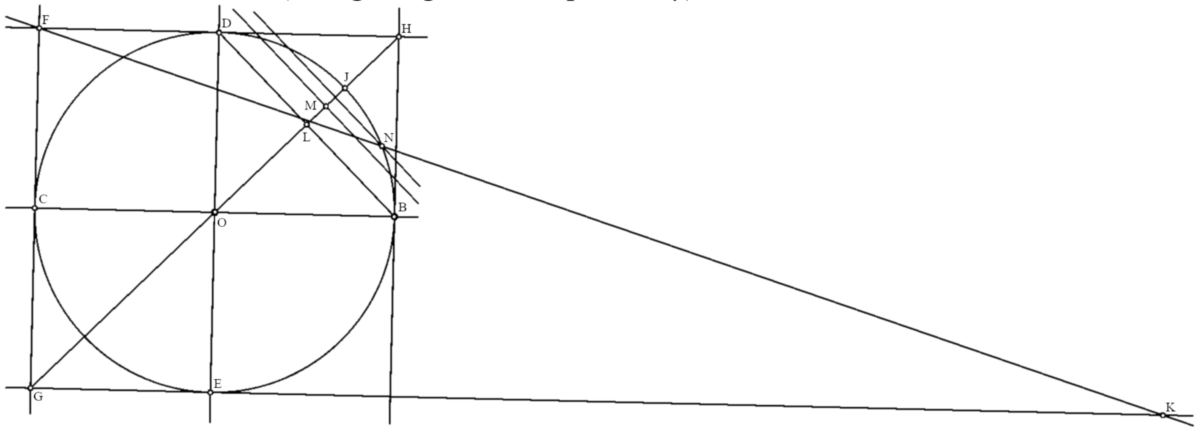
<u>**Method for Approximating the Circumference of Any Given Circle**</u>

Here is a method I devised to approximate the circumference of any given circle, strictly adhering to the Euclidean rule set. *(straightedge and compass only)*



Above is a diagram of the construction and below are the steps for reproducing it. **(OK)**

- Let **O=(0,0)**. Construct the unit circle centred at **O**. Let **B=(1,0)** and **C=(−1,0)** be the right/left cardinal points.
- Connect points **BC** and draw a perpendicular bisector using **BC** creating points **D=(0,1)** and **E=(0,-1)**.
- At point **B** draw the line perpendicular to **DE** *(this is the vertical line x=1 through* **B***)*. At point **C** draw the line perpendicular to **DE** *(the vertical line x=−1 through* **C***)*.
- At points **D** and **E** draw the lines perpendicular to **BC** *(these are the horizontal lines y=1 and y=−1)*. Extend the line through **E=(0,−1)** far to the right *(this extended horizontal line will act as the baseline for the final intersection)*. These four lines complete the circumscribing set of four squares about the circle.
- Draw the line **DB** joining **D=(0,1)** *(top-middle)* to **B=(1,0)** *(right-middle)*.
- Draw the ray **OH** where **H=(1,1)** is the top-right corner of the circumscribing square. Let **J=OH∩** *(circle) (the point where the ray meets the circle)* and let **L=OH∩DB**.
- Let **M** be the midpoint of segment **LJ**. Construct the perpendicular bisector of segment **MJ**.
- Let **N** be the **right-side** intersection point of that perpendicular bisector at **MJ**.
- Let **F** be the top-left corner of the circumscribing arrangement: **F=(−1,1)**. Draw the line **FN**.
- Let **K** be the intersection of line **FN** with the extended baseline *(the horizontal line through* **E***, i.e.* ***G*****=(−1,−1)***, extended far to the right)*.
- The constructed length **L** is the horizontal segment from the left-baseline anchor ***G*****=(−1,−1)** to the point **K**. Its numeric value for the unit circle is the coefficient **α** *(so for a circle of radius* ***r*** *the constructed length equals* ***α·r****)*.

Algebraic Formula, instructions and detailed comparison to 2π *(Derived by GPT-5)*.

$$\alpha = 2\,\frac{1+b+s_0}{1-b+s_0}, \quad b = \frac{1}{8} + \frac{3}{4\sqrt{2}}, \; s_0 = \tfrac{1}{2}\sqrt{2-4b^2}\,.$$

- **α ≈ 6.295357530626529** *(unit circle coefficient)*
- **2π ≈ 6.283185307179586**
- Absolute gap : **α − 2π ≈ 0.012172223446943%**
- Percentage of accuracy : **(GK / ₵) * 100 = 99.806647622550144561036880384858%**

(₵ denotes the actual circumference of the circle.)

Revised and rewritten by *Edward James Gordon*.