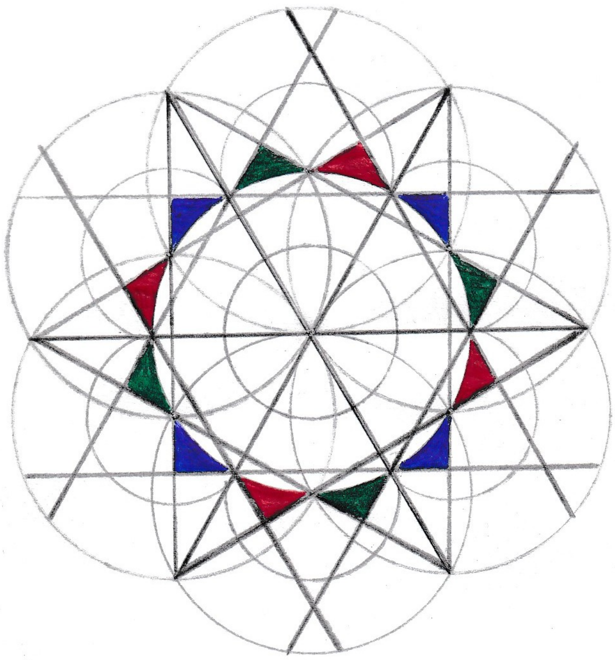
Question Reality ?.!



The Three Classic Geometry Problems

Introduction

When first confronting these problems, I was of course met head on with a wall of information that exclaimed them to be impossible and therefore any attempts to solve them, futile...

As well as many statements that would indicate that anyone who would attempt to do so, would be foolish, however pointless it may have been I chose to try anyway, I was not happy with the conclusion that computers could solve these problems, but we could not...

Why try the "Impossible"?

I figured even if I couldn't find a solution, that at the least I would have some fun drawing geometric patterns and maybe I could turn a few of them into interesting logo designs.

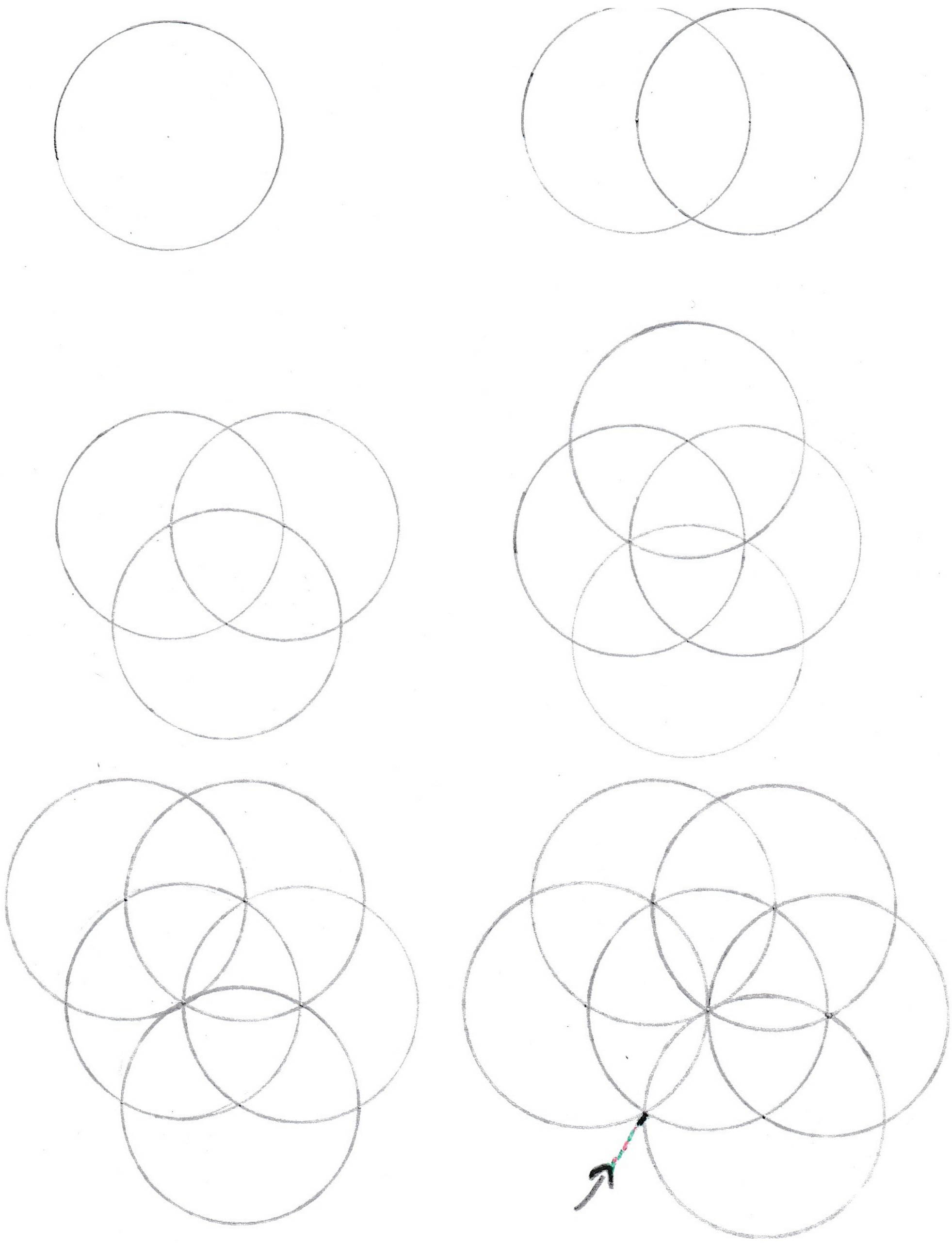
The more patterns I drew, the more I could see that a solution must exist for each of the three classic geometry problems the basis of each method shows that a circle can be divided into six equal segments and it is that relationship that I chose to utilise in each attempt.

Circling the Square v1.0

Squaring the circle or circling the square as I have named the method, when trying this I looked at an example of the problem solved by a computer and looked for similarities in the patterns I drew.

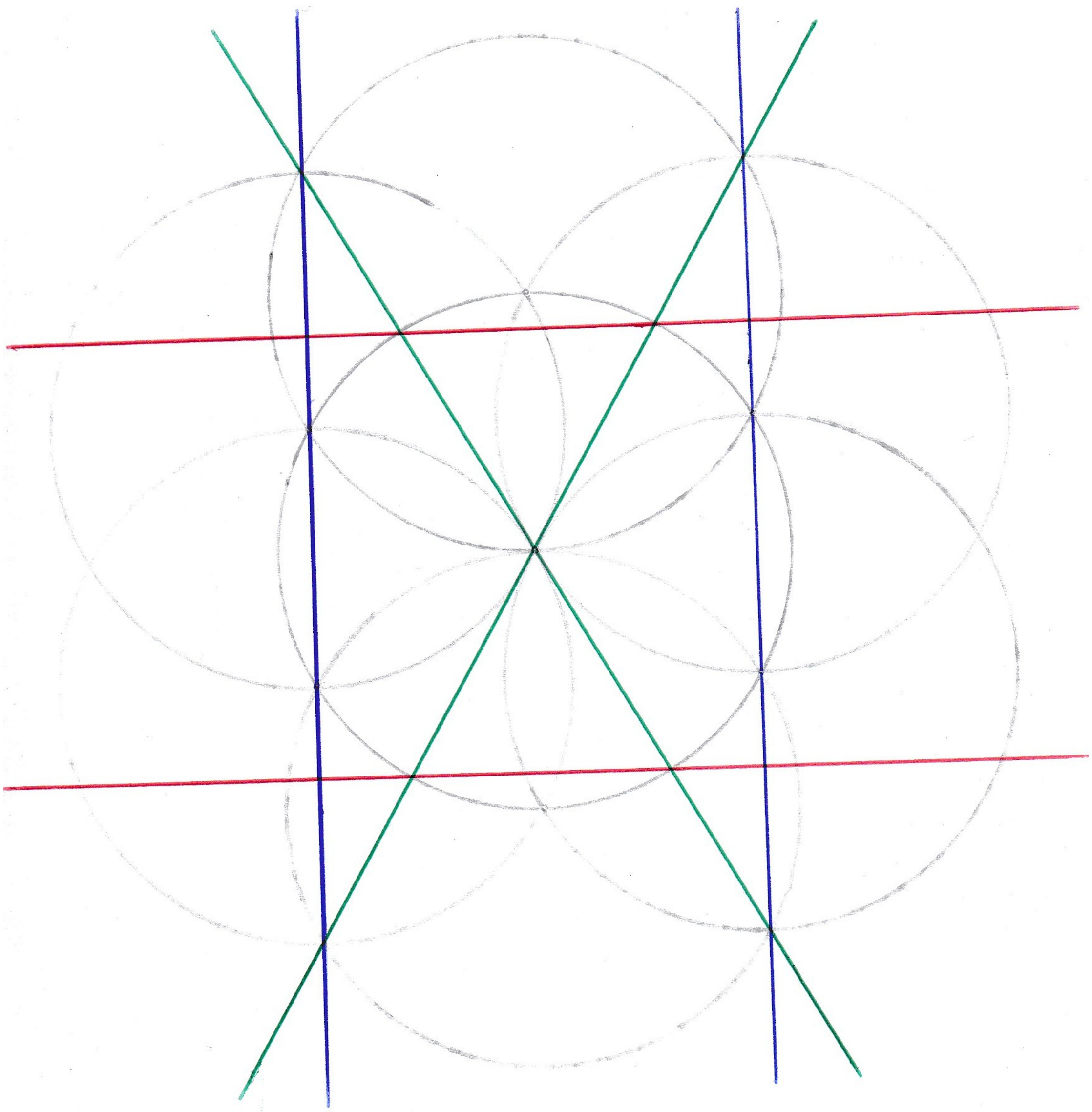
Once I had spotted a pattern, I took measurements to check and found the difference ranged between 0.03% and 3.5% approximately, at first I thought that the difference was ultimately caused by human error in my measurements and my drawings.

Start by drawing a circle of any size, then follow these steps.



Finish by drawing one last circle where the arrow points to.

Next draw the blue lines followed by the green lines and finally the red lines as seen below.

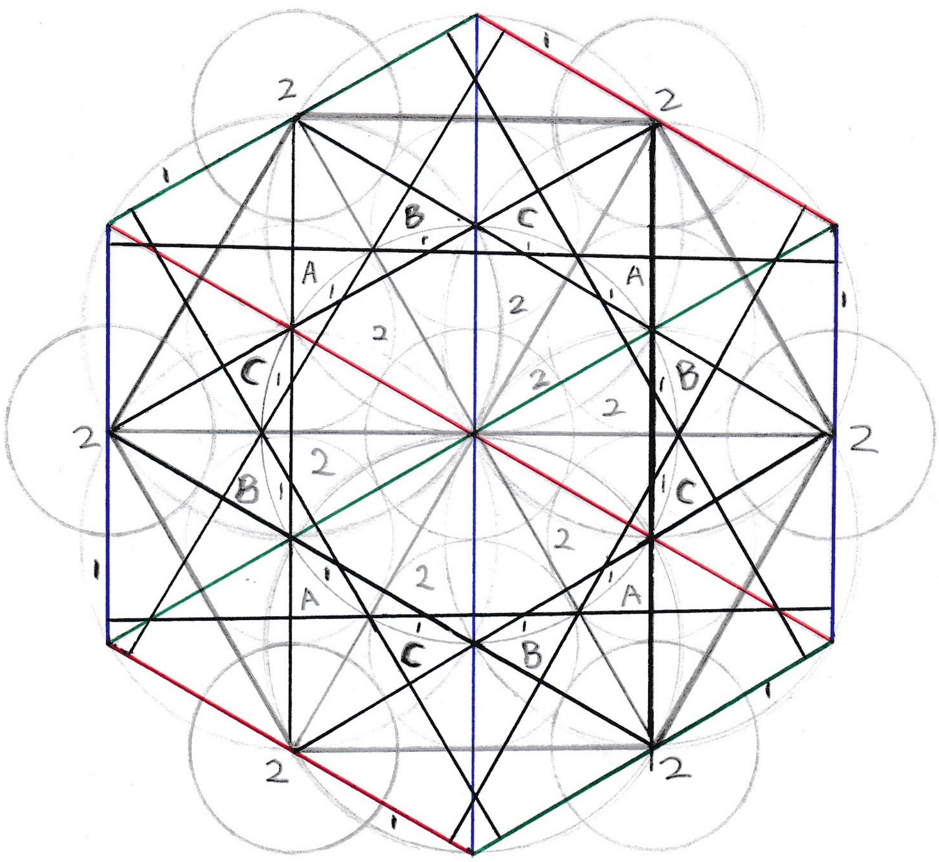


I took measurements upon completion with digital calipers in order to get the best measurements I could.

This method results in a construction that is 95.5% accurate.

Cube 2 Tesseract v1.0 (WIP Method Name)

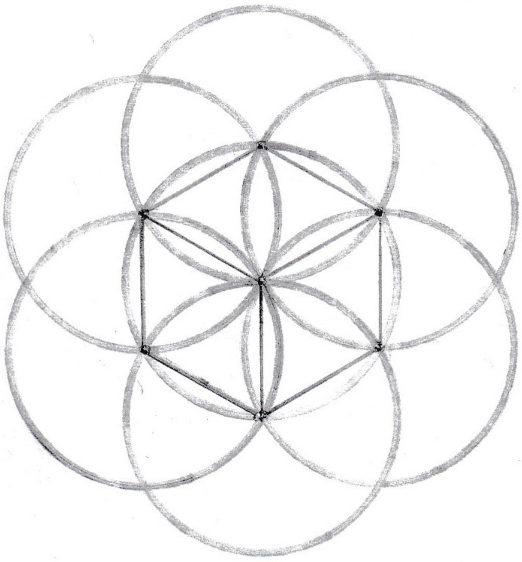
When looking for a solution to doubling the cube, I noticed that in one of the many patterns I drew for fun while squaring the circle, that I had made a pattern which made a cube with eight times the volume of a central cube, doubling the size of the cube, but not it's volume. *(see below)*



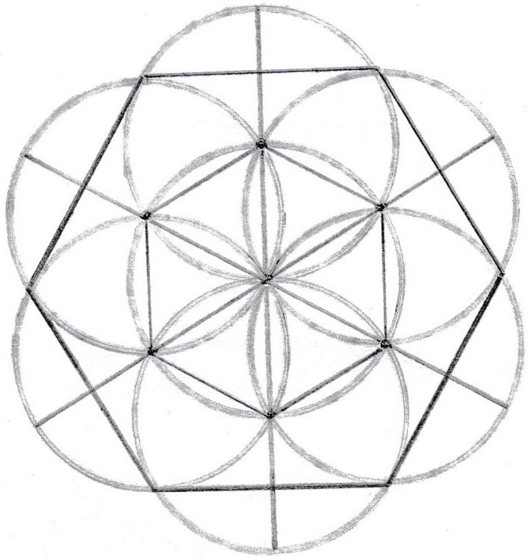
Because of that I figured that there must be a way to zero in on a pattern that would double it's volume.

Start by drawing a circle around the cube you wish to double the volume of, along with a series of circles like in my method for squaring the circle.

1

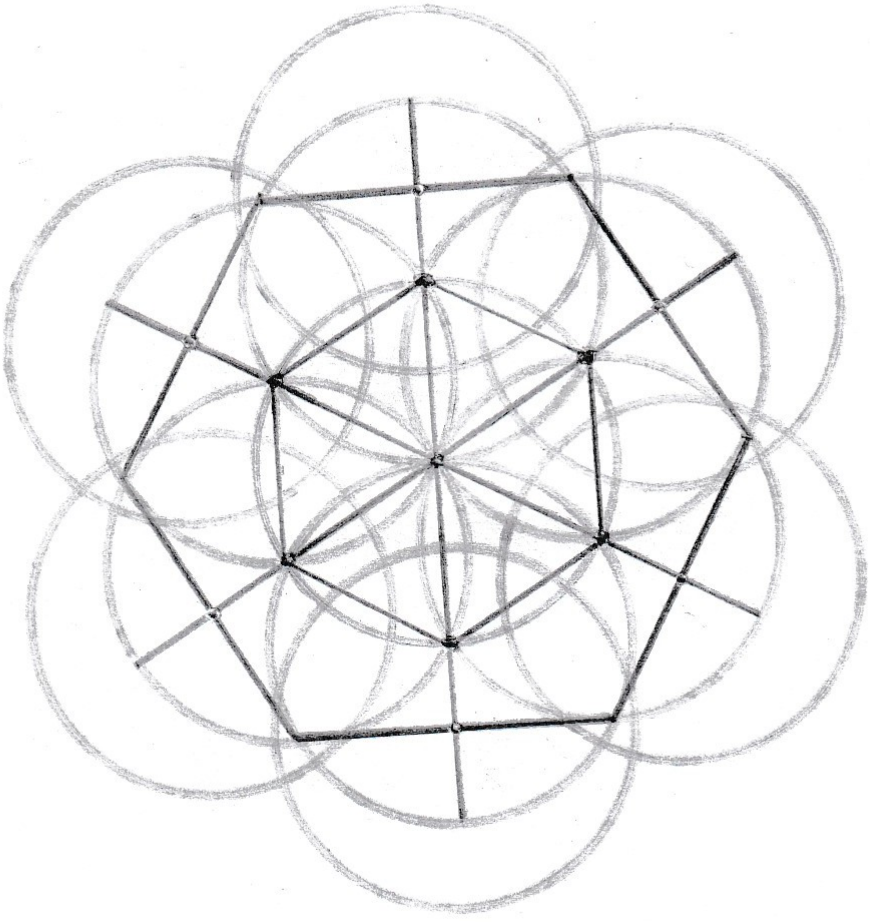


2



Then draw the nine lines added in step 2

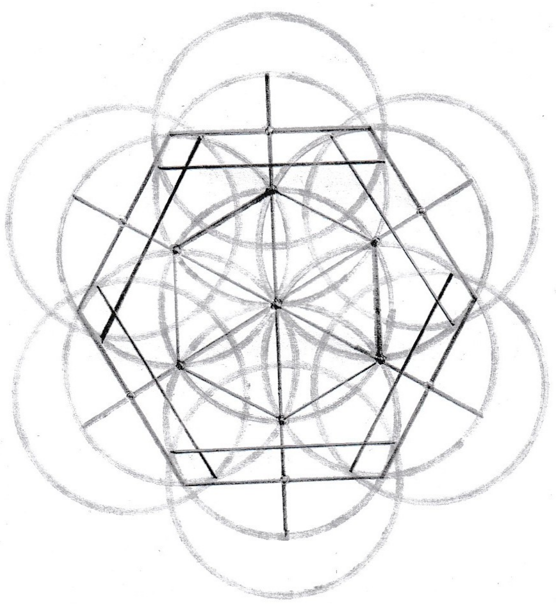
3



Followed by six more circles at the points marked by the lines drawn in step 2

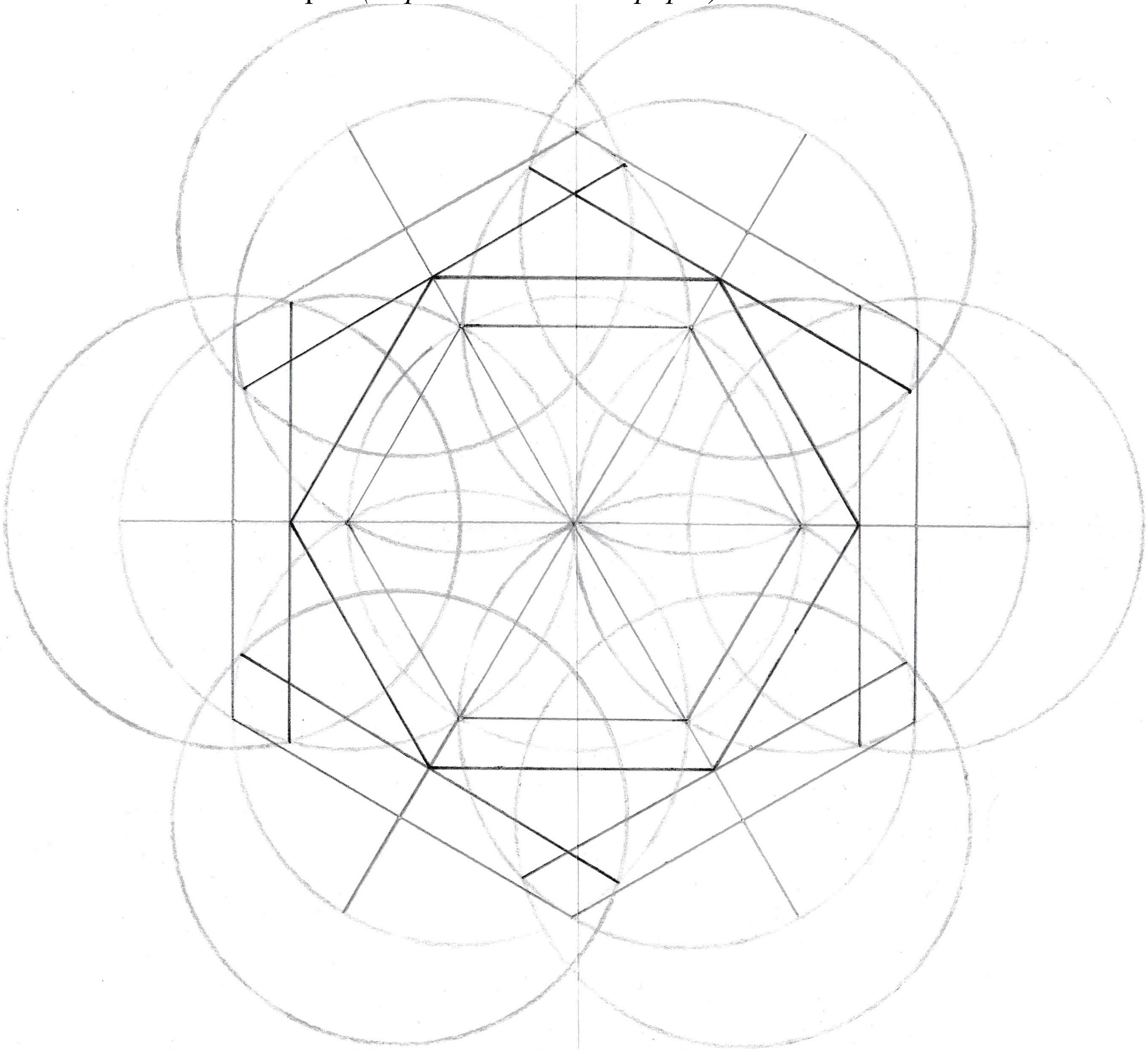
Connect the intersecting points as seen in step 4

4



Finally draw the doubled cube by connecting the points marked by those lines half way along each newly drawn line as seen in step 5. *(step 5 is done on A4 paper)*

5



This constructs a cube with approximately double the volume of the given cube.

Array Casting v1.0

In an attempt to solve this I used a similar method to the one I used to solve the other problems.

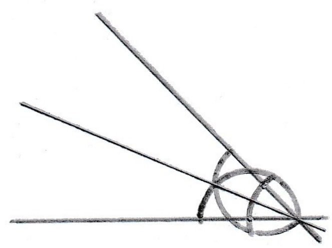
Having seen that I can divide a circle into six equal sections, I reasoned that it must be possible to use those same features to divide an angle into three.

While there is still no method to ascertain the initial circle size straight away, it is a simple process to determine the size with either circles that are too small or too large by dividing the excess into two.

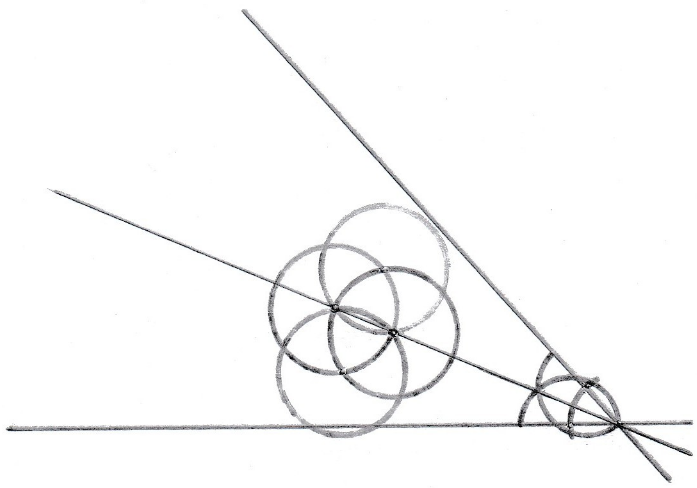
For angles under 180 this process only needs to be done once, but for angles over 180 the angle must be split into two and the process done twice.

This is because the process forms a line between the two sides of the angle and then uses the circles to divide it into three, the points that divide the circle can then be joined to the initial point of the angle to trisect the angle.

Start with an angle and bisect it.

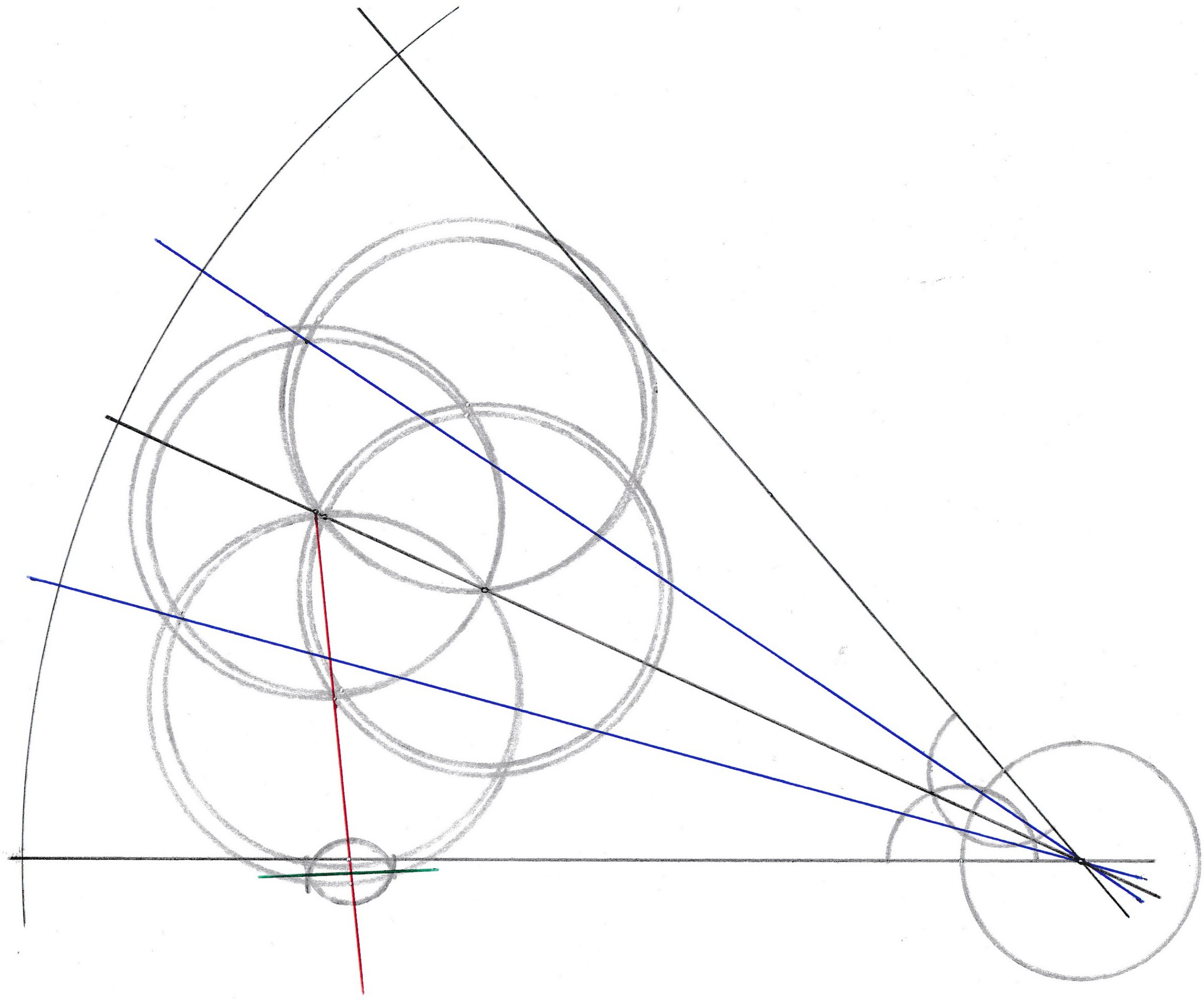


Then draw an array of circles along the bisected line.



Any inconsistency in the bisected line will result in the trisection being off by a factor of however much the initial bisection is off by. *(as seen in this example)*

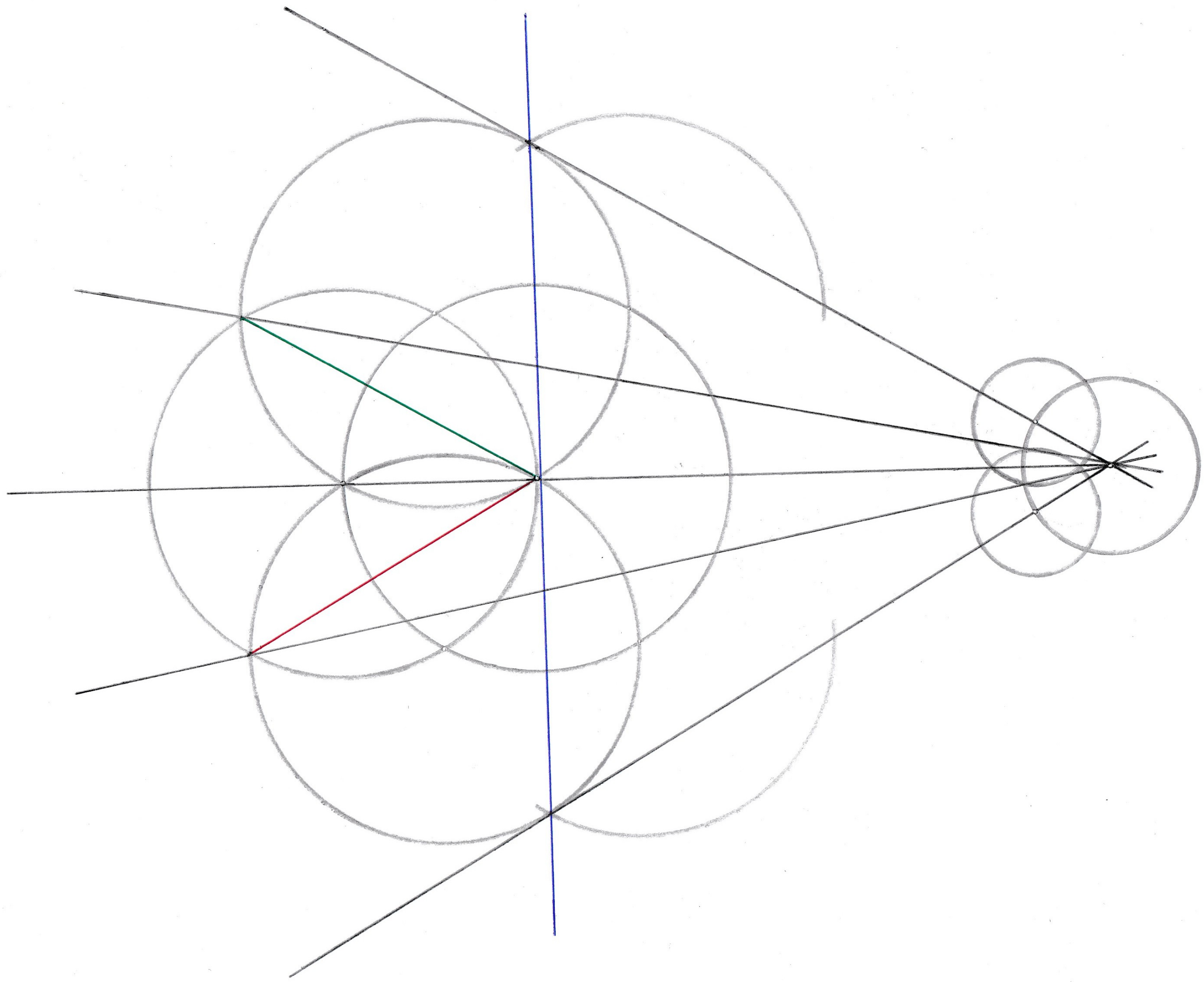
If the array of circles is too small or too large as is to be expected, divide the excess into half by bisecting the distance between the centre of the circle used to measure the excess and the halfway mark of the excess, this is the required circle size for the array. *(done one A4 paper)*



Re draw the array using the first circles centre as the starting point.

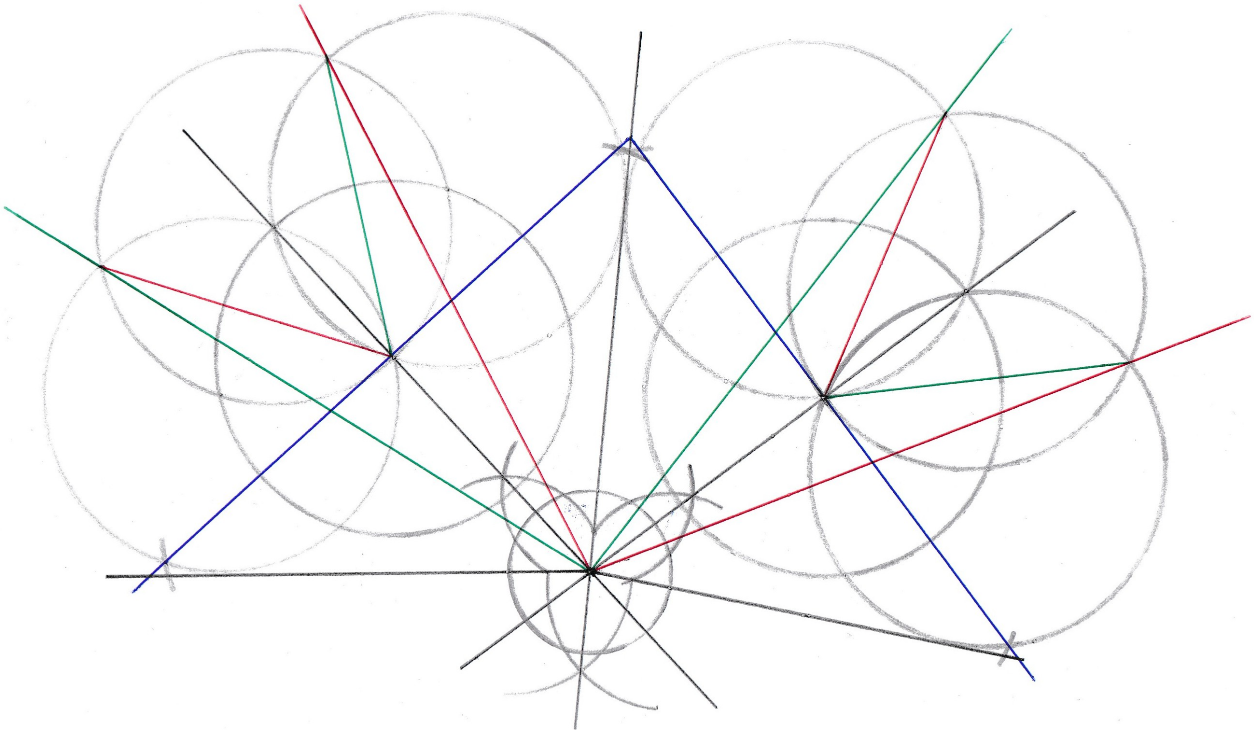
Finally connect the starting point of the angle to the intersections of the array of circles that mark a third of the circles angles facing away from the given angle.

The next drawing showcases how the method works, creating a line between the two edges of the angle that serves as a base to split the array of circles into three equal sections. *(done one A4 paper)*

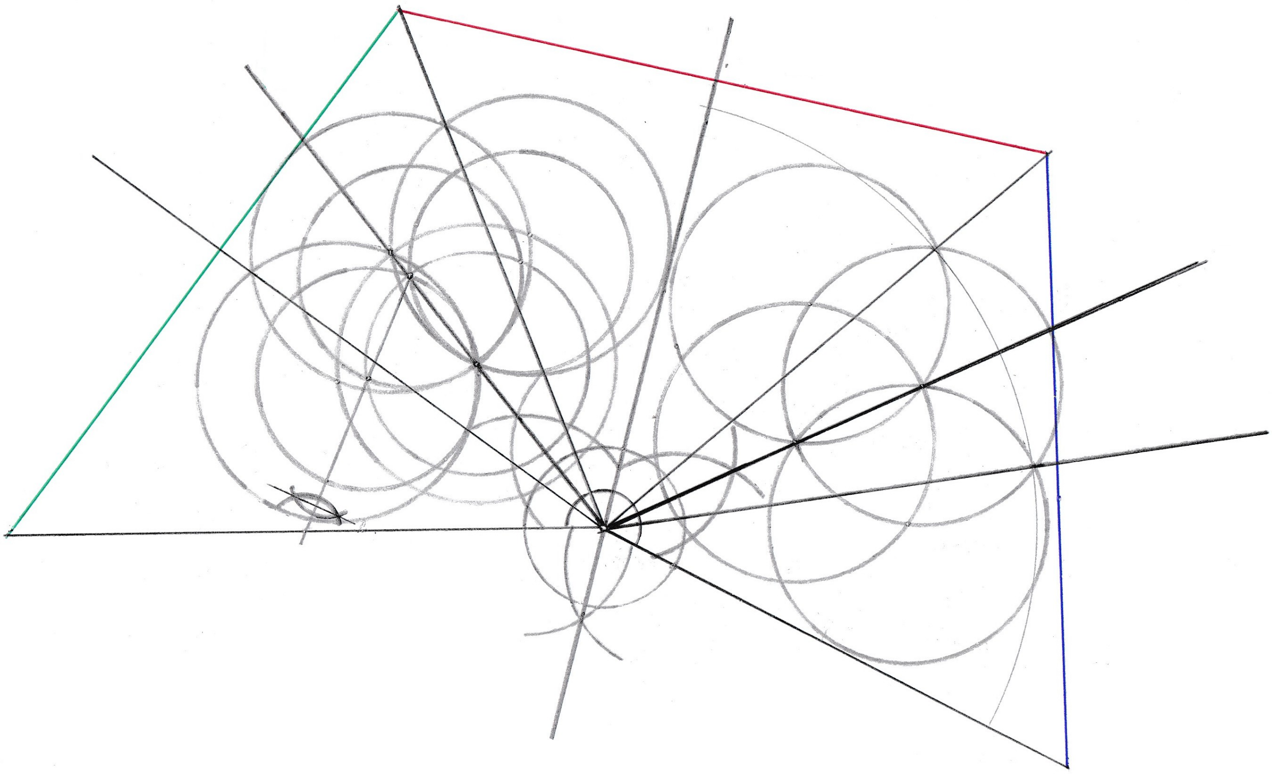


In this image, the blue line represents what I call the “projection plane” and the red and green lines are what I have referred to as the “proving pivot point” the projection plane defined by the intersections of the circles must rest exactly on either side of the given angle in order to properly split the angle into three.

The following images are examples of this process being done for angles that are over 180 degrees.



Another example.



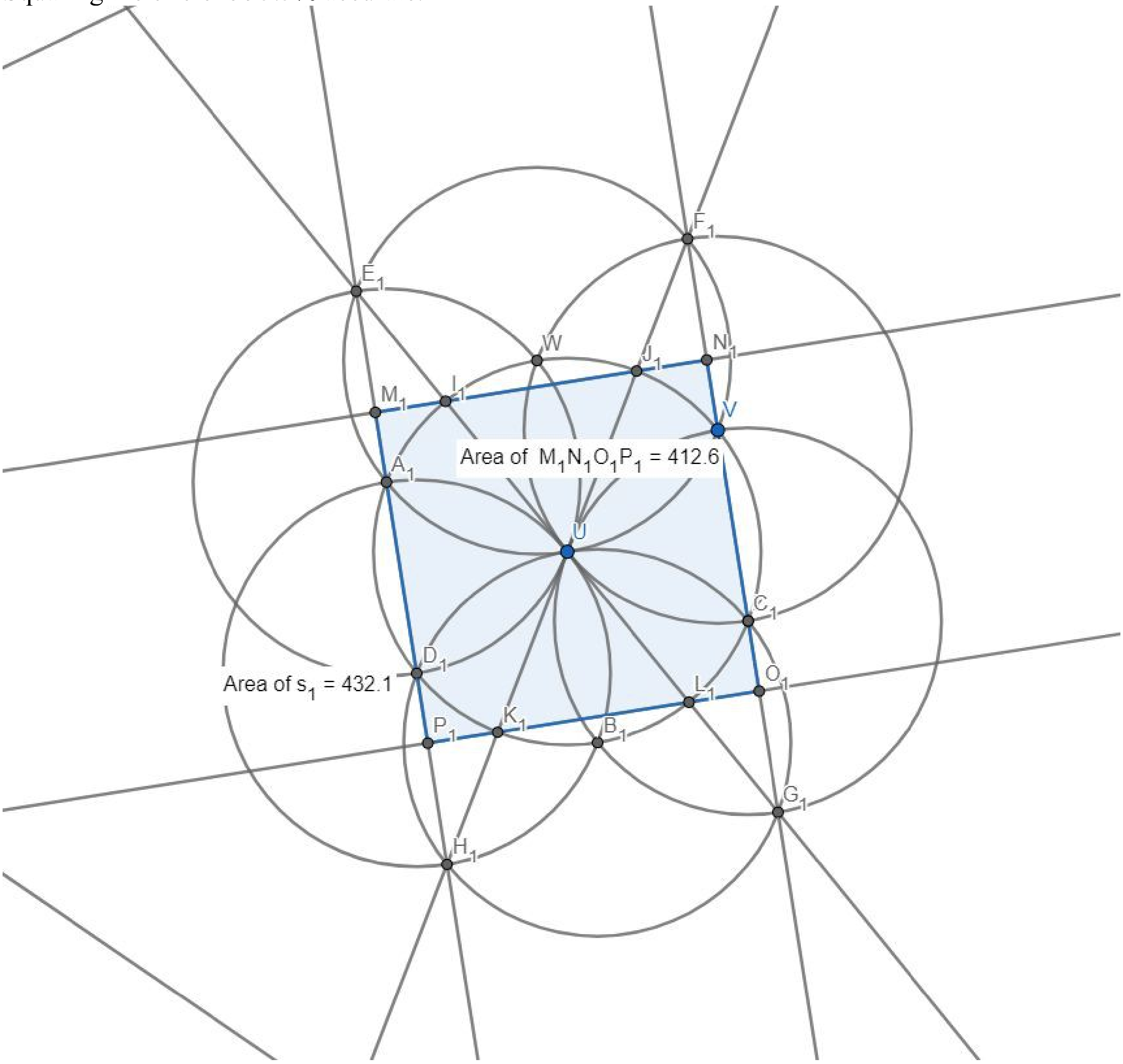
When doing this process for an angle over 180 degrees, things can get quite messy, quite fast, so I tend to work it out on one side and then use the distance between the first point of the angle and the centre of the first circle to mark the starting location of the array of circles on the other side.

(these two constructions were done on A4 paper)

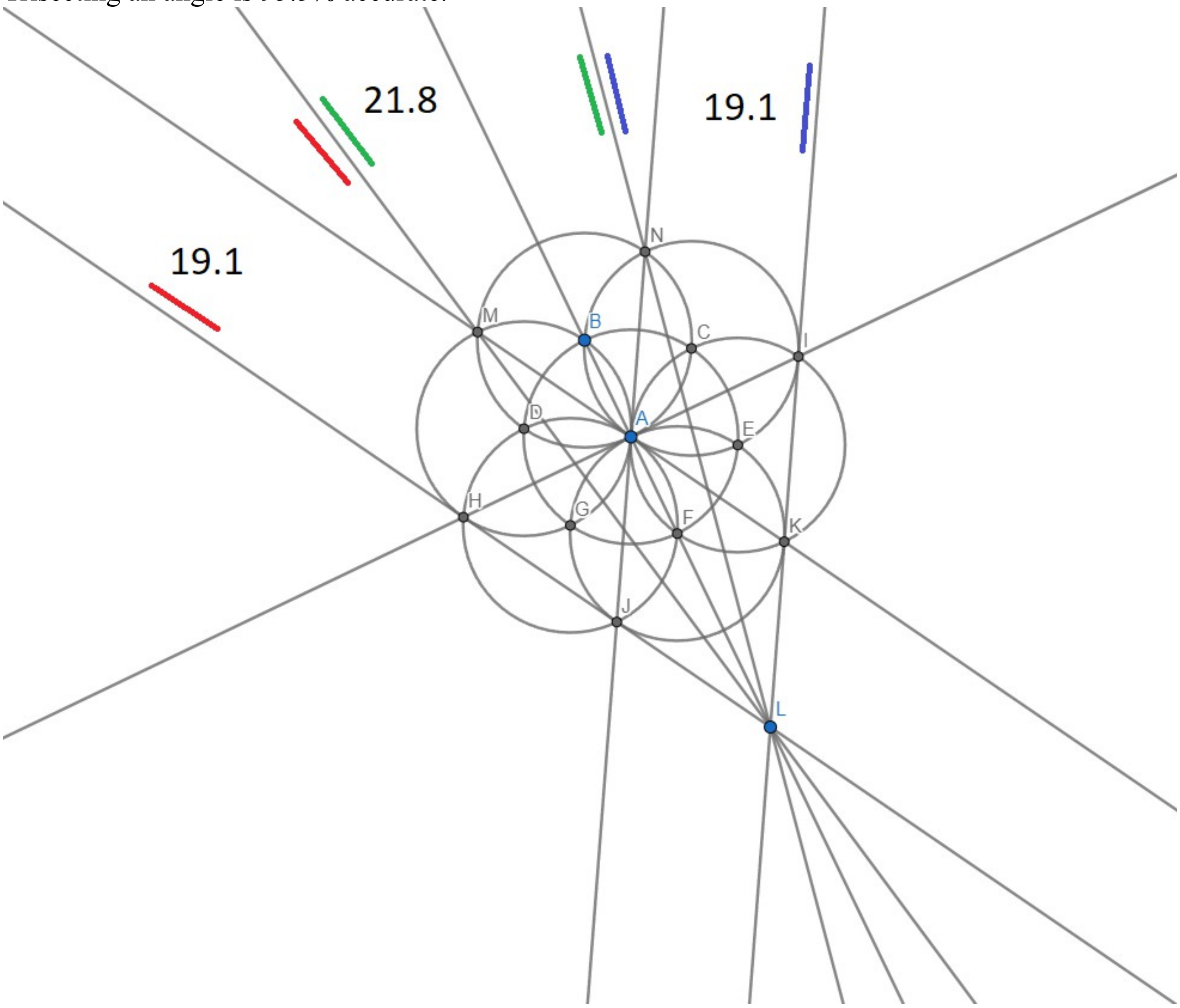
Approximations or Accurate?.!

Are these just approximations or are they accurate?

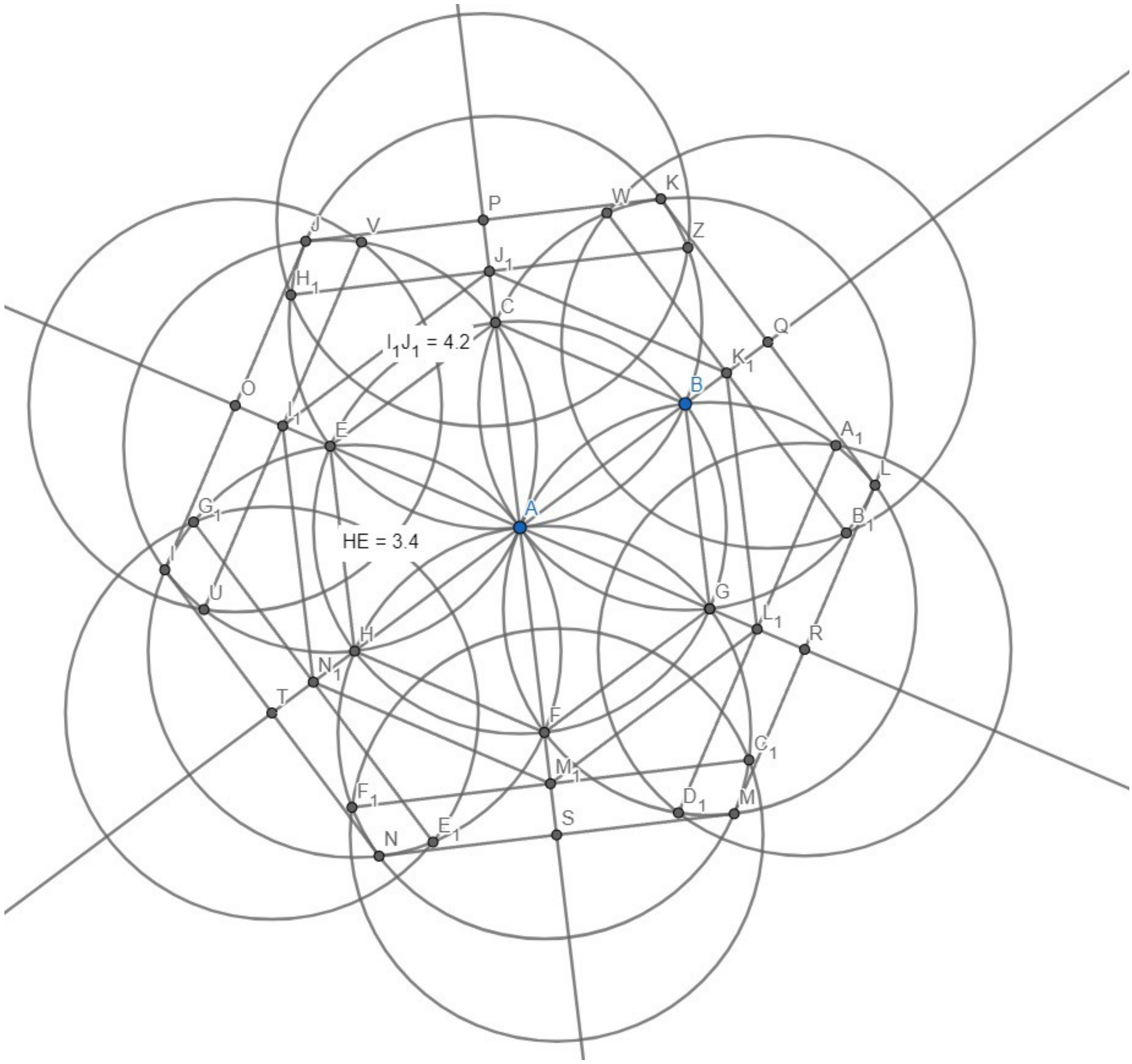
Squaring the circle is 95.5% accurate.



Trisecting an angle is 95.5% accurate.



Doubling the cube is 94.24994911459392% accurate.



Human Error

Any slight inconsistency in the constructions leads to a significant error in the final components of each construction.

The greater the initial inconsistency the larger the error in the final construction.

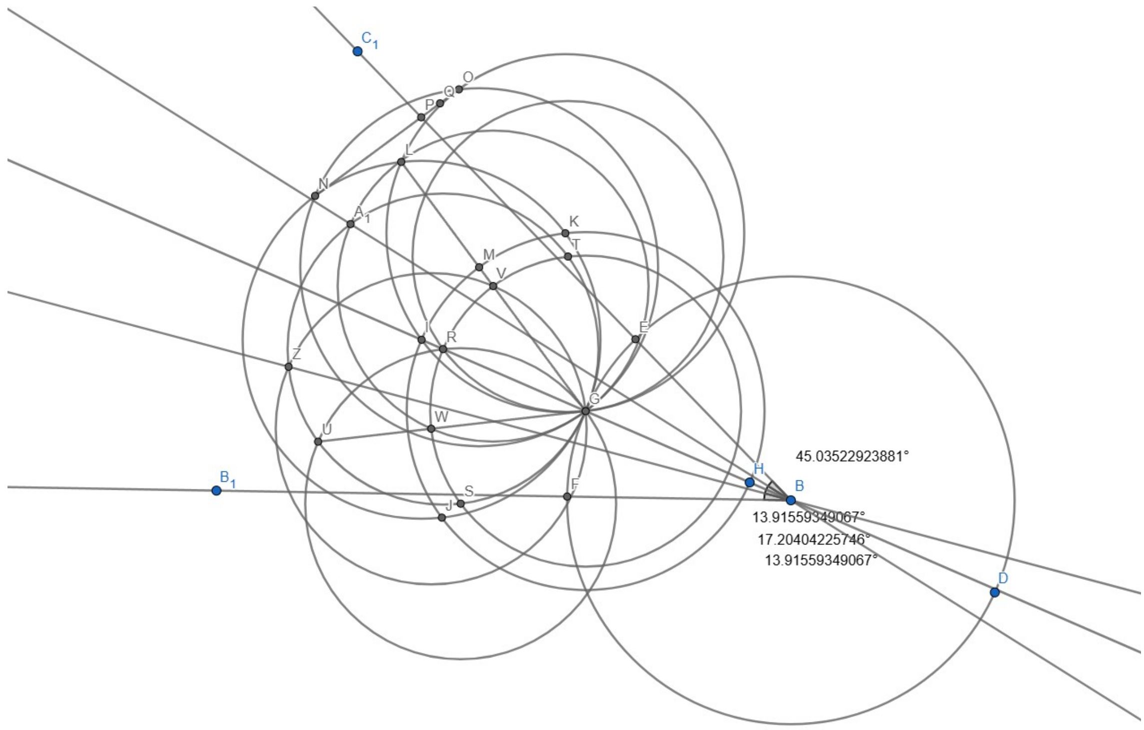
It is also possible to continually sub-divide the methods for squaring the circle and doubling the cube in order to get even closer with regards to it's accuracy.

While I thought this would be the end of the paper, I soon found myself returning to try to refine each method even further and I believe I have found an accurate method for trisecting an angle that easily approaches 99.99% accuracy and potentially even hitting 100% accuracy but ultimately being restricted by the fact that we count numbers in base 10.

Array Casting v2.0

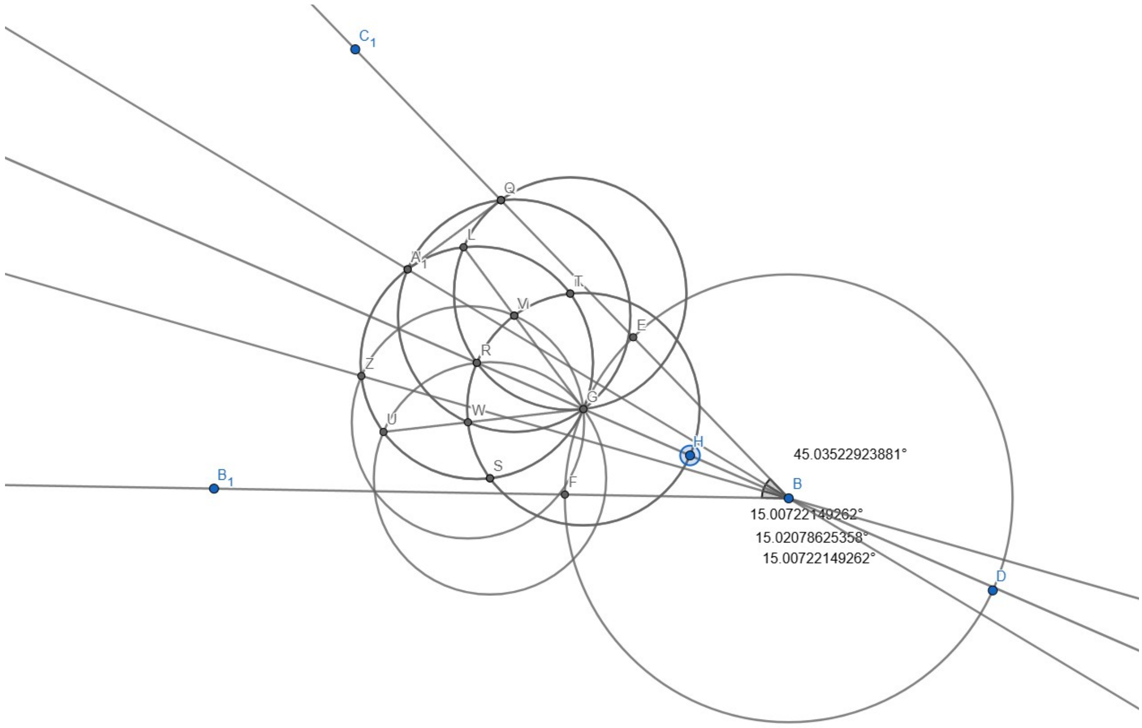
This method starts off very similarly to the other but differs slightly in it's approach, first off I will start by showcasing the method with some images constructed using the online software available at : https://www.geogebra.org/geometry

The first image shows two sets of circles that I set up so that I could accurately measure the efficiency of the method itself.

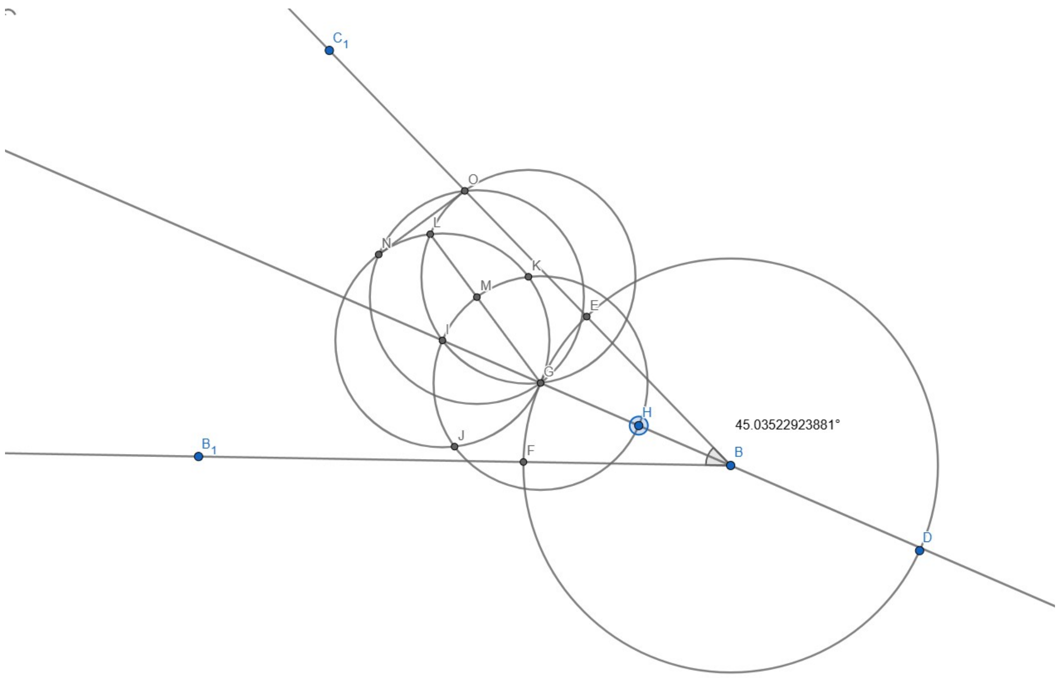


As I make the outer point *(I)* for the initial circle smaller, both circles gradually get smaller and close in on showing the accuracy for this method.

Here you can see just as I approach point Q resting on the upper/outer line of the initial angle, the angles of each section look to be practically identical and I believe are ultimately constrained by both the limits of counting in base 10 and potentially the distance point G is from point B which might even be negligible or not relevant at all.



As I make that circle even smaller and point Q passes over the upper/outer line of the initial angle, much of the construction disappears because of the way I put it together and while it might be wishful thinking, it really looks to me as if this method is capable of producing between 99.99% accuracy and possibly even 100% with regards to trisecting an angle that is less than 90 degrees.



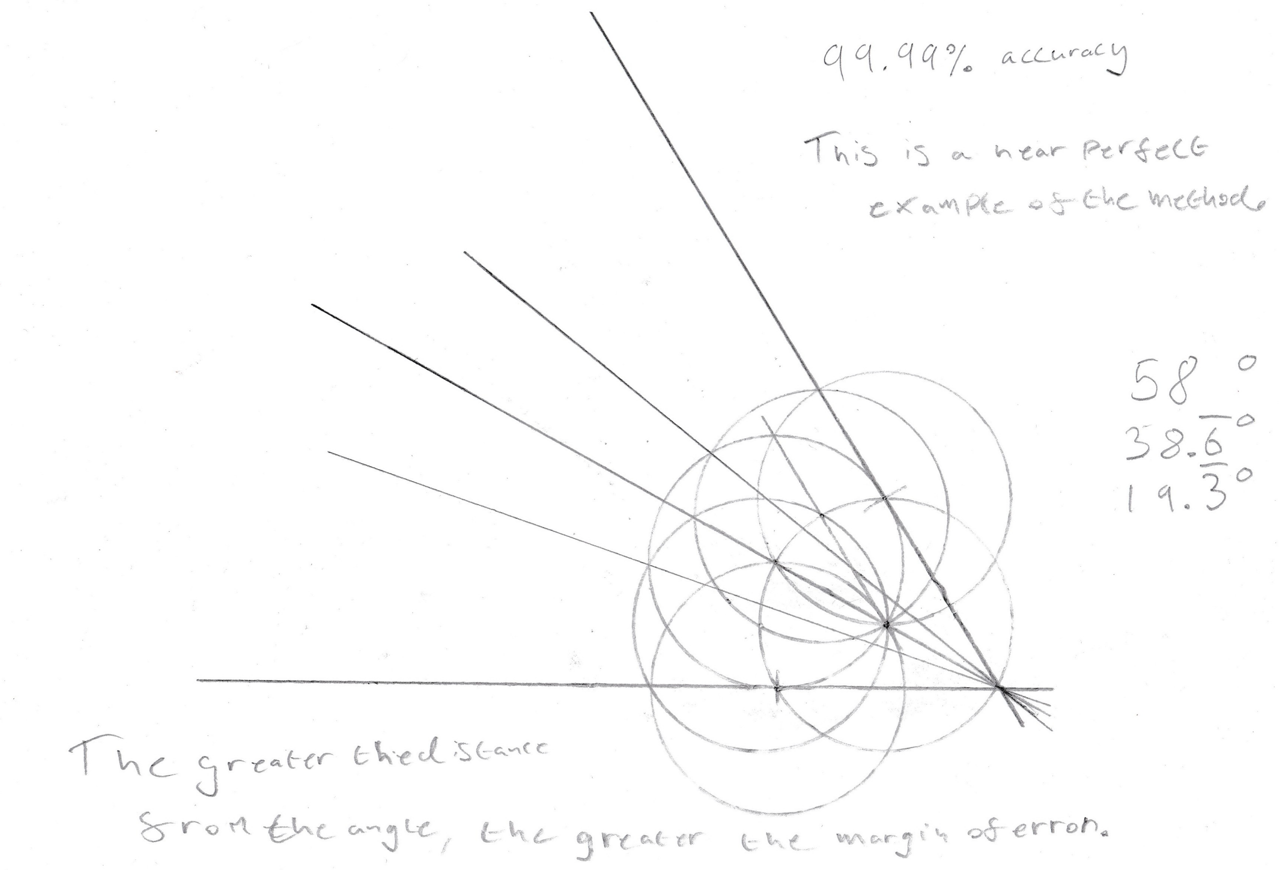
For angles over 90 degrees and under 180 degrees, this process must be done twice, as with my first attempt at the method, for angles over 180 degrees and under 360 degrees, it must be done four times.

I have tried this out on paper many times now and believe it is practically speaking perfect and while it might ultimately be 99.99% due to the limitations imposed upon it by counting in base 10 I think it would undoubtedly have been accepted as a viable method in the past.

Next I will demonstrate the entire method from beginning to end as I have drawn and prepared a set of steps on paper, though it is worth noting, there is no exact procedure for finding the central point of the initial circle, it must just create a construction or array of circles that is bigger than the initial angle compared to the desired array of circles that will be cast afterwards.

If this is being performed multiple times for large angles, the compass can be used to take these measurements and transfer the construction from one trisection to the others, which would make them look a lot cleaner than the initial construction. *(see examples at the end of this section)*

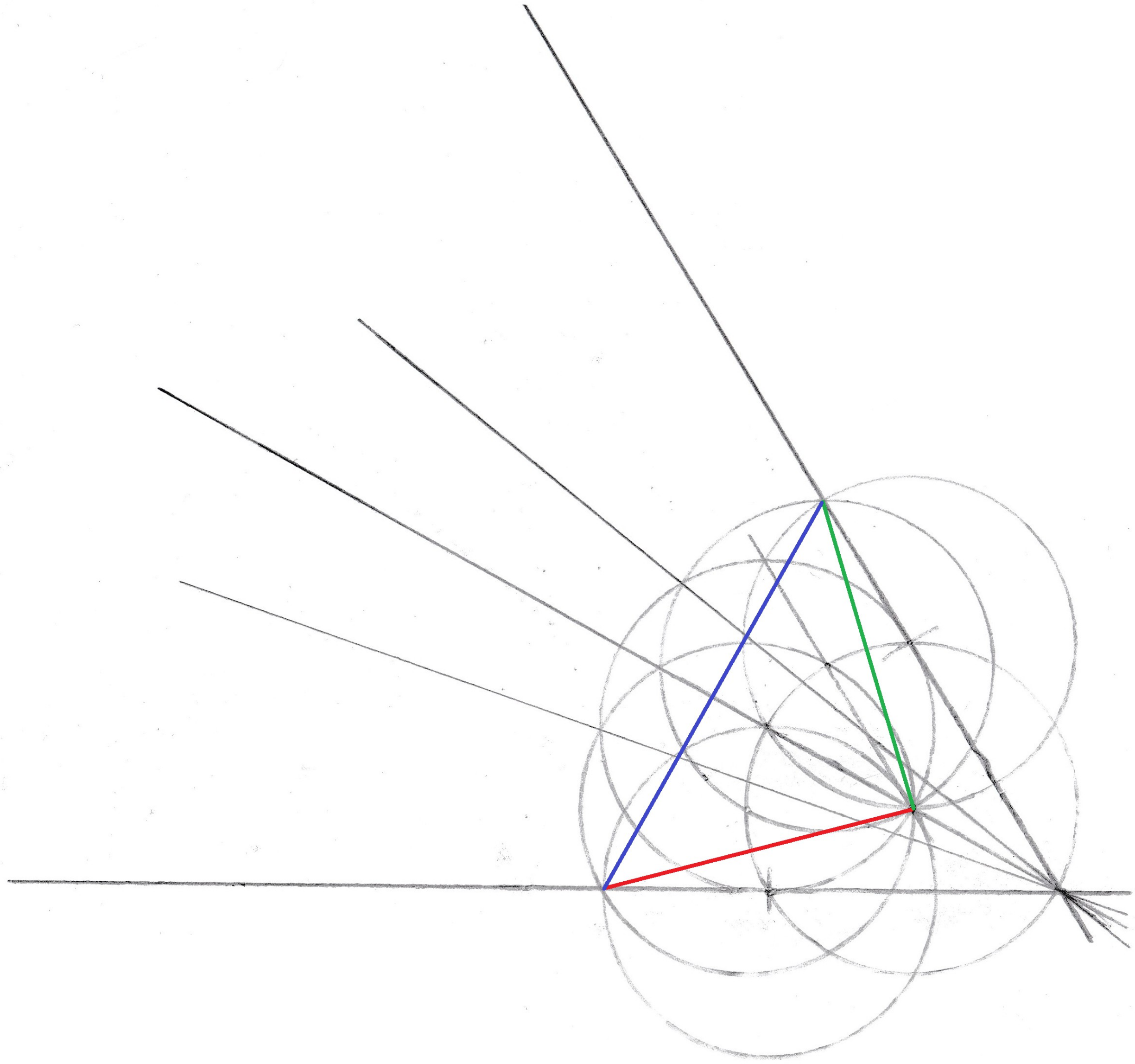
This first image shows a near perfect construction which can be found without needing to perform the full method if you are lucky, but it is not always the case.



As noted in the image, the closer the initial circle is to the inner corner of the angle, the more accurate I believe it would be, or the greater the margin of error if it is further away.

However while I am not certain yet, I think it might not matter as the method works very similarly to the first version of this method, but instead of casting a projection plane, it casts a projection angle, by creating a right angle inside the angle between the two edges of the angle.

This projection angle can be seen by connecting two lines to the relevant points in the construction as seen here.

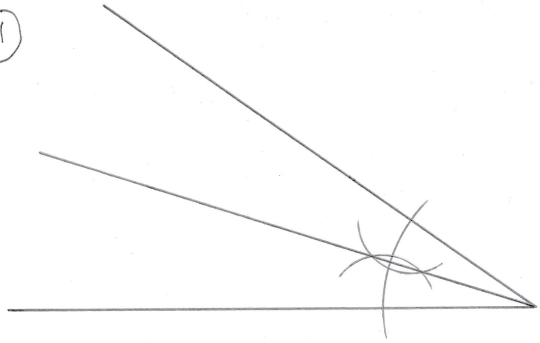


Which creates a perfect 90 degree angle for the circles to sit on, which means the three protruding bulges of the array of circles can be perfectly split into 30 degrees, though I suspect that the origin point of the initial circle might introduce a very small margin of error but I am still not sure of this yet, though my tests on GeoGebra seem to demonstrate that it is practically perfect.

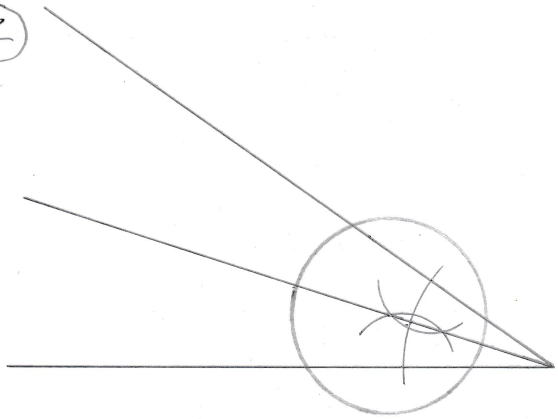
Now lets move onto the 17 odd steps required to perform this method, first give yourself an angle which is less than 90 degrees, as mentioned before if it is over 90 degrees or 180 degrees, different steps will have to be taken, but it is essentially just bisecting the larger angle and then performing this method multiple times.

Next bisect that angle which will leave you with the construction seen in the first step.

1

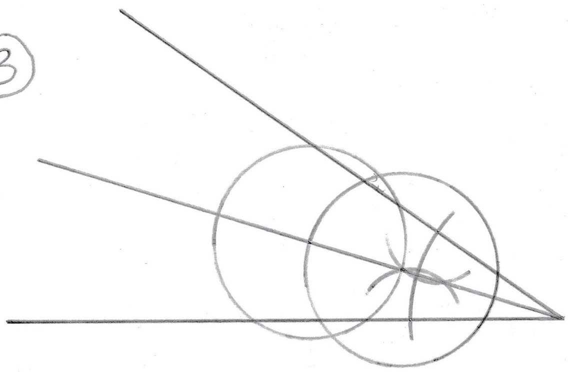


2

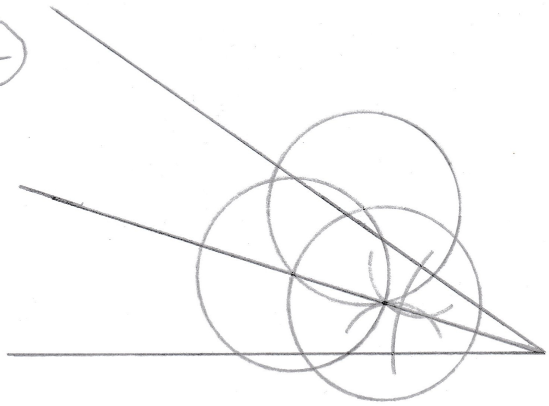


Find a point to draw the first circle from, I usually just use one of the points of the initial bisection but make sure the circle is larger than you expect might be required for the angle, ensuring it goes well out of the bounds of the angle itself.

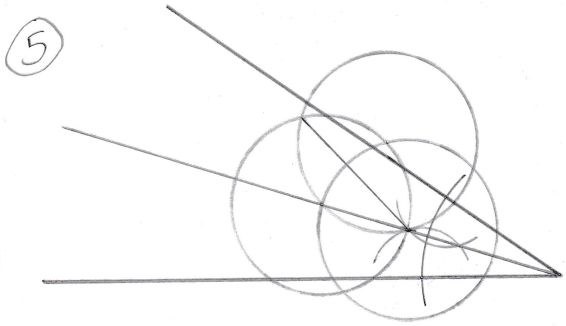
3

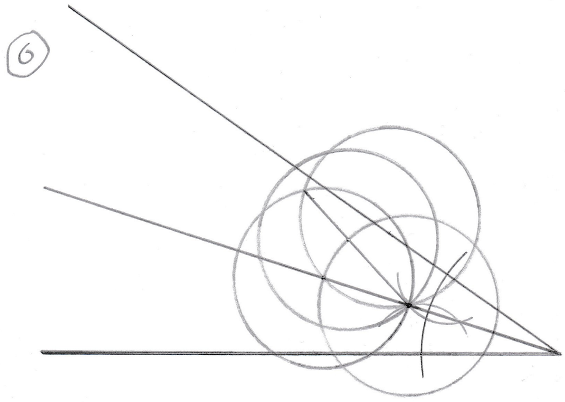


4

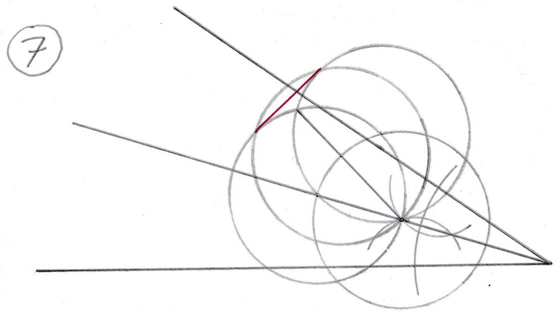


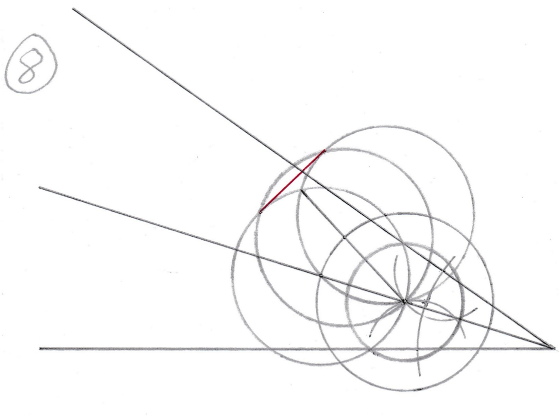
From here draw the next two circles as seen in steps 3 & 4 ensuring they use each other and the angle bisector as points of origin to form this triangular set of circles.
Next join the centre of the initial circle to the bisection of the second and third circles, this will mark the resting point for the next circle drawn in step 6.





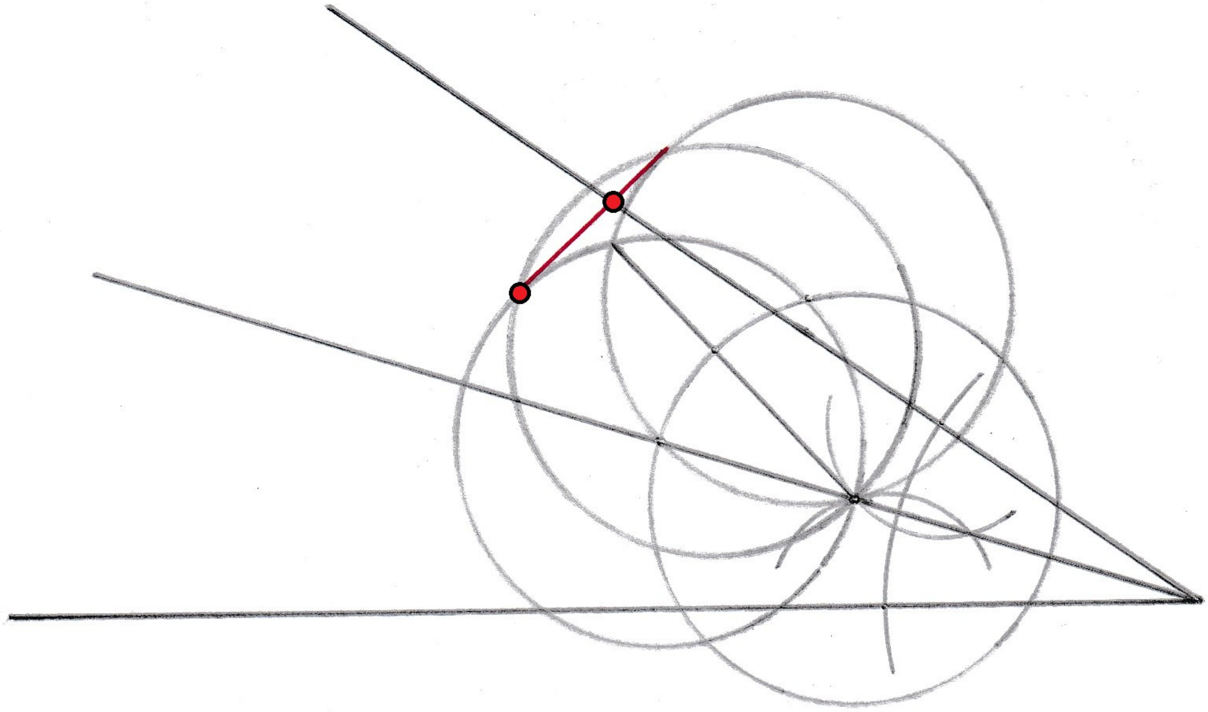
From here, draw a line connecting the two points that intersect the fourth circle with the second and third circles as seen in step 7 onwards. *(the red line highlights it's location)*





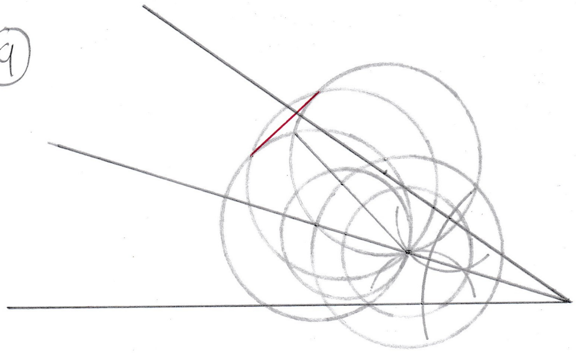
This point gauges the required size for the next set of circles, by measuring the points in between the outer line of the angle and the intersecting points of the second, third and fourth circles.

This image is used to better highlight the specific points to use for the size of the next set of circles used in step 8 onwards.

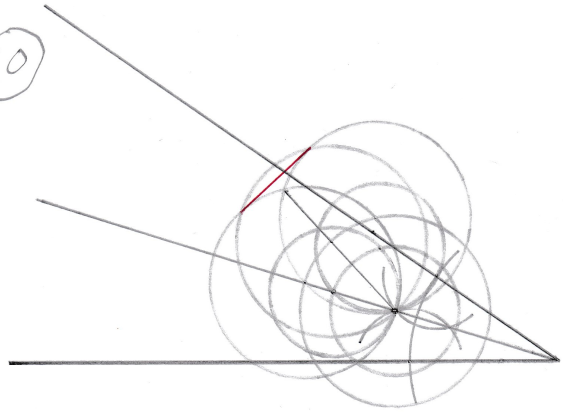


From here the rest of the construction is the same as the initial set of circles, but it uses the different size to create the final elements of the construction and two more circles.

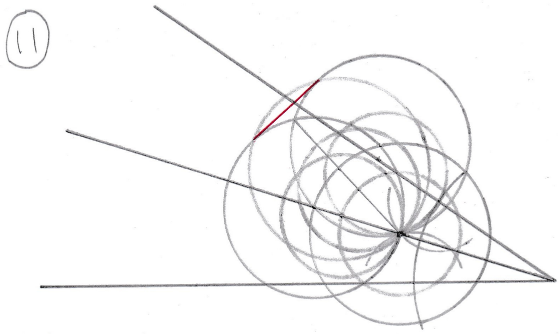
9

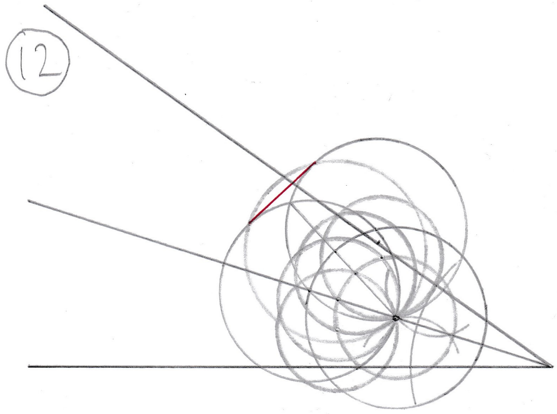


10

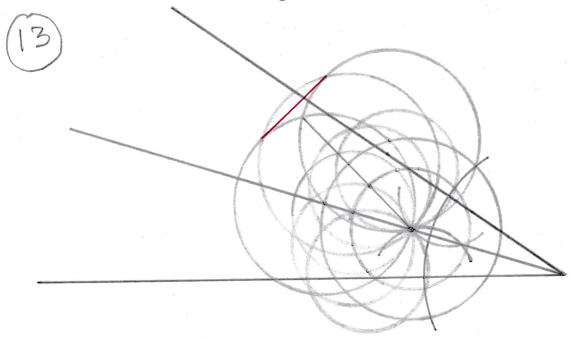


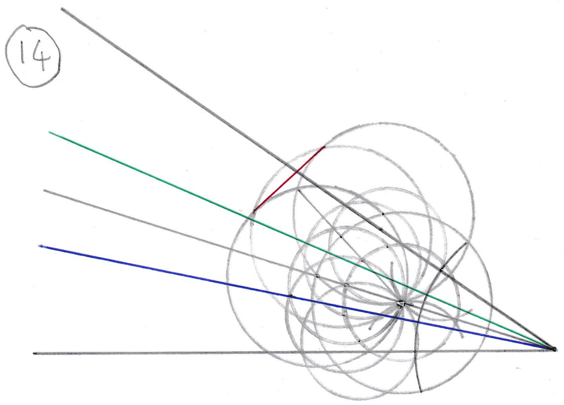
Once you have those three circles, you can use the bisection of the circles from step 5 to locate the origin point for the next circle in step 11, which will then also mark the point for the circle to be drawn in step 12 on the intersection of the initial circle of the second set of circles.





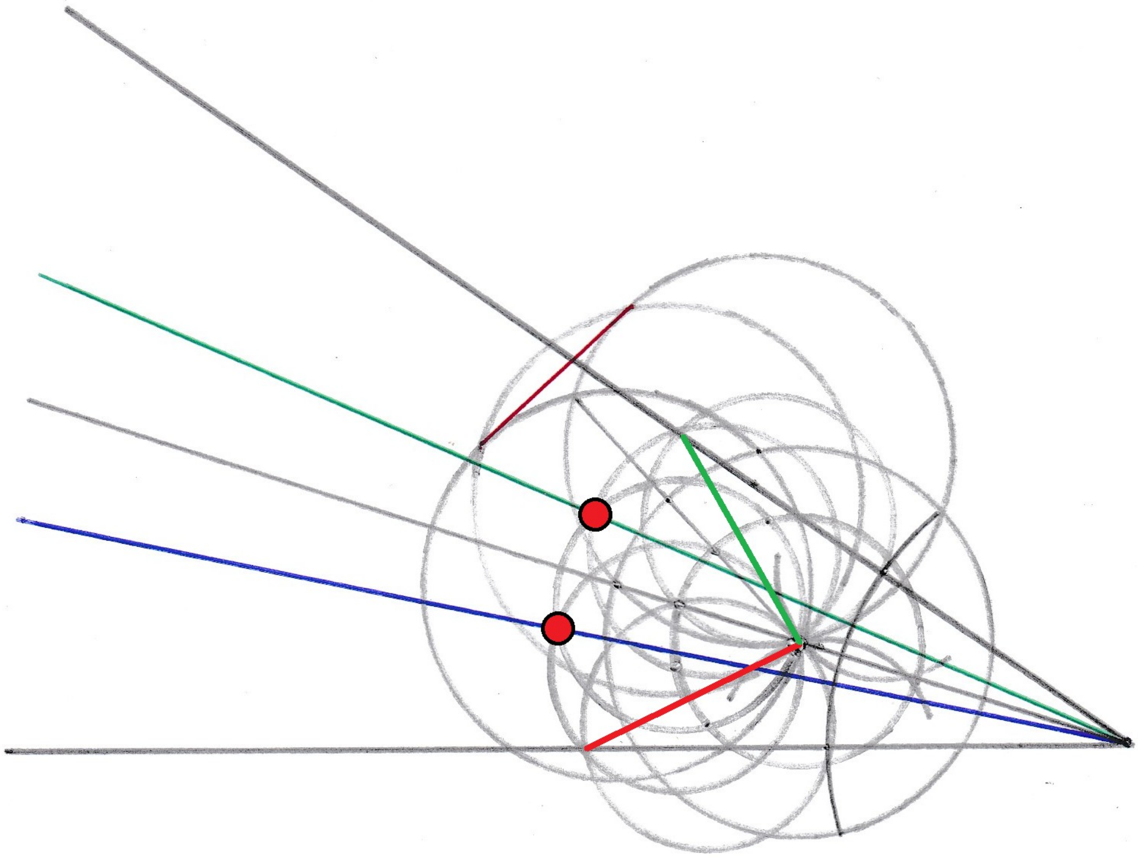
This leaves us with just one more circle to draw, which rests at the bottom intersecting point between the first two smaller circles that rest along the bisection of the angle.





And that finishes the construction and trisection of the angle.

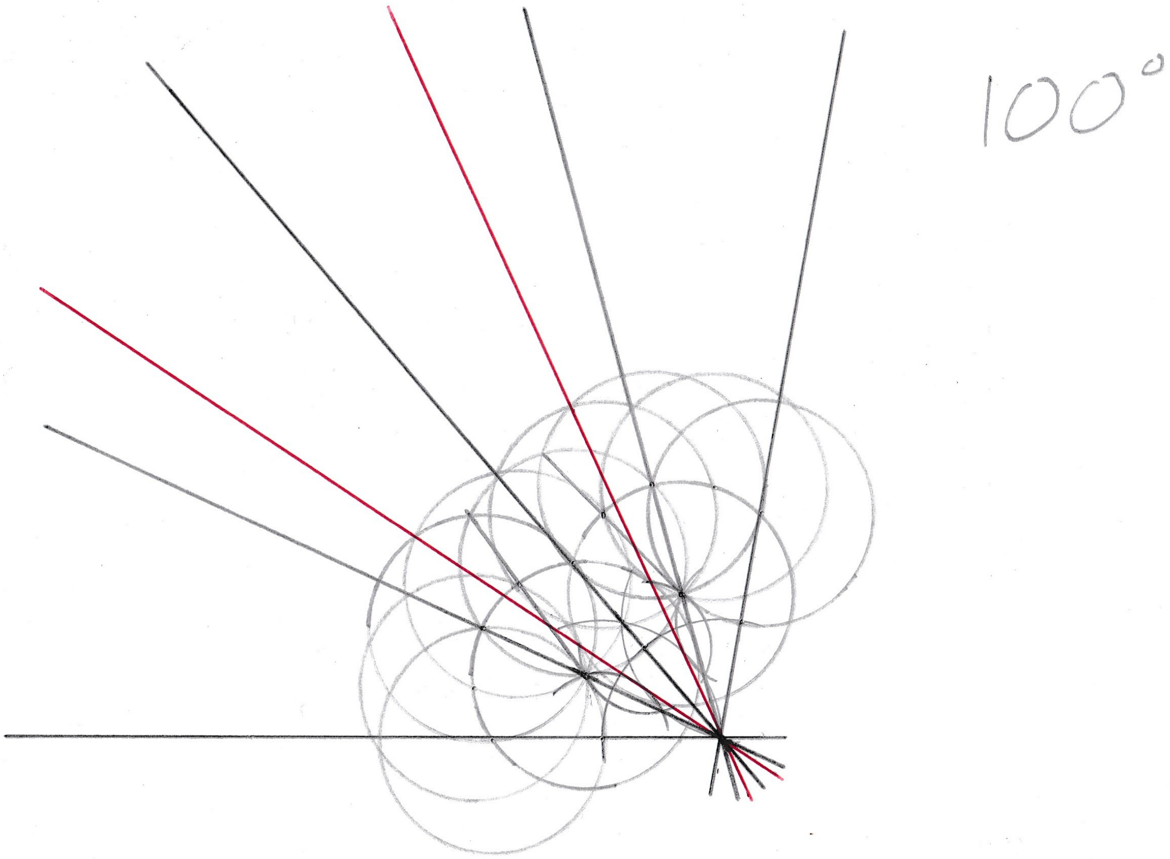
This will result in creating the three bulges of the array of circles as seen in my first examples, with that you can finish the construction by drawing lines between the main point of the angle and those points resting between the smaller circles.



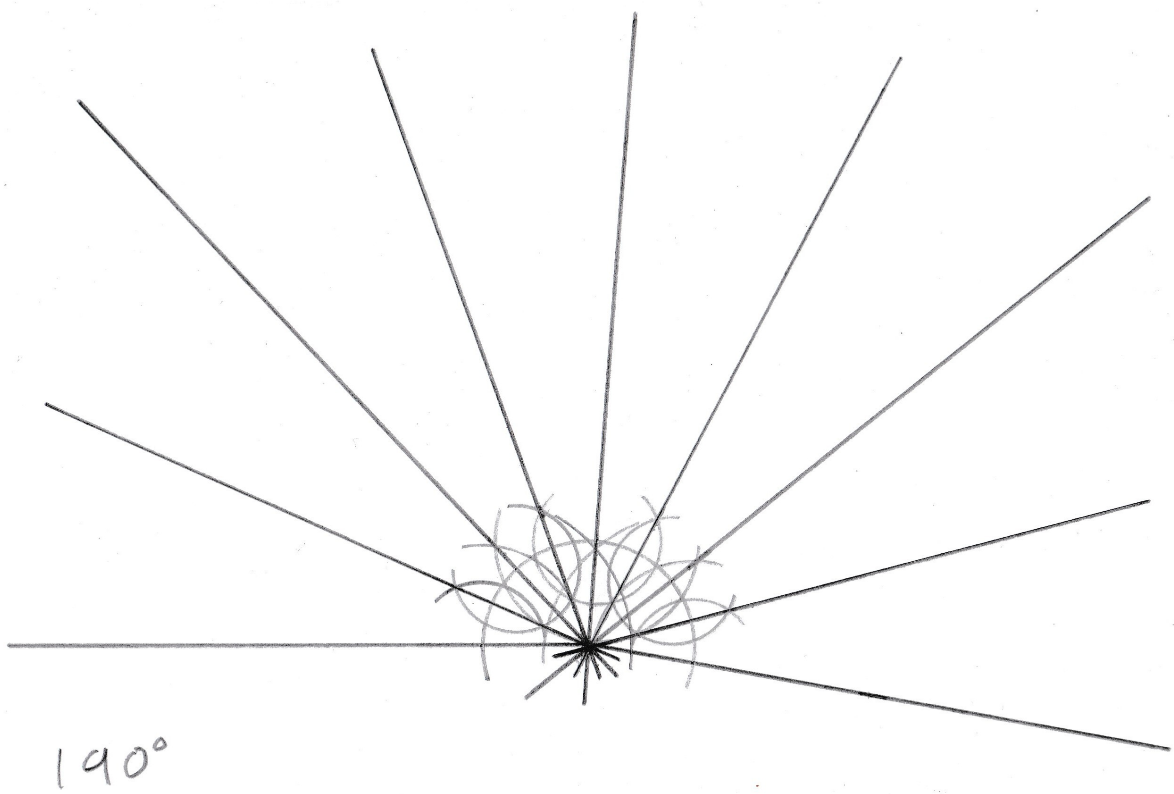
This final image shows the two points to connect, as depicted by the red dots and it also shows the projection angle cast by the array of circles, I hope this method might be of use to someone one day, though given the technological world we live in, it seems unlikely that will ever be the case, feel free to let me know if I have missed anything or if you can help prove or disprove these methods.

With that I will finish by showing some examples for angles that are over either 90 degrees or 180 degrees respectively.

The first example is 100 degrees, I did not need to do two sets of circles in order to find the correct measurement for the circles in this example because the second circle just happened to sit perfectly between the edges of the angle.

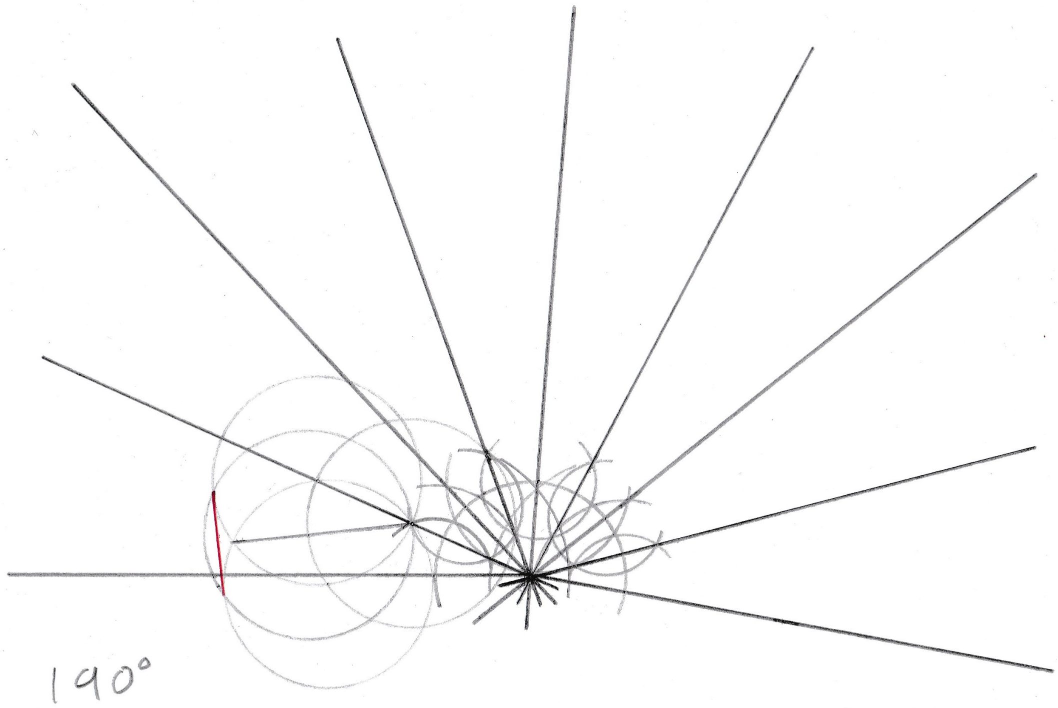


The following is an example of 190 degrees, but it only shows the relevant bisections required to start working on the construction.

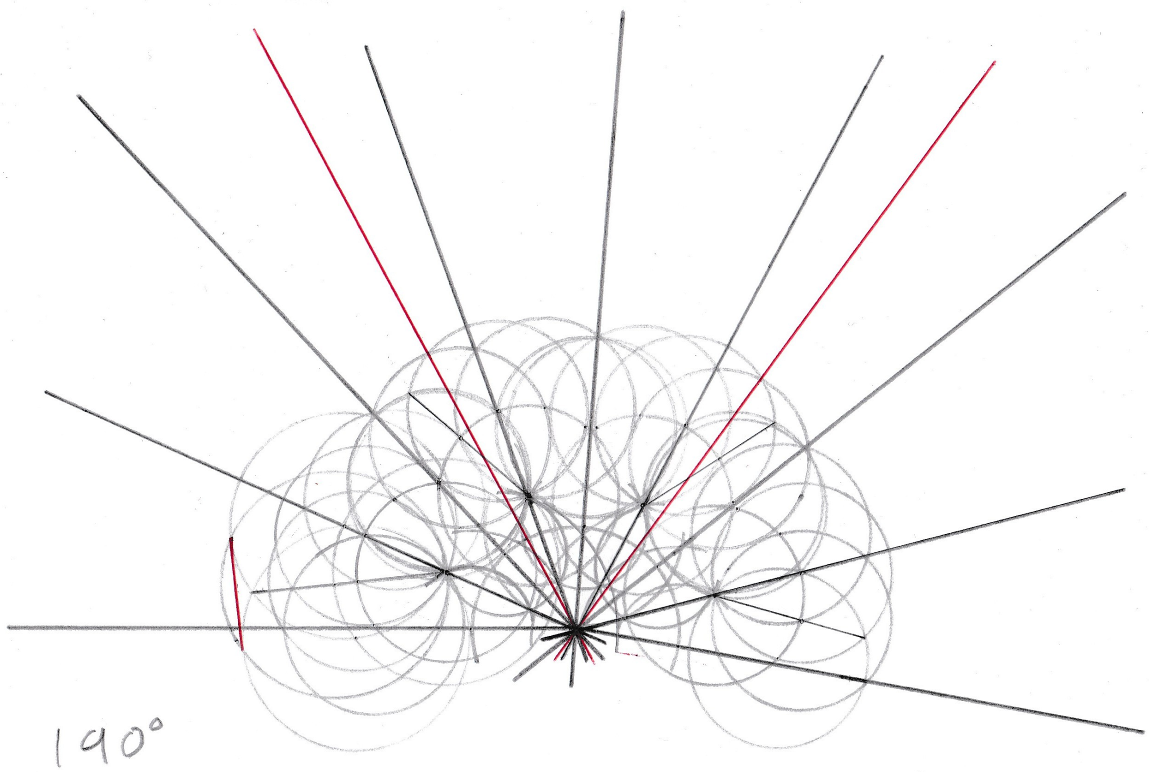


I ended up going back and completing the previous construction and these next two images will show just how easy it is for everything to fall ever so slightly out of line if any of the origin points are out of place.

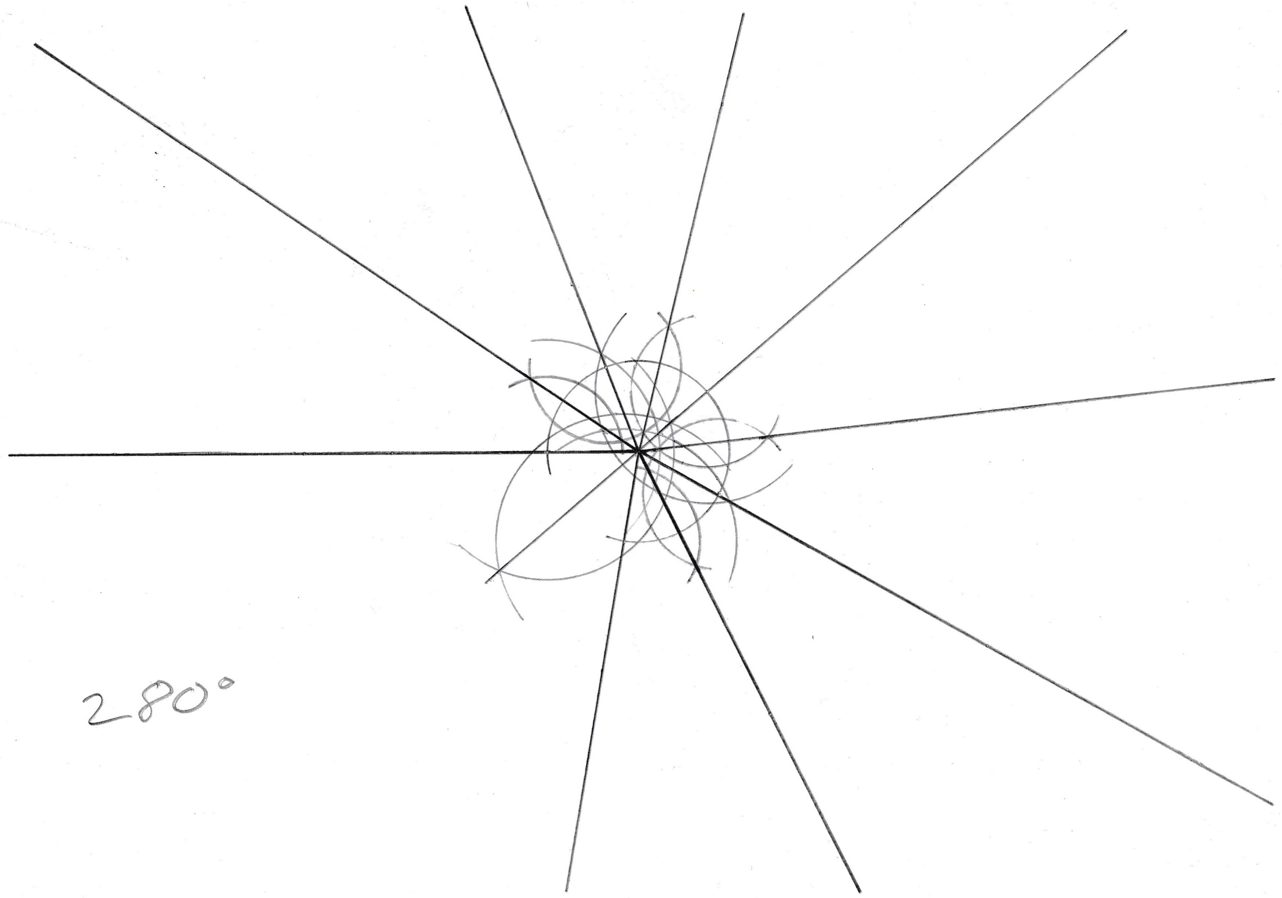
This first image shows the initial steps of the construction to get the proper circle size.



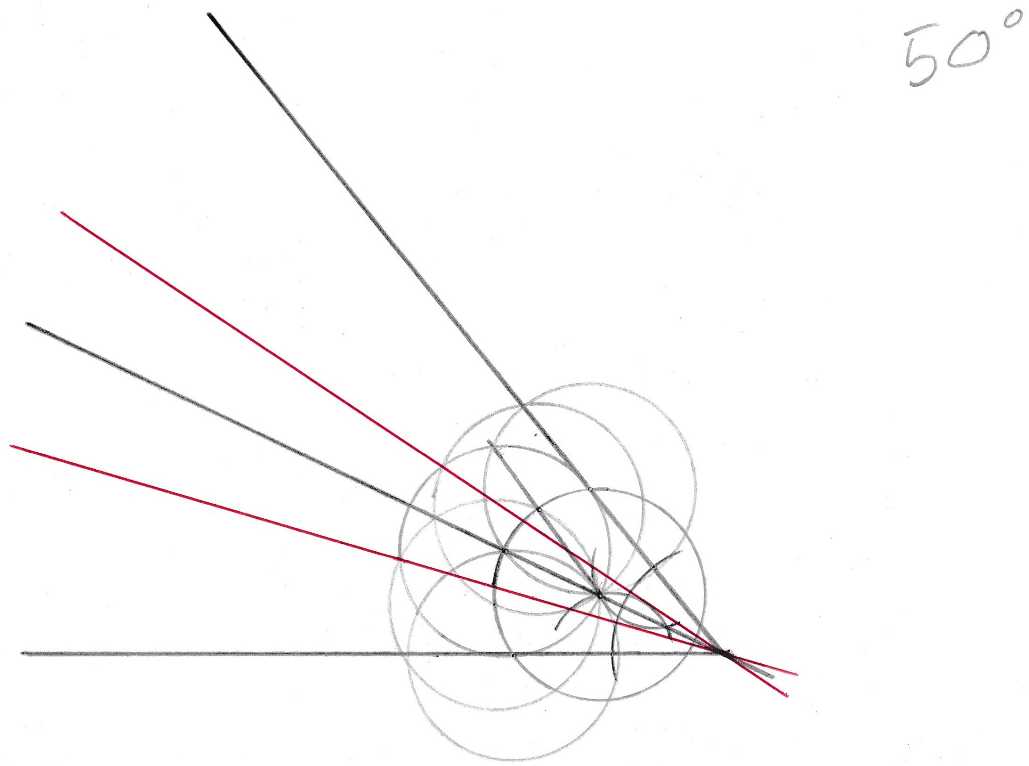
The second image shows the complete construction and trisected angle, unfortunately one of the set of circles on the right side fell ever so slightly out of line due to the difficulty in placing origin points on the construction. *(it is worth noting I was doing this on A5 paper most of the time)*



The final example shows an angle of 280 degrees and again it only shows the relevant bisections required to begin the construction. *(this is a construction done on A4 paper)*



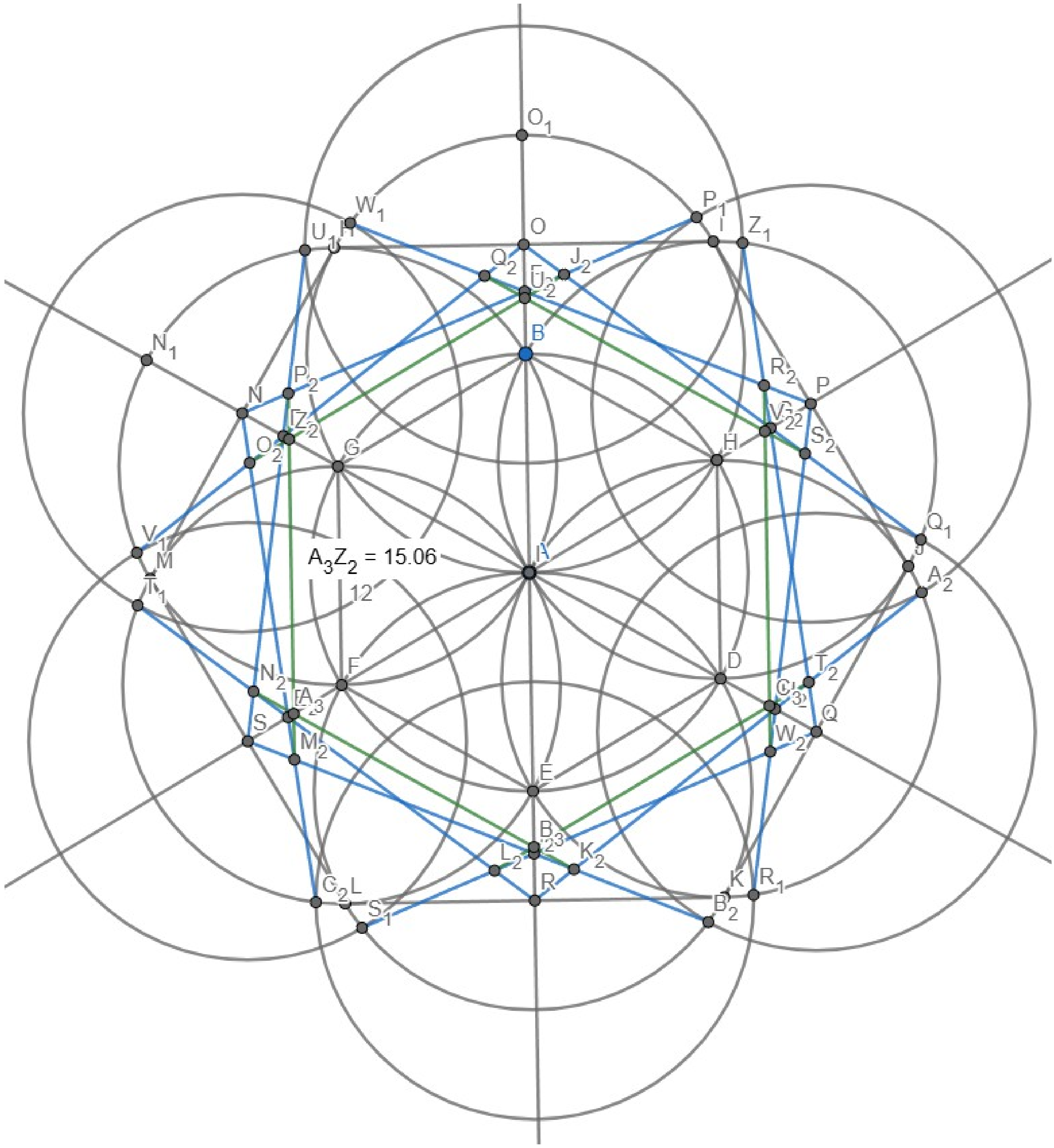
I have also created another example showing 50 degrees and will leave it here.



Hopefully this showcases the versatility of the method.

Cube 2 Tesseract v2.0

After a little trial and error I have also refined the method for doubling the cube that reaches 98.83% accuracy.



Using even more obscure points in the construction that use the intersections of the inner and outer circles as they alternate *(the blue lines)* I was able to create an intersecting point that lies on each central line *(the green lines)* which results in 98.83% accuracy.

Then joining these points results in a construction that is most likely about as close as one could get with a relatively simple construction, from here it might be possible to get a little closer by finding a set of points that create or can be used to create a line that is ever so slightly bigger than this, but it would no doubt take a lot of time to discover.

In this image the initial cube has edge lengths of 12, which results in a volume of 1728 while the newly constructed cube has edge lengths of 15.06 which results in a volume of 3415.662216 while the perfect edge length would be around 15.1190526 which would equal a volume of 3456.0000008651 though the perfect volume would be 3456.

The cube root of 3456 is actually 15.1190526 so that would likely be the edge length to aim for.

Using an excessive number of bisections from here on out can begin to approach this value, but that construction would be increasingly difficult to work with, with each added bisection.
For example these added lines at the corners of the newly constructed cube result in the following edge lengths.

15.1097 *(red line)*
15.1322 *(blue line)*
15.1209 *(green line)*

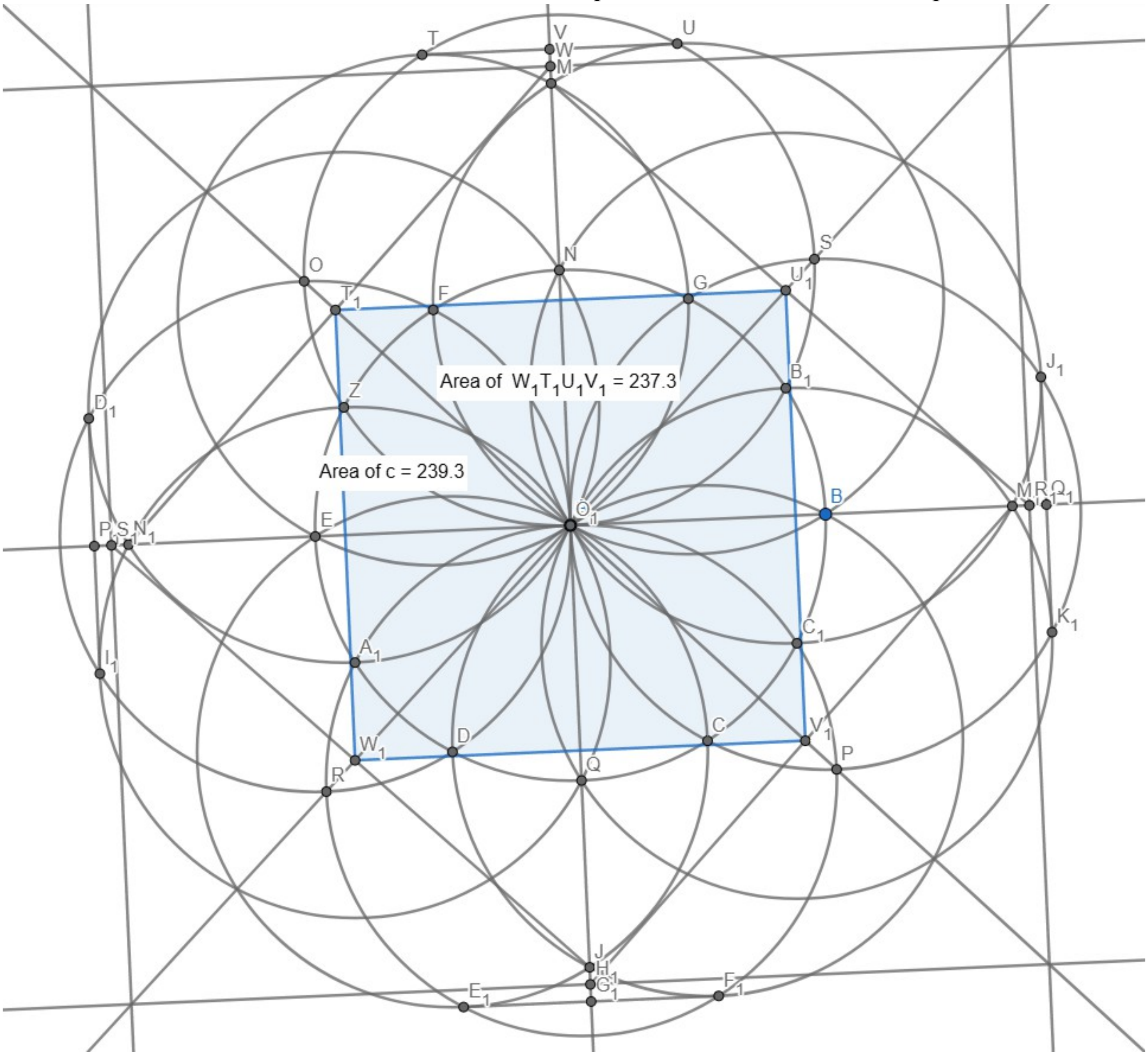
So perhaps with a few more bisections, one could approach 99.99% accuracy but doing so would essentially violate the rules that I followed, which were to construct these using a compass and straight edge, without the use of measurements and in a finite number of steps, as I followed the basic rules of squaring the circle for all three challenges.

Doing this I feel would violate the clause “in a finite number of steps” because of how many times this would need to be done, it would soon become cumbersome and practically impossible to work with.

That is not counting the fact that I used measurements to check the accuracy of the methods in each case of course.

Circling the Square v2.0

After trying a lot of different methods for Squaring the Circle I devised the following method which is 99.16422900125366% accurate and is not as complicated a construction as I expected it to be.

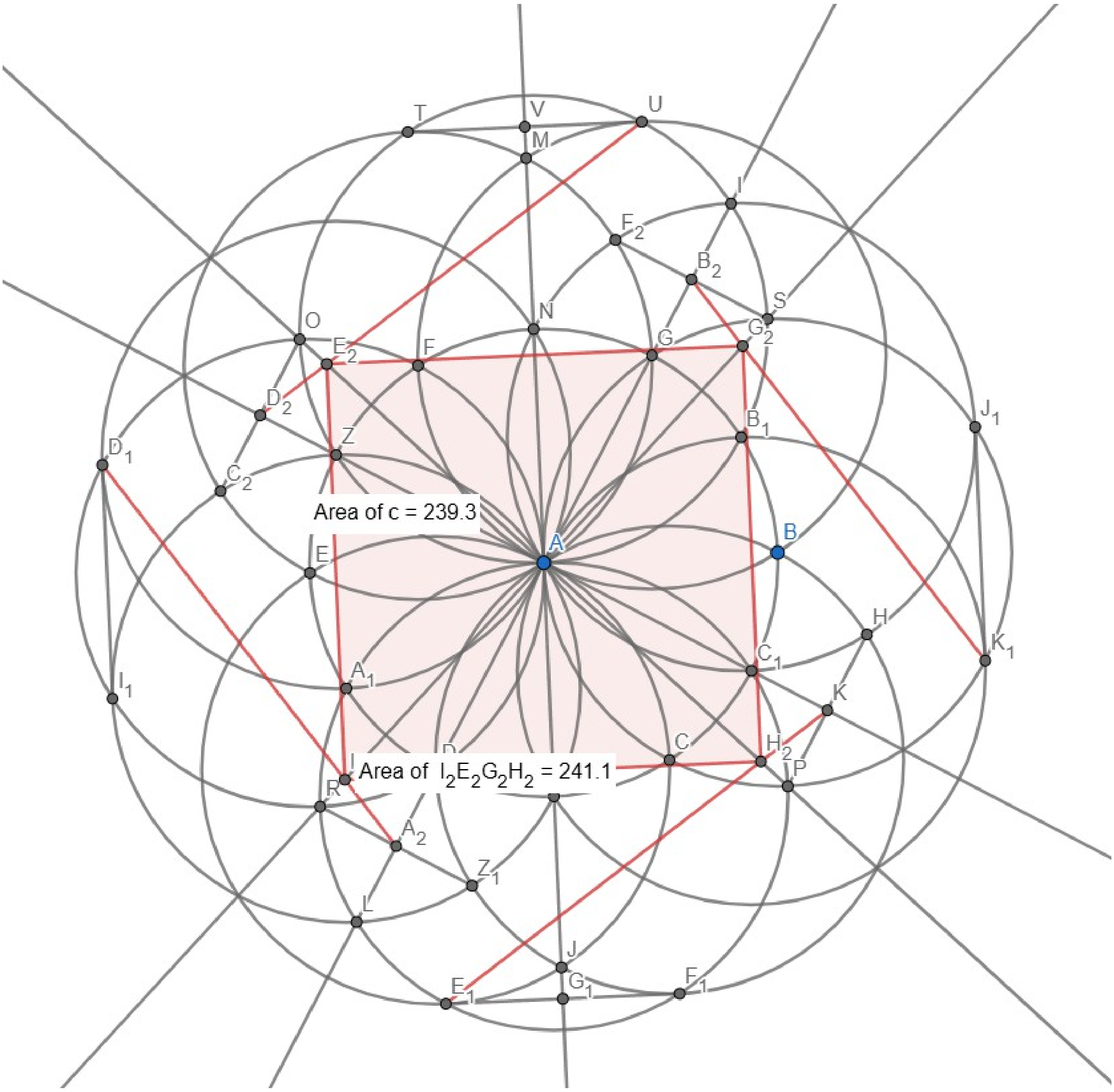


I think realistically this is about as close as I am going to get it.

Though, having said that I found myself trying again and found a method that uses a similar amount of lines and gets ever so slightly closer at 99.25342181667358% accuracy but is a construction larger than the desired one as opposed to smaller which is what I usually aim for.

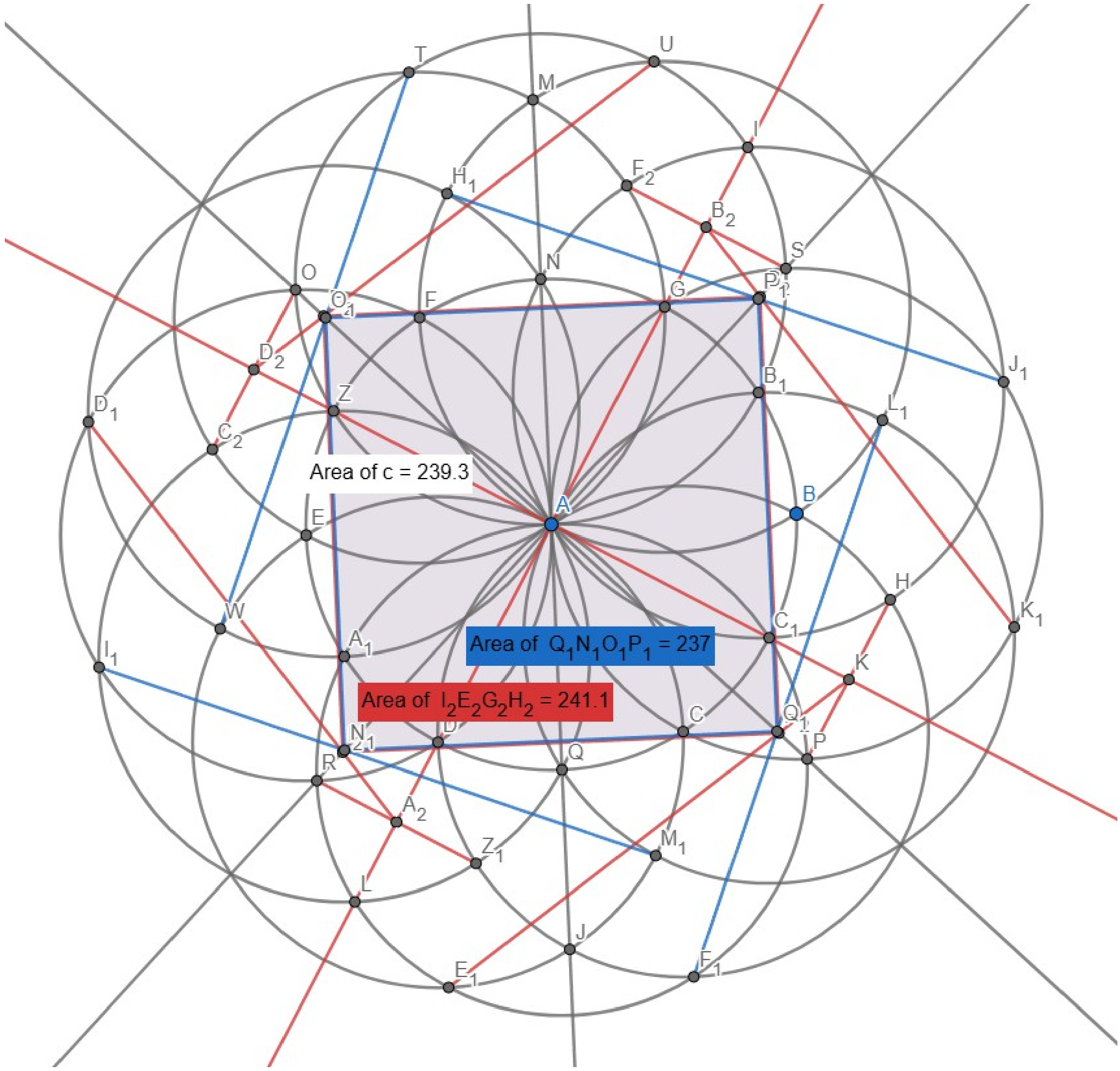
I also came up with a number of other versions that approach a higher degree of accuracy as shown on the following pages. *(these can be considered v3.0)*

This is that method, I keep looking for points innate to the base construction to use as guiding lines to get closer to the desired construction.



I don't know what's a better construction to aim for, smaller or larger than the desired one, but at the least it gives me a point to aim for in between them.

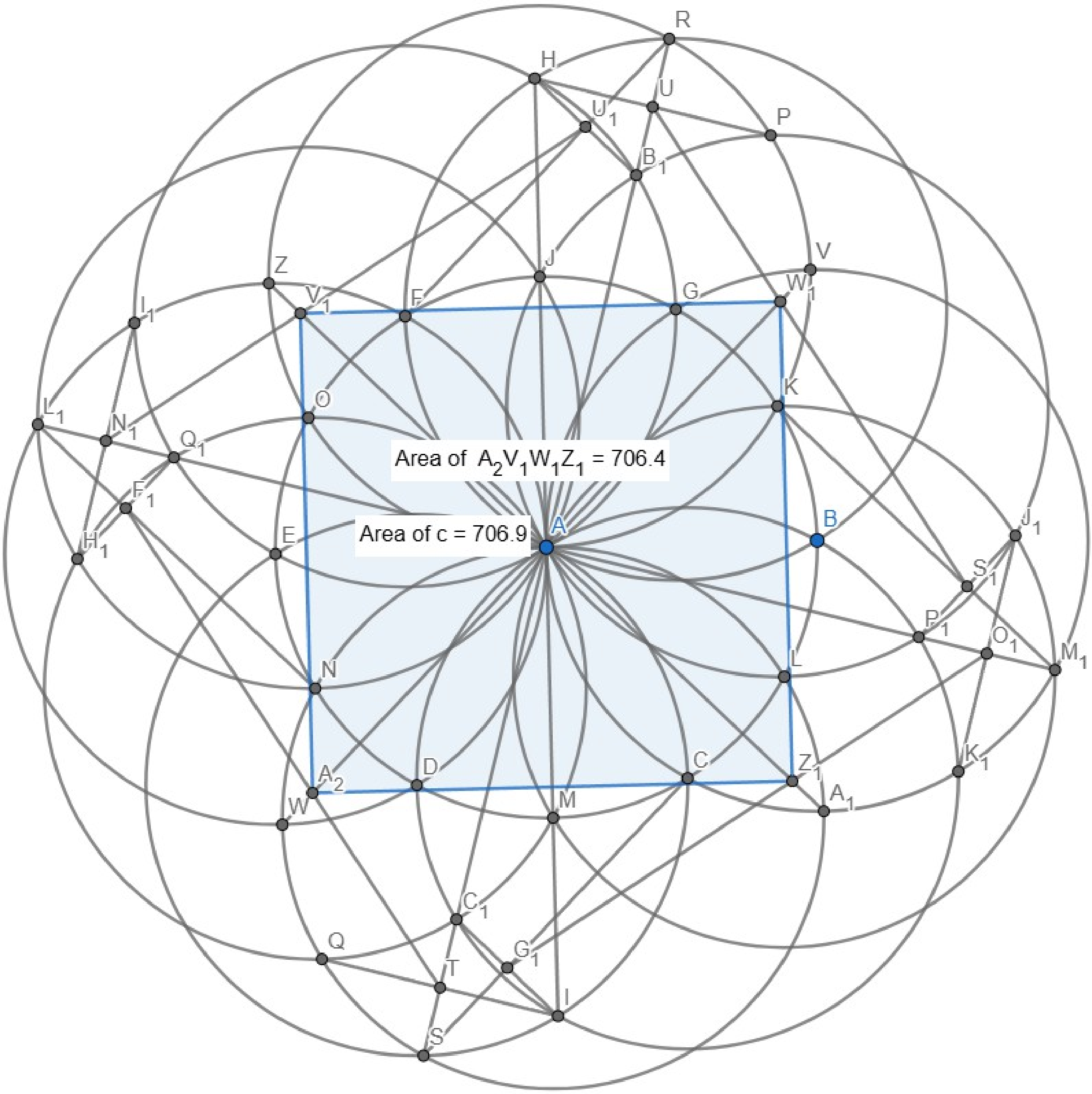
I found another and simpler construction which is not quite as close as the first method in this section but it gives me two points to try and aim between to find an even closer construction.



The blue lines mark the relevant lines required for the blue construction and the red lines mark the lines required for the red construction, grey lines and circles are required for either construction.

While the blue construction is one of the simplest methods I have found so far, the red is the closest with regards to percentage of accuracy but I feel I would rather have a construction slightly smaller as opposed to slightly larger than the desired construction.

After even more trial and error I have arrived at a construction which is 99.92926863771396% accurate and only 0.0707% smaller than the desired construction.



This construction actually manages to make use of the points that rest between the base construction of circles to form new intersecting points between the lines joined by those points, in order to arrive at a construction that is nearly indistinguishable from that which is desired.

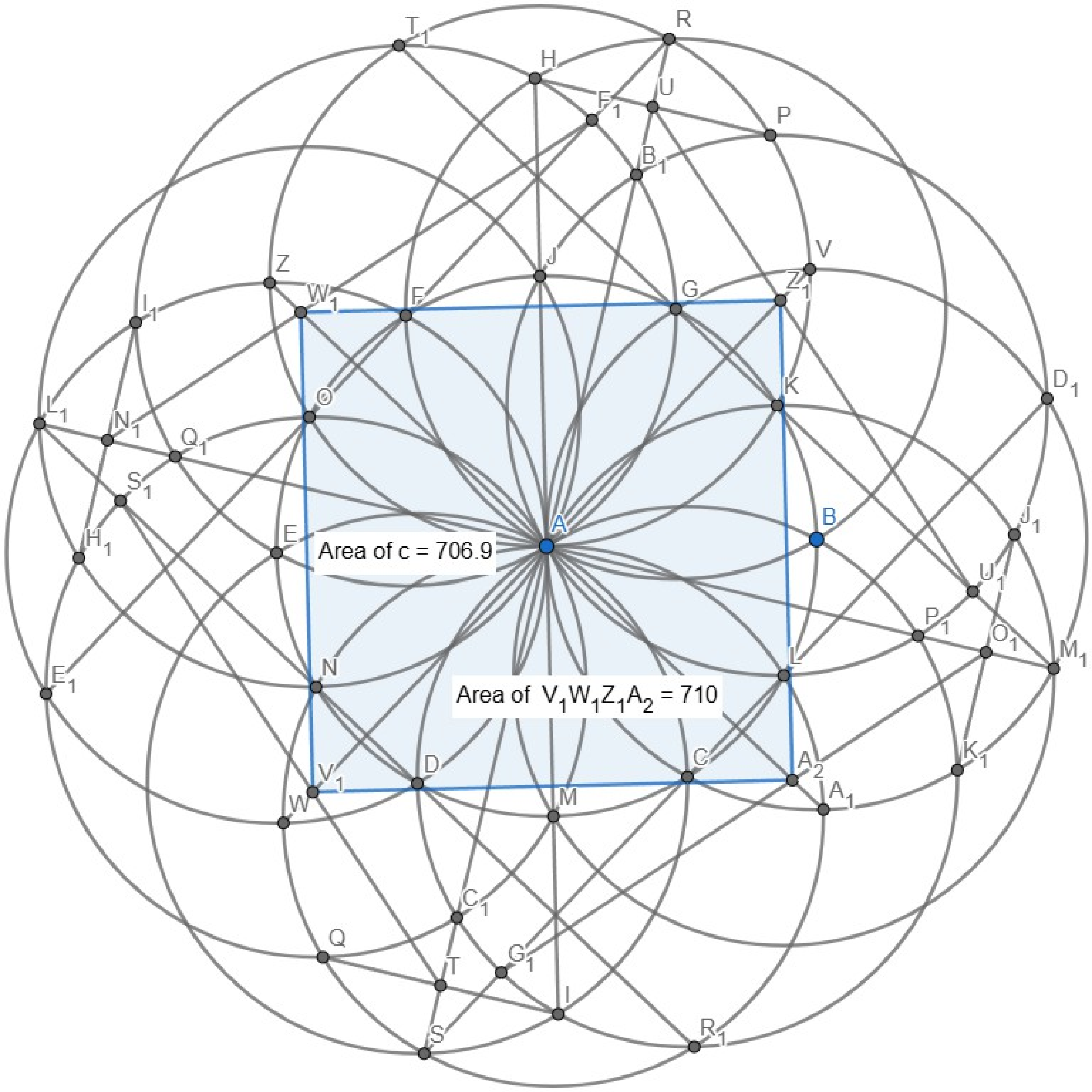
After trying so many different methods, I don't think I will ever be able to get any closer than this, though I would like to refine the methods so they are a little easier to follow and so their percentage of accuracy is charted.

Though it is possible that another point lies somewhere amongst the charted points that could be used to increase the size of the constructed square by a fraction of it's current size, but finding it will be difficult. However a 0.07% variance I think is more than acceptable.

With that I have provided a number of methods for squaring the circle to within a large range of percentage differences of accuracy.

It is possible to use the intersecting point of some of the lines to get a construction that is larger. This uses a few less points for it's construction and is still 99.5% accurate.

So whether it is acceptable for it to be smaller or larger, there is a method that approaches 100% accuracy.

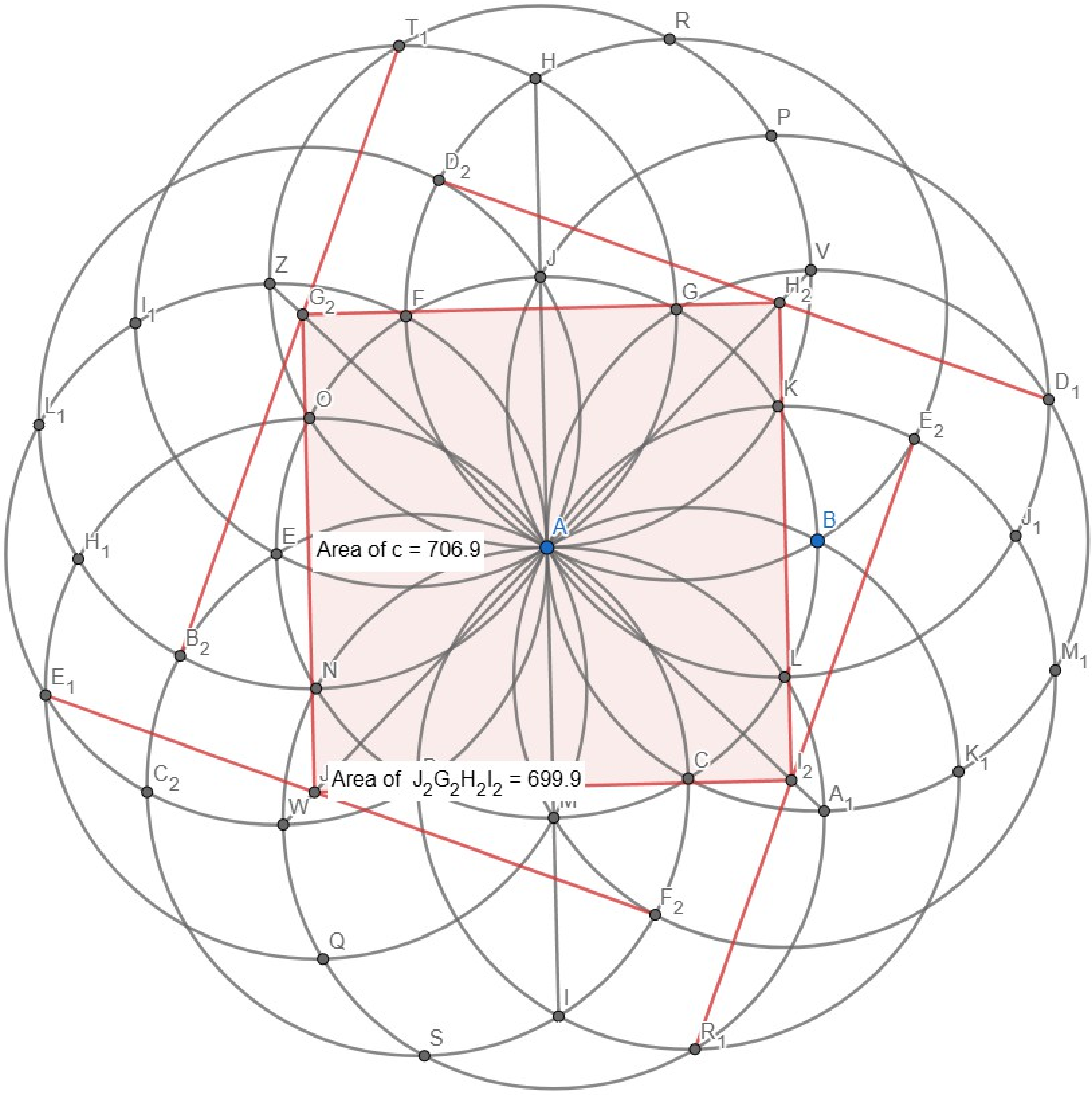


I have tried the majority of the points and intersections that lie between any of the points in this construction to try and get even closer but I think this is it.

Interestingly enough it forms a pattern that have arrow heads pointing in all four directions in both methods, however I am still left to wonder if there is a point I missed somewhere that could result in a construction that is even closer.

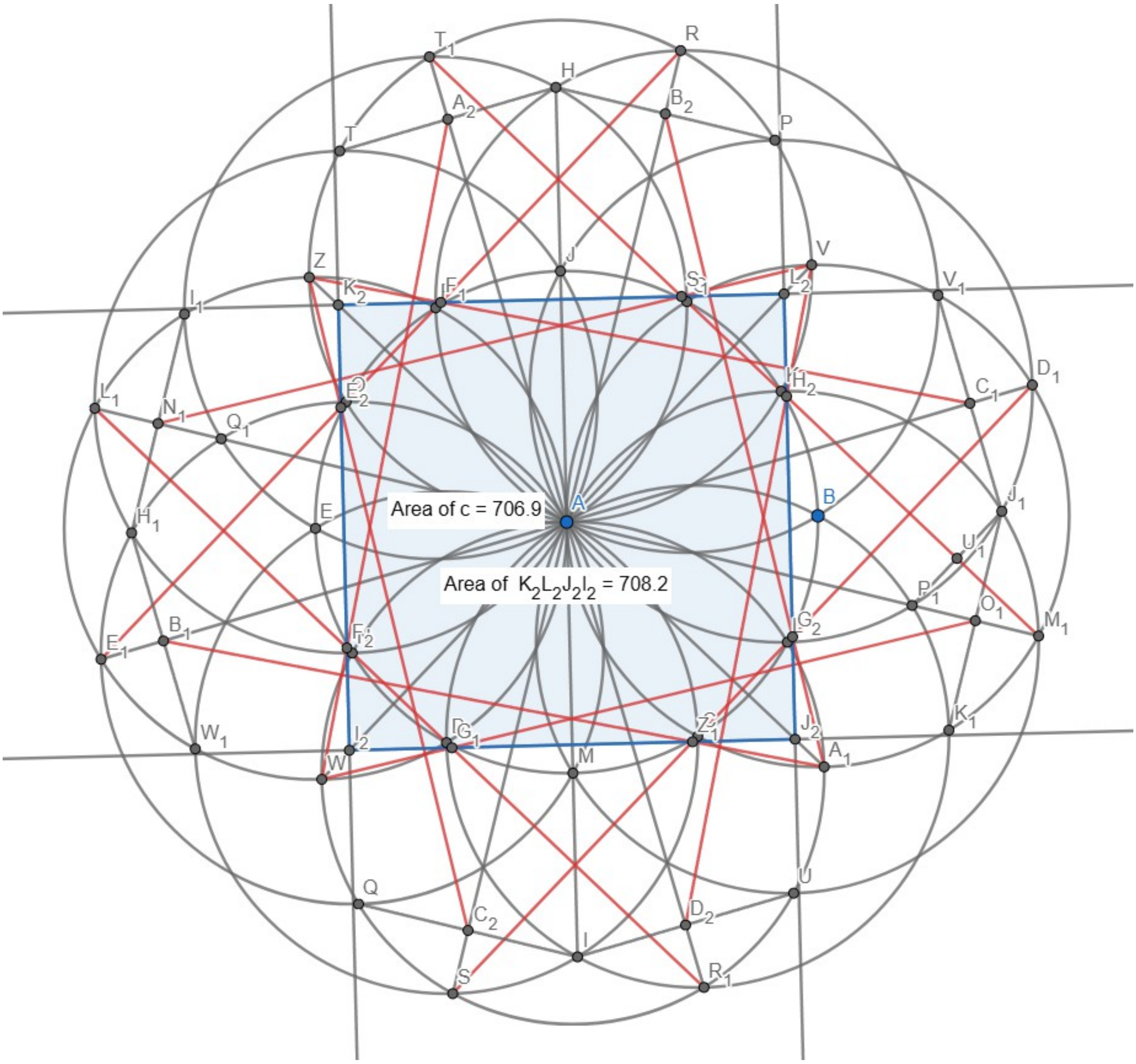
Perhaps one day I or someone else will find it.

I am thinking about making a chart that aims to define the required number of circles and lines as well as the accuracy of each method, this way depending on the desired accuracy and required ease or simplicity of the construction, a method could be easily chosen.



For example this method, also seen on page 30, uses only 11 lines and 13 circles for 99.00976092799547% accuracy.

Or this complicated construction which is 99.81643603501836% accurate but uses quite a number of construction lines and is over the required size rather than under the required size.

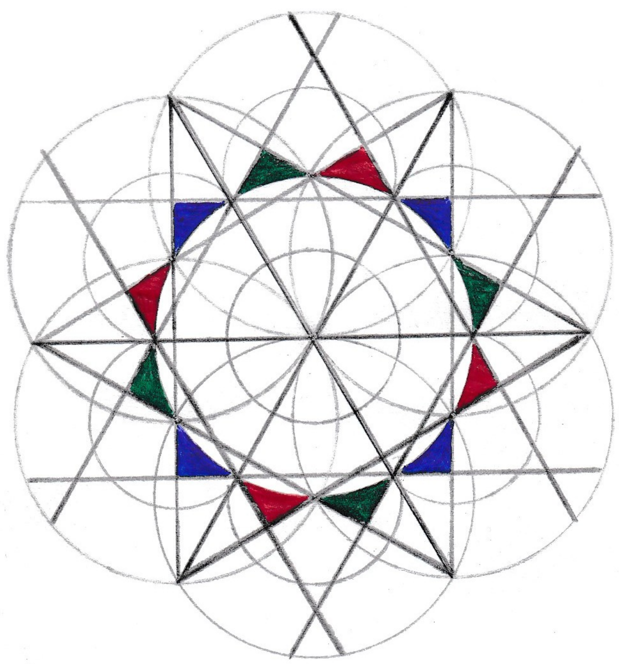


With that I have a method that is 99.8% accurate but is larger than the desired size and a method that is 99.9% accurate but smaller than the desired size.

Given enough time, I believe I can further refine the methods for squaring the circle and doubling the cube to a point where it is even closer to being perfect, though given the transcendental nature of Pi squaring the circle is still to be considered essentially impossible without strict measurements.

All drawings were done on A5 paper. *(unless otherwise noted in italicised brackets)*

Some constructions were made using the online software at : https://www.geogebra.org/geometry
I also used the following site to check the accuracy at : https://percentagecalculator.net/



By Edward James Gordon