Chair of Mathematical Finance
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Financial Mathematics 2
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Grade Bonus Task

In your grade bonus task, you will investigate what **Bonus Certificates with Cap** are and how to price them. You may ignore credit risk considerations. Moreover, assume any fraction of securities is tradable at any time and that the riskless return of the Bank account is given by $r = 2\%$ p.a.

Task 1

Upload the data set *Telekom.csv*, plot the evolution of the stock and the log-returns and investigate their mean, standard deviation, minimum, maximum and their quartiles.

Task 2

Fit a Black-Scholes model to the time series *Telekom.csv*.

Task 3

What are Bonus Certificates with Cap? Draw their payoff profile. Why would an investor buy any of those certificates? Which risks should be considered?

Task 4

Find a portfolio of products you know from the lecture and the exercises that replicate the certificate.

Task 5

Determine a fair price for each of the certificates and compute it explicitly for the market parameters from Task 2) with P_0 equal to the price quoted on 14.06.2024, barrier level 16, bonus level 24, maturity 1 year and Cap 28.
Hint: It holds that

$$\mathbb{E}_{\tilde{\mathbb{Q}}}\left[e^{-r\ T}(P_T - K)1_{\{\min_{t\in[0,T]} P_t > H\}}\right] = P_0\ Q^{(1)} - e^{-r\ T}K\ Q^{(2)},$$

where $Q^{(1)}$ and $Q^{(1)}$ are given by

$$Q^{(1)} = \mathcal{N}\left(\frac{\log\left(\frac{P_0}{H}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right) - \exp\left(-\frac{2\left(r + \frac{\sigma^2}{2}\right)\log\left(\frac{P_0}{H}\right)}{\sigma^2}\right)\mathcal{N}\left(\frac{-\log\left(\frac{P_0}{H}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right),$$

$$Q^{(2)} = \mathcal{N}\left(\frac{\log\left(\frac{P_0}{H}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right) - \exp\left(-\frac{2\left(r - \frac{\sigma^2}{2}\right)\log\left(\frac{P_0}{H}\right)}{\sigma^2}\right)\mathcal{N}\left(\frac{-\log\left(\frac{P_0}{H}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right),$$

where $\mathcal{N}$ denotes the cumulative distribution function of the standard normal distribution.

Task 6

Use Monte Carlo Pricing to confirm the pricing formulas you discovered in Task 5).

Task 7

Simulate the underlying and the payoff of the certificate under the physical probability measure. Compare

the distributions of the payoff to the distribution of the underlying. Compute for both of them the mean, variance, skewness, kurtosis and the 5% quantile (this is also called the Value-at-Risk). What do you observe?