Calculate the atmospheric CO_2 concentration in different units:

What is the mass m of the atmosphere?

The atmospheric pressure at sea level = the mass of the Earth's atmosphere x g / Earth's surface area:

$$p_0 = \frac{mg}{A} = \frac{mg}{4\pi R_j^2} \Leftrightarrow M = 4\pi R_j^2 \frac{p_0}{g} = 4\pi(6368 \cdot 10^3 m)^2 \frac{10^5 Pa}{\frac{9.81m}{s^2}} = 5.19 \cdot 10^{18} kg$$

What is the number of moles N in the atmosphere?

The mole weight is $M = 28.96 \cdot 10^{-3} kg/mole$

The number of moles in the atmosphere = the mass of the atmosphere / the mole weight:

$$N = \frac{m}{M} = \frac{5.19 \cdot 10^{18} kg}{28.96 \cdot 10^{-3} kg/mole} = 1.79 \cdot 10^{20} mole$$

What is the number of moles of CO_2, N_{CO_2} in the atmosphere?

The concentration of CO_2 is in units of ppm, and $1 \text{ ppm} = \frac{1}{1000000} = 1 \cdot 10^{-6} = 0.0001\%$.

We use a pre-industrial value, $c = 290 ppm$.

The concentration c is measured in ppm by volume =ppmV= volume% = %moles, i.e.

$$c = \frac{N_{CO_2}}{N} = \frac{290}{1000000} \Leftrightarrow N_{CO_2} = N \cdot 0.029\% = 5.20 \cdot 10^{-16} mole$$

How many kg of CO_2 does the atmosphere contain?

The mass m_{CO_2} of CO_2 can now be determined. The mole weight is $M_{CO_2} = 42.44 \cdot 10^{-3} kg/mole$

$$m_{CO_2} = N_{CO_2} \cdot M_{CO_2} = 5.20 \cdot 10^{-16} mole \cdot 42.44 \cdot 10^{-3} \frac{kg}{mole} = 2.25 \cdot 10^{15} kg$$

What it the outgassing of CO_2 in ppm?

The present outgassing rate of CO_2 is around: $\Delta m_{CO_2} = 100\, mill \frac{tonnes}{yr} = 100 \cdot 10^9 kg/yr$

The change in concentration per year is

$$\Delta c = \frac{\Delta N_{CO_2}}{N} = \frac{1}{N} \frac{\Delta m_{CO_2}}{M_{CO_2}} = \frac{100 \cdot \frac{10^9 kg}{yr}}{1.79 \cdot 10^{20} mole \cdot 42.44 \cdot \frac{10^{-3} kg}{mole}} \Leftrightarrow$$

$$\Delta c = 1.29 \cdot 10^{-8} = 1.29 \cdot 10^{-2} ppmV/yr$$