The 2016 HiMCM

Dr. Kathleen Snook, HiMCM Director

What a fantastic endeavor the High School Mathematical Contest in Modeling (HiMCM) continues to be for students, advisors, schools, and judges! We congratulate our 2016 Outstanding and National Finalist Teams as we celebrate our nineteenth year.

Outstanding Teams

Chesterfield County Math/Science High School at Clover Hill, Midlothian, VA, USA. Advisor: Pete Peterson.

Hangzhou NO. 14 High School, Hangzhou, Zhejiang, China. Advisor: Lin Xia.

Illinois Mathematics and Science Academy, Aurora, IL, USA. Advisor: Steven Condie.

Miss Porter's School, Farmington, CT, USA. Advisor: Matthew Poage.

North Carolina School of Science and Mathematics, Durham, NC, USA. Advisor: Daniel Teague (3 Teams).

The Affiliated High School of South China Normal University, Guangzhou, Guangdong, China. Advisor: Jingshu Huang.

The High School Affiliated to Renmin University, Beijing, China. Advisor: Yi Li.

National Finalist Teams

Archbishop Williams High School, Braintree, MA, USA. Advisor: Christopher Brunner.

Capital High School, Helena, MT, USA. Advisor: Dennis Peterson.

Chesterfield County Math/Science High School at Clover Hill, Midlothian, VA, USA. Advisor: Pete Peterson.

Friends' Central School, Wynnewood, PA, USA. Advisor: Julie Plunkett.

Illinois Mathematics and Science Academy, Aurora, IL, USA. Advisor: Steven Condie.

Mills E. Godwin High School, Henrico, VA, USA. Advisor: Miranda Watson.

Oregon Episcopal School, Portland, OR, USA. Advisor: Lauren Shareshian.

The High School Affiliated to Renmin University of China, Beijing, China. Advisor: Di Wu.

Woodbridge High School, Irvine, CA, USA. Advisor: David Gesk.

The solutions we received were truly impressive. The participating teams accomplished the vision of the HiMCM's founders by providing *unique* and *creative mathematical* solutions to complex open-ended real-world problems. As in the past, students chose from two problems, both representing "real-world issues." This year's problems allowed students to either plan for a triathlon in the *Swim, Bike, and Run* Problem A, or expand a company's warehouse locations to offer 1-day shipping in the *Shop and Ship* Problem B. The final judging results and 2016 statistics are shown in Figure 1.

Judging Results

The mathematical modeling ability of participating students and their advisors continues to be evident in the problem solutions and professional submissions we receive. As students and advisors engage in mathematical modeling at a higher level, we are happy and excited to assist their efforts.

Problem	Outstanding	%	National Finalist	%	Finalist	%	Meritorious	%	Honorable Mention	%	Successful Participant	%	Total
A	4	1%	5	1%	30	7%	47	11%	145	34%	198	46%	**429**
B	5	1%	4	1%	27	7%	54	13%	138	33%	183	45%	**411**
Total	9	1%	9	1%	57	7%	101	12%	283	34%	381	45%	**840**

Figure 1: 2016 HiMCM Statistics

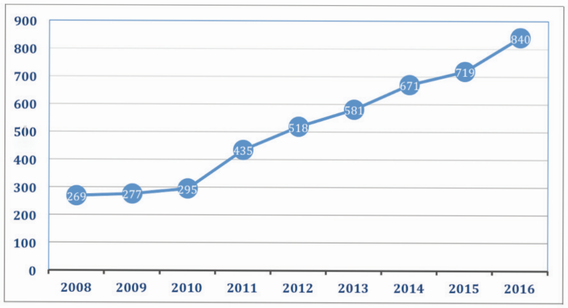


Figure 2: Number of HiMCM teams vs. contest year from 2008-2016

Overview

The HiMCM continues to grow. Figure 2 shows a plot of the growth over time. The trend continues to reflect an increasing engagement of high school students in mathematical modeling.

The 2016 contest had 950 registered teams resulting in 840 total submissions (88.4%), a significant registration increase of about 22% over last year. In total, 3569 students competed, representing an increase of 28%. A wide range of schools/teams competed including teams from the United States, New Zealand, Thailand, United Arab Emirates, Philippines, Singapore, Hong Kong SAR, and China. The 330 teams from the United States represented 23 states. This number was up 3.8% from last year. Submissions included 510 foreign teams, representing a 27% growth. China represented about 91% of the foreign entries.

Of the 3569 student participants this year, 1334 or about 37.4% were female, 2186 were male, and 49 did not specify gender. Since the start of the HiMCM in 1999, females represent 36.7% (9595) of the 26,175 total participants. This participation has been fairly consistent over a number of years. We hope that all competing students will enjoy their contest experience and continue to pursue further STEM education.

Problem Choice

We congratulate all students and advisors for their varied and creative mathematical efforts. Again this year, it appears that teams enjoyed developing solutions to their chosen problem. We continue to encourage all registered teams to submit a solution in order to experience the learning impact and satisfaction of fully completing this challenging contest. Of the 840 submissions, 429 completed Problem A: *Swim, Bike, and Run*, and 411 completed Problem B: *Shop and Ship*. The 36 continuous hours to work on the problem provided for high quality papers for both problems.

Judging

All submissions this year were electronic which allowed us to expand our judging pool. In December 2016, we used two regional judging sites and a third group of remote judges. The regional sites were located at Francis Marion University in Florence, SC and Carroll College in Helena, MT. Remote judges were located in Alabama, California, Georgia, Illinois, Massachusetts, Virginia, and Pennsylvania.

All judging was blind with respect to any identifying information about the participants or their schools. Judges ranked papers as Finalist, Meritorious, Honorable Mention, and Successful Participant. Each site judged papers for both problems A and B. Judges sent all papers ranked as "Finalist" to the National Judging in Atlanta, Georgia. This year, 75 papers were sent to Atlanta for ten judges, from both academia and industry, to consider. At this level, judges first selected the best of the Finalist papers as National Finalists, and then, from this set, chose the "*best of the best*" as Outstanding papers. Nine papers earned the Outstanding award and nine were deemed National Finalists. The national judges commended the regional judges for their efforts in selecting the high quality Finalist papers forwarded to Atlanta. We feel that the structure of Regional and National Judging provides a good process for identifying the best papers.

The Future

The HiMCM attempts to give all high school students, and especially underrepresented students, an opportunity to compete and achieve success in mathematics. Our efforts are always toward meeting this important goal. Mathematical modeling is growing within the curricula at the high school level. This contest provides a vehicle for using mathematics to build models that allow students to represent and to understand real world behavior in a quantitative way. It enables student teams to look for patterns and think logically about mathematics and its role in our lives. We continue to strive to improve the contest, and want the contest accessible to all students. Any school and team can enter, as there are no restrictions on the number of schools or the numbers of teams from a school. A regional judging structure offers flexibility to accommodate the number of teams.

Mathematics continues to be more than just learning skills and operations. Mathematics is a language that involves our daily lives. Applying the mathematical principles and concepts that one learns is key to individual and societal future success. The ability to recognize problems, formulate a mathematical model, use technology, and communicate and reflect on one's work are important skills to develop. Students gain confidence by tackling ill-defined problems and working together to generate a solution. We are excited that in our contest, applying mathematics is a team sport!

Mathematical modeling is an art and a science. Advisors need only be motivators and facilitators to encourage students to be creative and imaginative. Through modeling, students learn to think critically, communicate effectively, and be confident, competent problem solvers. Success is not only about the procedural technique used, but the conceptual understanding in discovering the role of assumptions and model development in driving those techniques to a valid solution and conclusion. We encourage all high school mathematics faculty to get involved, to encourage students to be problem solvers, to make mathematics relevant, and to open the doors to students' future success.

Contest Dates

Mark your calendars for the next HiMCM to be held October 27 –November 20, 2017. Registration for the 2017 HiMCM will open in September. Teams will choose a consecutive 36-hour block within the extended contest window to complete the problem and electronically submit a solution. Teams can learn more and register via the Internet at: **www.himcmcontest.com.**

MathModels.org

Powered by COMAP content, Mathmodels.org has been reimagined as a new resource to make math modeling a year round activity. Teachers can use the materials found on this membership-based site to enrich their classes and to help prepare students for mathematical modeling competitions: **www.mathmodels.org**.

Awards

The award format has changed slightly in the past two years as the contest continues to grow internationally.

After final judging, the HiMCM papers are designated in the categories below. The top approximately 20% of submitted papers receives a designation of Meritorious or above. The top approximately 2% is designated as National Finalist or Outstanding.

Successful Participant
Honorable Mention
Meritorious
Finalist
National Finalist
Outstanding

The International Mathematical Modeling Challenge (IM2C)

We are in the preliminary years of a new international secondary school mathematical modeling competition. The purpose of the IM^{2}C is to promote the teaching of mathematical modeling and applications at all educational levels for all students. It is based on the firm belief that students and teachers need to experience the power of mathematics to help better understand, analyze and solve real world problems outside of mathematics itself – and to do so in realistic contexts.

All USA teams that successfully compete in the HiMCM and are awarded a designation of Meritorious or above (Meritorious, Finalist, National Finalist, or Outstanding) are invited to compete in The International Mathematical Modeling Challenge, IM^{2}C. From these participants, we select two teams to move on and represent the USA in the IM^{2}C international round.

The third annual IM^{2}C is set to take place March 13th through May 8th, 2017. To learn more visit **www.immchallenge.org.**

Problem Discussions and Judges' Commentary

The problem discussions below provide general comments on how teams addressed each problem. Following this section, we break down the various parts of a submission and provide judges' comments about the solutions and the presentation of the solutions. The complete problems are available to the public at: **www.mathmodels.org**.

Problem A: Swim, Bike, and Run

Author: Kathleen Snook, COMAP Inc.

In Problem A, teams were asked to work with their town's Mayor to organize an Olympic triathlon to support a local youth organization. An Olympic triathlon begins with a 1500m swim, followed by a 40k bike ride, and finally a 10K run. There is a transition area where athletes change gear between each event. The organizers expected some number of professional and premier athletes, but as an open triathlon, participants did not have to qualify and athletes of all levels were welcomed. Teams planned for 2000 triathletes and the problem materials included a database of the results of a recent triathlon, which included participants' gender, age, status (professional, premier, open), race event times, transition times, and total times.

The race sponsor wanted to ensure that congestion on the course was minimized so that participant athletes would be able to proceed without hindrance during each phase of the triathlon. The mayor was concerned about traffic in town and wanted to have the roads closed for no more than 5.5 hours.

The problem required that teams use the data set to determine the various athlete divisions in the race and to develop a schedule of wave start times to minimize congestion and road closure time. They then explored any advantages achieved in congestion and road closure time if they adjusted race distances of one or more of the events. Finally, teams wrote a letter to the Mayor summarizing their analysis and presenting the race day event schedule.

Many teams started this triathlon problem by using basic statistics to organize and perform some initial analysis on the provided data. Judges were surprised, however, that more teams did not continue additional data exploration into the various groups and subgroups, and apply their data analysis findings to the problem. For example, the difference in average race times for men of various ages in the open division was not significant. The same was true for women of various ages. Significant differences did occur between the professional/premier athletes and the open athletes, and between the open athletes of each gender with the Clydesdales and Athenas. Because of this, the organization of the starting waves within the Open category weren't as important as the decision of when to start the Professionals, the Premiers, the Clydesdales, and the Athenas. These were the categories that would most impact the two requirements: minimizing congestion and minimizing road closure time.

Conducting some cursory research on triathlons or other types of races to understand the organization and the concept of the starting wave schedule benefited some teams. For example, it is physically impossible to have several hundred participants start the swimming event at the same time. The schedule required many wave start times at reasonable intervals.

Competitive papers did not assume away or oversimplify the constraints of the problem. Prior to applying any mathematical models, teams should have recognized the data mandated the Clydesdales and Athenas start fairly early in the wave schedule or the road closure time would become a significant issue. Additionally, the data suggested that unless the Professionals and Premiers started the race early in the day, congestion would become an issue for these participants. Better papers did not eliminate the slowest racers or reduce the course length, but they scheduled start times to minimize road closure time and congestion.

Better papers recognized the need for some quantification of congestion either by modeling or by graphical analysis. They applied the results of their congestion analysis to determine the start waves. Many teams had difficulty with this and did not address the congestion issue completely. The fastest participants need to start first or they will be impeded on the course. The slowest participants might have to be spread out among several waves to not impact congestion or road closures.

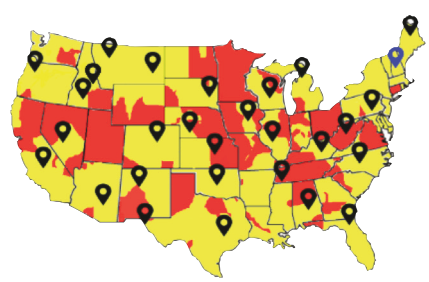
When addressing road closures, the best papers took into account the difference in the bike and run course lengths and suggested creative ways to minimize road closures. For example, closing and opening roads in a rolling manner helped to minimize time of closures.

In examining adjustments to distances, the better papers conducted sensitivity analysis on the various events of the race. In investigating the impact of adjustments to various events for all or some of the participants, congestion and road closure times could be further addressed.

Numerical and graphical modeling and analysis resulted in successfully addressing this problem. The best papers showed a bit of both. Competitive papers were well organized, addressed all requirements, and offered a reasonable and supported plan.

Problem B: Shop and Ship

Author: John Tomicek, COMAP Inc.



Problem B had the teams working for a recreation equipment company with one warehouse located in New Hampshire, USA. With an increase in online shopping, the company is interested in building additional warehouse locations within the 48 continental United States in order to provide all customers in those states with 1-day United Parcel Service (UPS) ground shipping service. The problem included an example shipping map from the UPS web site.

The problem required that teams find the minimum number of warehouses needed to provide 1-day shipping service to the continental United States and to specify the location of those warehouses. They then had to analyze the tax implications of warehouse locations for their equipment sales and the tax implications if they added clothing and apparel to their line of products. Finally, teams wrote a letter to the company's president summarizing their final recommendations and justifications.

We saw many viable and varied approaches to our shipping problem. Teams were extremely creative in their solutions which include the following analysis methods.:

- Geometric: Teams used various shapes representative of an average 1-day shipping area and overlaid these shapes on a United States map to determine warehouse locations.
- UPS Map Analysis: Teams used the UPS web site maps to systematically start from one side of the United States and work across the country to choose locations to cover the area with 1-day shipping.
- Enhanced UPS Map Analysis: After analyzing population centers and the United States interstate highway system, teams chose initial locations and then used the UPS web maps to hone in on the optimal locations for 100% coverage of 1-day shipping.
- Matrix Analysis: Through setting up and analyzing a 48x48 matrix representing the ability to ship from each of the 48 Continental United States to the others, teams honed in and eliminated double coverage to minimize the number of warehouses needed for 100% coverage of 1-day shipping.
- Pixel/Color UPS Map Analysis Programming: Teams wrote programs to cull out data for 1-day shipping from the color-coding of the UPS web site maps and then minimized warehouses through the process of eliminating overlap.

The enhanced map analysis, matrix analysis, and pixel/color analysis methods all proved viable and more competitive solutions when done well. No matter the analysis method, the better papers used their analysis method along with the UPS map data to provide specific warehouse locations and ensure 100% 1-day shipping coverage (or explain why some areas could not be covered; e.g. mountainous or unpopulated areas). The geometric methods did not work independent of the UPS web site. The better papers addressed minimizing the number of warehouses by minimizing any overlap of 1-day delivery services.

Better papers recognized the difficulty in determining the location of warehouses in the more rural areas and in the larger states. For example, California and Texas both required more than 1 servicing warehouse. The best papers specifically identified warehouse locations (city and state) and justified their locations with analysis. Some teams only listed the states and/or simply marked a dot on a map without explanation.

In addressing the questions on tax implications, the better papers developed some sort of tax impact formula. In applying the formula, teams then determined whether or not warehouses should be moved from their initial locations to limit tax liabilities to consumers. The best papers considered and analyzed the tradeoffs between 1-day shipping convenience and lowest cost (reducing taxes).

Numerical and graphical modeling and analysis resulted in successfully addressing this problem. The best papers were a combination of both. Competitive papers were well organized and logical, addressed all requirements, and offered specific recommendations.

Judges' Comments

While the problem discussions above provide comments on the solutions to this year's problems, here we examine the sections of a submission and provide comments about the solutions and the presentation of the solutions from a judge's point of view. We have included excerpts from several papers. All the unabridged versions of the Outstanding papers are available to mathmodels.org members at: **www.mathmodels.org**.

Overall

Participants must ensure their papers follow the contest rules posted on the contest site. Papers need to be coherent, organized, clear, and well written. Teams should present their solution and analysis in fewer than 30 pages using at least 12-point font. While students may want to include some background research on the problem topic, this information should be brief. It is not the number of pages, but the ability to complete all contest requirements and communicate the solution in a concise and articulate fashion that will merit recognition. Students should use spelling and grammar checkers before submitting a paper. Foreign papers should insure that all symbols in tables and graphs are in English.

Papers recognized at the National level start with a clear summary that describes the problem. They then preview their paper with a table of contents. They present assumptions with justifications, explain the development of the model and its solutions, support the results mathematically and communicate them clearly, address strengths and limitations, and finally, close by stating overall conclusions.

Summaries

Judges are best able to analyze a paper when students restate the problem in their own words and clearly preview the focus and organization of their paper. Executive summaries should be written last as they summarize the entire contents of the paper. Teams should consider a three to five paragraph approach for their summary: a restatement of the problem and questions in their own words, a short description of their method and solution to the problem (in words and not in mathematical expressions), and the conclusions providing the numerical answers in context. The executive summary should entice the reader, in our case the judge, to read the paper. Although written last, ensure you spend time on this important part of your submission. Your executive summary provides the first impression of your paper. Here is an example of a good summary for Problem A: *Swim, Bike, and Run*.

Summary

On September 25, 1974, the first triathlon was held at Mission Bay in San Diego, California. This triathlon consisted of six miles of running (with the longest continuous stretch being 2.8 miles), a five-mile bike ride, and a 500 yard swim (with the longest continuous stretch being 250 yards). Since then, the sport has grown in popularity and was eventually awarded Olympic status and featured for the first time during the 2000 Olympic games in Sydney, Australia.

In collaboration with the Mayor, a triathlon was organized in order to support the local youth organization. The city decided that the race should be an Olympic triathlon, meaning that the race would consist of a 1.5 kilometer swim, a 40 kilometer bike ride, and a 10 kilometer run. Because the race was to be a world -class event, it attracted Professional (the best in the nation and the world), Premier (the best amateurs in previous races), and Open (everyone else) level participants. The goal was to effectively organize the event so the roads would be closed for no more than 5.5 hours. At the same time, minimizing the amount of congestion on the course was important in order to gain the continual sponsorship of the Super Tread Race Company.

Using data from a previous triathlon, a model was created to determine the most effective time to start the racers. Results of past the triathlon were broken up into the following subcategories: professionals (and their gender groups), premiers (and the gender groups), opens (and their gender and age groups), Clydesdales (men over 220 pounds), and Athena (women over 165 pounds). Using this data, average times and standard deviations for each sub group were calculated. These two components were then used to make a model that shows how the triathlon participants were released from the beginning line. The fastest groups were released first in order to prevent an influx of congestion in certain areas of the event. Transition areas (areas between events) were a designated park between swimming and cycling, and a parking lot between cycling and running to ensure minimal road closure time.

Team 7031: Chesterfield County Math/Science High School at Clover Hill

The comments above for summaries also apply to any required memos or letters. Your letter might briefly describe your model or process, but do so in a non-technical manner. Your letter should focus on why your model and its results are applicable and important to the reader. The key in your letter is to answer the question(s), and to interpret and communicate the results clearly. The following is an example of a good letter for Problem B: *Shop and Ship*.

November 13, 2016

Dear Esteemed Company President,

We have decided that your business needs to expand. Extending your market to the entire nation will help you surpass the competition, and, thus, we have decided to pursue this strategy. We will achieve this by placing warehouses across the contiguous United States. Many problems arise, such as building costs, taxes, which raise our prices and thus decrease our demand, and the lack of clothing tax cuts in certain states. After 36 hours of deep consideration, we have figured out the optimal business strategy. Using a computer program, we have considered area covered, number of warehouses, state taxes, and tax exemptions for apparel in order to find the optimal number of warehouses and locations.

When we use 32 warehouses, area coverage is 83.89% of continental U.S., and the average tax rate is 2.27%. The ZIP codes for the locations are as follows:

59223, 82922, 89701, 57002, 59313, 81321, 97010, 03570, 59001, 59831, 56208, 97010, 23004, 31772, 97101, 63828, 43512, 14009, 80020, 85135, 73052, 03046, 59223, 59058, 97710, 59214, 59641, 59435, 97828, 69135, 76634, 59701

However, we saw that while the tax rates were pleasantly low, the area covered was only 83.89% of the total landmass of the continental United States. An appealing option that disregards tax fitness is as follows: If 23 warehouses are built, area coverage will be 99.56% and the average tax rate will be 5.07% The ZIP codes are

49710, 44017, 42021, 30122, 87008, 83325, 58620, 68005, 27343, 77331, 12108, 85324, 76008, 54106, 71004, 98220, 93601, 66402, 57051, 59010, 33825, 80020, 57650

As you can see, the first solution gives more weight to tax while the second gives weight to area while using less warehouses. The second solution raises the scope of our shipping coverage. It also will decrease demand and sales because of the higher tax rate. We have also included a formula that will tell you which option is better if you input the number of warehouses you plan to build. These are the most optimized solutions we came up with, and we hope they help.

Sincerely,

Team 7211: Illinois Mathematics and Science Academy

Assumptions with Justifications

Modeling assumptions should include only those that come to bear on the solution (this can be part of simplifying the model). Long lists of assumptions that do not play directly in the context of model development or its solution are not considered relevant and deter from a paper's quality. Assumptions that oversimplify the problem do not allow for a full solution. You should include a short justification to show the assumption is reasonable and necessary. On the right are several good assumptions for Problem A: *Swim, Bike, and Run*.

Mathematical Model

Papers should explain the development of the mathematical model(s), and define all variables. Teams that merely present a model without explaining or showing the development of that model do not generally do well. Presenting multiple models, without identifying the most appropriate model to answer the questions, is detrimental to your paper's success. Your team may have considered several models that did not work before developing a good model. Do not include all of your trials, but instead clearly present the development and results of your successful model. Judges do value creativity and thinking "outside of the box" in your modeling process. This varies from problem to problem, but we usually see more than one appropriate solution method to our problems.

Assumptions:

1. The cycling, running, and transition segments of the triathlon take place on a six-meter-wide road and the swimming course is 30 meters wide.

   Justification: We assume that most of the race takes place on two-lane roads. The National Association of City Transportation Officials **(http://nacto.org/publication/urban-street-design-guide/street-design-elements/lane-width/)** recommend that lanes are no wider than eleven feet; rounding down to meters gives a width of three meters per lane. Because the swimming portion of the triathlon takes place in open water, its bounds are less restricted than the other road-based parts of the race. We assume a broad course to accommodate the large numbers of athletes.

2. All athletes maintain constant speeds within each of the swimming, cycling, running, and transition segments, though their speeds may vary between different segments.

   Justification: The acceleration and deceleration times between each part of the race are relatively quick, especially in relation to the length of each stage. We assume that the effects of fatigue and other factors that might influence the speed of an athlete are negligible.

3. The given data set is representative of the athletes who will register for this triathlon.

   Justification: It is difficult to predict the demographics of race participants, especially since most will be open registrants. We assume that relative percentages of professional, premier, and amateur athletes for both males and females remain the same as the provided triathlon data.

4. The triathlon course never overlaps or intersects with itself.

   Justification: This is a simplifying assumption as overlapping or intersecting points cause difficulty in modeling congestion for those particular areas.

5. The transition area is not on the roads, is 500 meters long, and congestion is negligible in these areas.

   Justification: By having the transition area off the road, no road closures are necessary in the hours prior to and after the race while triathletes prepare for and recover from the race. The roads can close immediately following the last athlete. The length of the transition area, 500 meters, has no impact on our model, but is reasonable considering that two thousand bicycles and the accompanying transition gear that must be stored in each area. Additionally, it allows for most athletes in the transition area to be on the sidelines transitioning and so congestion is unlikely to occur in the main paths in this area.

Team 7055: The NC School of Science and Mathematics

Strengths:

1. We apply the methodology of **0-1 programming** in many aspects of our model. We construct an associated 0-1 matrix to demonstrate whether any given two states are related in a regular UPS service. We also utilize 0-1 planning to solve our problems in Part 2 and 3. This method of 0-1 planning can be effectively used to make connections between graph theory problems and linear (or nonlinear) planning problem. The model is appropriate and highly applicable because 0-1 simplifies the graph as a whole. In addition, the simplified version of our problem is amenable to computer programming. The help of computers have rendered our answers accurate and precise.
2. **Greedy Algorithm** is used in our solving in Part 1. We optimized states most likely to be the final solution, and calculate whether that is real. With *0-1 Matrix*, we are able to come up with a program that eventually leads to the optimal solution set.
3. We apply **Graph theory** to simplify the map as a whole. Our *Vector Delivery Model* has so greatly simplified the problem that our subsequent methods become easy to be programmed by computers.
4. Our approach in Part 2 and 3 can be widely used to address problems of the same kind. For instance, setting warehouses around the world is also amenable to this approach. As long as we have data in hand, with the help of computer programming, we are able to eventually come up with an optimal solution.

Weaknesses:

1. Our approach in Part 1 is applicable when addressing the entire United States. However, it is no longer the optimal method when addressing larger and more complex areas such as Asia, or the entire world. This is because, for the US, we have used computers to simulate every possible outcome. In the case of the world, we are challenged to simulate more than possibilities. Unless our approach is modified, this mind-boggling number will not be addressed easily.
2. In our model, we apply the population of each state as a key index to predict the population of digital shoppers in each state. However, this is not always true. It is better if we can take into account different types of variables, such as average GDP in each state, and number of higher-than-typical households.

Team 6475: The Affiliated High School of South China Normal

Strengths and Limitations

In evaluating your model and solution, be sure to address strengths and limitations. Include model extensions or sensitivity analysis of your solution. Validate your model, even if by numerical example or intuition. An example of a good strengths and limitations section for Problem B: *Shop and Ship* is shown on the following page.

Conclusion

A clear conclusion and answers to specific scenario questions are also key components to an outstanding paper. Attention to detail and proofreading the paper prior to final submission are vital as the judges look for clarity and style. See the example conclusions on the next page.

Conclusions

Using Model 1, we found the most efficient warehouse locations when considering only unique area and distance (see locations listed in Results of Model 1). We had 35 warehouse locations, however, some of these locations, such as Eureka, CA and Meeker, CO, covered a very small area that did not contain many people. If we excluded the warehouse locations that only served areas that are mostly wilderness or are sparsely populated - Meeker, CO, Grand Junction, CO, Eureka, CA, Caribou, ME, and Syracuse, NY - the number of warehouse locations could be reduced to 30.

We realized that sales tax may change the favorability of certain warehouse locations over others, since the different states had different tax rates. Our model was designed so that taxes affect the smallest population possible. This did not affect most warehouse locations, since the population outside of the state in which each warehouse was located generally exceeded that of the state itself. Nevertheless, with Models 2-4, St. Louis, MO scored higher than its competing location, Springfield, IL (31.4 > 23.3; see Appendix T2), since Missouri had a lower tax rate than Illinois. Though sales tax was incorrectly implemented in Model 2, the score of St. Louis remained greater than that of Springfield even after the issue was rectified in Model 3. Thus, after final implementation of sales tax rates, the warehouse locations were unchanged from Model 2.

No changes occurred in the most efficient warehouse locations from Model 3 to Model 4, despite the implementation of changing of tax rate based on apparel tax. This may be because few states outside of the East Coast lacked apparel tax. Therefore, few calculations of the tax term were affected. Even though some calculations were affected, it was not enough to change the score difference, potentially because the proximity of compared location resulted in some compared groups residing in the same state. Apparel tax may have had a larger impact if the locations resided in different states.

Our three models effectively chart the ideal locations of warehouses to cover the contiguous United States with 1-day shipping. However, further development can refine our model to make the process more efficient and effective. While manually using population to choose potential locations allowed us to cover as many customers as possible, we should develop a less arbitrary method. We must also compare the cost of building and maintaining warehouses, other infrastructure, and whether that cost is justified by potential sales in each area.

While our models focused on the space efficiency of warehouse placement and incorporated factors such as sales tax, a true realistic model would require many more factors. Construction and shipping in reality are complicated processes, and calculating logistics must be taken into account. It is likely that many companies will face this challenge as the online world of shopping and consumers continues to expand.

Team 6779: Mills E. Godwin High School

Citations and References

Citations are very important within the paper, as well as either a reference list or bibliography page at the end. Teams that use existing models should cite their source during the exposition and not just include a reference citation in the back of the paper. This is true for all graphs and tables taken from the literature. Use "in line" documentation with footnotes or endnotes to give proper credit to outside sources. All data, figures, graphs, and tables that come from outside sources require documentation. We have noticed an increase in use of Wikipedia. Teams need to realize that although useful, information from Wikipedia might not be accurate. Teams should recognize and acknowledge this fact.

Dr. Kathleen Snook is a Mathematics Education Consultant and HiMCM Director. She can be reached by email at info@comap.com.