediate, and 40% difficult rated slopes. Teams then ranked their proposed ski area against existing ski areas/resorts in North America.

Finally, teams wrote a two-page memo to Ms. Mogul reporting the results of their design and ranking.

We saw many viable and creative solutions to our ski slope problem. Most teams conducted some initial research on ski slopes, and on Olympic venues and events. While having skied or visited a ski resort may have provided some teams more familiarity with the scenario, the focus was on terrain and slope. Teams used the materials and maps provided to begin their design. In addressing all requirements, competitive papers offered a reasonable ski slope design for the Wasatch Ranch. Specifically, these papers identified the actual slopes by difficulty rating on a map and measured those slopes so that they met the design criteria, ranked their proposed ski area against existing ski areas/resorts in North America, and wrote a clear letter to Ms. Mogul.

In designing their ski slopes, most teams analyzed the terrain and slopes in various areas on the mountain. Some teams designed and drew their slopes by hand on the provided maps based on analysis of the terrain. Other teams used available terrain and satellite maps found on the Internet, along with terrain analysis to identify possible trails based on the gradient (slope) of the mountain. Both methods provided reasonable solutions with better papers explaining how they analyzed the terrain and determined where to place the slopes and trails. Some graphing programs assisted teams in electronically marking the designed slopes and trails. The better papers discussed choices and decisions about slope placement over the entire mountain. The best papers also addressed integration of the main ski runs with connecting trails, as well as noting where ski lifts might be located.

In developing a ranking system, most teams used the provided North American ski slope comparison data, along with web sites that rank ski slopes. Competitive papers used these data to compare with their ski slope design and to rank Wasatch among these other slopes. Better papers accounted for variables available for all ski slopes, explained and weighted the variables, developed a comparison model, and ranked the resorts. The best papers included a well-developed mathematical comparison model which included the provided data along with additional information such as average snow fall, snow making ability, lodging ability, local transportation system, and ease of travel to Wasatch (e.g. location of closest airport).

Judges' Comments

While the Problem Discussions above provide comments on the solutions to this year's problems, here we examine the sections of a submission and provide comments about the solutions and the presentation of the solutions from a judge's point of view. We have included examples from several papers at the end of this article. Mathmodels.org members can view all the unabridged versions of the Outstanding papers online.

Overall

Participants must ensure their papers follow the contest rules posted on the contest site. Papers that are coherent, organized, clear, and well written provide a great impression to the judges. The logic and mathematics of these papers are easy to follow. Teams should present their solution and analysis in fewer than 30 pages using at least 12-point font. While students may want to include some background research on the problem topic, this information should be brief. It is not the number of pages, but the ability to complete all contest requirements and communicate the solution in a concise and articulate fashion that will merit recognition. Students should use spelling and grammar checkers before submitting a paper. Foreign papers should insure that all symbols in tables and graphs are in English.

Papers considered for Meritorious and Outstanding start with a clear summary that describes the problem. They then preview their paper with an organized Table of Contents. They present assumptions with justifications, explain the development of the model and its solutions, support the results mathematically and communicate them clearly, address strengths and limitations, and finally, close by stating overall conclusions.

Executive Summary

Judges are best able to analyze a paper when students restate the problem in their own words and clearly preview the focus and organization of their paper. Teams should write these executive summaries after they finish their solutions as they summarize the entire contents of the paper. Teams should consider a three to five paragraph approach for their summary: a restatement of the problem and questions in their own words, a short description of their method and solution to the problem (in words and not in mathematical expressions), and the conclusions providing the numerical answers in context. The executive summary should entice the reader, in our case the judge, to read the paper. Although written last, ensure you spend time on this important part of your submission. Your executive summary is the first page the judges will read and provides the first impression of your paper. Example 1 is a good summary for Problem A: *Drone Clusters as Sky Light Displays*.

The comments above for summaries also apply to any required memos or letters. Your letter might briefly describe your model or process, but do so in a non-technical manner as the reader may not have a mathematical background. Your letter should focus on why your model and its results are applicable and important to the reader. The key in your letter is to answer the question(s), and to interpret and communicate the results clearly. Example 2 is a good 2-page memo for Problem A: *Drone Clusters as Sky Light Displays*.

Assumptions with Justifications

Modeling assumptions should include only those that come to bear on the solution (this assists with simplifying your model). Long lists of assumptions that do not play directly in the context of model development or its solution are not considered relevant and deter from a paper's quality. Assumptions that oversimplify the problem too much do not allow for a full solution. You should include a short justification to show the assumption is reasonable and necessary. Example 3 displays several good assumptions for Problem A: *Drone Clusters as Sky Light Displays*. Note that within the paper, this team has used in-text citations. The team then listed these sources in a Reference section at the end of their paper.

Mathematical Model

Papers should explain the development of the mathematical model(s), and define all variables. Teams that merely present a model without explaining or showing the development of that model do not generally do well. Presenting multiple models, without identifying the most appropriate model to answer the questions, is detrimental to your paper's success. Your team may have considered several models that did not work before developing a good model. Do not include all of your trials, but instead clearly present the development and results of your successful model. It is helpful to the judges to have the definitions of your variables near your model so they can follow the logic of your model. If you include a long list of variables in a table early in your paper, consider reminding the reader of the variable definitions as you present each equation in your model.

Judges do value creativity and thinking "outside of the box" in your modeling process. There is always more than one appropriate solution method to our HiMCM problems, so be creative. Example 4 is a description of how one team developed a model for the dragon image of their light show for Problem A: *Drone Clusters as Sky Light Displays*.

Strengths and Limitations

Teams should address strengths and limitations in evaluating their model and solution. Include model extensions or sensitivity analysis of your solution. Validate your model, even if by numerical example or intuition. Example 5 displays a good strengths and limitations section for Problem A: *Drone Clusters as Sky Light Displays*. Example 6 shows a nice sensitivity analysis for Problem B: *Ski Slope*

Conclusion

A clear conclusion and answers to the specific scenario questions are key components to an Outstanding paper. Attention to detail and proofreading the paper prior to final submission are vital as the judges look for excellence in your submission. Example 7 is a good conclusion from Problem B: *Ski Slope*.

Citations and References

Citations are very important within the paper, as well as either a reference list or bibliography page at the end. Teams that use existing models should cite their source within the paper at the point they present the model and not just include a reference citation in the back of the paper. This is also true for all graphs and tables taken from the literature. Use "in line" documentation with footnotes or endnotes to give proper credit to outside sources. All data, figures, graphs, and tables that come from outside sources require documentation at the point in the paper where they appear. Lack of documentation will result in a lower score. We have noticed an increase in use of Wikipedia. Teams need to realize that although useful, information from Wikipedia might not be accurate. Teams should recognize and acknowledge this fact.

The quality of HiMCM submissions continues to improve each year. We enjoy reading all of the papers submitted for review and are truly impressed by the work of the student teams. We encourage teams to review the comments and guidance provided in this article and to visit: **mathmodels.org** in preparation for next year's contest.

List of Following Examples:

1. Summary (Problem A, Team 8040: Shenzhen Vanke Meisha Academy)
2. 2-Page Memo (Problem A, Team 7359: YK Pao School)
3. Assumptions (Problem A, Team 7879: Raffles Institution)
4. Model Development (Problem A, Team 8028: Shenzhen College of International Education)
5. Strengths and Limitations (Problem A, Team 8206: NC School of Science and Mathematics)
6. Sensitivity Analysis (Problem B, Team 8121: Columbus North High School)
7. Conclusion (Problem B, Team 8201: NC School of Science and Mathematics)

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Summary Sheet

The problem of a sky display with Drones can be interpreted as designing visible and safety locus of drone lights to form image of three different objects, which are Ferries wheel, dragon, and a self-designed pattern (a smiling face) respectively. In this paper, we successfully developed a few comprehensive mathematical models to determine the number of drones required and model their initial location and the flight paths mathematically in both static and animate state.

We addressed this problem mainly through space coordinate frame which allowed us to formulate the motions of all the drones in all animations. In the first part, we designed the Ferris wheel as a composition of several geometrical shapes, a circle with spokes and two identical triangles. Considering the space coordinate of every points and using the idea of matrix, we constructed the model based that on the thought of substituting every points own coordinate with its consecutive former one in the same circular motion path.

In the second part, we made a two dimensional dragon that has detailed features such as head, tail, claw, and scale. We made the body of the dragon as the shape of sin(x), and the movement model was created, enabling the dragon to move smoothly and periodically during the animation.

For the third part, our team decided to create a calm face that would eventually turn into a smile face. We made a few functions to express straight lines that represent expressionless eyes and mouth were bent to a smile face by changing the coordinates of the points in the straight line to coordinates of points that fit in parabolas.

After finishing all three models, our team gave out the evaluation for each of them with MATLAB programming. Based on the calculation, we determined to use up to 427 drones to create three sky displays. And the required launch area would take up 9m×9m, and the air space of 100m in length, 50m in width and 80m in height was required. The duration for the light show would be 30 minutes.

In summary, our mathematical model could successfully determine the requirements for this aerial light show.

Example 1: Summary from Team 8040: Shenzhen Vanke Meisha Academy.

Memo

To: Mr. Mayor

From: 2017 HIMCM Team #7359

Date: 2017/11/18

Topic: Report on possibility of drone light performance in New Year.

Dear Mr. Mayor:

You must have heard or seen the astonishing drone light show on the sky of Florida. In order to add unique festive atmosphere to the upcoming New Year of 2018, we suggest a drone light show as the best option. Please have a look at our report on the plan and possibility of such a light show below:

I. Plan and Design

Our general plan is to use a total of 420 drones arranged in a 20*22 order initially. They will take off in 1 minutes and form a vivacious and dynamic Ferris Wheel in the sky. With specific movement of the fleet of drones, the Ferris Wheel figure will start to spin. Three minutes later, a ferocious dragon will appear in the sky and perform a dance in front of the people. Finally, with a 5-second-count-down clock, a figure “2018” will appear in the sky to celebrate the New Year and put an end to the 10-minute drone light show. Below are the design for figures used in the light show:

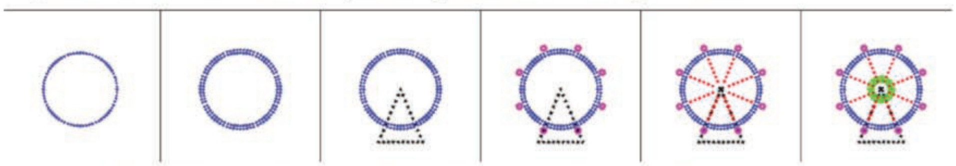


Fig1: Design of taking off and formation of Ferris Wheel

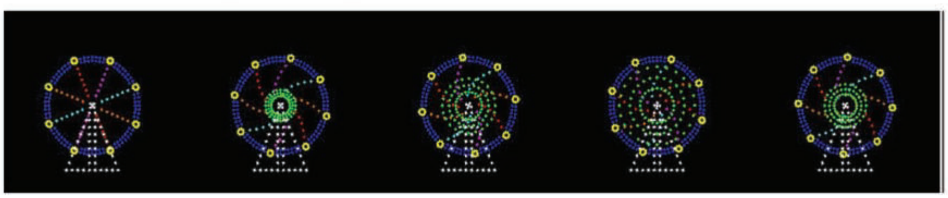


Fig2: Design of Rotation of the Ferris Wheel

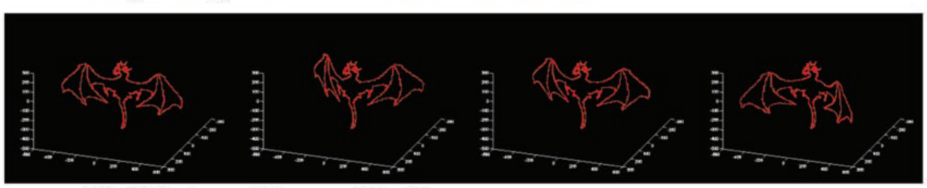


Fig3: Design of Dance of the Dragon

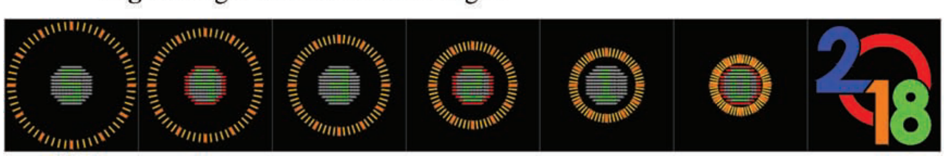


Fig4: Design of the Count-Down Clock

Example 2: Memo from Team 7359: YK Pao School

Mr. Mayor,We hope that our proposal has intrigued you. You might ask why use drones to do a light show and our answer is:

II. Reasons for Using Drones

First of all, a drone light show is a perfect combination of art and technology. Drones can control their trajectory precisely, and they are capable of showing various colors to create vivid and even moving figures. This will be a novel visual experience for the people and will thus bring a spirit of modern time to the city.

Also, a drone light show is environmentally friendly. Fireworks might be as well beautiful, but they produce huge amount of harmful gases such as Sulfur Dioxide, Nitrogen Monoxide, and Nitrogen Dioxide. There is no such problem with a drone light show and drones also can be recycled after performance.

Moreover, our team has already developed an advanced system to control the drones' flight. All you have to do is to provide an image, then we can finish off by designing, planning of trajectory, and determining number of drones.

Last but not least, we have also considered safety issues which you must have concerned with. We calculated taking of time for every drone according to its position in the sky and prevented collision issues during transition between two figures by designing a dense scatter diffusion algorithm. The possibility and safety of our plan is confirmed by simulations on the computer.

III. Plan for the Site for Performance

We will be using *Intel Shooting Stars* Drones in this performance. The size of each drone is 384*384*93 mm. 420 drones will be used in the performance and 10 for backup, so the total number of drones used will be 430. They will take off in a formation of 20*22. Assume that each drone takes 1m^2 of space, the area of the site for taking off should be at least around 430 m^2. We suggest the site to be located at the top of Times Square or a carrier boat on Hudson River in New York. We also have few suggestions for site for the audience:

1. Previous audience sites of firework shows: e.g. Times Square or Prospect Park, etc.
2. Open space near the Hudson river so figures in the sky can reflect: e.g. South Street Seaport, Williamsburg, Green point, and Dumbo, etc.

IV. Cost and Budget

We suggest to rent drones instead of purchasing them. For renting, every drone costs $30, and the total fee will be $12900. On the other hand, a similar Fireworks show will cost about $28500. As a result, a drone light show is also economically friendly.

We sincerely wish that the drone light show we designed could bring joy to the New Year and we hope to earn your support, thank you!

Example 2 Continued: Memo from Team 7359: YK Pao School

2.2 Assumptions

The following assumptions are made:

Assumption 1: The model of the drones used in this show is the Intel® Shooting Star™drone, with the following specifications (“Intel® Shooting Star™Drones Featured in First-Ever Drone Integration during Pepsi Zero Sugar Super Bowl LI Halftime Show”, 2017):

1 Each drone has a flight time of up to 20 minutes.
2 Each drone can travel at the speed of $3ms^{-1}$ at all times.
3 Each drone has a size of $384 \times 384 \times 93$ mm

Justification: Intel® Shooting Star™drones have already been used for aerial light shows in the past (as mentioned in the question) and are hence appropriate to be used as drones in the outdoor aerial light show for our citys annual festival.

Assumption 2: The show will be part of an annual festival of a city in the United States of America, and will have to comply with rules related to drone operation stated by the Federal Aviation Administration (FAA) of the United States Department of Transportation (Federal Aviation Administration, 2017):

No.	Rule	Adherence
1	Drone must fly under the height limit of 121.92 m*	Not adhered to (refer to Assumption 4)
2	Drone must fly during the day and not at night*	Not adhered to (The show will be carried out at night.)
3	Drone must fly at or below $160 \cdot 934kmh^{-1}$	Adhered to
4	Drone must not fly over people	Adhered to

Justification: We chose United States of America as the country for our model as they have clear and quantitatively specified rules on drone operation that we can adhere to in our model. Note that items marked with * are negotiable.

Assumption 3: The show is carried out at night (Organizers need to obtain waiver for rule 2, i.e. Drone must fly during the day and not at night), with a clear, dark sky as the background, and there will be negligible light pollution in the vicinity of the show, such that the audience will be able to see a drone if and only if the light of that drone is turned on.

Justification: Given that the annual festival of a large city should be a major event that is carried out well, the show should be carried out under optimal viewing conditions for the audience. Intel has managed to obtain the

Example 3: Assumptions from Team 7879: Raffles Institution waiver for flying drones at night ("Intel Receives Drone Waiver from the FAA", 2016). Since this show is similar in nature to the one organised by Intel, the organizers should also be able to receive a waiver for rule 2 in Assumption 2.

Assumption 4: Minimum distance between drones is 1.5m, i.e. the distance between any two drones should not be within 1.5m at any point of time in air. It will not be a concern when the drones are on the ground.

Justification: A minimum distance between drones has to be set to prevent any collision between drones, which may result in damage to the drones with the drone or any broken parts of the drone becoming a falling-object hazard to the audience on the ground. In a previous aerial light show, five hundred Intel® Shooting Star™drones were spaced at least 1.5m apart from each other (aeronewstv.com, 2016) (Kaplan, 2016). We will adhere to this minimum distance as it has been tried and shown to be safe. It is reasonable to assume this distance need not be maintained by drones still on the ground as they are not interfering with moving drones.

Assumption 5: The drones will remain in the sky throughout the show. In other words, they will not return to the ground during the transition between consecutive displays. They will only do so after they finish the third and final display - the double-headed eagle.

Justification: For simplicity of the model and to minimise transition time between consecutive displays, the drones shall remain in the sky throughout the show. Instead, the light on each drone can be turned on or off (refer to Assumption 3) to give the audience the illusion that the number of drones in the sky is changing during the show. Furthermore, in Coachella Drone Light Show Delights Concert (KG, 2017), an aerial light show held in the past, the drones stayed in the air throughout the show, showing that it is indeed feasible to have such an arrangement.

Assumption 6: All drones will always fly in a straight line, except when two drones that are approaching each other are less than 2 metres apart, those two drones will move around each other in a circular path.

Justification: It is reported that this drone is pre-programmed to avoid collisions. (Glaser, 2016) For simplicity, we assume they can safely avoid any collision.

Assumption 7: All drones function normally and are equipped with motion sensors that can sense presence of birds in the sky and fly around any bird in its path via a circular path.

Justification: For simplicitys sake, we assume so.

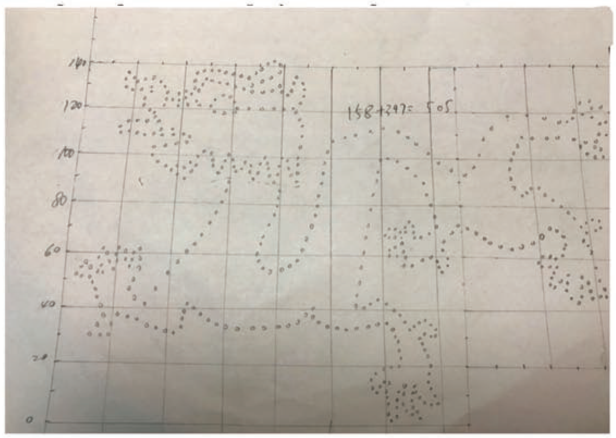
Assumption 8: The safe distance between audience and drones will be 155m.

Justification: This safe distance is used in Walt Disney Light shows. ("4 Things You Need To Know Before Using Drones At Your Event", 2017)

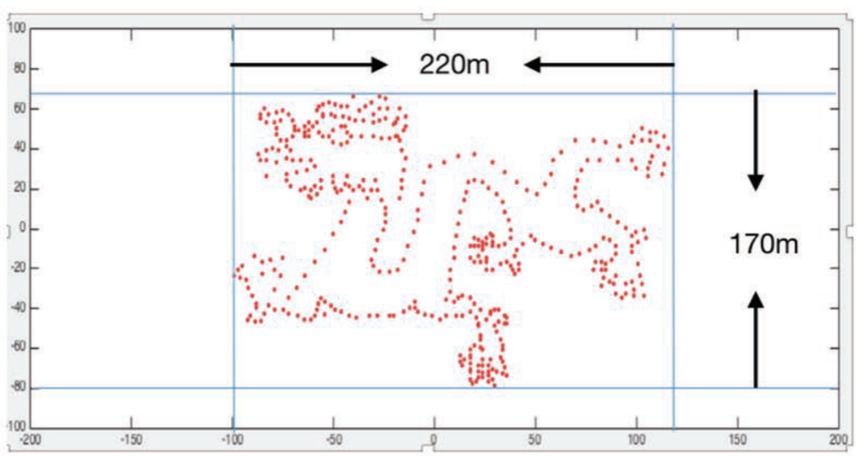
Example 3 Continued: Assumptions from Team 7879: Raffles Institution

Model Development

We made a graph of the dragon by hand and selected key points from the graph. There are 505 dots in the initial hand-made graph of our dragon. Considering the safety distance between drones, and other displays had lower requirement of the number of drones, we decided to reduce the number of dots in the dragon figure without too much losing quality. Finally, we deleted 79 dots and leaving 426 dots in the fire. The actual xy-coordinates of each dot is measured and recorded. The center of mass of the figure is selected as the origin$(0,0)$ of the coordinate system. The dragon figure was displayed using Matlab, which looks very good.



The total dimension of the dragon is 220 m $\times$ 170 m, and the distance between each pair of two drones is at least 3 m. We are going to use 426 drone in each of the following display.



Example 4: Model description from Team 8028: Shenzhen College of International Education

10 Strengths & Weaknesses

10.1 Strengths

- Our model mathematically describes the linear path (including initial and final positions) for each drone for both transitions and animations. We are able to calculate a definitive start and end point by optimizing each drone's destination in the display using the Hungarian algorithm. This concept can be witnessed within Figure 7. The blue drones, which start towards the left of the x-axis move to the left of the Ferris wheel. Similarly, the red drones, which start towards the right of the x-axis move to the right of the Ferris wheel. The brown drones, which are closest to the center of the Ferris wheel, form the wheel's center. This notion is also seen in the Dragon and Balloon models, indicating our optimization of shortest-distance linear paths is highly functional.
- Our model outputs the velocity for each drone during all transitions and animations. Using our path functions and collisions avoidance program, a drone (such as a DJI Phantom 3) could be easily loaded with a matrix of velocity commands and can follow the path accurately.
- The pathing algorithm prevents drone collisions without sacrificing overall fluidity by optimizing drone transition formations and finely tuning drone velocities such that all drones will move to their target locations as efficiently as possible while not colliding.
- Safety analysis predicts the highest wind speed in which our drones may fly so that event organizers can plan a safe light show date based on weather forecasts.
- The collision avoidance algorithm to move the drones such that the interaction between them is minimized. This algorithm is based on the idea that the drones move to their target points such that the sum of the distances collectively traveled is minimized, otherwise known as the Extremal Principle.

10.2 Weaknesses

- Our model does not account for turbulence between drones. Turbulence is a significant real word phenomenon that does occur between aircraft. Though the drones are spaced significantly away, there is a chance that turbulence could interfere with the drone flights when they are more densely packed (such as during takeoff or landing).
- Our model does not account for luminous flux; essentially, it does not account for the fact that objects get less bright the further away they are seen. We are assuming that the drones' lights will always be seen, but should take into account factors like light pollution and diminishing intensity.
- The pathing algorithm is very computationally expensive: the runtime increases rapidly with increasing data set complexity, increasing collision radius, decreasing time step size, and decreasing change in velocity with each trial. However, if the time step or velocity change factor is too large, the drones will choke up on each other and the simulation will jam. If the collision radius is too small, the simulation may not be sufficient to account for real-world drone collisions.
- The algorithm cannot calculate the collision avoidance routes for the drones' transition from the 14x14 grid to the Dragon, due to the program's excessive runtime for complex designs. This could be remedied by using a more powerful computing platform or having a longer period of time for computations.

Example 5: Strengths & Limitations from Team 8206: NC School of Science and Mathematics

5.3 Ranking Sensitivity Analysis

Our ranking system, like all models, is not perfect. Our confidence in the model is high, however, as sensitivity analysis shows that small changes do not significantly affect the rankings. For example, snow quality is measured as:

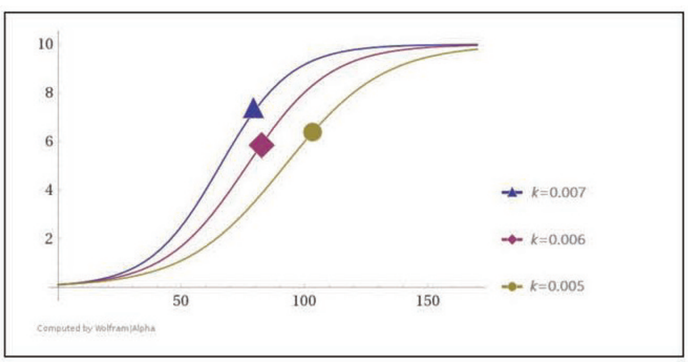
	Annual Average Snowfall in meters (w)	Difference in Proportion Facing N/E and Proportion Facing S (d)	Snow Quality Rating (Q)
Actual Wasatch Values	10.20	66.7%	9.0
Slightly increasing *d* (increasing N/E facing, decreasing S facing)	10.20	**73.3%**	9.5
Maxing *d*	10.20	**100.0%**	10.0
Halving *d*	10.20	**33.3%**	2.3
Slightly decreasing *w*	**9.5**	66.7%	8.5
Doubling *w*	**20.00**	66.7%	10.0
Halving *w*	**5**	66.7%	2.2

Changing average annual snowfall w creates large changes but only when there are drastic changes; otherwise, the snow quality rating changes appropriately to reflect changing snowfall. Similarly, changing the d also can have significant change (where halving d will decrease rating by more than half, as such a large decrease puts the difference at well below the average d of 51.6%).

Similarly, the fit rating F_{skill} also varies as the different inputs of the number of slopes for that skill level n, length of trails for that specific skill level (km) t, total length of trails (km) l, and vertical drop (m) v change. For F_b (fit rating for beginners), the rating increases proportionally to t and l and inversely proportionally to v:

$$F_b = \frac{1}{0.1 + 9.9e^{-10 \times k_b \times n \times t \times \frac{11l}{10v}}}$$

As k_b is our adjusting constant so that we can change the sensitivity of our model to n, t, l, and v.



The chart shows the variation of F_b while k_b varies.

The above chart shows the variation of F_b for different values of k_b. We set $k_b = 0.006$ in our calculations as for the data set we used, $k_b = 0.006$ gave a wide range of reasonable F_b ratings. Using the same optimization strategy (of getting the largest range of fit ratings for a given skill level), we had $k_i = \frac{1}{5500}$ and $k_e = \frac{1}{26400}$.

Example 6: Sensitivity Analysis from Team 8121: Columbus North High School

Conclusion

Using both mathematical and computational models, we evaluated potential ski slopes on the Wasatch Peaks Ranch property. We drew topological and altitude data from US Geological Survey databases, and used it to create a series of maps, including a slope gradient, solar aspect, and a vector field fall line. Overlaying the three graphs on top of each other allowed us to identify 199.4 km of ski slope, with a distribution of 22.72% green circle, 38.50% blue square, and 38.77% black diamond. Applying our ranking metric to the Wasatch Peaks Ranch indicates that it would rank as the fourth best ski resort out of the ones sampled in North America. However, our ski slope prides itself on being extremely ecologically friendly, using natural fall lines and preserving as much of the topography and landforms (rivers, basins, creeks, etc.) as possible, keeping costs low and showcasing the beauty of the American landscape for all to see at the 2026 Winter Olympics.

We also developed a computational agent-based model in NetLogo, which allows us to input a series of parameters and obtain an output graph detailing the average time spent waiting for lifts as well as the amount of people on the slopes compared to people riding the lifts. This analysis provides useful data for us to judge the amount of ski lifts required for a given park, with the goal being to minimize time spent waiting and to maximize the ratio of people on the slopes to people on lifts.

Example 7: Conclusion from Team 8201: NC School of Science and Mathematics