#### Course 02402 Introduction to Statistics Lecture 13:

#### A course summary

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#### Agenda - the 12 lectures

- Ochapter 1: Simple Graphics and Summary Statistics
- Chapter 2: Discrete Distributions
- Chapter 2: Continuous Distributions
- One sample confidence intervals
- 6 Chapter 3: One sample hypothesis testing
- 6 Chapter 3: Two Sample statistics
- Chapter 4: Statistics by simulation
- Chapter 5: Simple linear Regression Analysis
- Ochapter 6: Multiple linear Regression Analysis
- © Chapter 8: One-way Analysis of Variance
- Chapter 8: Two-way Analysis of Variance
- Chapter 7: Inferences for Proportions

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#### Chapter 1: Simple Graphics and Summary Statistics

Look at data as it is! (descriptive statistics)

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- Summary Statistics
  - Mean  $\bar{x}$
  - Standard deviation s, variance  $s^2$
  - Median, upper- and lower quartiles

### Chapter 1: Simple Graphics and Summary Statistics

- Look at data as it is! (descriptive statistics)
- Summary Statistics
  - Mean  $\bar{x}$
  - Standard deviation s, variance  $s^2$
  - Median, upper- and lower quartiles
- Simple graphics
  - Scatter plot (xy plot)
  - Histogram, cumulative distribution
  - Boxplots, Bar charts, Pie charts

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#### Chapter 2: Discrete Distributions

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- General concepts:
  - Definition of a stochastic variable
  - Density function
  - Distribution function
  - Mean and variance

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  - Definition of a stochastic variable
  - Density function
  - Distribution function
  - Mean and variance
- Specific distributions:
  - The binomial distribution
  - The hypergeometric distribution
  - The Poisson distribution

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#### Chapter 2: Continuous Distributions

- General concepts:
  - Density function, distribution function
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  - Calculation rules for stochastic variables

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- General concepts:
  - Density function, distribution function
  - Mean, variance
  - Calculation rules for stochastic variables
- Specific distributions:
  - Normal
  - Log-Normal, Uniform, Exponential

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#### Chapter 3: One sample confidence intervals

## General concepts

- Estimation, confidence intervals
- Population and a random sample
- Sampling distributions  $(t \text{ and } \chi^2)$
- Central Limit Theorem

### Chapter 3: One sample confidence intervals

# General concepts

- Estimation, confidence intervals
- Population and a random sample
- Sampling distributions  $(t \text{ and } \chi^2)$
- Central Limit Theorem
- Specific methods, one sample:
  - Confidence intervals for the mean
  - Confidence intervals for the variance (and standard deviation)

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#### Chapter 3: One sample hypothesis testing

- General concepts:
  - Hypotheses, p-value, Significance level
  - Type I and Type II error, Power
- Specific methods, One sample:
  - t-test for mean difference
  - Sample size for wanted power
  - Normal qq-plot

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## Chapter 3: Two Samples

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  - Test and confidence interval for the mean difference (t-test)

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  - "Take difference"  $\Rightarrow$  "One sample"

## Chapter 3: Two Samples

- Specific methods, two samples:
  - Test and confidence interval for the mean difference (t-test)
- Specific methods, two PAIRED samples:
  - "Take difference"  $\Rightarrow$  "One sample"
- Planning for precision and/or power
  - One-sample Confidence interval: sample size for wanted precision
  - One-sample hypothesis test: sample size for wanted power (or other combinations)
  - Two-sample hypothesis test: sample size for wanted power (or other combinations)

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#### Chapter 4, Statistics by simulation

- Introduction to simulation
- Error propagation rules

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- Introduction to simulation
- Error propagation rules
- Bootstrapping
  - Parametric
  - Non-parametric
  - Confidence intervals (and hence also hypothesis testing)

## Chapter 4, Statistics by simulation

- Introduction to simulation
- Error propagation rules
- Bootstrapping
  - Parametric
  - Non-parametric
  - Confidence intervals (and hence also hypothesis testing)
- Specific situations: (4 versions of confidence intervals)
  - One-sample and Two-sample data
  - Parametric and Non-parametric

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#### Chapter 5: Simple linear Regression Analysis

- Two quantitative variables, x and y.
- Calculating least squares line

## Chapter 5: Simple linear Regression Analysis

- Two quantitative variables, x and y.
- Calculating least squares line
- Inferences for a simple linear regression model
  - Statistical model:  $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$
  - Interval estimation and test for  $\beta_0$  and  $\beta_1$ .
  - Confidence interval for the expected line.
  - Prediction interval.

## Chapter 5: Simple linear Regression Analysis

- Two quantitative variables, x and y.
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  - Confidence interval for the expected line.
  - Prediction interval.
- r and  $r^2$ 
  - r describes the strength of a linear relation.
  - $r^2$  expresses the proportion of the y variability explained by the linear relation.

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## Chapter 6: Multiple linear Regression Analysis

- Many quantitative variables,  $x_1$ ,  $x_2$  and y.
- Calculating least squares fit

## Chapter 6: Multiple linear Regression Analysis

- Many quantitative variables,  $x_1$ ,  $x_2$  and y.
- Calculating least squares fit
- Inferences for a the multiple linear regression model
  - Statistical model:  $y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_{2,i} \varepsilon_i$
  - Interval estimation and test for  $\beta_0$  and  $\beta_i$ .
  - Confidence interval for the expected fit.
  - Prediction interval.

# Chapter 6: Multiple linear Regression Analysis

- Many quantitative variables,  $x_1$ ,  $x_2$  and y.
- Calculating least squares fit
- Inferences for a the multiple linear regression model
  - Statistical model:  $y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_{2,i} \varepsilon_i$
  - Interval estimation and test for  $\beta_0$  and  $\beta_i$ .
  - Confidence interval for the expected fit.
  - Prediction interval.
- $r^2$  expresses the proportion of the y variability explained by the linear relation.

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#### Chapter 8: One-way Analysis of Variance

• Specific methods, *k* INDEPENDENT samples

## Chapter 8: One-way Analysis of Variance

- Specific methods, *k* INDEPENDENT samples
- One-way analysis of variance
  - Compares the means of the groups
  - ANOVA-table: SST = SS(Tr) + SSE
  - F-test.
  - Post hoc test: pairwise t-test with/without Bonferroni correction

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## Chapter 8: Two-way Analysis of Variance

- Block design two-way analysis of variance
- ANOVA-tabel: SST = SS(Tr) + SS(Bl) + SSE
  - SST, SS(Tr) and SS(Bl) calculated as one-way ANOVA
  - SSE = SST SS(Tr) SS(Bl)
- F-test.
- Post hoc test: pairwise *t*-test with/without
  Bonferroni correction

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## Chapter 7: Inferences for Proportions

- Specific methods, one, two and k > 2 samples
  - Binary/categorical response
- Estimation and confidence interval of proportions
  - Large sample vs. small sample methods

## Chapter 7: Inferences for Proportions

- Specific methods, one, two and k > 2 samples
  - Binary/categorical response
- Estimation and confidence interval of proportions
  - Large sample vs. small sample methods
- Hypotheses for one proportion
- Hypotheses for two proportions
- Analysis of contingency tables ( $\chi^2$ -test) (All expected > 5)

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