

統計的機械学習レポート

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宿題1 $L(\mu, \sigma) = \frac{1}{(2\pi\sigma^2)^{d/2}} \exp\left(-\frac{(x-\mu)^T(x-\mu)}{2\sigma^2}\right)$ とすると

$$\log L(\mu, \sigma) = -\frac{nd}{2} \log(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^d \sum_{j=1}^n (x_j^{(i)} - \hat{\mu}_{ML}^{(i)})^2$$

最尤推定量 $\hat{\mu}_{ML}, \hat{\sigma}_{ML}^2$ は

$$\frac{\partial}{\partial \mu} \log L(\mu, \sigma) = -\frac{1}{2\sigma^2} (2n\mu - 2 \sum_{i=1}^n x_i) = 0$$

を満たすので

$$\hat{\mu}_{ML} = \frac{1}{n} \sum_{i=1}^n x_i$$

また

$$\frac{\partial}{\partial \sigma} \log L(\mu, \sigma) = -\frac{nd}{2} \frac{4\pi\sigma}{2\pi\sigma^2} + \frac{1}{\sigma^3} \sum_{i=1}^d \sum_{j=1}^n (x_j^{(i)} - \hat{\mu}_{ML}^{(i)})^2 = 0$$

より

$$\hat{\sigma}_{ML}^2 = \sum_{i=1}^d \sum_{j=1}^n (x_j^{(i)} - \hat{\mu}_{ML}^{(i)})^2$$

宿題2 フィッシャーの線形判別分析より

$$\begin{aligned} a &= \hat{\Sigma}^{-1}(\hat{\mu}_1 - \hat{\mu}_2) \\ &= \frac{1}{9} \begin{pmatrix} 5 + 4 \cos 2\beta & -4 \sin 2\beta \\ -4 \sin 2\beta & 5 - 4 \cos 2\beta \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} \\ &= \frac{1}{9} (20 + 16 \cos 2\beta / -16 \sin 2\beta) \end{aligned}$$

一方

$$b = 0$$

となるので境界は

$$x_2 = \frac{4 \sin 2\beta}{5 + 4 \cos 2\beta} x_1$$

となる

宿題3

```
clear all
n=600; alpha=0.1;
n1=sum(rand(n,1)<alpha); n2=n-n1;
x1=[randn(1,n1)+2; 3*randn(1,n1)];
x2=[randn(1,n2)-2; 3*randn(1,n2)];

mu1=[2;0]; mu2=[-2;0]; S=[1 0;0 9];
p1=mu1'*inv(S)*x1-mu1'*inv(S)*mu1/2+log(alpha);
p2=mu2'*inv(S)*x1-mu2'*inv(S)*mu2/2+log(1-alpha);
result=sign(p1-p2);
sum(result== -1)
sum(result~= -1)

p1=mu1'*inv(S)*x2-mu1'*inv(S)*mu1/2+log(alpha);
p2=mu2'*inv(S)*x2-mu2'*inv(S)*mu2/2+log(1-alpha);
result=sign(p1-p2);
sum(result== -1)
sum(result~= -1)
```

結果

```
octave:28> sample
ans = 4
ans = 67
octave:29> sample
ans = 6
octave:30> sample
ans = 4
ans = 63
ans = 530
ans = 3
octave:31> sample
ans = 4
ans = 43
ans = 552
ans = 1
octave:32> sample
ans = 4
ans = 64
ans = 527
ans = 5
octave:33> sample
ans = 8
ans = 57
ans = 532
ans = 3
octave:34> sample
ans = 6
ans = 52
ans = 541
ans = 1
octave:35> sample
ans = 4
ans = 56
ans = 537
ans = 3
octave:36> sample
ans = 4
ans = 60
ans = 531
ans = 5
octave:37> sample
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```
ans = 7
ans = 64
ans = 527
ans = 2
```

実装するとこのようになった。

宿題 4

```
clear all
load digit.mat

mu1=mean(X(:,:,1),2);
mu2=mean(X(:,:,2),2);
mu3=mean(X(:,:,3),2);
mu4=mean(X(:,:,4),2);
mu5=mean(X(:,:,5),2);
mu6=mean(X(:,:,6),2);
mu7=mean(X(:,:,7),2);
mu8=mean(X(:,:,8),2);
mu9=mean(X(:,:,9),2);
mu10=mean(X(:,:,10),2);

S=(cov(X(:,:,1))'+cov(X(:,:,2))'+cov(X(:,:,3))'+cov(X(:,:,4))'+
+cov(X(:,:,5))'+cov(X(:,:,6))'+cov(X(:,:,7))'+cov(X(:,:,8))'+
+cov(X(:,:,9))'+cov(X(:,:,10))'))/2000;

t=T(:,:,1);
invS=inv(S+0.000001*eye(256));
p1=mu1'*invS*t-mu1'*invS*mu1/2;
p2=mu2'*invS*t-mu2'*invS*mu2/2;
p3=mu3'*invS*t-mu3'*invS*mu3/2;
p4=mu4'*invS*t-mu4'*invS*mu4/2;
p5=mu5'*invS*t-mu5'*invS*mu5/2;
p6=mu6'*invS*t-mu6'*invS*mu6/2;
p7=mu7'*invS*t-mu7'*invS*mu7/2;
p8=mu8'*invS*t-mu8'*invS*mu8/2;
p9=mu9'*invS*t-mu9'*invS*mu9/2;
p10=mu10'*invS*t-mu10'*invS*mu10/2;
result=sign(p1-max([p2,p3,p4,p5,p6,p7,p8,p9,p10]));
sum(result==-1)
sum(result~= -1)

t=T(:,:,2);
invS=inv(S+0.000001*eye(256));
p1=mu1'*invS*t-mu1'*invS*mu1/2;
p2=mu2'*invS*t-mu2'*invS*mu2/2;
p3=mu3'*invS*t-mu3'*invS*mu3/2;
p4=mu4'*invS*t-mu4'*invS*mu4/2;
p5=mu5'*invS*t-mu5'*invS*mu5/2;
p6=mu6'*invS*t-mu6'*invS*mu6/2;
p7=mu7'*invS*t-mu7'*invS*mu7/2;
p8=mu8'*invS*t-mu8'*invS*mu8/2;
p9=mu9'*invS*t-mu9'*invS*mu9/2;
p10=mu10'*invS*t-mu10'*invS*mu10/2;
result=sign(p2-max([p1,p3,p4,p5,p6,p7,p8,p9,p10]));
sum(result==-1)
sum(result~= -1)

t=T(:,:,3);
invS=inv(S+0.000001*eye(256));
p1=mu1'*invS*t-mu1'*invS*mu1/2;
p2=mu2'*invS*t-mu2'*invS*mu2/2;
p3=mu3'*invS*t-mu3'*invS*mu3/2;
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p4=mu4'*invS*t-mu4'*invS*mu4/2;
p5=mu5'*invS*t-mu5'*invS*mu5/2;
p6=mu6'*invS*t-mu6'*invS*mu6/2;
p7=mu7'*invS*t-mu7'*invS*mu7/2;
p8=mu8'*invS*t-mu8'*invS*mu8/2;
p9=mu9'*invS*t-mu9'*invS*mu9/2;
p10=mu10'*invS*t-mu10'*invS*mu10/2;
result=sign(p3-max([p2,p1,p4,p5,p6,p7,p8,p9,p10]));
sum(result== -1)
sum(result~= -1)

t=T(:, :, 4);
invS=inv(S+0.000001*eye(256));
p1=mu1'*invS*t-mu1'*invS*mu1/2;
p2=mu2'*invS*t-mu2'*invS*mu2/2;
p3=mu3'*invS*t-mu3'*invS*mu3/2;
p4=mu4'*invS*t-mu4'*invS*mu4/2;
p5=mu5'*invS*t-mu5'*invS*mu5/2;
p6=mu6'*invS*t-mu6'*invS*mu6/2;
p7=mu7'*invS*t-mu7'*invS*mu7/2;
p8=mu8'*invS*t-mu8'*invS*mu8/2;
p9=mu9'*invS*t-mu9'*invS*mu9/2;
p10=mu10'*invS*t-mu10'*invS*mu10/2;
result=sign(p4-max([p2,p3,p1,p5,p6,p7,p8,p9,p10]));
sum(result== -1)
sum(result~= -1)

t=T(:, :, 5);
invS=inv(S+0.000001*eye(256));
p1=mu1'*invS*t-mu1'*invS*mu1/2;
p2=mu2'*invS*t-mu2'*invS*mu2/2;
p3=mu3'*invS*t-mu3'*invS*mu3/2;
p4=mu4'*invS*t-mu4'*invS*mu4/2;
p5=mu5'*invS*t-mu5'*invS*mu5/2;
p6=mu6'*invS*t-mu6'*invS*mu6/2;
p7=mu7'*invS*t-mu7'*invS*mu7/2;
p8=mu8'*invS*t-mu8'*invS*mu8/2;
p9=mu9'*invS*t-mu9'*invS*mu9/2;
p10=mu10'*invS*t-mu10'*invS*mu10/2;
result=sign(p5-max([p2,p3,p4,p1,p6,p7,p8,p9,p10]));
sum(result== -1)
sum(result~= -1)

t=T(:, :, 6);
invS=inv(S+0.000001*eye(256));
p1=mu1'*invS*t-mu1'*invS*mu1/2;
p2=mu2'*invS*t-mu2'*invS*mu2/2;
p3=mu3'*invS*t-mu3'*invS*mu3/2;
p4=mu4'*invS*t-mu4'*invS*mu4/2;
p5=mu5'*invS*t-mu5'*invS*mu5/2;
p6=mu6'*invS*t-mu6'*invS*mu6/2;
p7=mu7'*invS*t-mu7'*invS*mu7/2;
p8=mu8'*invS*t-mu8'*invS*mu8/2;
p9=mu9'*invS*t-mu9'*invS*mu9/2;
p10=mu10'*invS*t-mu10'*invS*mu10/2;
result=sign(p6-max([p2,p3,p4,p5,p1,p7,p8,p9,p10]));
sum(result== -1)
sum(result~= -1)

t=T(:, :, 7);
invS=inv(S+0.000001*eye(256));
p1=mu1'*invS*t-mu1'*invS*mu1/2;

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p2=mu2'*invS*t-mu2'*invS*mu2/2;
p3=mu3'*invS*t-mu3'*invS*mu3/2;
p4=mu4'*invS*t-mu4'*invS*mu4/2;
p5=mu5'*invS*t-mu5'*invS*mu5/2;
p6=mu6'*invS*t-mu6'*invS*mu6/2;
p7=mu7'*invS*t-mu7'*invS*mu7/2;
p8=mu8'*invS*t-mu8'*invS*mu8/2;
p9=mu9'*invS*t-mu9'*invS*mu9/2;
p10=mu10'*invS*t-mu10'*invS*mu10/2;
result=sign(p7-max([p2,p3,p4,p5,p6,p1,p8,p9,p10]));
sum(result== -1)
sum(result~= -1)

t=T(:, :, 8);
invS=inv(S+0.000001*eye(256));
p1=mu1'*invS*t-mu1'*invS*mu1/2;
p2=mu2'*invS*t-mu2'*invS*mu2/2;
p3=mu3'*invS*t-mu3'*invS*mu3/2;
p4=mu4'*invS*t-mu4'*invS*mu4/2;
p5=mu5'*invS*t-mu5'*invS*mu5/2;
p6=mu6'*invS*t-mu6'*invS*mu6/2;
p7=mu7'*invS*t-mu7'*invS*mu7/2;
p8=mu8'*invS*t-mu8'*invS*mu8/2;
p9=mu9'*invS*t-mu9'*invS*mu9/2;
p10=mu10'*invS*t-mu10'*invS*mu10/2;
result=sign(p8-max([p2,p3,p4,p5,p6,p7,p1,p9,p10]));
sum(result== -1)
sum(result~= -1)

t=T(:, :, 9);
invS=inv(S+0.000001*eye(256));
p1=mu1'*invS*t-mu1'*invS*mu1/2;
p2=mu2'*invS*t-mu2'*invS*mu2/2;
p3=mu3'*invS*t-mu3'*invS*mu3/2;
p4=mu4'*invS*t-mu4'*invS*mu4/2;
p5=mu5'*invS*t-mu5'*invS*mu5/2;
p6=mu6'*invS*t-mu6'*invS*mu6/2;
p7=mu7'*invS*t-mu7'*invS*mu7/2;
p8=mu8'*invS*t-mu8'*invS*mu8/2;
p9=mu9'*invS*t-mu9'*invS*mu9/2;
p10=mu10'*invS*t-mu10'*invS*mu10/2;
result=sign(p9-max([p2,p3,p4,p5,p6,p7,p8,p1,p10]));
sum(result== -1)
sum(result~= -1)

t=T(:, :, 10);
invS=inv(S+0.000001*eye(256));
p1=mu1'*invS*t-mu1'*invS*mu1/2;
p2=mu2'*invS*t-mu2'*invS*mu2/2;
p3=mu3'*invS*t-mu3'*invS*mu3/2;
p4=mu4'*invS*t-mu4'*invS*mu4/2;
p5=mu5'*invS*t-mu5'*invS*mu5/2;
p6=mu6'*invS*t-mu6'*invS*mu6/2;
p7=mu7'*invS*t-mu7'*invS*mu7/2;
p8=mu8'*invS*t-mu8'*invS*mu8/2;
p9=mu9'*invS*t-mu9'*invS*mu9/2;
p10=mu10'*invS*t-mu10'*invS*mu10/2;
result=sign(p10-max([p2,p3,p4,p5,p6,p7,p8,p9,p1]));
sum(result== -1)
sum(result~= -1)

```

結果

```
octave:47> m1
ans = 200
ans = 0
ans = 197
ans = 3
ans = 199
ans = 1
ans = 199
ans = 1
ans = 198
ans = 2
ans = 199
ans = 1
ans = 198
ans = 2
ans = 200
ans = 0
ans = 199
ans = 1
ans = 198
ans = 2
```

実装するとこのような結果が得られた。