統計的機械学習レポート

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宿題 1 不偏性より

$$\mathbb{E}(\hat{\theta}) = \theta^*$$

これを θ^* で微分して、

$$\begin{aligned}
1 &= \frac{\partial}{\partial\theta^*}\int \hat{\theta} q(\mathbf{x};\theta^*)d\mathbf{x} \\
&= \hat{\theta}\int \frac{\partial}{\partial\theta^*} q(\mathbf{x};\theta^*)d\mathbf{x} \\
&= \hat{\theta}\int \frac{\partial \log q(\mathbf{x};\theta^*)}{\partial\theta^*}d\mathbf{x} \\
&= \hat{\theta}\mathbb{E}[\frac{\partial}{\partial\theta^*}\log q(\mathbf{x};\theta^*)] \\
&= \mathbb{E}\big[\hat{\theta}\sum_{i=1}^{n}\frac{\partial}{\partial\theta}\log q(x_i;\theta)|_{\theta=\theta^*}\big]
\end{aligned}$$

を得る。

宿題 2 まず

$$\begin{aligned}
\frac{\partial^2}{\partial\theta\partial\theta^T}\log q(\mathbf{x};\theta) &= \frac{\partial}{\partial\theta}(\frac{1}{q(\mathbf{x};\theta)}\frac{\partial}{\partial\theta^T}q(\mathbf{x};\theta)) \\
&= \frac{\frac{\partial^2}{\partial\theta\partial\theta^T}q(\mathbf{x};\theta)\cdot q(\mathbf{x};\theta) - \frac{\partial}{\partial\theta}q(\mathbf{x};\theta)\frac{\partial}{\partial\theta^T}q(\mathbf{x};\theta)}{(q(\mathbf{x};\theta))^2} \\
&= \frac{\frac{\partial^2}{\partial\theta\partial\theta^T}q(\mathbf{x};\theta)}{q(\mathbf{x};\theta)} - \frac{\frac{\partial}{\partial\theta}q(\mathbf{x};\theta)\frac{\partial}{\partial\theta^T}q(\mathbf{x};\theta)}{(q(\mathbf{x};\theta))^2}
\end{aligned}$$

より

$$
\begin{aligned}
\mathbf{F}(\theta) &= \int (\frac{\partial}{\partial\theta}\log q(\mathbf{x};\theta))(\frac{\partial}{\partial\theta}\log q(\mathbf{x};\theta))^T q(\mathbf{x};\theta)d\mathbf{x} \\
&= \int \frac{1}{q(\mathbf{x};\theta)}\frac{\partial}{\partial\theta}q(\mathbf{x};\theta)\frac{\partial}{\partial\theta^T}q(\mathbf{x};\theta)d\mathbf{x} \\
&= \int \Big(\frac{\partial^2}{\partial\theta\partial\theta^T}q(\mathbf{x};\theta) - q(\mathbf{x};\theta)\frac{\partial^2}{\partial\theta\partial\theta^T}\log q(\mathbf{x};\theta)\Big)d\mathbf{x} \\
&= \frac{\partial^2}{\partial\theta\partial\theta^T}\int q(\mathbf{x};\theta)d\mathbf{x} - \int \Big((\frac{\partial^2}{\partial\theta\partial\theta^T}\log q(\mathbf{x};\theta))\Big)q(\mathbf{x};\theta)d\mathbf{x} \\
&= -\int \Big((\frac{\partial^2}{\partial\theta\partial\theta^T}\log q(\mathbf{x};\theta))\Big)q(\mathbf{x};\theta)d\mathbf{x}
\end{aligned}
$$

となる。

宿題 3 python で実装した。

```
import math
import random
import matplotlib.pyplot as plt
t = 0.5 θ#
n = 10 標本数#
l = 1000 繰り返し数#
likelihood = []
def coin(t):
    if random.random()<t:
        return 1
    else:
        return 0
for i in range(0, l):
    m = 0
    for j in range(0, n):
        m += coin(t)
    likelihood.append(m/n)
plt.hist(likelihood)
plt.show()
```

結果

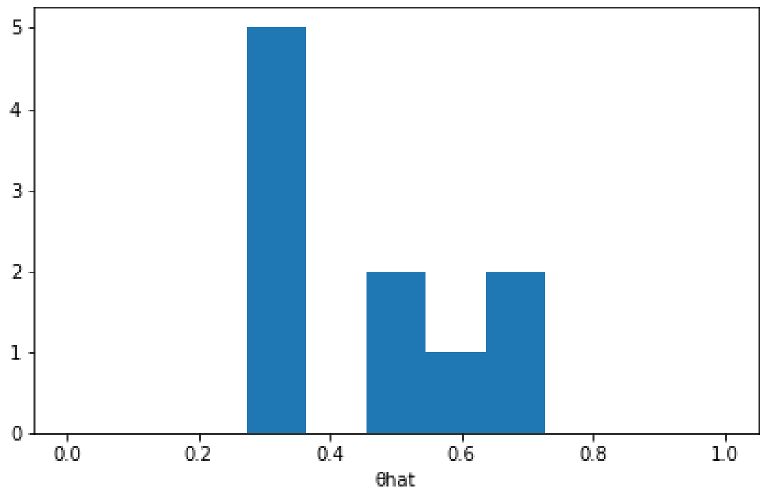

Figure 1: n=10

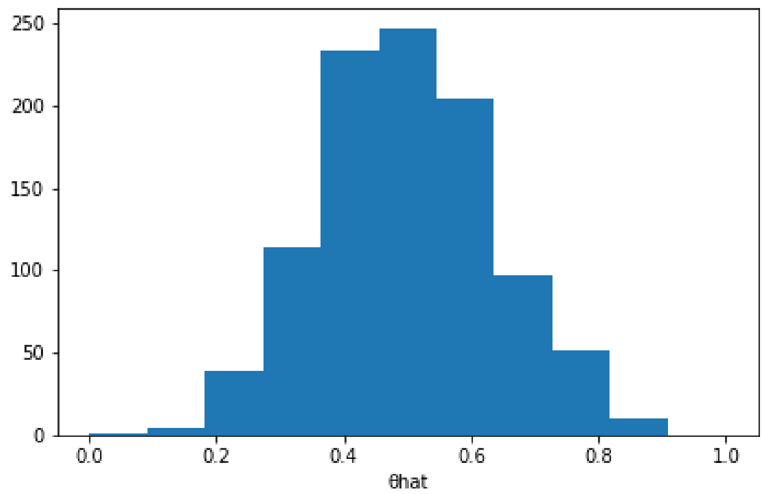

Figure 2: n=1000

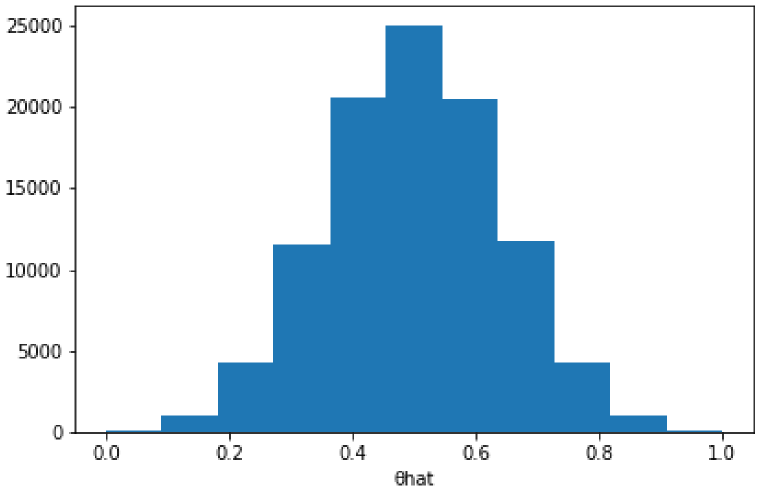

Figure 3: n=1000000

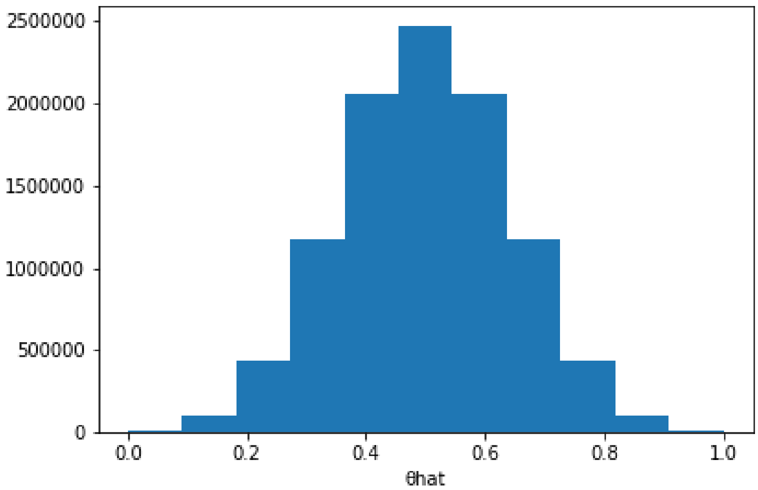

Figure 4: n=10000000

実装するとこのようになった。確かに正規分布に近づいている。