A more efficient method of verifying the Collatz conjecture

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1 Abstract

The Collatz conjecture in mathematics asks whether repeating two simple arithmetic operations will eventually transform every positive integer into one. It concerns sequences of integers in which each term is obtained from the previous term as follows: if the previous term is even, the next term is one half of the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture is that these sequences always reach 1, no matter which positive integer is chosen to start the sequence.[1]

The Collatz conjecture remains open for nearly 85 years, initially introduced by Lothar Collatz.

This conjecture is verified to be true until 2^{68}.

In this paper, we propose a simpler method to check the Collatz conjecture by interpreting it geometrically.

2 An introduction to the idea using integers

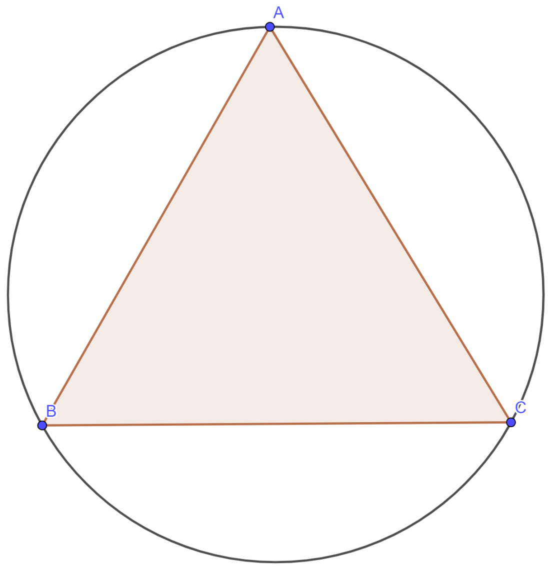
Consider the number line.

Now, consider this in $\pmod 3$ to get the sequence

$$0, 1, 2, 0, 1, 2, 0, 1, 2 \ldots$$

where $0, 1, 2$ is the recurring part.

We can interpret this geometrically by considering three equally spaced points on a circle, A, B, C, where A represents 0, B represents 1, and C represents 2. Then, we travel from one point to another based on the modulo sequence. Ultimately, we are cycling over the three points.



We can do this for any modulo m, and the shape inside the circle changes.

3 Extending the idea to the Collatz sequence

Define collatz(n) as the sequence following the Collatz function for the starting number n until it (hypothetically) reaches 1.

Now, we replace the numbers on our considered number line with collatz(n) for some $n \in \mathbb{N}$. Then, we take all the numbers in this sequence modulo $m \in \mathbb{N}$.

Also, define graph $\left(\frac{n}{m}\right)$ as the design that results when the sequence collatz(n) for some $n \in \mathbb{N}$ it taken modulo $m \in \mathbb{N}$ and is then interpreted geometrically (see section 1). Also, suppose that $m > 2$. As a sidenote, this shows that infinitely many such designs are possible as there are infinitely many rational numbers $\frac{n}{m}$.

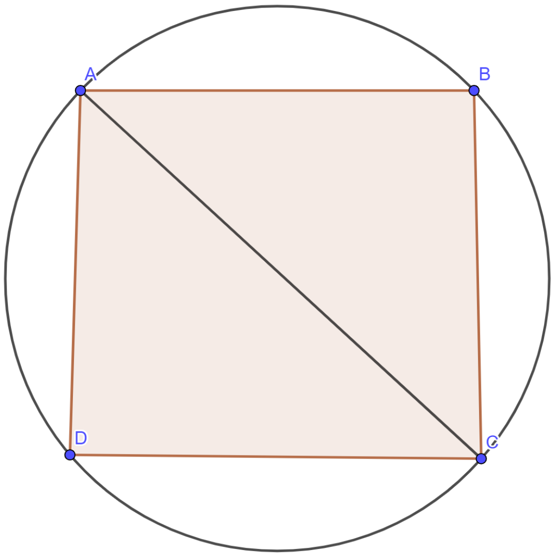
For example,

$$\text{collatz}(6) = 6, 3, 10, 5, 16, 8, 4, 2, 1.$$

Taking this modulo 4, we get the sequence

$$2, 3, 2, 1, 0, 0, 0, 2, 1.$$

Now we interpret this geometrically to get the figure below, where A represents $0 \pmod 4$, B represents $1 \pmod 4$, C represents $2 \pmod 4$, and D represents $3 \pmod 4$.



We can make such graph $\left(\frac{n}{m}\right)$ for every n, m.

4 Using designs to simplify verifying the Collatz conjecture

Some of these graph $\left(\frac{n}{m}\right)$ are symmetric, while other are not (see Further work).

(I NEED TO MAKE A COMPUTER PROGRAM THAT MAKES SUCH DESIGNS FOR A GIVEN n, SO THAT I CAN SEE WHEN THEY ARE SYMMETRICAL AND WHEN THEY ARE NOT.)

For the designs that are symmetric, we can develop a key statement. Notice that when collatz(n) reaches 1, it must reach 2 right before. So, since the design is symmetric, there must be a corresponding segment between two points that corresponds with the segment between $1 \pmod m$ and $2 \pmod m$.

Using this, we can simplify the verifying process of the conjecture. Rather than going all the way to $1 \pmod m$, one can simply check until the number that is $x \pmod m$ and corresponds with $1 \pmod m$. We can find the corresponding number that is $x \pmod m$ by using the symmetry. In other words, if we know that a specific design is symmetric, we can follow the following steps to reduce the verifying process.

1) Identify the corresponding symmetrical segment of the segment between $1 \pmod m$ and $2 \pmod m$. Let this segment be between $x \pmod m$ and $y \pmod m$.

2) Keep drawing the segments of graph $\left(\frac{n}{m}\right)$ for the m that makes the design symmetrical until you reach the segment from $y \pmod{m}$ to $x \pmod{m}$. If you reached it, then you don't have to check further, and it means that the conjecture holds true for this n.

We can simplify the verifying even more, and this part, unlike the symmetry simplification, also applies to the designs that aren't symmetrical. Some designs are nearly identical (this is not concrete), with just one, two, or a few extra segments added. If two designs graph $\left(\frac{n_1}{m_1}\right)$ and graph $\left(\frac{n_2}{m_2}\right)$ (without lxoss of generality, let $n_1, m_1 < n_2, m_2$) are "nearly identical", then, rather than starting from scratch again and checking the conjecture for collatz(n_2), we can use graph $\left(\frac{n_1}{m_1}\right)$ and simply build on a few segments from there, essentially not having to check anything.

5 Further work

In the future, if it is found and proved that graph $\left(\frac{n}{m}\right)$ is symmetric for some cases and not for others, then this method can actually be implemented into programs to make verifying much, much faster.

Also, if it is concretely found and proven that some cases of designs are nearly identical, then we can implement that as well into the programs.

6 References

[1] https://en.wikipedia.org/wiki/Collatz_conjecture