The Howland Current Pump

January 07, 2019 by Dr. Sergio Franco (/author/dr.-sergio-franco)

The Howland current pump, invented by MIT's Professor Bradford Howland in the early 1960s, consists of an op-amp and a balanced resistor bridge and puts out current in either direction.

The Howland current pump, shown in Figure 1a, is a circuit that accepts an input voltage v_I, converts it to an output current $i_O = Av_I$, with A as the transconductance gain, and pumps i_O to a load LD, regardless of the voltage v_L developed by the load itself. To see how it works, label it as in Figure 1b, and apply Kirchoff's Current Law (https://www.allaboutcircuits.com/textbook/direct-current/chpt-6/kirchhoffs-current-law-kcl/) and Ohm's Law (https://www.allaboutcircuits.com/textbook/direct-current/chpt-2/voltage-current-resistance-relate/).

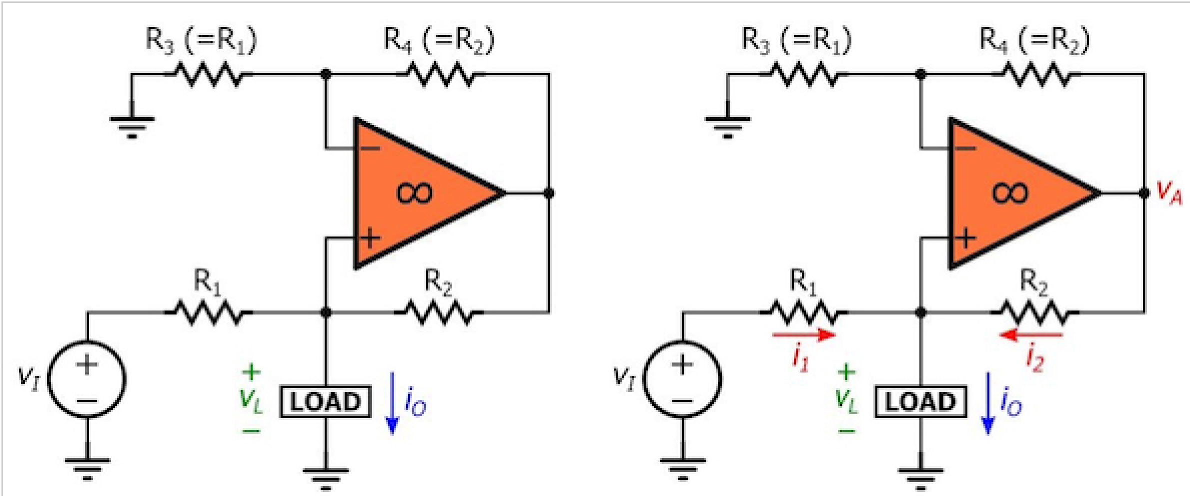


Figure 1. *(a) The Howland pump. (b) Properly labeling the circuit for its analysis.*

$$i_O = i_1 + i_2 = \frac{v_I - v_L}{R_1} + \frac{v_A - v_L}{R_2}$$

Equation 1

The op-amp, together with R_3 and R_4, forms a non-inverting amplifier with respect to v_L, thus giving

$$v_A = \left(1 + R_4 / R_3\right) v_L$$

Equation 2

Substituting v_A into Equation 1 and collecting, we put i_O into the insightful form

$$i_O = Av_I - \frac{v_L}{R_o}$$

Equation 3

where A is the transconductance gain, in A/V,

$$A = \frac{1}{R_1}$$

Equation 4

and where R_o is the output resistance presented by the circuit to the load,

$$R_o = \frac{R_2}{R_2 / R_1 - R_4 / R_3}$$

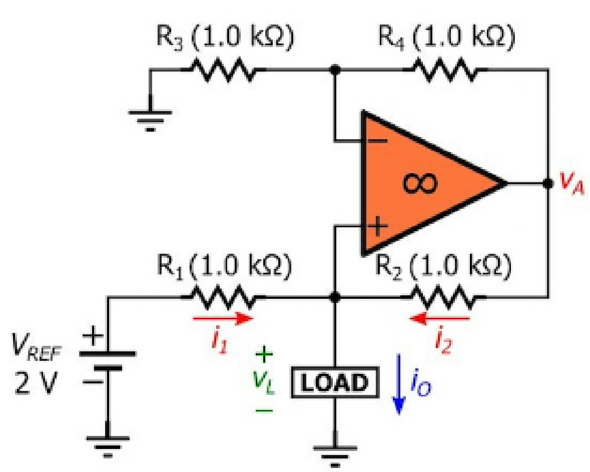
Equation 5

To make i_O independent of v_L we must impose $R_o \to \infty$, or the balanced-bridge condition.

$$\frac{R_4}{R_3} = \frac{R_2}{R_1}$$

Equation 6

Take a look at the example in Figure 2 and observe, row-by-row, how the op-amp adjusts i_2, via v_A, so as to ensure the same current i_O regardless of v_L.



v_L	i_1	v_A	i_2	i_O
0	2	0	0	2
1	1	2	1	2
2	0	4	2	2
3	−1	6	3	2
−1	3	−2	−1	2
−2	4	−4	−2	2

Figure 2. *(a) A 2 mA current source, and (b) its inner workings for different values of vL (voltages in volts, currents in milliamps; a negative current value means that current flows in the direction opposite to the arrow).*

With the polarity of V_{REF} as shown, the pump sources i_O to the load. Inverting the polarity of V_{REF} will cause the pump to sink i_O from the load. Note that for the pump to work properly v_A must always be confined within the linear range of op-amp operation. If the op-amp is driven into saturation, the pump will cease to operate properly.

The Effect of Resistance Mismatches

A practical bridge is likely to be unbalanced because of resistance tolerances, so R_o is likely to be less than infinity. Denoting the tolerances of the resistances in use by p, we note that the denominator D of Equation 5 is maximized when R_2 and R_3 are maximized and R_1 and R_4 are minimized. For $p << 1$, we write

$$D_{\max} = \frac{R_2(1+p)}{R_1(1-p)} - \frac{R_4(1-p)}{R_3(1+p)} \cong \frac{R_2}{R_1}(1+p)^2 - \frac{R_4}{R_3}(1-p)^2 \cong \frac{R_2}{R_1}\left[(1+2p)-(1-2p)\right] \cong \frac{R_2}{R_1}4p$$

Here we have incorporated the relationship of Equation 6, applied approximation

$$1/(1 \mp p) \cong 1 \pm p$$

and ignored quadratic terms in p. Substituting into Equation 5 gives

$$R_{o(\min)} = \frac{R_2}{D_{\max}} \cong \frac{R_1}{4p}$$

Equation 7

As an example, using 1% (p = 0.01) resistances in Figure 2a can lower R_o from ∞ to as little as 1,000/(4×0.01) = 25 kΩ, thus making i_O depend upon v_L, by Equation 3. If the bridge is unbalanced in the opposite direction of above, then the worst-case condition for R_o is –25 kΩ. So, depending on the mismatch, R_o may lie anywhere from +25 kΩ to ∞ to –25 kΩ.

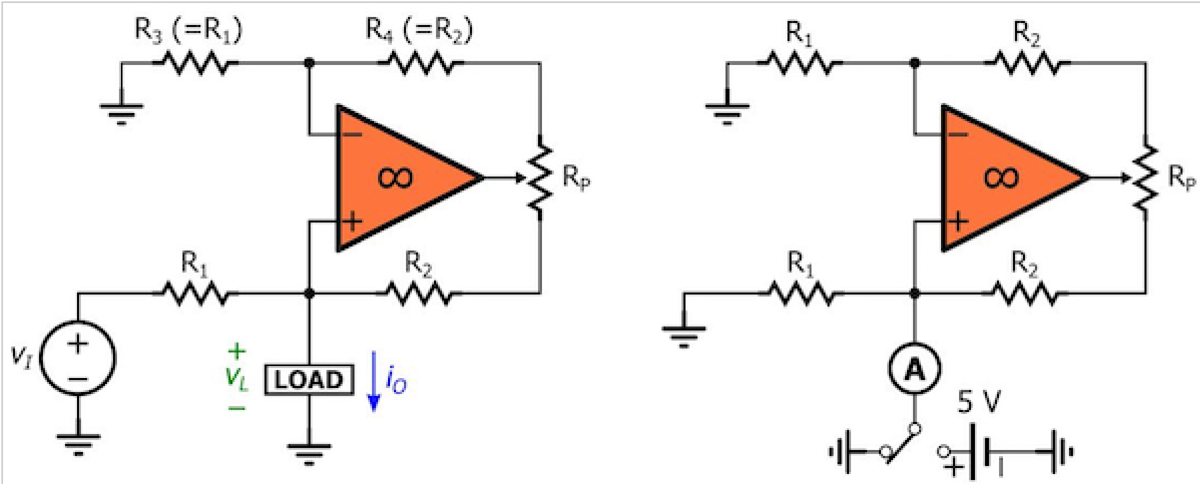


Figure 3. *(a) Using a potentiometer R_p to balance the resistive bridge. (b) Calibration set up.*

For improved performance, we must either use lower-tolerance resistances or balance the bridge using a potentiometer R_p, as in Figure 3a. To calibrate the circuit, ground the input as in Figure 3b and use an ammeter A. First, flip the switch to ground, and if necessary, zero the op-amp's input offset voltage until the ammeter reads zero. Then flip the switch to a known voltage, such as 5V, and adjust R_p until the ammeter reads again zero. By imposing that i_O with v_L = 5 V be equal to i_O with v_L = 0 V, we are making i_O independent of v_L, in effect driving R_o to infinity, by Equation 3.

The Effect of Op-Amp Nonidealities

Common-Mode Rejection Ratio

A practical op-amp is sensitive to its common-mode input voltage, a feature that is modeled with a small internal offset voltage in series with the noninverting input. In the case of the Howland pump, this offset voltage can be expressed as v_L/CMRR, where CMRR is the common-mode rejection ratio as reported in the op-amp's datasheet. With reference to Figure 4a, we note that Equation 1 still holds, but Equation 2 changes to

$$v_A = \left(1+\frac{R_4}{R_3}\right)\times\left(v_L+\frac{v_L}{\text{CMRR}}\right) = \left(1+\frac{R_2}{R_1}\right)\times v_L\times\left(1+\frac{1}{\text{CMRR}}\right)$$

Substituting into Equation 1, solving for i_O, and putting i_O in the form of Equation 3 gives

$$R_o = \left(R_1 \parallel R_2\right)\times \text{CMRR}$$

Equation 8

As an example, using an op-amp with CMRR = 60 dB (=1000) in the above example will lower R_o from ∞ to $(10^3||10^3)\times1000$ = 500 kΩ. With an arrangement of the type of Figure 3b, we can use the potentiometer to compensate for the cumulative effect of bridge imbalance as well as non-infinite CMRR.

Open-Loop Gain

So far we have assumed the op-amp to have infinite open-loop gain. The gain *a* of a practical op-amp is finite, so let us now see how this affects circuit behavior.

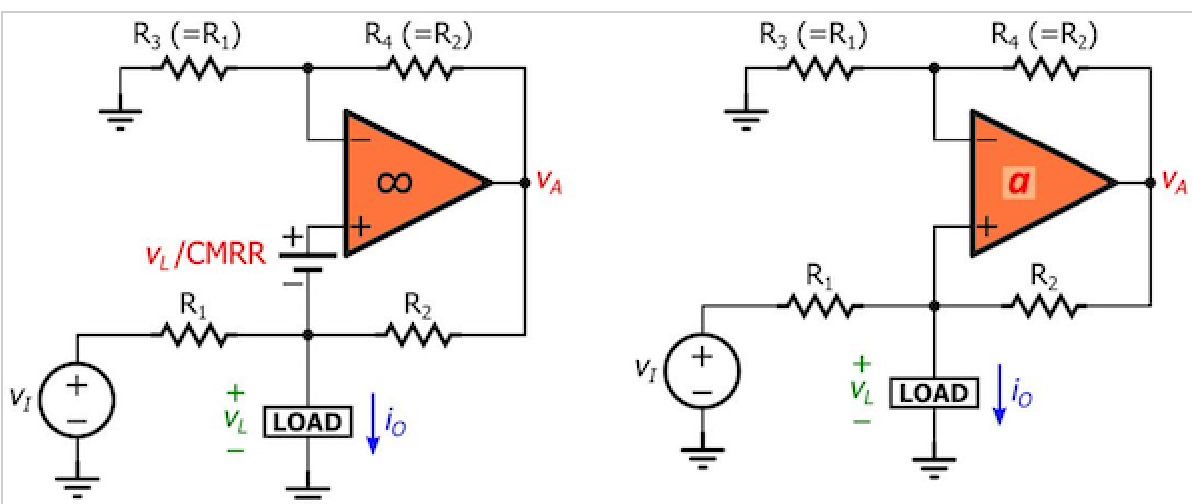


Figure 4. *Circuits to investigate the effect of (a) non-infinite common-mode rejection ratio and (b) non-infinite open-loop gain.*

With reference to Figure 4b, we now have

$$v_A = a\left(v_L - \frac{R_3}{R_3 + R_4} v_A\right)$$

Solving for v_A, substituting into Equation 1, solving for i_O, and putting i_O in the form of Equation 3 gives

$$R_o = (R_1 \parallel R_2) \times \left(1 + \frac{a}{1 + R_2 / R_1}\right)$$

Equation 9

As an example, using an op-amp with a DC gain of 100 dB (=100,000 V/V) will lower R_o from ∞ to $(10^3 \| 10^3) \times (1 + 100{,}000/2) \cong 25$ MΩ. With an arrangement of the type of Figure 3b, we can use the potentiometer to compensate for the cumulative effect of bridge imbalance, non-infinite CMRR, and non-infinite open-loop DC gain, and raise R_o as close as possible to ∞.

However, as we increase the frequency of operation, the gain a rolls off with frequency, leading to a progressive deterioration of R_o. For example, if an op-amp with a DC gain of 100 dB has a gain-bandwidth product (https://www.allaboutcircuits.com/technical-articles/negative-feedback-part-2-improving-gain-sensitivity-and-bandwidth/) of 1 MHz, its open-loop gain vs. frequency (assuming a single-pole response) will look like this:

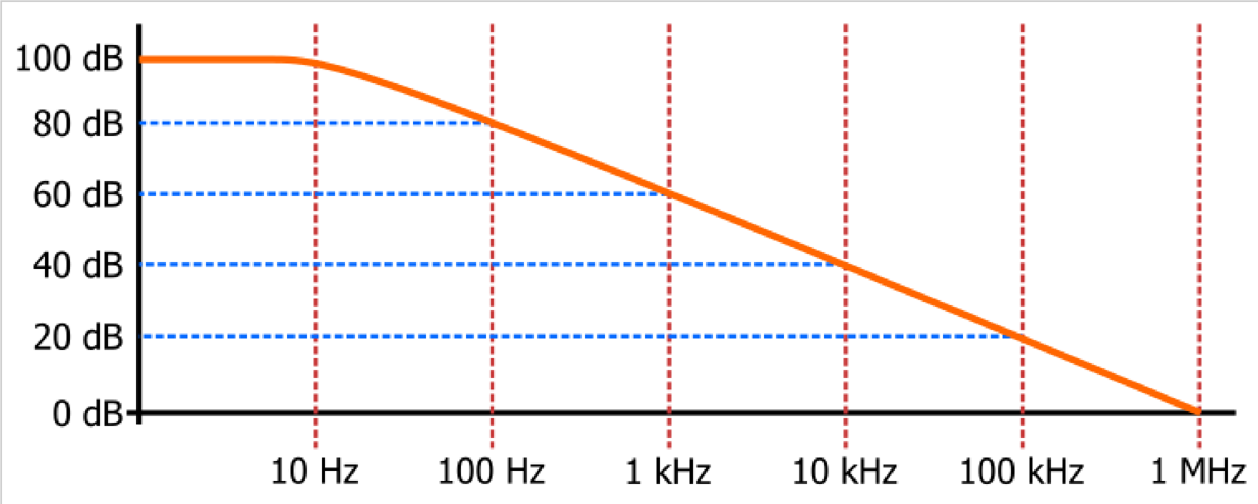


Figure 5. *Single-pole frequency response of a 1 MHz op-amp with a DC open-loop gain of 100 dB.*

Thus, the gain *a* drops to 60 dB (=1000 V/V) at 1 kHz, and the value of R_o will drop to 500×(1 + 1000/2) ≅ 250 kΩ. At 10 kHz R_o drops to 500×(1 + 100/2) ≅ 25 kΩ, and so on.

Further Reading

A Comprehensive Study of the Howland Current Pump (https://www.ti.com/lit/an/snoa474a/snoa474a.pdf) (PDF): an application note published by Texas Instruments.